GEOS 397/597

Lecture #19: Polynomial roots

use **practical_19.m**

1) Update from github

2) polyfit, polyval, corrcoef, roots

MATLAB provides several built-in functions to fit curves

- Many require the “Curve Fitting Toolbox”, or other toolboxes.
- We will only use the basic curve fitting functions that are part of standard MATLAB
- We will focus on:

1) polyfit

2) polyval

3) roots

4) fzero -- to find roots when function is not a polynomial

5) corrcoef

3) Review of polynomials with polyval, polyfit

4) Review roots of polynomials

Roots of a function are where the function equals 0 (i.e. f(x)=0)

Some polynomials have no real roots, but do have roots with imaginary numbers -- the discriminant tells us the roots.

$b^2 - 4ac$

The discriminant is the name given to the expression that appears under the square root (radical) sign in the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The discriminant tells you about the *nature* of the roots of a quadratic equation given that a, b and c are rational numbers. It

quickly tells you the number of real roots associated with a quadratic equation.

Three cases

1) $b^2 - 4ac > 0$, there are two real roots

2) $b^2 - 4ac = 0$, there is one real root

3) $b^2 - 4ac < 0$, there are two complex roots

Let's look at an example:

$x^2 - 6x + 8$, we can factor this to find $x^2 - 6x + 8 = (x - 2)(x - 4)$

Discriminant = $6^2 - 4 * (1) * (8) = 4$

Discriminant is greater than zero so the function has *real* roots: $x = 2$ and $x = 4$. Let's plot this polynomial in MATLAB.

- 1^{st} order: 1 root
- 2^{nd} order: 0, 1, or 2 roots
- 3^{rd} order: up to 3 roots
- n^{th} order: up to n roots

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roots() -- find a polynomial's roots given the polynomial coefficients

Try first practical exercise.

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bisection method to find roots of a function on the interval [a,b]

What about when we have a function (e.g. a data time series) and we do not know the polynomial coefficients, but we want to find the roots in a given interval?

As an example let's find the roots of $x^2 = 2$. We know analytically that the roots are $x = \pm\sqrt{2}$. Let's use a computer program to find the root $x = \sqrt{2}$.

See the two practical exercises.

First exercise demonstrates bisection to find x when $x^2 = 2$.

Second exercise demonstrates how to modify the bisection method to find two points (one negative, one positive) that bound a zero.

Other methods used to find roots (i.e. zeros) of a function

1) Newton's Method

2) Secant Method

3) Inverse Quadratic Interpolation

5) How to find roots of general functions

In MATLAB **fzero()** is the function we want to use; it combines the above methods for high accuracy and speed.

6) Student exercises in class