4.5 <u>Objective:</u> Find a and b

using: $\hat{x}^p = a\hat{y} + b$

$u = \hat{x} - \hat{x}^p$ (error-per-sample)

<u>Assume:</u> mean($\{u\}$) = 0 and to minimize var($\{u\}$)

a. Using the mean of the error terms, find b.

$$\text{mean}(\{u\}) = 0 \Rightarrow \text{mean}(\{\hat{x} - \hat{x}^p\}) = 0$$

We know that mean($\{\hat{x}\}$) = 0, as is mean($\{\hat{y}\}$)

$$\Rightarrow \text{mean}(\{\hat{x}\}) - \text{mean}(\{\hat{x}^p\})$$

$$\Rightarrow -\text{mean}(\{\hat{x}^p\}) = -\text{mean}(\{a\hat{y} + b\})$$

$$= -a\,\text{mean}(\{\hat{y}\}) - b$$

So, $b = 0$

b. We want to minimize the variance to find a.

$$\text{var}(\{u\}) = \text{mean}(\{(u - \text{mean}(\{u\}))^2\})$$

$$= \text{mean}(\{u^2\})$$

$$= \text{mean}(\{(\hat{x} - \hat{x}^p)^2\})$$

$$= \text{mean}(\{(\hat{x} - (a\hat{y}))^2\})$$

$$= \text{mean}(\{\hat{x}^2 - 2a\hat{x}\hat{y} + a^2\hat{y}^2\})$$

$$= \underbrace{\text{mean}(\{\hat{x}^2\})}_{1} \underbrace{- 2a\,\text{mean}(\{\hat{x}\hat{y}\})}_{-2a \cdot r} \underbrace{+ a^2\,\text{mean}(\{\hat{y}^2\})}_{+a^2 \cdot 1}$$

$$= 1 - 2a \cdot r + a^2 \cdot 1$$

var($\{u\}$) $\Rightarrow 1 - 2ar + a^2$, so $\frac{\partial\, \text{var}(\{u\})}{\partial a} = -2r + 2a = 0$

So, $a = r$

4.6 We know that $\mu_Y = 1988.5$, $\sigma_Y = 14$, $r = 0.882$

$\mu_T = 0.175$, $\sigma_T = 0.231$,

(Y = year, T = Earth Temp.)

$T_0^P(y=2014) \approx 0.550$

$T_1^P(y=2028) \approx 0.756$

$T_2^P(y=2042) \approx 0.962$

4.7 We know that $\mu_T = 0.175$, $\sigma_T = 0.231$

$\mu_N = 316$, $\sigma_N = 30.8$, $r = 0.471$

(N = Number of tornadoes, T = Earth temp.)

$N_0^P(T=0.5) \approx 51.21$

$N_1^P(T=0.6) \approx 57.49$

$N_2^P(T=0.7) \approx 63.77$