Discrete Distributions

Bernoulli

$0 < p < 1$

$$f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$$

$$M(t) = 1 - p + pe^t$$

$$\mu = p, \qquad \sigma^2 = p(1-p)$$

Binomial

$b(n, p)$

$0 < p < 1$

$$f(x) = \frac{n!}{x!(n-x)!}\, p^x(1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$$

$$M(t) = (1 - p + pe^t)^n$$

$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Geometric

$0 < p < 1$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$$

$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \qquad t < -\ln(1-p)$$

$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

Hypergeometric

$N_1 > 0, \quad N_2 > 0$

$N = N_1 + N_2$

$$f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, \qquad x \le n, x \le N_1, n - x \le N_2$$

$$\mu = n\left(\frac{N_1}{N}\right), \qquad \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Negative Binomial

$0 < p < 1$

$r = 1, 2, 3, \ldots$

$$f(x) = \binom{x-1}{r-1} p^r(1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$$

$$M(t) = \frac{(pe^t)^r}{[1-(1-p)e^t]^r}, \qquad t < -\ln(1-p)$$

$$\mu = r\left(\frac{1}{p}\right), \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson

$0 < \lambda$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$$

$$M(t) = e^{\lambda(e^t-1)}$$

$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Uniform

$m > 0$

$$f(x) = \frac{1}{m}, \qquad x = 1, 2, \ldots, m$$

$$\mu = \frac{m+1}{2}, \qquad \sigma^2 = \frac{m^2-1}{12}$$

Continuous Distributions

Beta
$0 < \alpha$
$0 < \beta$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha+\beta}, \qquad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Chi-square
$\chi^2(r)$
$r = 1, 2, \ldots$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \qquad t < \frac{1}{2}$$

$$\mu = r, \qquad \sigma^2 = 2r$$

Exponential
$0 < \theta$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \qquad t < \frac{1}{\theta}$$

$$\mu = \theta, \qquad \sigma^2 = \theta^2$$

Gamma
$0 < \alpha$
$0 < \theta$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}, \qquad t < \frac{1}{\theta}$$

$$\mu = \alpha\theta, \qquad \sigma^2 = \alpha\theta^2$$

Normal
$N(\mu, \sigma^2)$
$-\infty < \mu < \infty$
$0 < \sigma$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty$$

$$M(t) = e^{\mu t + \sigma^2 t^2/2}$$

$$E(X) = \mu, \qquad \mathrm{Var}(X) = \sigma^2$$

Uniform
$U(a, b)$
$-\infty < a < b < \infty$

$$f(x) = \frac{1}{b-a}, \qquad a \le x \le b$$

$$M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \qquad t \ne 0; \qquad M(0) = 1$$

$$\mu = \frac{a+b}{2}, \qquad \sigma^2 = \frac{(b-a)^2}{12}$$