FOCS Notes
1/21/2020

Agenda:

- Precise statements: propositions
- Compound propositions: using conjunction
- Predicates: statements about sets
- Quantifiers: restrict the range of predicates

Statements can be ambiguous
i.e. Everyone loves someone.
2+2=4 2+2=5 $\leftarrow$ Precise but false If you study you will get an A in FOCS

Propositions

- p: You studied
- q: You got an A in FOCS
- r: Kilam is an American
- s: y^2 is even

Compound Propositions

Conjunction	Symbol	Example
Not	$\neg$	$\neg p$: You did not study
If..then	$\rightarrow$	$p \rightarrow q$: If you studied then you get an A in FOCS.
And	$\wedge$	$p \wedge q$: You studied and you got an A in FOCS
Or	$\vee$	$p \vee q$: You studied or you got an A in FOCS

Truth Tables

p	q	r	¬p	$q \wedge r$
T	T	T	F	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

Truth table for implications
$p \rightarrow q$: if p then q
if $p \rightarrow q$ and p is T, then q is T. If p is F, then q can be anything
if $p \rightarrow q$ is false and p is T, then q is F

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Contrapositive
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
Note:

- $p \to q \not\equiv \neg p \to \neg q$
- $p \to q \not\equiv q \to p$ (Converse)

Algebra for Propositions

- $\neg(\neg p) = p$
- $\neg(p \wedge q) = \neg p \vee \neg q$
- $\neg(p \vee q) = \neg p \wedge \neg q$
- $p \to q \equiv \neg p \vee q$
- $\neg(p \to q) = p \wedge \neg q$

Example

- $(q \wedge \neg r) \to \neg p \equiv (p \wedge q) \to r$
- $q \wedge \neg r) \to \neg p \equiv \neg(q \wedge \neg r) \vee \neg p$ (implication)
  - $\equiv (\neg q \vee r) \vee \neg p$ (negation)
  - $\equiv \neg q \vee (r \vee \neg p)$ (associativity)
  - etc...

Revisit Implications
Suppose $p \to q$ is true, what can I say about q?

Statements about sets
Everyone has some gray hair
Everyone loves someone
Every even integer is a sum of two primes (Goldbach conjecture)
Someone failed FOCS
In every group of 6+ people, there are either three people who are mutual friends, or three people who are mutually not friendly

Predicates: a statement parametrized by an argument
$P(x)$: x studied for FOCS exam $\exists x \in F : P(x)$ where $F = \{x | x \textit{ is a student in FOCS spring } 2020\}$

Qualifiers

- $\exists$: "there exists"
- $\forall$: "for all"
- $Q(x)$: x got an A in FOCS
- $\forall x \in F : P(x) \wedge Q(x)$

Negating Qualified Propositions

- $\neg(\forall x : P(x)) \equiv \exists x : \neg P(x)$
- $\neg(\exists x : Q(x)) \equiv \forall x : \neg Q(x)$
- $\neg(\forall x : P(x) \wedge Q(x)) \equiv \exists x : \neg(P(x) \wedge Q(x))$
  - $\equiv \exists x : \neg P(x) \vee \neg Q(x)$

Clarify "Everyone loves someone"
$P(a, b)$: a loves b
$\exists b : (\forall a : P(a, b))$
$\forall a : (\exists b : P(a, b))$

Falsifying statements
$p \to q$ is false if there exists a person who studied but failed FOCS
Likewise, if there is no such case, then $p \to q$ is always true

Claims:

1) $\forall n : 2^{2^n} + 1$ is primes

2) $\neg \exists (a, b, c) \in \mathbb{N}^3 : a^3 + b^3 = c^3$

3) $\forall (a, b, c) \in \mathbb{N}^3 : a^3 + b^3 \neq c^3$

Note: 2) and 3) above are equivalent