Digital Sampling and Analysis

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Abstract

In this lab we digitally sample and process radio signals to help us understand how theory maps to the real world. We explore the Nyquist Criterion, Fourier voltage and power spectra, as well as noise and mixers, finding that while real world data is imperfect it is an extremely useful tool in understanding the universe.

1 Introduction

Central to the field of radio astronomy is of course radio waves, which in addition to being essential for wireless communication and entertainment on Earth, allow us to study a wide assortment of astronomical objects. Before one can hope to study these signals there must first be a solid practical and theoretical understanding of electromagnetic waves, how to digitally sample them, and how to analyze them once sampled.

Consequently, in Lab 1 we studied the mechanics of radio astronomy such as the Nyquist Criterion and its relation to aliasing, the lab equipment, our class python package `ugradio`, as well as useful mathematical concepts such as the discrete Fourier transform.

2 Methods and Experiments

2.1 Nyquist Rate and First Samples

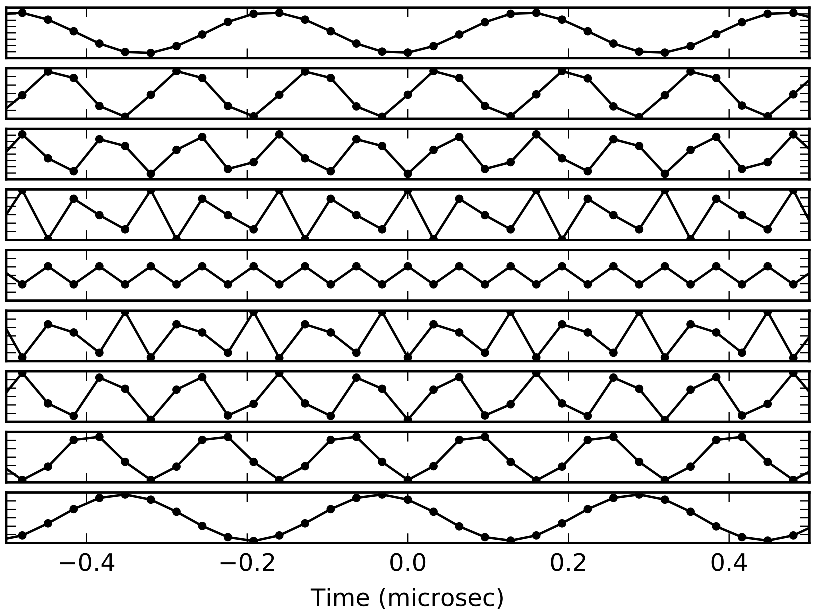
As an introduction we began by attempting to generate then sample a simple sine wave. From the Nyquist criterion we knew the sampling frequency must be chosen carefully to ensure that $\nu_{sample} \geq 2\nu_{signal}$ otherwise, the signal will be *aliased*, meaning it will not appear as the true signal but will instead be distorted.

We chose a sampling frequency of 31.25MHz then scaled the signal frequency accordingly because it can be set with high precision. The data was collected from the pico sampler using `ugradio.pico.capture_data` and saved with `numpy.save` as suggested in the lab. From here we were able to move into plotting and analysis.

2.1.1 Outcome

After taking 9 samples, each a different multiple of the sample frequency ($\nu_{sig} = (0.1, 0.2, 0.3, \ldots, 0.9)\nu_{samp}$), we noticed that at $\nu_{sig} = 0.5\nu_{samp}$ the waveform we observed no longer represented the true signal as it was increasingly aliased beyond this point (See Figure 1). In addition, the power spectra also confirmed that the frequency calculated from the data is increasingly aliased as the spikes get further and further from the true value we set.

Figure 1: Aliasing increasing as the sampling frequency decreases



2.2 Fourier Analysis

In the context of this lab Fourier analysis refers to our use of `ugradio.dft` to compute and plot voltage spectra, power spectra, and discrete Fourier Transforms of our data we collected.

The first obstacle to overcome was understanding Fourier transforms, which at the basic level can be understood as a way to decompose a wave into its component frequencies. Mathematically it is defined as an integral:

$$\tilde{E}(\nu) = \int_{-T/2}^{T/2} E(t) e^{2\pi i \nu t} dt \ . \tag{1}$$

Importantly this process is invertible, meaning we can go back and forth between time and frequency domains. In the real world of the lab however we need the Discrete Fourier transform because once sampled, these waves are no longer continuous. Essentially this means the integral becomes a sum:

$$f(t) = 1/N \sum_{i=0}^{N-1} f(\omega_i) e^{i\omega_i t} \tag{2}$$

In this lab we used `ugradio.dft.dft` on our signal using a time interval running from $\frac{-N}{2}/\nu_{samp}$ to $(\frac{N}{2} - 1)/\nu_{samp}$.

2.2.1 Fourier Voltage and Power Spectra

In section 4.3 we plotted the voltage spectra of our signals and noticed that there was a symmetry in the peaks about the center $\nu = 0$.Some also had opposide symmetry with one peak with positive amplitude and one peak with negative amplitude on either side. (See Figure 2)

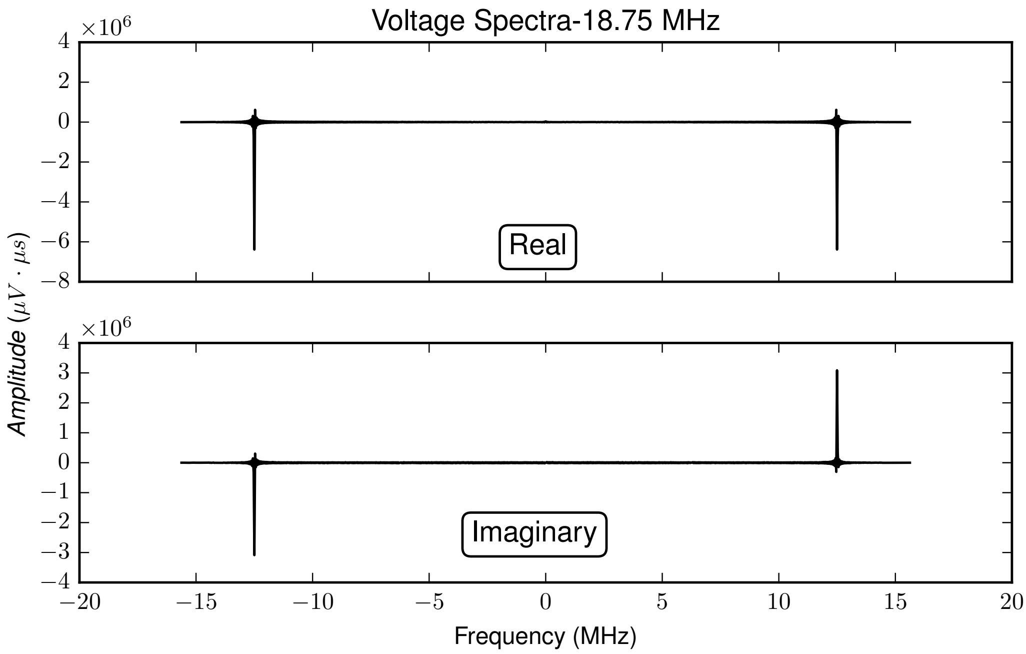


Figure 2: Real and Imaginary parts of the voltage spectra for a signal at 18.75MHz where the symmetries can be clearly seen

This alludes to the fact that the sine wave we generated has a negative *and* a positive frequency which is apparent in the math: $\sin(\omega t)\frac{1}{2i}(e^{i\omega t} - e^{-i\omega t})$. These negative frequencies are no less real than the real-valued frequencies.

We also noticed the output of the DFT function is an array of frequencies which are real valued as well as a complex valued array of voltages. These voltages are complex and by taking the absolute square we obtained the corresponding power spectra. We would continue to use these concepts in the following sections of the lab.

2.3 Spectral Leakage

New to me was the concept of spectral leakage. As I understood it, leakage occurs as a result of windowing and can be seen as a non-zero power at a frequency other than ν_{sig}. This happens because the DFT function takes a finite number of samples(N) with a certain sampling rate(ν_{sig}) and then outputs N values. If ν_{sig} is an integer multiple then it will all be recorded at one peak value, however if it is not it will be pushed to the next and appear as a smaller peak. We observed and plotted this leakage power in Figure 3

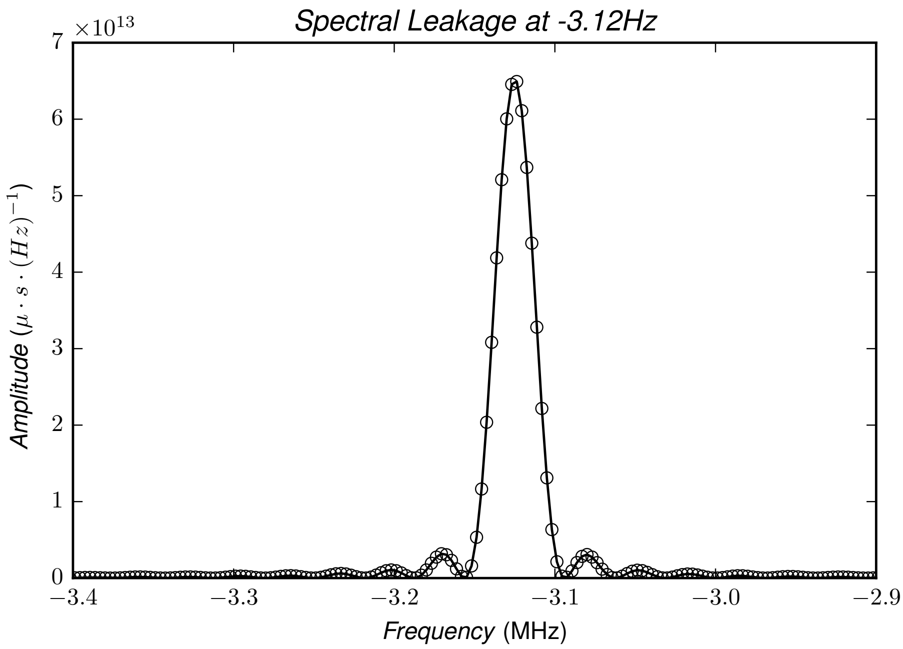


Figure 3: Spectral leakage observed at a negative frequency

2.4 Frequency Resolution and Nyquist Windows

2.4.1 Frequency Resolution

Frequency Resolution depends on the distance in frequency space between data points, meaning the Hz in each DFT bin. This is given by ν_{samp}/N where N = number of samples.

If the peaks of two signals get too close we will be unable to distinguish them.

2.4.2 Nyquist Windows

When plotting the Nyquist window we observed that each time we increased N, there were more and more peaks symmetric across $\nu = 0$. However

I am still not sure how to interpret this.

2.5 Noise

We sampled the noise using the generator as described in the lab. After figuring out how to properly set it up we computed the average power spectra for $N = [1, 2, 4, 8, 16]$ blocks then plotted it against the average of all the blocks. We noticed that as we increased the number, the noise shrunk down to match the total.

2.6 Mixers

A mixer is a device which allows a signal to be shifted from one frequency range to another, a process which is also called heterodyning. Mixers allow radio astronomers to take signals from space which may be in the GHz range and shift them into a range that can be comfortably handled by the lab equipment.

In this lab we constructed both a single sideband (SSB) mixer and a double sideband (DSB) mixer For the SSB mixer We used the two signal generators with our RF at 21MHz and our local oscillator at 21 ± 1MHz. We then used the pico sampler to collect data from both ports, one corresponding the real and the other imaginary. We observed a waveform with two frequencies just as we saw on the oscilloscope trace (Figure 4). We then added a delay using the $\frac{\lambda}{4}$ cable, which as expected produced two waves with an approximately 90° phase delay.

We plotted the power spectra and found that *are* able to tell them apart from this alone as the peaks are clearly on opposite sides of $\nu = 0$

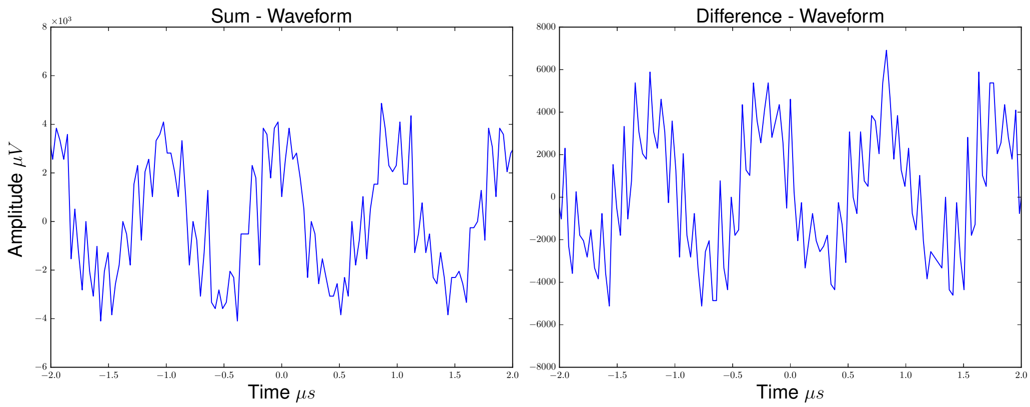


Figure 4: Mixer output for 21MHz ± 1MHz

3 Results and Discussion

There are may physical phenomena studied in this lab and i believe that for the most part we have represented them accurately through our experiments and plots. Deviations from theory are still clearly present, however it is mostly due to the imperfect nature of the equipment and the real world in general. See for example the intermodulation or *birdies*

There were also imperfections as a result of our methods. Though in the end we were able to correct many of these, they were certainly major obstacles to our progress. For example, we had to collect the mixer data several times due to an incorrect setup of the delay wire. Overall this was very beneficial as it led to a deeper understanding of both the equipment and the theory.

4 Conclusion

In doing this lab we have begun our journey into Radio Astronomy. While there was no specific scientific objectives, we have begun to familiarize ourselves with the core techniques we'll need going forward. Along with that we have encountered the fact that real machinery is not the same as the idealized theory we are used to. Our mixers do not mix perfectly, we do not have infinite resolution, and we certainly do not have infinite time. However with careful planning and patience we can begin to approach the level of precision desired and observe some interesting properties of our universe through radio astronomy.

5 Acknowledgements

In this Lab I mainly helped with initial troubleshooting of the code and equipment as well as setting up and collecting data for the mixers with Matt Lundy. Matt also helped with the setup of the other equipment used. Andrew Hsu and Christine Nguyen wrote most of the initial code though I rewrote sections on my own in order to generate my own plots for this report and to ensure I was following the material.