Radio Interferometry Lab Report

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Abstract

This lab serves as our introduction to radio interferometry. We describe our use of coordinate transforms and the fundamental equations to perform simple observations of a point source, the Orion Nebula, and an extended source, the Sun. We then extract some of the useful information from the interferometer fringe, which allows us to measure the Sun's angular radius to be $0.3 \pm 0.2°$ and our North-South and East-West Baselines to be approximately 1 m and 18 m respectively.

1 Introduction

In radio interferometry the signal from pairs of antennas are being *correlated*, which allows us to look anywhere on the sky that the component antennas can see and effectively adds up the smaller telescopes into a much larger telescope. The output, known as the fringe, contains a wealth of information and in this paper we explain how we used these concepts to make accurate measurements of objects of interest both on Earth and beyond. We start with an overview of our observing setup and basics of pointing in §2, our first solar observation and what we learned from it in §3, our analysis of the fringe from a horizon to horizon sun observation in §4, how we calculated the angular radius of the sun in §5, and finally our observation of the Orion Nebula, and how we calculated the baseline in §6 and §7.

2 Hardware

For all our observations we used the two dish multiplying interferometer mounted on the roof of Campbell Hall (latitude: N 37° 52' 23.52" longitude: W 122° 15' 25.43") which operates at about 11 GHz. As previously mentioned the output of the interferometer, called the fringe, contains all the information in its phase, amplitude, and frequency (more on this in §4) but ultimately after mixing and filtering, we read it out as a voltage from the multimeter.

2.1 Coordinates and Pointing

This telescope operates using a Topocentric coordinate system to mark positions of an object. Each position has an *altitude*, which is an angular elevation above the equator, and an *azimuth* which is the number of degrees clockwise from due North (usually). However, stars and other astronomical objects are usually catalogued using Equatorial coordinates, right ascension (R.A) and declination (DEC), which are like a projection of terrestrial latitude and longitude. To convert from one system to the other we can use a rotation matrix which takes in one pair of coordinates and transforms to the other. In the final program however, we used the `ugradio.get_altaz` function to do this. We also needed to track the source as the Earth rotates so we wrote a Python script which takes in the desired RA and DEC then, in a loop, calculates the position in azimuth and altitude every few seconds and sends it to the interferometer.

3 Getting Some Sun

With the machinery in place, we begin with a short observation of the sun to test our pointing. Knowing that the fringe amplitude is given by:

$$R(h_s) = \underbrace{F(h_s)}_{Point-Source\ Fringe} \times \underbrace{\int I(\Delta h)\cos(2\pi f_f \Delta h)\ d\Delta h}_{Fringe\ Modulator} \tag{1}$$

where the point-source fringe is a factor to be discussed later dependent on the baseline, hour angle and declination, and the Fringe modulator is the Fourier transform of the source intensity distribution on the sky, we expect to see a sinusoidal signal with a recognizable fringe pattern. As shown in Figure 1, this is exactly what we see, the fringe oscillates quickly over time while its amplitude oscillates at a different, lower frequency. Confident in our pointing abilities we can now move on to longer observations.

4 Full Day of Sun

We again observed the Sun, this time for a full day. Just as in the first observation, we expected a sinusoidal fringe, but importantly we also expected to prominently see the full *envelope*, which is the overall amplitude variation in Figure 2. To understand this we consider the "measurement equation" for interferometry:

$$V(u,v) \approx e^{-2\pi i\omega} \int\int A(l,m)I(l,m)e^{-2\pi i(ul+vm)}\ dl\ dm \tag{2}$$

where u is the East-West component of the baseline, and v is the North-South component. The first exponential term, known as the "phasor", is responsible for the fast sinusoidal variations we saw in our first observation, and the integral term is responsible for the envelope which we now see clearly in Figure 2. However, we did expect

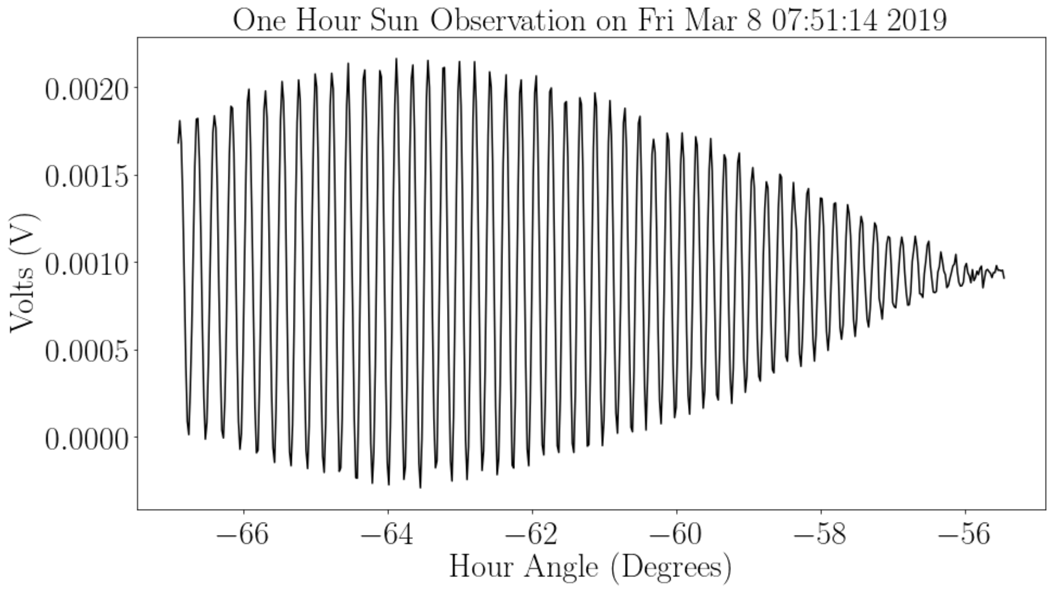

Figure 1: Raw data from a one hour observation of the sun on Mar 8 2019. Clearly visible are the fast oscillations if the fringe frequency which occur as a result of of the "phasor" in the measurement equation, and the decreasing amplitude which is a result of the varying baseline of the interferometer from the sources point of view.

so have left-right symmetry about zero hour angle in the envelope, and the fact that we do not could mean there was something odd about the Sun on that day (perhaps a Sun spot) or there was some unknown complication with the equipment or our line of sight.

We then ask, why should the power received from the telescope go to zero if we are always pointed at it? The answer is the sun is an extended source! Since an interferometer essentially projects a Sine wave onto the sky, and as a disk like the sun or moon moves through it, it can only fit a certain number of wavelengths within. So when a whole number fit, they equally cancel, giving a zero in the plot. This is similar to a Bessel function of the first kind, with each end being a start, so we exploit this in the following section to calculate the angular diameter of the Sun.

5 Angular Radius

Marked by the vertical lines in Figure 2 are the zero crossings we could accurately locate in our data. By matching each of these to a root of a Bessel function of the first kind, then using $\theta_r = \frac{x\lambda}{2\pi\cos{(HA)}}$ we use each to solve for an angular radius. The resulting radii were all about 0.3° with an average value of $0.303 \pm 0.2°$ which gives a diameter close to the suns true angular diameter of 0.53°.

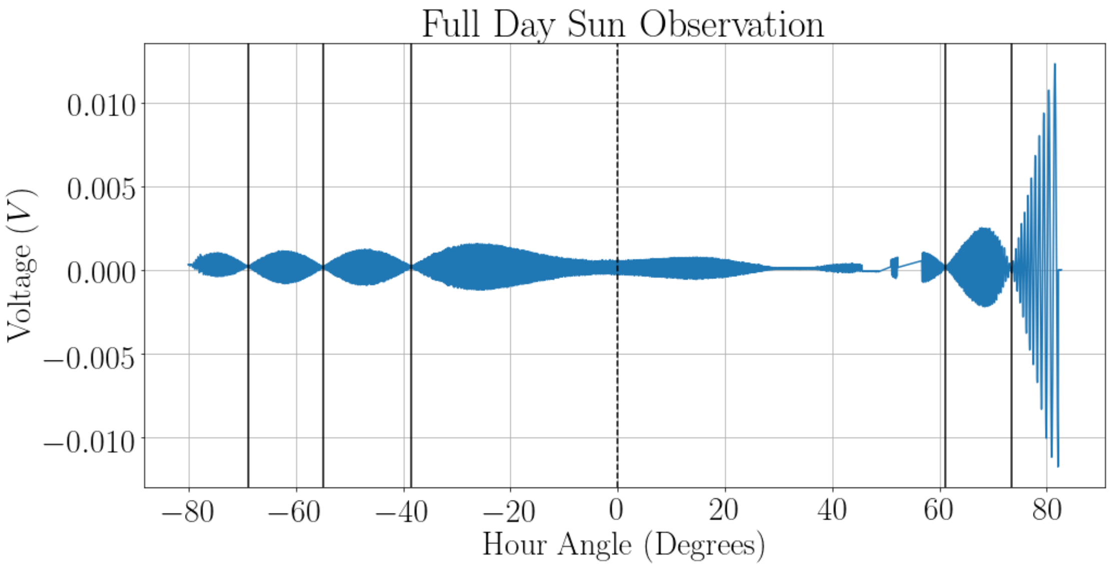

Figure 2: The raw data from a horizon to horizon observation of the sun. Most notable is the *envelope*, which oscillates and flattens out near 0, as expected. The Several null are here marked by vertical lines. Lastly, two small discontinuities near 50° were a result of pointing errors which we stopped collecting to correct, they had no discernible effect.

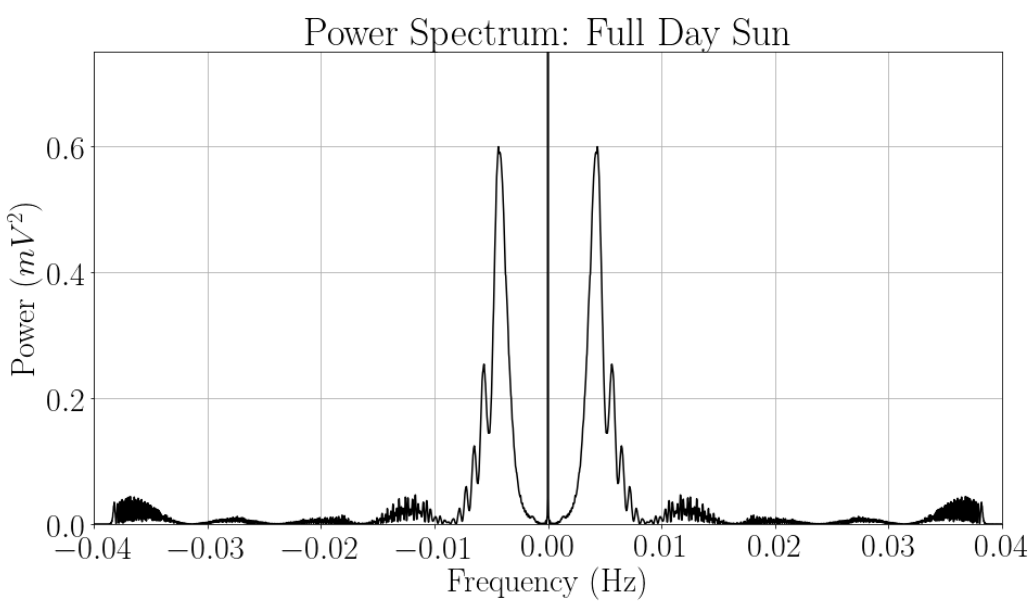

Figure 3: The power spectra from the sun observation shows multiple peaks which are evidence of the fringe frequency moving from slow to fast and back again which is as expected because telescopes when closer will move through the nodes of a Bessel function faster.

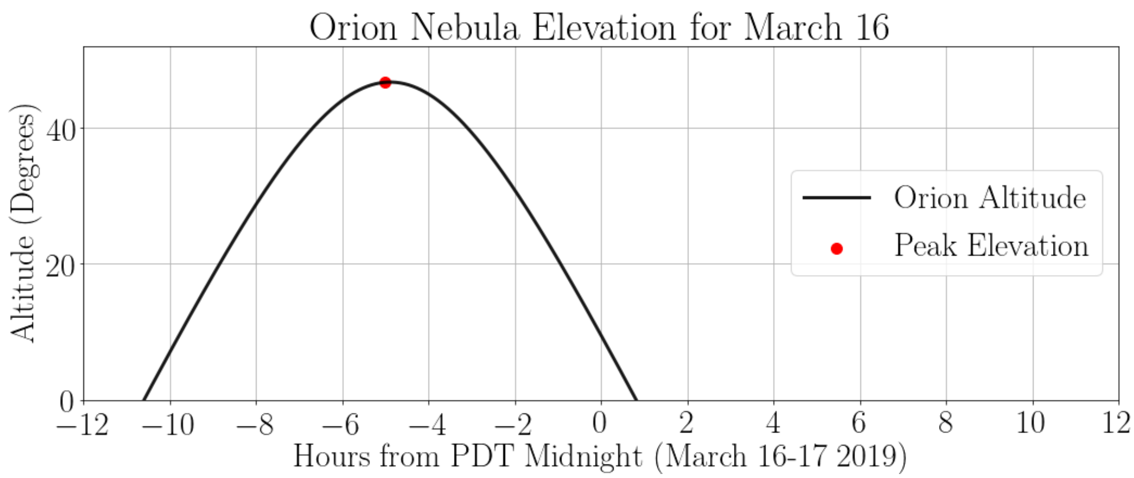

Figure 4: A plot of the Orion Nebula altitude on the night of March 16. Created using `Astropy` this served as our rough indication of when we would need to observe.

6 Point Source Observation

We observed the Orion Nebula at R.A(2000): $05^h35^m17.3s$, DEC(2000): $-05°23'28''$ on Saturday March 16 2019. Our goal was to track it from horizon to horizon so we first plotted its elevation for the night to determine when to start and stop pointing (See Figure 4).

Plotting the data (Figure 5) we notice it is increasing with time, which we did not expect, however we think this is due to the built up charge on the processing equipment so it can safely be removed by fitting a polynomial. We also notice a large dip which, due to its short duration, we did not remove. From this data alone it is not immediately clear we got a detection but a plot of the power spectrum (Figure 6) reveals two symmetric peaks off zero, confirming that we were on target. If we had not been, the twin peaks would not have been present.

7 Baseline Fitting

Using the data collected thus far, we can calculate the baseline of the interferometer. From the lab manual we know:

$$F(h_s) = A\cos\left(2\pi\nu\tau_g'\right) + B\sin\left(2\pi\nu\tau_g'\right). \tag{3}$$

where, $\nu\tau_g' = \left[\frac{B_{ew}}{\lambda}\cos\delta\right]\sin h_s + \left[\frac{B_{ns}}{\lambda}\sin L\cos\delta\right]\cos h_s$, B_{ew} and B_{ns} are our East-West and North-South baselines, and δ is the source's declination. Solving this is a non-linear least squares problem as the argument (hour angle) is inside a trigonometric function. If we make the substitution $Q_{ew} = \left[\frac{B_{ew}}{\lambda}\cos\delta\right]$ and $Q_{ns} = \left[\frac{B_{ns}}{\lambda}\sin L\cos\delta\right]$ we convert it to a a linear least squares problem to solve for the A and B parameters. Our initial

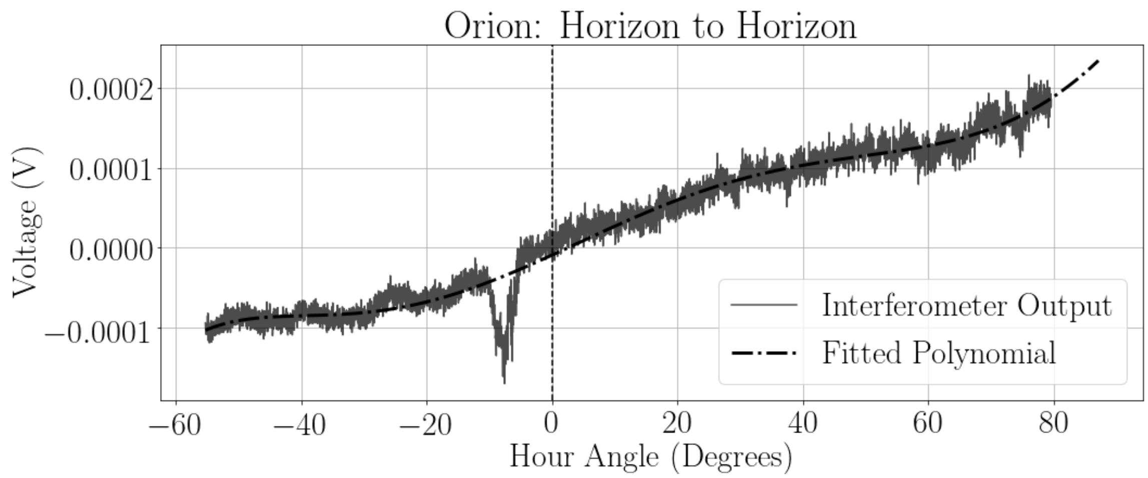

Figure 5: Raw data for our Orion observation. We note the fringe is increasing with time and is not centered around zero as it should be. To correct this we fit a polynomial (also plotted). A dip, which due to its short duration was not removed is also visible, but its effect on our analysis seemed to be minimal.

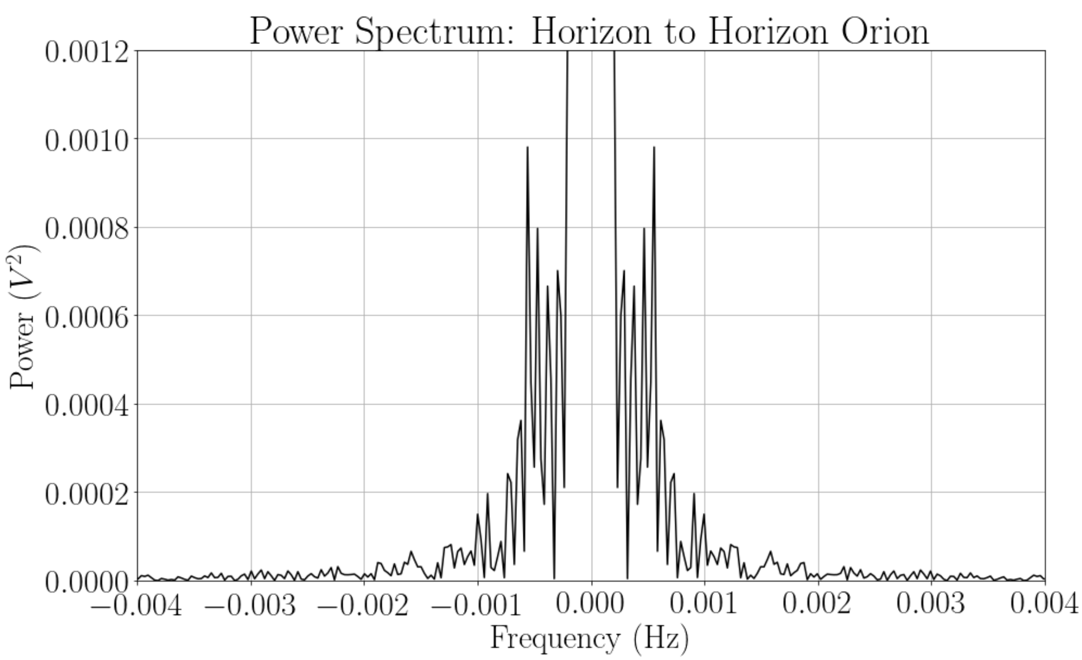

Figure 6: This unfiltered power spectrum of our Orion data, while seemingly weak, confirms we did in fact get the detection of Orion which we were expecting as it shows two peaks away from zero frequency.

solution for this was to use Scipy's curve fit function by creating a large array Q_{ew} and Q_{ns} then feeding each into equation 3, and finding the pair of Q values for which the sum of square residuals is a minimum. This method produced a North-South baseline of 0.92 m and an East-West baseline of 18.9 m which is slightly longer than the true baseline of 15.2 m[1].

8 Conclusion

The theme of this lab was increasing accuracy and precision. Unlike the days of the Big Horn and pointing it in "that direction" radio interferometry requires a finer touch to glean such useful things such as the angular radius of the sun which we calculated to be $0.303 \pm 0.2°$ and the baseline of our interferometer which we calculated to be about 18 m. Our analysis mostly fell within our expectations though given more time we would have like to improve our methods making them more accurate and properly calculate the error on our baseline fit.

9 Acknowledgments

As always, I would like to thank my group I♡Radio for their help in completing this lab.

10 Code

```
def orionaltaz():
    '''
    Computes the alt az for Orion:
    designed to be called repeatedly and sent out to the
    telescope for pointing

    Returns:
    oralt(float): The current altitude in degrees
    oraz(float): The current azimuth in degrees
    '''

def unix2LHA(utstamp):
    '''
    Converts a unix time stamp to an hour angle by first
    calculating the corresponding Julian Date and LST
```

[1]Due to lack of time, these numbers are presented without error bars. Given more time We would have found the error by taking a difference between the adjacent points and fitting a Gaussian to determine the error.

```
    Parameters:
    utstamp(Integer):unix time stamp in seconds

    Returns:
    LHA: the local hour angle in degrees
    '''
def u(ha,_baseline=baseline,_lambda=lam):
    '''
    Changes a crossing time in hour angle into a u coordinate
    Parameters:
    ha(float):Given hour angle in degrees
    _baseline(float):telescope east-west
        baseline value in meters
    _lambda(float): wavelength in meters

    Returns:
    u: coordinate in radians
    '''
def theta(u,x):
    '''Calculates the angular radius from given u coordinate
    and known x value
    Parameters:
    u:coordinate in radians
    x:dimensionless x coordinate corresponding
    to a bessel root

    Returns:
    _theta: angular radius in degrees
    '''
```