Using salinity variance budgets to quantify numerical mixing in a coastal ocean model

Dylan Schlichting, Lixin Qu, Robert Hetland, Daijiro Kobashi

2022 Ocean Sciences Meeting

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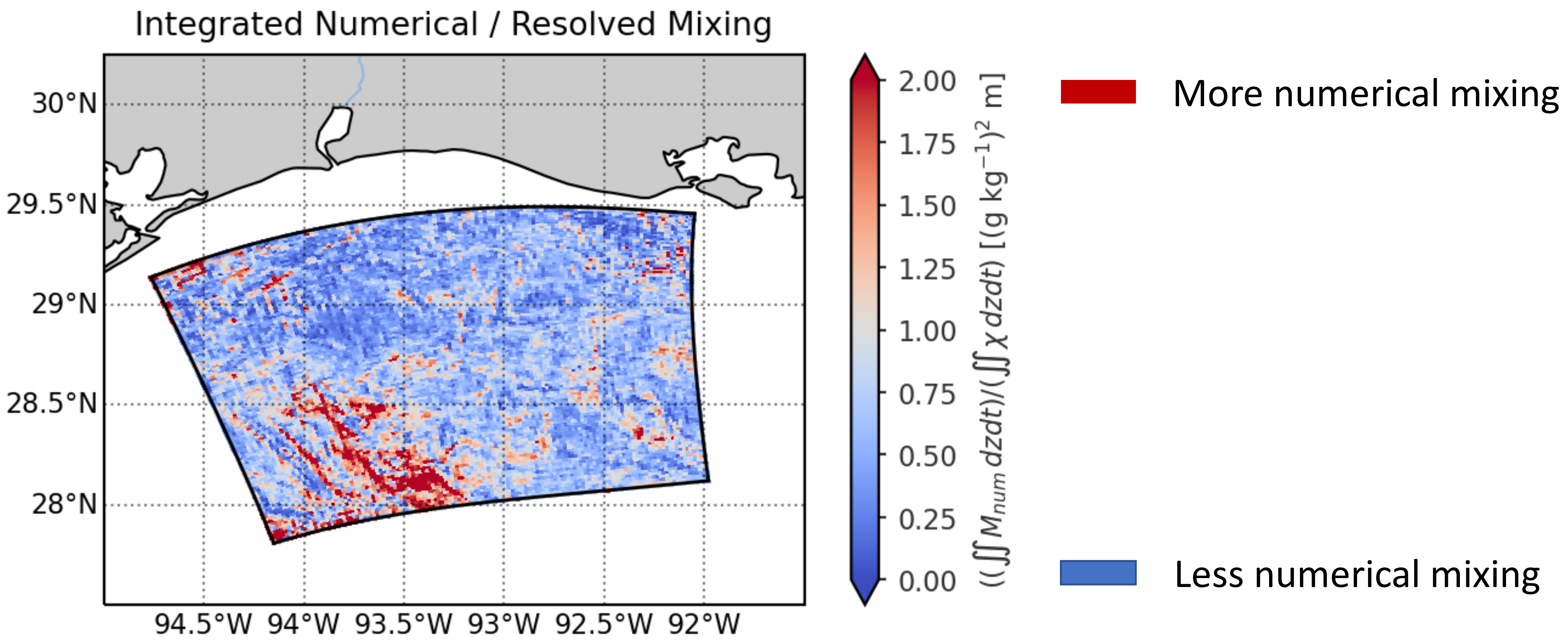
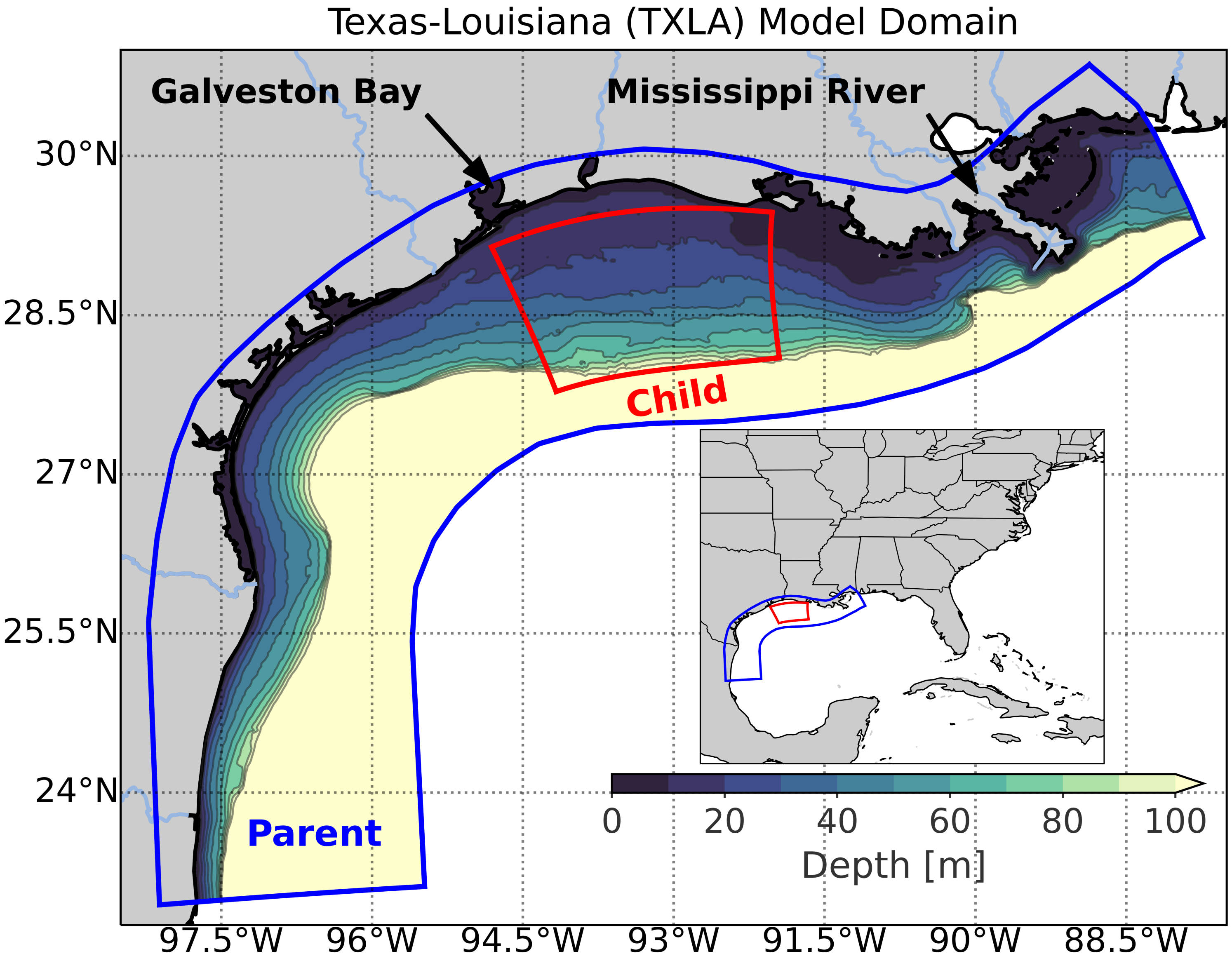
TEXAS A&M UNIVERSITY Oceanography

Numerical mixing = mixing generated by the discretization of advection schemes.



- Can quantify online via Burchard and Rennau (2008) or Klingbeil et al. (2014), but have to rerun model
- Can quantify offline using tracer variance budgets (MacCready et al., 2018; Burchard et al., 2019)

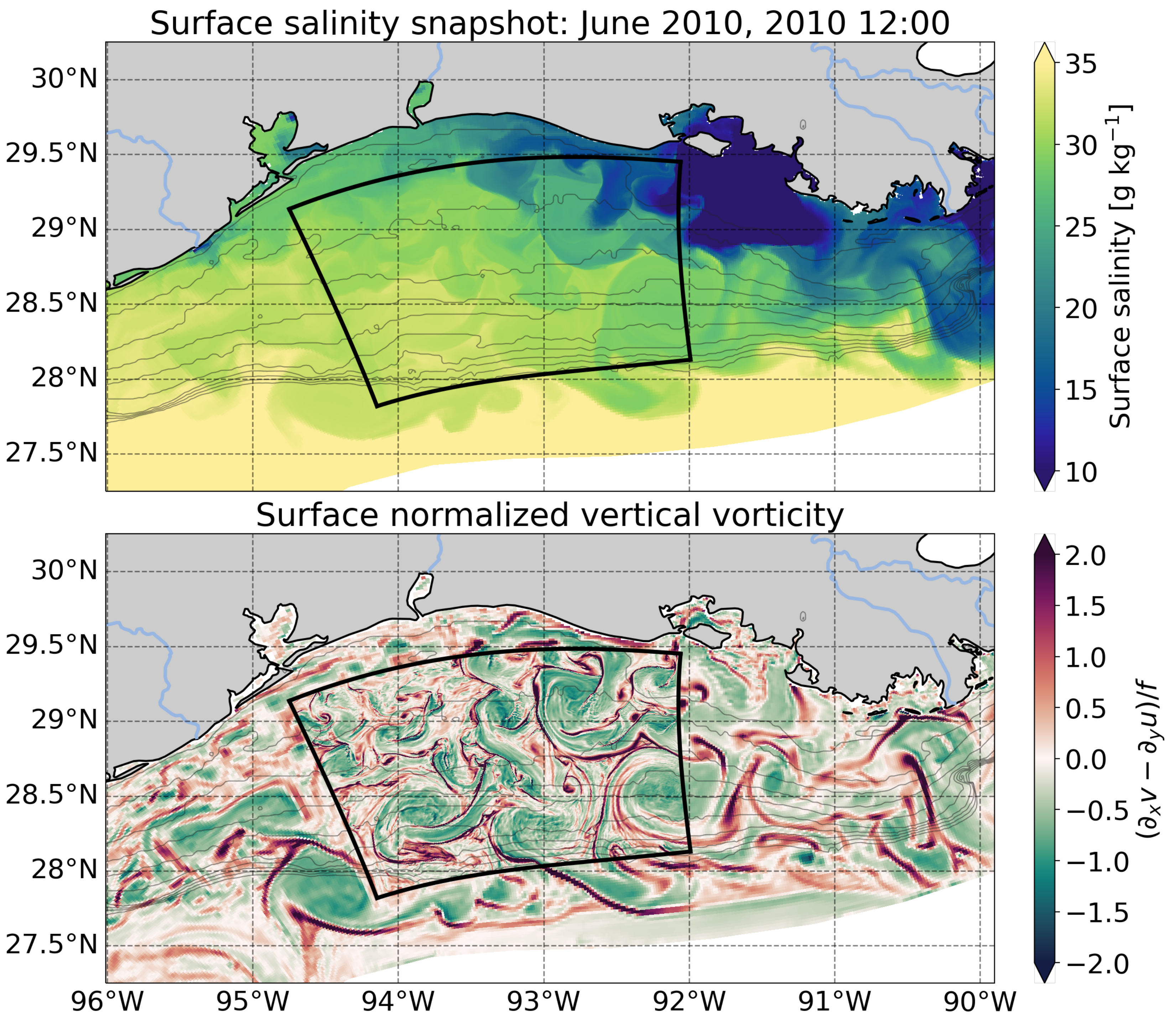


Zhang et al. (2012)
Thyng and Hetland (2017)
Kobashi and Hetland (2020)

- Realistic implementation of ROMS
- Mean resolution of 1.6 km in nested region
- 30 vertical layers
- Forced with NARR atmospheric data, 9 regional rivers
- Nested within HYCOM
- Skillful at reproducing salinity field; unbiased errors

Nested model:

- Two-way exchange
- 5X Parent - Mean resolution 300 m
- Surface fluxes, ICs, and boundary info interpolated from parent
- Hourly output
- Run from June 3 – July 14, 2010



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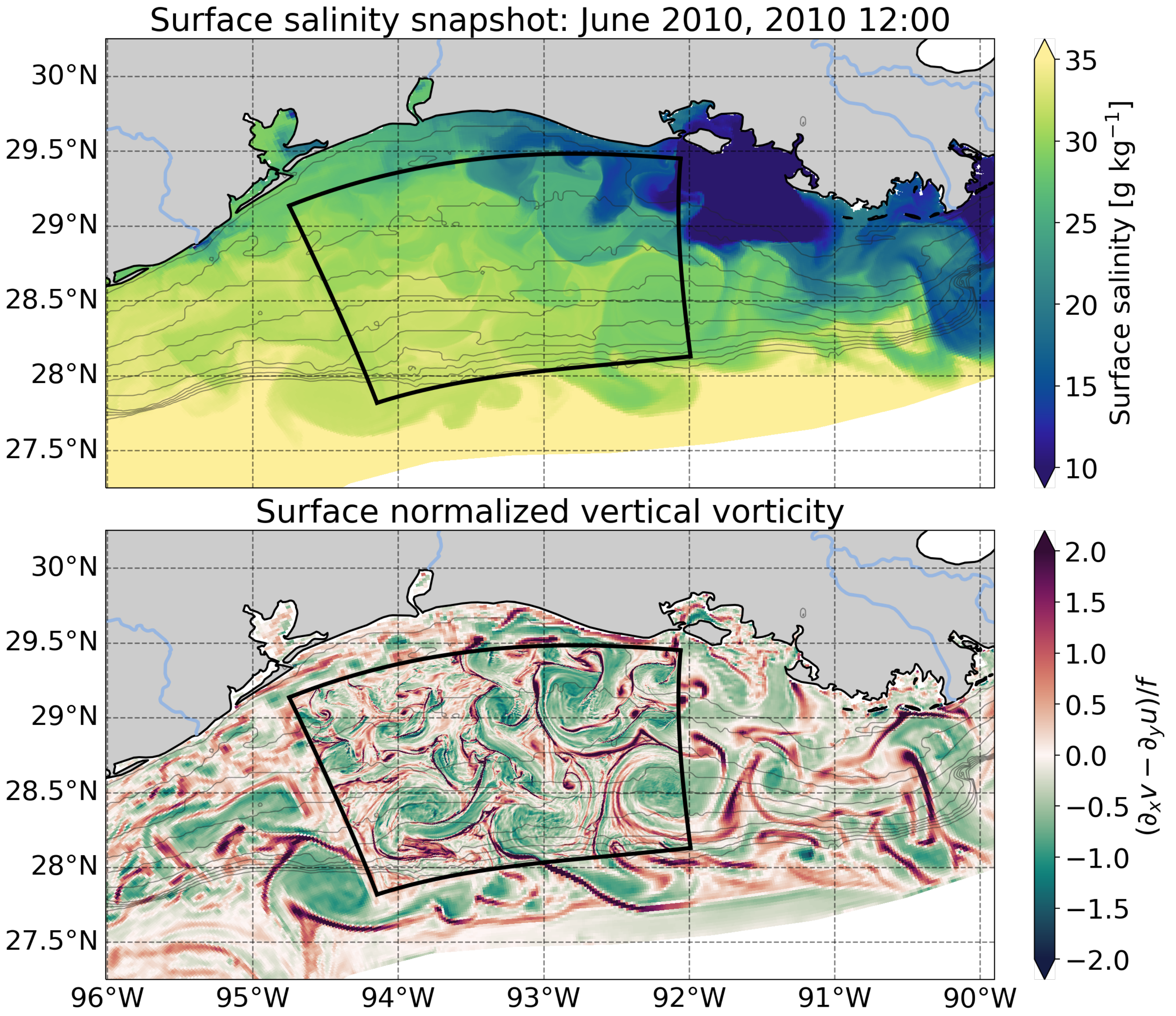
Research Questions

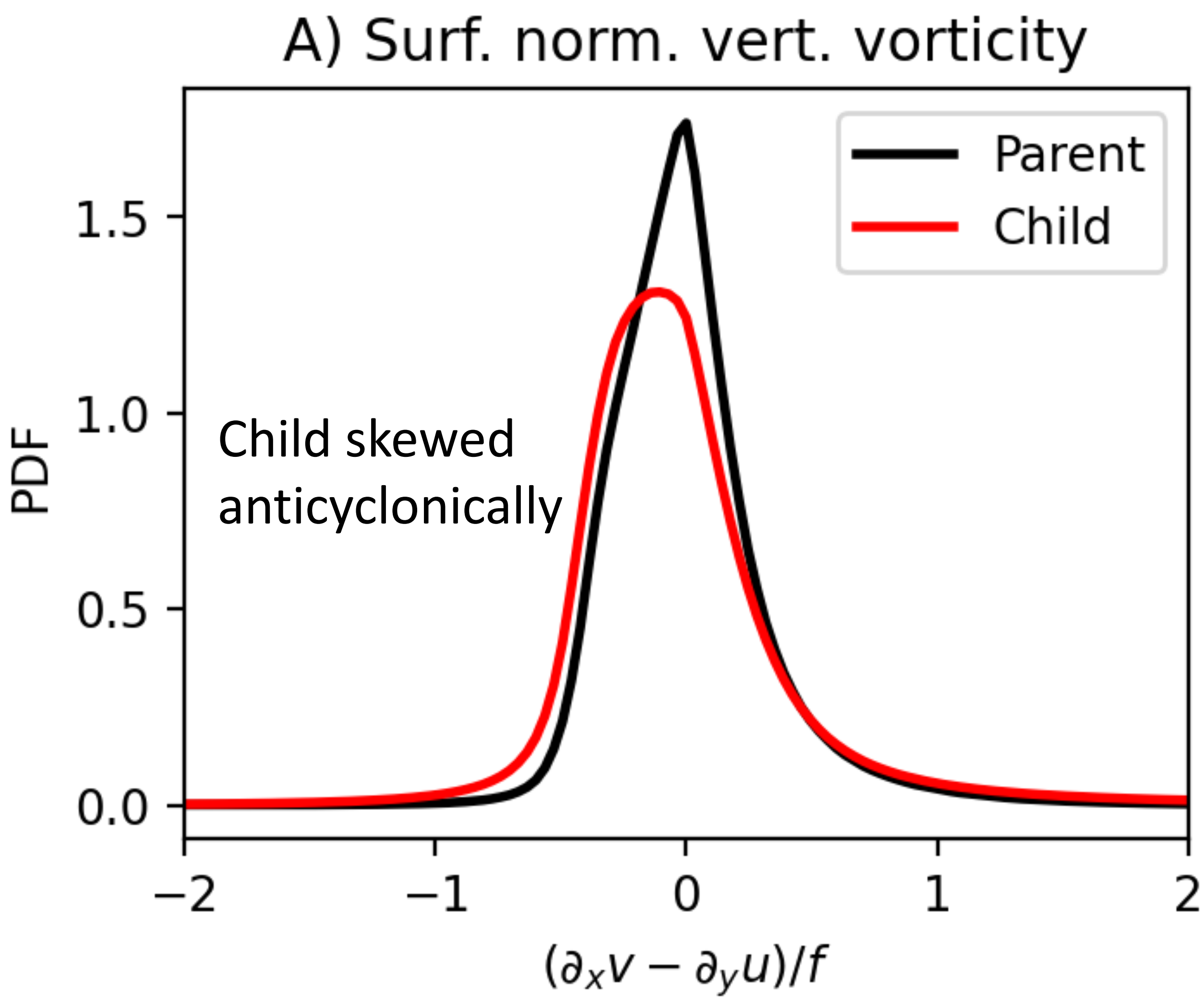
1. How do the parent and child models differ?
2. How does the numerical mixing differ between the models?

↓

Use salinity squared and salinity anomaly squared budgets, Burchard and Rennau (2008) algorithm



Velocity and salinity gradients are sharpened in the nested model!



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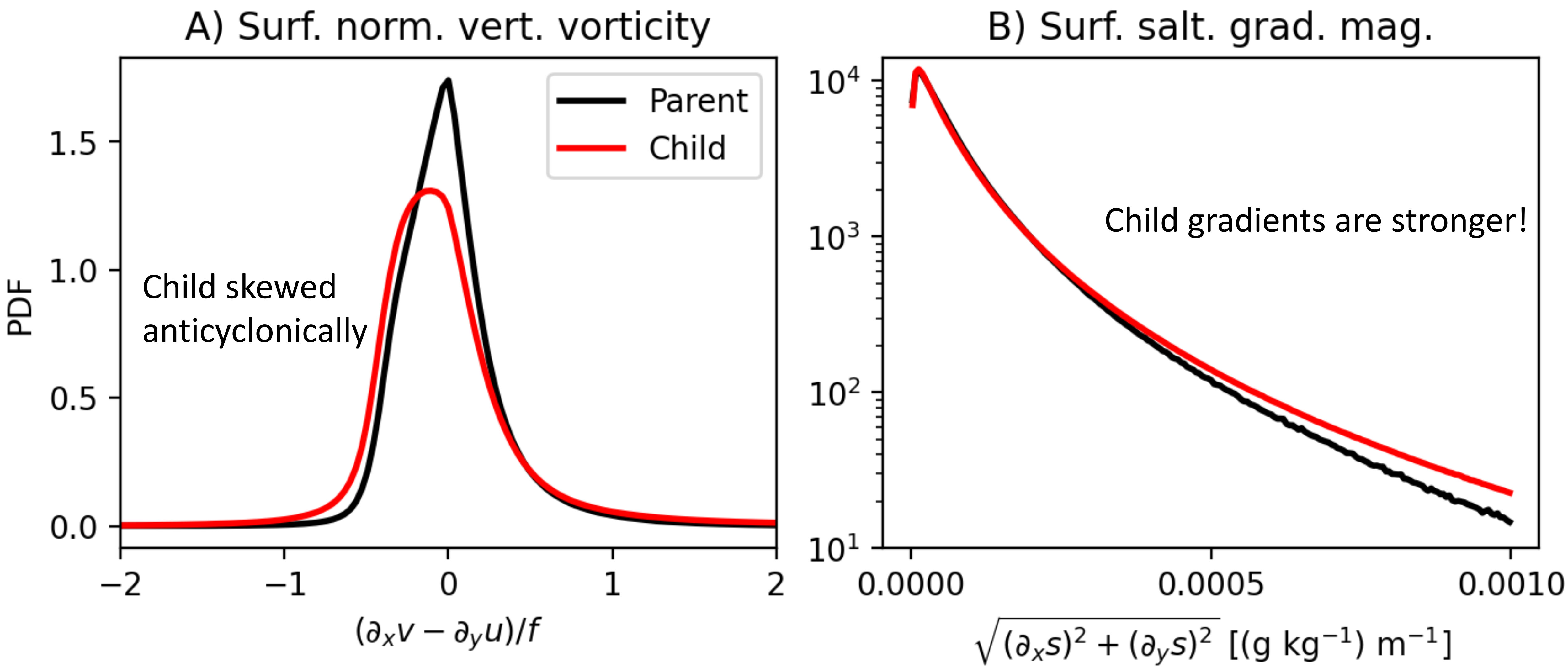

$$\boldsymbol{s^2}\ \textbf{Budget:}\quad \underbrace{\iiint_V \frac{\partial s^2}{\partial t}\, dV}_{Tendency} + \underbrace{\iint_A (\boldsymbol{u}s^2)\cdot d\boldsymbol{A}}_{Advection} + \underbrace{\iint_{A_{surf}} 2s^2(E-P)\, dA}_{Surface\ fluxes} = \underbrace{-2\iiint_V K_v \left(\frac{\partial s}{\partial z}\right)^2 dV}_{Resolved\ mixing} - M_{num,s^2}$$

MacCready et al. (2018) JPO
Lorenz et al. (2021) JPO

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s'^2 Budget: Define $s' = (s-\bar{s}),\ \bar{s} = \frac{1}{V}\iiint_V s\, dV$

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Online numerical mixing: $$M_{num,online} = \frac{A\{s^2\} - A\{s\}^2}{dt}$$

Burchard and Rennau (2008) OM

How does the s'^2 budget differ from the s^2 budget?

Rewrite salt squared in terms of salinity anomaly squared

$s' = (s - \bar{s}) \longrightarrow \quad \mathrm{s} = \bar{s} + s'$ Plug into salt conservation and boundary condition:

$$\frac{\partial(\bar{s} + s')}{\partial t} + \boldsymbol{u} \cdot \nabla(\bar{s} + s') = \frac{\partial}{\partial z}\left(K_v \frac{\partial(\bar{s} + s')}{\partial z}\right), \qquad -K_v \frac{\partial(\bar{s} + s')}{\partial z} = -(\bar{s} + s')(E - P) \ @z = \eta,$$

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Multiply by $2(\bar{s}+s')$ and volume integrate:

$$\boxed{\frac{d\bar{s}^2}{dt}V + 2\bar{s}\iiint \frac{\partial s'}{\partial t}dV} + \boxed{\iiint \frac{\partial s'^2}{\partial t}dV} + \boxed{\bar{s}^2\iint_A (\boldsymbol{u})\cdot d\boldsymbol{A} + 2\bar{s}\iint_A (\boldsymbol{u}s')\cdot d\boldsymbol{A}} + \boxed{\iint_A (\boldsymbol{u}s'^2)\cdot d\boldsymbol{A}}$$

$$+\boxed{\iint_{A_{surf}} (-2\bar{s}^2 - 2\bar{s}s')(E-P)\,dA} + \boxed{\iint_{A_{surf}} (-2\bar{s}s' - 2s'^2)(E-P)\,dA}$$

$$= \boxed{-2\iiint K_v\left(\frac{\partial s'}{\partial z}\right)^2 dV - M_{num,s^2}}$$

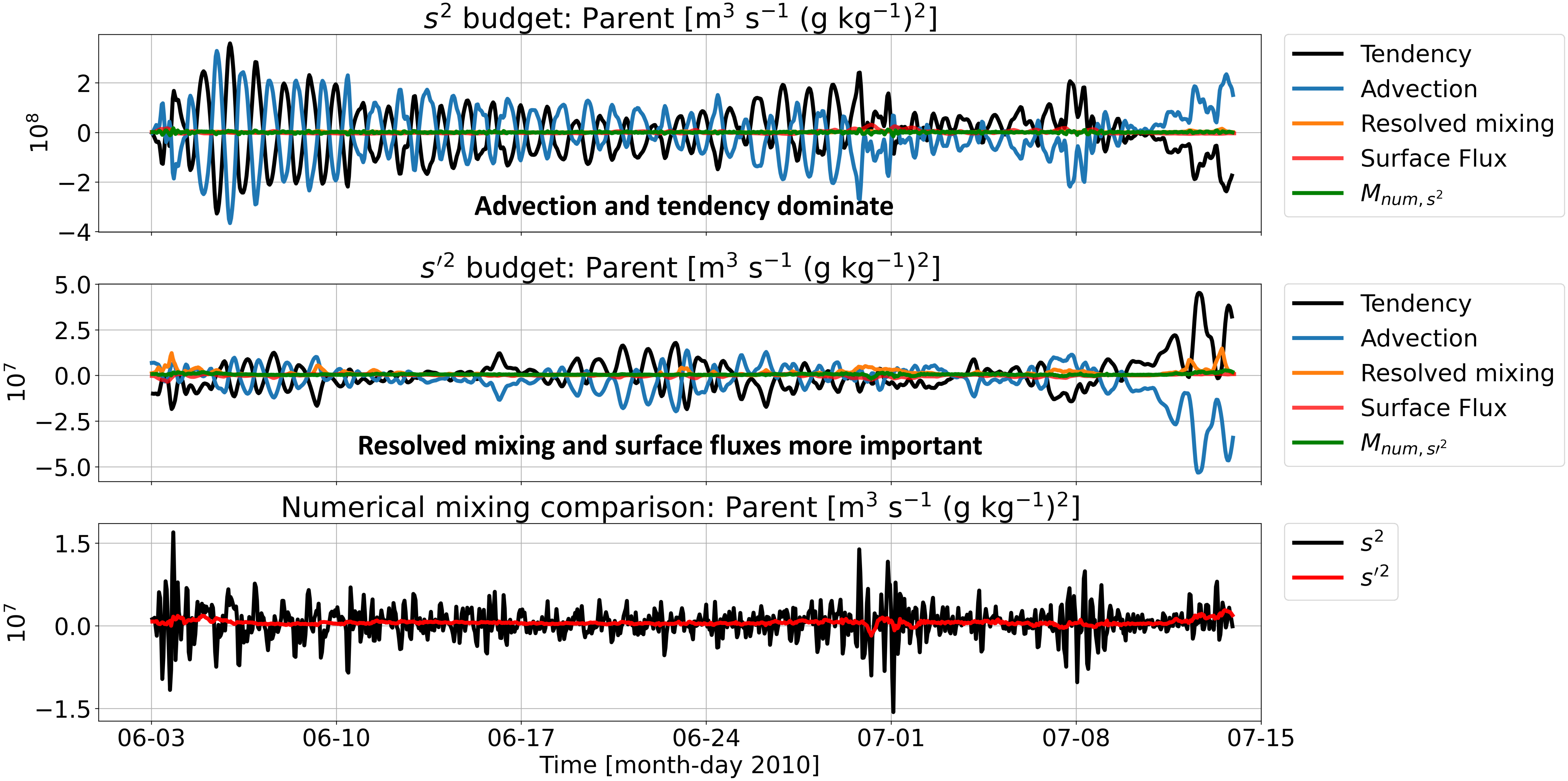
Extra tendency

Same

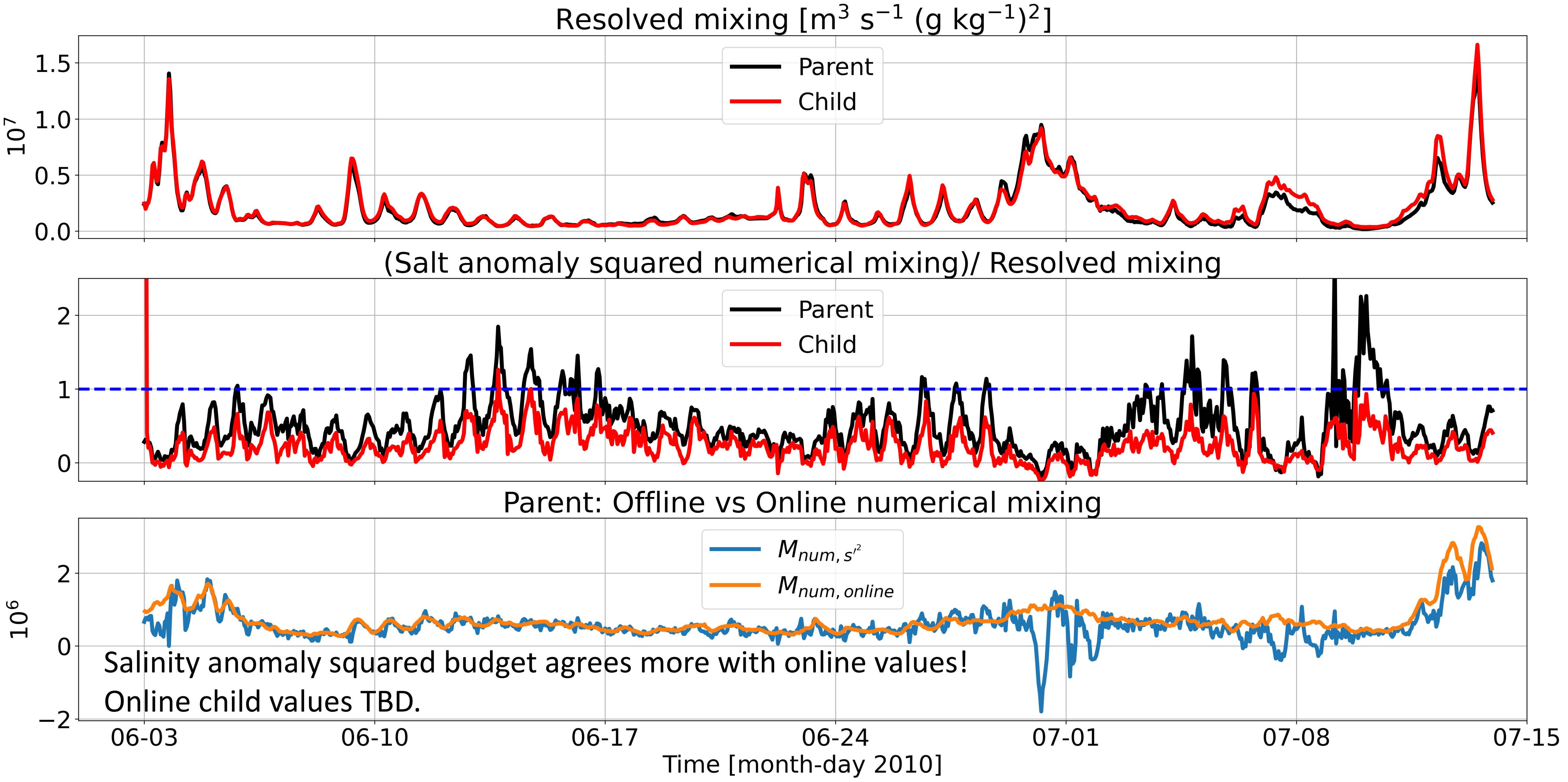
Extra advection

Extra surface

Numerical mixing estimates differ without extra terms!



Resolved mixing increases in child, numerical mixing decreases.



Conclusions

1. The salt squared and anomaly squared budgets can both quantify numerical & total mixing, but they will give different estimates unless extra terms are included

2. The salt anomaly squared budget is closer to the true numerical mixing created by the advection scheme because it includes extra processes

3. The numerical mixing in the child model is nearly halved relative to the parent, and we believe that is due to newly resolved frontal processes

For more information, visit https://dylanschlichting.github.io/