BLENDING SIR AND PREDATOR-PREY MODELS TO PREDICT THE LABOR MARKET

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Abstract. The labor market, including the unemployment rate and the amount of workers looking for jobs, can have a large impact on the economy. The more people employed means more money being spent, which in turn means more money being made. Furthermore, rise in unemployment can lead to a recession. Being able to predict the labor market can help us prepare for a recession and help us understand the economy better. In this paper, we adapt an SIR model to characterize the dynamics of employed, unemployed, and retired individuals in the labor market. Additionally, we employ a quasi predator-prey model to illustrate the oscillatory behavior observed in the white-collar and blue-collar industries. By comparing the SIR model to the predator-prey model, we aim to enhance our understanding of the complex interactions within the labor market, providing potential insights for recession prediction and economic analysis. Plz

1. Background/Motivation

One thing that is certain in life is that people will always need jobs. Not only this, but people will often lose their jobs. Furthermore, people will (eventually) retire from their jobs. The focus of our project is modeling this situation.

In recent studies exploring the complexities of employment trajectories and occupational sectors, researchers have employed various modeling approaches such as Agent-Based Modeling [8], and Markov Chains [12]. However, in a departure from conventional methodologies, our investigation takes an innovative turn by adapting the SIR (Susceptible-Infectious-Recovered) model [5], typically utilized for studying disease dynamics, to the realm of employment dynamics. This unique application aims to unravel the intricate propagation of employment statuses, specifically delving into the transitions between being employed, unemployed, and retired.

By employing the SIR model, we aim to comprehend the propagation of employment statuses—specifically, the transitions between being employed, unemployed, and retired. A discernible trend has emerged in recent times, notably influenced by the technological revolution. The surge in interest and demand for tech-oriented careers has prompted a significant shift away from traditional blue-collar professions. This migration has led to a dual challenge: a scarcity of skilled workers in the blue-collar sector and an oversaturation of the tech industry [3]. To capture this relationship between white and blue collar jobs,

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we incorporate elements of a quasi-predator-prey framework inspired by ecological models, which offers insights into the cyclical dynamics between these sectors.

Motivated by the imperative to comprehend and address the consequences of this evolving employment landscape, our research aims to contribute valuable insights for informing strategic policies and industry interventions. By synthesizing the strengths of both the predator-prey framework and the SIR model, we aspire to provide a comprehensive understanding of the intricate dynamics shaping the contemporary employment scenario.

2. Modeling

2.1. **Theoretical Framework.** The Susceptible-Infectious-Recovered (SIR) model, developed by Kermack and McKendrick in 1927 [5], is a foundational mathematical framework for understanding the spread of infectious diseases in populations. It divides individuals into susceptible, infectious, and recovered compartments, capturing the dynamics of disease transmission. In our model, we adapt the SIR model to represent the dynamics of the employment market through the labor force, unemployed, and retired populations.

2.2. **Previous Work.** We begin by building off the work of ElFadily et. al. [2]. In their work, ElFadily et. al. proposed a model representing the labor force and unemployed populations. They begin by defining their equations as

$$\frac{dL}{dt} = \gamma U - (\sigma + \mu)L, \quad \frac{dU}{dt} = \rho\left(1 - \frac{L_\tau + U_\tau}{N_c}\right)L_\tau + \sigma L - (\gamma + \mu)U, \tag{1}$$

where L is the labor force, U is the unemployed population, with initial conditions for (1) defined as:

$$\begin{gathered} L(0) > 0, \quad U(0) > 0, \\ (L(\theta), U(\theta)) = (\varphi_1(\theta), \varphi_2(\theta)), \quad \theta \in [-\tau, 0], \end{gathered} \tag{2}$$

where $\varphi_i \in C([-\tau, 0], \mathbb{R}^+)$, $i = 1, 2$.

The parameters are defined as follows:

- γ: employment rate
- σ: rate of job loss
- μ: mortality rate
- ρ: maximum population growth rate
- N_c: population carrying capacity
- τ: time lag needed to contribute in the reproductive process of a new individual looking for a job

2.3. **Modifications: The Retirement Group.** With this information in mind, we can begin to adapt this model to fit our needs. We begin by adding a third population, the retired population, R. We can then define our new equations as

$$
\begin{aligned}
\frac{dL}{dt} &= \gamma U - (\sigma+\mu)L - \left(\frac{\Sigma}{L+U}\right)L + \omega\left(\frac{\Sigma}{L+U}\right)R + \rho L,\\
\frac{dU}{dt} &= \rho\left(1-\frac{L+U}{N_c}\right)L + \sigma L - (\mu+\gamma)U,\\
\frac{dR}{dt} &= \left(\frac{\Sigma}{L+U}\right)L - \omega\left(\frac{\Sigma}{L+U}\right)R \ - \ \mu R,
\end{aligned} \tag{3}
$$

which simplify to

$$
\begin{aligned}
\frac{dL}{dt} &= \gamma U - (\sigma+\mu)L + (\omega R - L)\left(\frac{\Sigma}{L+U}\right) + \rho L,\\
\frac{dU}{dt} &= \rho\left(1-\frac{L+U}{N_c}\right)L + \sigma L - (\mu+\gamma)U\\
\frac{dR}{dt} &= (L-\omega R)\left(\frac{\Sigma}{L+U}\right) \ - \ \mu R.
\end{aligned} \tag{4}
$$

One of the first things to note from our equations is the removal of the time lag τ. This is because, instead of factoring in people when they are born, we are instead factoring them in when they turn of working age (16). This reduces unnecessary complexity in our model. Additionally, we make two assumptions about unemployed people, being they will neither retire directly from looking for work, nor contribute to the growth of the population. A final thing to note is that we have added two new parameters, ω and Σ. We define Σ to be the number of people who retire each year in the United States, and ω to be the rate at which retired people enter back into the full-time workforce (which is a dimensionless constant). We can then define our new initial conditions as:

$$
\begin{gathered}
L(0) > 0, \quad U(0) > 0, \quad R(0) > 0,\\
(L(\theta), U(\theta), R(\theta)) = (\varphi_1(\theta), \varphi_2(\theta), \varphi_3(\theta)), \quad \theta \in [-\tau, 0],
\end{gathered} \tag{5}
$$

where $\varphi_i \in C([-\tau, 0], \mathbb{R}^+)$, $\quad i = 1, 2, 3$. Incorporating nuanced dynamics into our model, we introduce the following terms and elucidate their significance within the equations:

- $\pm\left(\frac{\Sigma}{L+U}\right)L$: Captures the dynamic interplay of individuals retiring from the workforce at each time step. The term considers the annual number of retirees (Σ) in relation to the total workforce population ($L+U$), calculating the percentage of individuals retiring and multiplying by the number of individuals actively engaged in the workforce (L), excluding unemployed individuals.
- $\pm\omega\left(\frac{\Sigma}{L+U}\right)R$: Accounts for individuals retiring and re-entering the workforce. The dimensionless constant ω denotes the rate at which retired individuals transition back to full-time work. The term calculates the expected number of retired individuals returning to the workforce.

- ρL: Represents the natural growth of jobs, proportional to the number of employed people. Assumes economic well-being and the creation of new jobs.
- $-\mu R$: Represents the natural attrition of retired individuals through mortality at each time step. Acknowledges the inevitable departure of individuals due to mortality, contributing to a comprehensive understanding of the model's temporal evolution.

2.4. **Modifications: The White-Collar and Blue-Collar Groups.** We can further model the labor market by examining the relationship between two industries, commonly referred to as the white-collar and blue-collar industries. In recent years, we have seen an explosion of jobs and interest in the white-collar field, specifically in the tech industry, while the blue-collar industry has seen a decline in interest. This has led to an oversaturation of the white-collar industry and a scarcity of skilled workers in the blue-collar industry, which in turn has slowed growth in the white-collar industry and led to increased growth in the blue-collar industry. This cyclical relationship mirrors that of a predator-prey relationship, and we can model it as such.

Choosing a predator-prey model to represent blue-collar and white-collar jobs is justified by its ability to capture dynamic interactions and cyclic behavior between the two job categories. This modeling approach incorporates feedback loops, reflecting the influence each job type has on the other, and naturally represents population dynamics in response to economic, educational, or technological factors. Additionally, the model's adaptability allows for the inclusion of additional factors, offering flexibility in addressing the multifaceted dynamics of the labor market. This choice opens avenues for research and exploration of hypothetical scenarios, providing a structured framework to analyze and understand the interplay between different job categories over time.

The classical predator-prey model is given by the following equations:

$$\frac{dx}{dt} = \rho x - axy, \quad \frac{dy}{dt} = -\mu y + \varepsilon axy. \tag{6}$$

where x is the prey population, y is the predator population, and ρ, a, μ, and ε are parameters. We can adapt this model to fit our needs by defining the following:

$$\frac{dx}{dt} = \rho x \left(1 - \frac{x}{k}\right) - axy, \quad \frac{dy}{dt} = -\mu y + \varepsilon axy + \beta y \left(1 - \frac{y}{C}\right). \tag{7}$$

where

- ρ: Growth rate of blue-collar jobs
- a: Rate at which people switch from blue collar to white collar jobs
- μ: Decay rate of white-collar jobs
- ε: The rate at which new job growth in converting from blue-collar to white-collar jobs
- k: Carrying capacity for blue-collar jobs
- C: Carrying capacity for white-collar jobs.
- β: Growth rate of white-collar jobs

Thus, we can interpret the new additions to our model as:

- $\rho x(1-\frac{x}{k})$: This term adds a carrying capacity to the "prey" population (or blue collar population). This is because, while the blue-collar industry is growing, it is not growing at an exponential rate. Instead, it is growing at a rate that is proportional to the number of blue-collar workers.
- $\beta y(1-\frac{y}{C})$: This term adds a carrying capacity to the "predator" population (or white collar population). This is because the white-collar industry, unlike a traditional predator, cannot grow indefinitely. Instead, it is limited by the number of people in the workforce. Furthermore, the white-collar industry can grow independent of the blue-collar industry, which is also a modification from the traditional predator population.

The reason that only white-collar jobs at a decay rate is meant to model the current market situation. The white-collar market is becoming oversaturated so as it gets a higher population it decays, whereas the blue-collar market has not reached this point and never does with the initial conditions given. We also made the assumption that the white-collar market converts whereas the blue-collar market does not, this was a modeling choice to model the specific circumstance we wanted to attempt to model, in the real world, this could go both ways with varying degrees, but for simplification we made it only one way.

2.5. **Labor Force, Unemployement, and Retirement Simulations.** We now give the specific values for hyperparameters for our model and how we came to those values. We define the following:

- $\sigma = 0.013905$: We derived this value by analyzing comprehensive data on total layoffs and discharges in the United States from 2000 to 2023, as documented by the Federal Reserve Economic Data (FRED) [4]. The calculated average resulted in $\sigma = 0.013905$.
- $\rho = 0.014577$: The maximum growth rate, denoted as ρ, was determined by examining an Excel spreadsheet provided by MacroTrends [6]. After averaging the growth rates from 2000 to 2022, we incorporated three standard deviations to arrive at $\rho = 0.014577$.
- $\gamma = 0.6062$: The average employment rate from 2000 to 2022, obtained from the Bureau of Labor Statistics [10], led to the computation of $\gamma = 0.6062$.
- $\mu = 0.008498$: Derived from an analysis of mortality data from 2000 to 2022 [11], the mortality rate was calculated by taking the average number of deaths per 100,000 people. This resulted in $\mu = 0.008498$.
- $N_c = 260,000,000$: The population of individuals aged 18 and above in the United States in 2022 was calculated to be $260,000,000$, as reported by Kids Count [1].
- $\Sigma = 775,045$: The annual number of retirees in the United States, determined by examining Social Security Administration data from 2000 to 2021 [9], was calculated using the formula:

$$\Sigma = \frac{1}{2021-2001}\sum_{i=2001}^{2021}(x_i - x_{i-1})$$

where x_i is the number of retirees in year i. This resulted in $\Sigma = 775,045$.

- $\omega = 0.063$: The rate at which retired individuals re-enter the full-time workforce, denoted as ω, was identified as 6.3% or 0.063 based on research by Maestas [7], resulting in $\omega = 0.063$.

We began testing our model by running it for 60 years with the current numbers for the United States (see figure 1). As you can see, the model very quickly reaches what appears to be an equilibrium. Additionally, we can see the total population grow as time goes on, reflecting the expected population growth of the United States.

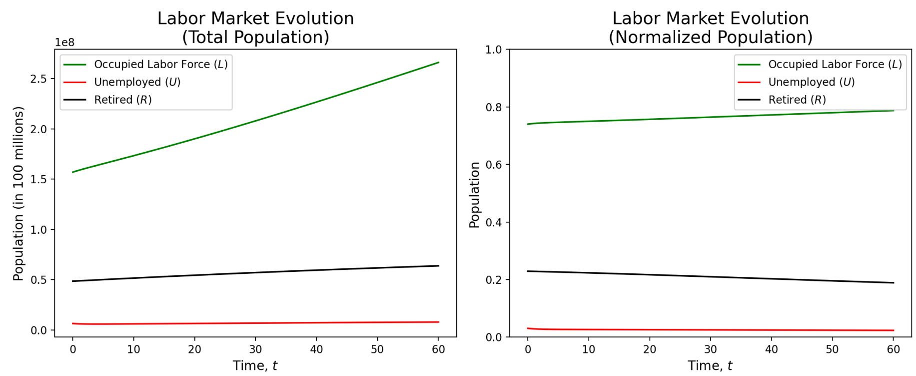

FIGURE 1. Initial conditions: $L(0) = 157,000,000$, $U(0) = 6,500,000$, $R(0) = 48,590,000$.

To test the robustness of our model, we ran it with different initial conditions that do not represent the current situation in the United States. We first decreased the number in the labor force, increased the number of unemployed, and decreased the number of retired. We made these changes rather conservative by only slightly perturbing the real numbers. We then ran the model for 60 years (see figure 2). Parallel to figure 1, we can see that the model still reaches an equilibrium, despite the initial conditions being skewed from their true values.

We ran our model, once again, against a different set of initial conditions. This time, we significantly decreased the number of people in the occupied labor force, significantly increased the number of unemployed (ensuring that the number of unemployed was much greater than the number of people in the labor force), and moderately decreased the number of retired. We then ran the model for 60 years (see figure 3). As you can see, the model still reaches an equilibrium, despite the initial conditions being heavily skewed from their true values.

We ran a final test, this time having the number of retired people set as greater than the number of people in the labor force and unemployed combined. We then ran the model for 60 years (see figure 4).

Unlike the previous graphs, we can see that the model does not reach an equilibrium in 60 years. While the number of people in the labor force rises and the number of retired

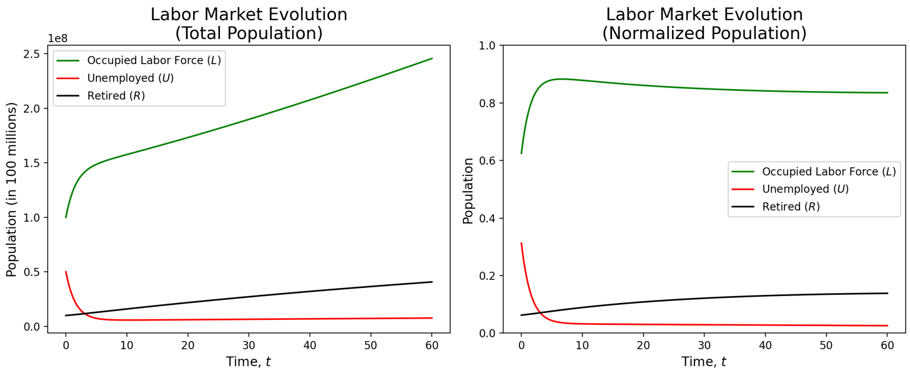

FIGURE 2. Initial conditions: $L(0) = 100,000,000$, $U(0) = 50,000,000$, $R(0) = 10,000,000$.

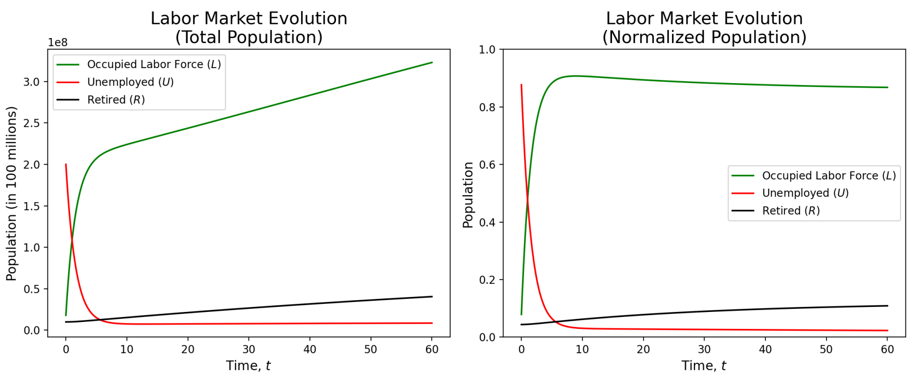

FIGURE 3. Initial conditions: $L(0) = 18,000,000$, $U(0) = 200,000,000$, $R(0) = 10,000,000$.

people falls, this chance does not appear to be significant enough to reach an equilibrium. However, when ran again for $T = 100$ years, we can see that the model gets closer to an equilibrium, but still does not reach one (see figure 5).

2.6. **White-Collar and Blue-Collar Simulations.** For our white- and blue-collar model, we experimented with different hyperparameters to see how they affected the model.

In our first run of the model, we used parameters $\rho = 7$, $a = 5$, $\mu = 1$, $\varepsilon = .2$, $k = 3$, $\beta = 1$, $C = 1.5$. As we see, the model oscilates slightly in the beginning, and then settles into a stable equilibrium (see figure 6). The initial conditions come from data on the US Labor marekt and percentage of workers in white-collar or blue-collar jobs [10].

Consider two different sets of initial conditions, namely $\rho = 7$, $a = 5$, $\mu = 2$, $\varepsilon = .2$, $k = 3$, $\beta = 1$, $C = 1.5$ and $\rho = 7$, $a = 5$, $\mu = 1$, $\varepsilon = .2$, $k = 3$, $\beta = 1$, $C = 1.5$. Note that

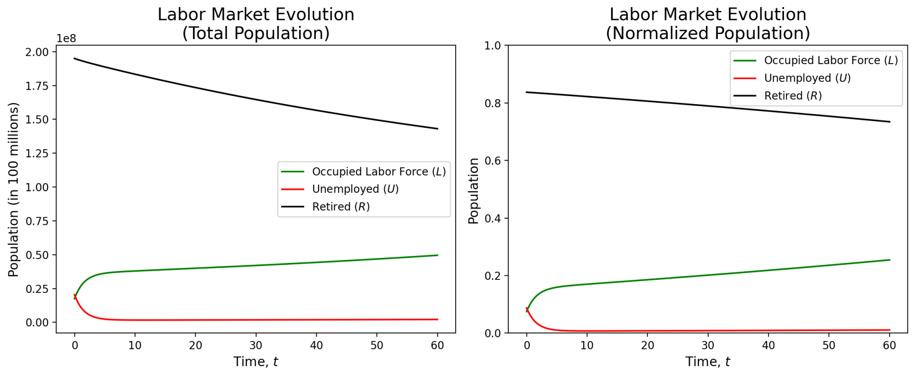

FIGURE 4. Initial conditions: $L(0) = 17,500,000$, $U(0) = 20,400,000$, $R(0) = 195,000,000$.

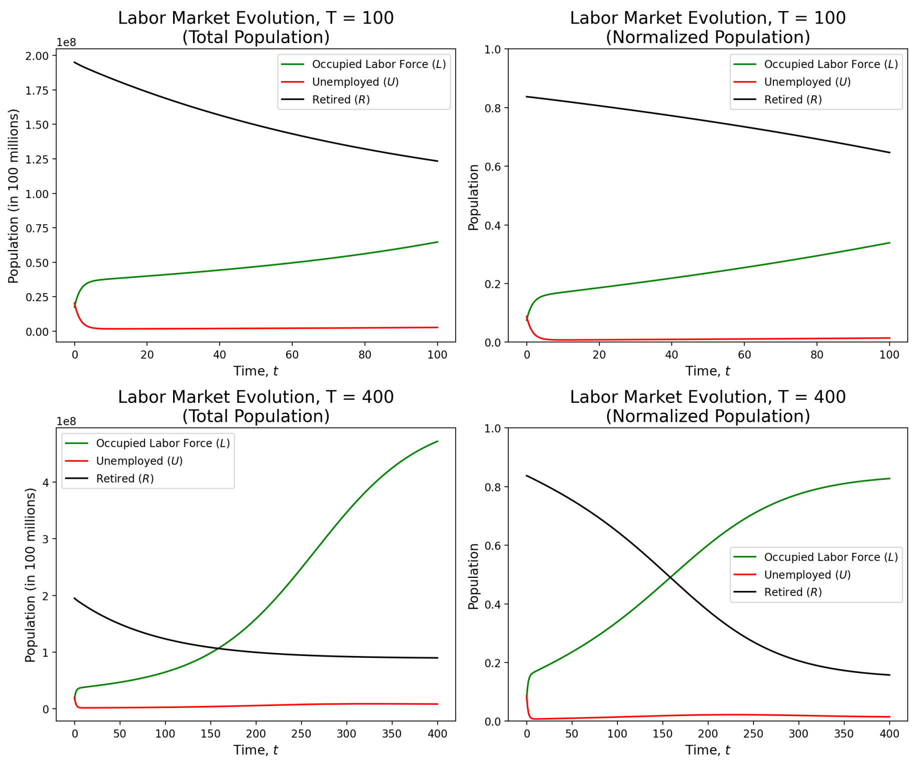

FIGURE 5. Initial conditions: $L(0) = 17,500,000$, $U(0) = 20,400,000$, $R(0) = 195,000,000$.

the only difference between these two initial conditions is how μ differs by a 1. Despite this, our model predicts completely different results (see figure 7).

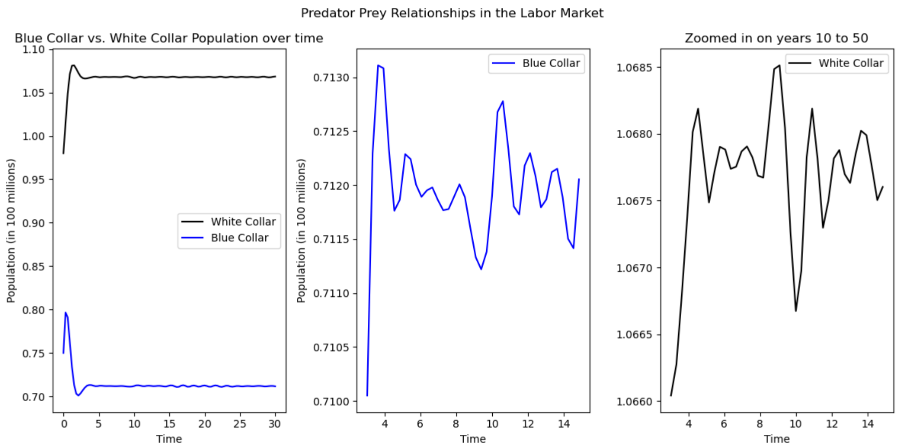

FIGURE 6. Parameters $\rho = 7$, $a = 5$, $\mu = 1$, $\varepsilon = .2$, $k = 3$, $\beta = 1$, $C = 1.5$. Zooming in between years $10 - 50$, we can see the oscillations more clearly.

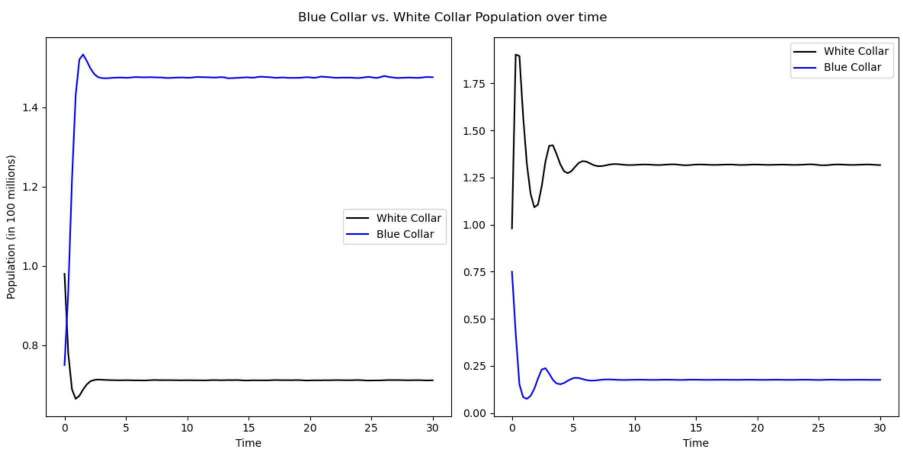

FIGURE 7. The left graph corresponds to $\mu = 2$, while the right graph corresponds to $\varepsilon = 1$, others parameters are kept the same.

3. Results

The SIR and predator-prey models in the context of employment dynamics offers a comprehensive framework for understanding the complex interactions within the labor market. Here are some key observations and conclusions drawn from the presented model:

The SIR model is very sensitive to changes in the hyperparameters, and even a small change can cause the model to behave very differently. We see in figure 7 that a change of 1 in μ causes the populations to entirely flip. Additionally, changing ε from .2 to 1 causes the white-collar population to swing wildly and see booms and busts equating to almost

100 million jobs. That massive of a swing is not realstic and not very useful for real world application.

The predator prey white-collar and blue-collar model, while not as robust as the SIR labor force model, still shows some interesting results. It is interesting to see how the relationships in the model caused oscillations in the different populations. The oscillations are small enough that they are not visible on the graph, but they are still present, and can mimic the overall labor force where a swing of thousands of jobs is noticed by the economy as a whole. A strength of this second model is exactly that, being able to see the oscillations while keeping the oscillations to a scale that would be realistic in the real world.

Overall, our model of the labor market shows remarkable stability. We can see that, regardless of the initial conditions, the model reaches an equilibrium, with the number of employed, unemployed, and retired individuals remaining relatively constant. When we used initial conditions that reflected the current numbers for the United States, the model saw relatively little change as time went on (see figure 1). With initial conditions that represented a larger than average unemployed population, the model corrected itself and reached a similar equilibrium as the previous model (see figure 2). Finally, when presented with initial populations that were flipped, the model still stabilized to the same equilibrium (see figure 3).

4. Analysis/Conclusions

In our model, modified from ElFadily et. al. [2], we have extended their classical SIR framework, traditionally used to understand the spread of infectious diseases, to capture the dynamics of the labor market. In our adaptation, we introduced an additional compartment for Retired (R) individuals, reflecting the life cycle of employment. The key modifications involve incorporating terms that represent the natural attrition of the retired population, their re-entry into the workforce, and the growth of job opportunities proportional to the number of employed individuals. These adjustments provide a nuanced representation of the labor market's temporal evolution, accounting for retirement dynamics, mortality, and the cyclical nature of job creation and re-entry. This enhanced model allows for a more comprehensive understanding of the complex interactions within the labor market over time.

Despite the strengths of our model, there are weaknesses present. One weakness is that changing the initial conditions can cause the results to differ significantly between each other results during the first few years. While it is true that the solutions end up reaching similar values as T grows, those first few years of difference can pose a problem. Another more significant weakness is that this model only considers how the labor markets interact with each other. One major factor in the labor market is how the economy is, and our model does not take that into account. Thus, one improvement that can be made is finding a way to include present economic conditions. Another weakness is that, while hard to see with the scale on the graph, the retired population does increase over time. This is because more people retire as population grows and we see a slow but steady growth, as expected.

Despite its weaknesses, the model provides insights into the long-term stability and equilibrium of the workforce. The inclusion of retirement-related terms allows policymakers and economists to analyze the impact of demographic shifts on employment trends and anticipate workforce fluctuations. Moreover, the explicit consideration of job creation and re-entry mechanisms offers a more realistic representation of economic dynamics, enabling better predictions of labor market behavior. Understanding the cyclical nature of job opportunities and retiree contributions provides valuable insights for economic planning, workforce management, and policy development. This modified SIR model, by bridging epidemiological principles with labor market dynamics, contributes to a holistic framework for studying the interplay between demographic factors and economic trends, supporting informed decision-making in the real world.

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