# Optimal Taxation with Political Externalities

Dylan T. Moore*

[Click here for latest version]

November 10, 2021

**Abstract**

When should tax policy be used to influence political donation behavior? In a model of electoral politics where campaign spending is financed by citizen donations, inequality of political influence favoring the "donor class" can arise. Adopting the normative stance that such inequality is undesirable, characterizations of optimal linear and nonlinear taxation of political donations are presented. Sufficient statistics for optimal policy include not only donation demand elasticities, but also the marginal efficacy of campaign spending, and the effect of taxes on the sensitivity of donations to candidate policy platforms. The results help to rationalize some observed policies. For example, taxing donations reduces campaign spending, driving up the marginal return to campaign spending and potentially increasing political inequality. Nonlinear subsidies targeting small donors - such as those seen in Canada - can decrease the relative influence of large donors without decreasing campaign spending.

*PhD Candidate, University of Michigan, Department of Economics. Email: dtmoore@umich.edu. I am grateful for helpful comments on earlier drafts received from Ashley C. Craig, James R. Hines Jr., Nirupama Rao, and Joel Slemrod. I also thank seminar participants at the University of Michigan as well as conference participants at the 2019 Annual Meeting of the National Tax Association and the 2020 IIPF Annual Congress.

Politics in modern representative democracies is a resource-intensive enterprise. Political candidates and parties in these societies must somehow acquire the substantial funds needed to credibly compete for citizens' votes. For many, this state of affairs raises concerns about whether and how the financing of political activity distorts political outcomes relative to some democratic ideal. Perhaps fundraising incentives cause politicians to place disproportionate weight on the policy preferences of wealthy donors, special interest groups, or other actors who provide political funds. Writing about the modern politics in the United States Overton [2004] suggests that "a small, wealthy, and homogenous donor class... effectively determines which candidates possess the resources to run viable campaigns" and that this has the effect of enhancing the relative political influence of this "donor class" in a way which undermines democratic principles. Scarrow [2018] suggests that such phenomena contravene ideals of political equality; specifically, the principle of "citizen parity", which is the idea that citizens in a democracy ought to have equal "opportunities to influence the political process".

If these concerns are warranted (and deemed normatively important) then political finance activities might be said to produce a kind of externality which calls for a corrective policy response. Scarrow [2018], in a review of the effects of political finance regulation on political equality, highlights a range of possible policy responses, such including public subsidies for political parties/campaigns, regulating/restricting campaign spending, limiting the size of contributions, or subsidizing small donors. It is this last proposal might stand out most to tax economists as it raises a natural question: can these "political externalities" be corrected using standard tax policy tools? Should a social planner simply levy Pigouvian taxes/subsidies on certain political finance transactions? What is the marginal social cost/benefit of these transactions? Existing work in tax analysis does not provide answers to these questions. This is a striking gap in view of the fact that many political finance regulatory measures that have been proposed or implemented around the world make explicit use of the standard tools of tax policy. For example, in Canada, there are substantial tax credits available for political contributions at both the federal and provincial/territorial level [Elections Canada, 2017]. These credits vary in their design but most commonly entail a 75% credit on the first dollar of donations, with a declining marginal rate of subsidy at higher levels. Is this a sensible tax policy?

The absence of a formal theoretical treatment of the tax policy implications of "political externalities" seems even more glaring in light of the fact that some prominent public economists have cited concerns about political inequality as justification for instituting a wealth tax and/or raising top marginal income tax rates. Saez and Zucman [2019] argue that wealth inequality generates social costs by inducing inequality of political power (among other mechanisms). They suggest that these costs are externalities analogous to the costs of pollution, and thus corrective taxation of wealth and/or high incomes may be justified. Kopczuk [2019] notes that standard theoretical arguments call for directly addressing the source of an externality through taxation or regulation and suggests that it might be preferable to address concerns that wealth concentration leads to concentration of political power via "suitable reforms of political system"? Scheuer and Slemrod [2020] raise similar concerns. Viewed through the lens of optimal tax theory, these authors are engaged in speculation about whether Sandmo's "Principle or Targeting" or "Additivity Property" applies to some ambiguously defined political externalities. The literature on optimal tax theory has explored the conditions under which this property holds but absent a clear definition of what political externalities are this work provides no guidance this question [Micheletto, 2008].

In this paper, I take a first step towards expanding normative tax analysis to account for concerns about the effects of tax policy on the functioning of democratic institutions. At its core, this theory rests on three components:

1. A tractable model of political institutions, which maps from citizen preferences and actions to equilibrium policy outcomes.

2. A normative criteria for evaluating the equilibrium policy outcomes of the model.

3. A normative thought experiment which specifies which policy instruments are endogenously controlled by the model of political institutions and which are exogenously controlled by a social planner.

Section 1 accomplishes the first task by introducing a simple extension of probabilistic voting models which incorporates campaign spending financed by individual donors. Equilibrium policy choices in this model can be characterized the maximizer of an implicit social welfare function, clarifying how campaign finance motives may influence policy outcomes. In classic probabilistic voting models, a citizen's implicit welfare weight is proportional to their relative importance as a marginal source of votes. In my model, this result is augmented so that a citizen's implicit welfare weight also depends on their relative importance as a *marginal source of donations*, reflecting the candidates' incentive to alter their policy platforms as a way to finance campaign expenditures which can increase their vote share. The candidates weight this new marginal donations incentive in proportion to the *marginal effect of campaign spending* on vote share. This intuitive result may seem straightforward, but it is significant in its own right. The idea that a donor's influence on may not depend on the relative size of their donation so much as their importance as a marginal donor has received little discussion in the political finance literature. As I demonstrate later, this distinction has important implications for what kind of externalities political donations generate and, consequently, for optimal taxation of donations.

Section 2 presents two possible normative criteria by which one might assess equilibrium policy outcomes and compare them against alternative outcomes which would prevail if a particular tax (or other policy) reform were adopted. I first present a welfare metric which I call *political welfare.* This approach requires a commitment to a normatively relevant welfare function for use in evaluating equilibrium policy outcomes. The discrepancy between this welfare function and the implicit welfare function that characterizes equilibrium policy choices in my elections model may result in a policy outcome which is not political welfare-maximizing. The "political externality" can then be defined as the welfare loss associated with implementing policies chosen by the elections model rather than imposing the political welfare-maximizing policy. I also consider a non-welfarist metric which I call *political inequality.* This metric is intended to capture the idea that inequality of political influence constitutes a direct normative cost, irrespective of the ultimate choice of policy. It requires choosing a specific measure of distributional inequality and applying it to the implicit welfare weights that characterize a particular equilibrium of the elections model. The "political externality" can then be defined as the measured inequality in implicit welfare weights. I discuss the properties of each in the context of my model and how they relate to one another.

Section 3 describes a specific normative thought experiment in which the policy space in the elections model is composed of economically and politically irrelevant policy instruments. For example, this means the policies the election is being fought over do not include things like income taxes, political contribution taxes/subsidies, or campaign finance regulations. The social planner chooses the tax treatment of political contributions to maximize an objective function adds one of my political externality measures to a traditional economic welfare function. This thought experiment has the advantage of simplicity, but this comes at the cost of a level of abstraction which may seem unpalatable. I briefly discuss possible implications of this approach relative to possible alternatives.

I first apply my normative thought experiment to the derivation of a linear tax on political contributions. Such a tax can influence political externalities through two mechanisms. First, it can alter the amount of campaign spending candidates do. Whether the marginal dollar of campaign spending has a positive or negative effect on political externalities depends on how the spending affects the distribution of implicit welfare weights. For example, if such spending enhances voter turnout amongst relatively high-turnout groups, it may be that marginal spending has a negative effect on political externalities. On the other hand, if inequality of importance as a marginal source of donations is the primary source of political externalities, and there are diminishing marginal returns to campaign expenditure increased campaign spending may have a positive effect, since driving down the marginal effect of campaign spending reduces the candidates' valuation of marginal donations. However these various factors net out, the effect of the campaign spending mechanism on political externalities can be addressed via the standard Pigouvian remedy: a tax/subsidy on political contributions equal to the social cost/benefit of the marginal dollar of campaign spending. This result reflects the fact that this campaign spending effect is a standard atmospheric externality: it depends only on the total donations made and is invariant to the composition of the contributors to that total.

The second way a donation tax can affect political externalities is by changing the how responsive citizen donations are to the policies candidates propose. This is analogous to the effect that taxation of a specific commodity might have on the relationship between a consumer's willingness-to-pay for a particular product and its quality. By changing this responsiveness of citizen donation behavior—what I call the *donation sensitivity*—taxation can change the relative importance of citizens as a source of marginal donations. It is not obvious whether this mechanism will in general have a positive or negative effect on political externalities: this depends on how the tax affects different types of citizens' donation sensitivity and how relative political influences covaries with such differences. But it is important to note that this effect cannot be treated as a standard atmospheric externality, as it does not depend on the total amount of donations. Indeed, it does not it fit the standard description of a non-atmospheric externality either as it does not directly depend on the donation quantities at all. This result casts doubt on the validity of deriving intuitions about corrective policy for political externalities from the results that pertain to standard types of externalities.

The effect of taxation on political externalities via its effect on donation sensitivity suggests a possible rationale for a nonlinear political contribution tax schedule. If citizens with differing relative political influence tend to systematically locate on different parts of the donation distribution, a nonlinear tax schedule may allow the planner the flexibility to enhance the donation sensitivity of some groups while reducing it for others, while simultaneously manipulating the level of total donations in order to control political externalities via campaign spending effects. In section 4 I formalize this intuition by deriving the optimal nonlinear tax on donations, providing a plausible justification for the nonlinear donation subsidy schedules seen in some countries.

I conclude by discussing possible extensions of the model and briefly considering the prospects for applied welfare analysis. Empirical application of my optimal tax formulae requires the estimation of novel sufficient statistics which have received no prior discussion or empirical investigation in the political finance literature: the effect donation taxation on donations sensitivity. In fact, it is not enough even to estimate these effects of taxation: applied welfare analysis using my formulae also requires knowing how these effects are differ across citizens with different relative political influence or policy preferences. These high informational requirements suggest that more work is needed to translate the insights of this paper into applied policy recommendations.

The reader should note that this work is meant to be illustrative rather than comprehensive. I hope that, in detailing one approach to constructing a theory of optimal taxation with political externalities, this paper highlights a set of general issues which must be confronted in order to treat this topic seriously. This is a task which demands further attention if tax economists and political finance scholars seek to provide informed insights about how tax policy should be designed when it affects the functioning of political institutions.

# 1 A Spatial Model of Elections with Individual Donors

Here I develop a probabilistic voting model which accounts for the role of individual donors and campaign expenditures in electoral competition.[1] The probabilistic voting theory literature develops spatial models of elections in which candidates who are uncertain about citizen voting behavior strategically choose policy commitments with the goal of winning votes.[2] Prominent contributions include Hinich et al. [1972, 1973], Hinich [1976, 1978], Lindbeck and Weibull [1987, 1993], and Coughlin [1992]. Banks and Duggan [2005] provide a unified treatment of probabilistic voting models in two candidate majority-rule elections.

Note, the version of the elections model discussed here is highly stylized. This is intended to simplify the discussion of my primary topic of interest: the optimal tax implications of political externalities.[3]

## 1.1 Setup

**Agents** Two *candidates* ($A$ and $B$) compete in an election by simultaneously choosing policy positions. Both candidates announce a *policy position* $x \in X \subset \mathbb{R}$ where $X$ is a compact and convex policy space. For any candidate $c \in \{A, B\}$, let $x^c$ be their policy position. Let $-c \equiv \{A, B\} \setminus \{c\}$ denote candidate $c$'s opponent. Both candidates also engage in campaign activities designed to raise voter turnout amongst their supporters. These activities are financed by donations the candidates receive.[4]

The voters and donors in the election are a unit mass of *citizens.* Each citizen is characterized by a *type* $\theta \in \Theta$ and a *preference shock* $\xi \in \mathbb{R}$ which jointly determine their political behavior. Citizen types are distributed according to some measure $\mu$. Citizen preference shocks are independent of type and distributed according to $G$, a symmetric, mean zero distribution.

I will sometimes refer to a type $\theta$ citizen with a preference shock $\xi$ as a $(\theta, \xi)$-citizen.

[1] To my knowledge, the extension of probabilistic voting theory I present here has no precedent in the literature with the notable exception of Hettich et al. [1999, p. 136], who provide a very brief sketch of such a model in a their comprehensive treatise on the political economy of taxation, which makes extensive use of probabilistic voting models.

[2] Candidate uncertainty about citizen behavior is what distinguishes these models from the other predominant framework for modeling strategic candidate policy choice: deterministic or "Downsian" models of elections. These assume candidates know citizen policy preferences with certainty [Hotelling, 1929, Downs, 1957, Black, 1958].

[3] In ongoing work, I am exploring a more general version of the model.

[4] This is just one way to introduce campaign finance motives into a probabilistic voting model. In ongoing work I am exploring the implications of alternative assumptions regarding the role of money in politics.

**Policy Preferences** All type $\theta$ citizens are assumed to have the same preferences over the policy that will be implemented after the election. These preferences are represented by a *policy utility function* $u_\theta : X \to \mathbb{R}$ which is bounded on $X$.

I assume that citizen political behavior on depends on policy utilities through the difference between candidate policy utilities:[5]

$$\Delta u_\theta \left(x^c, x^{-c}\right) \equiv u_\theta \left(x^c\right) - u_\theta \left(x^{-c}\right).$$

For convenience, I will sometimes use the shorthand $\Delta u_\theta^c \equiv \Delta u_\theta \left(x^c, x^{-c}\right)$. Note, these definitions imply that $\Delta u_\theta^c = -\Delta u_\theta^{-c}$.

**Preferred Candidate** Citizens assume that if a given candidate is elected, they will implement their announced policy and thus evaluate candidate policy platforms according to their policy preferences. However, citizens do not vote based on policy preferences alone: their preference shock $\xi$ also plays a role in their decision.

Let $\xi^A \equiv \xi$ and $\xi^B \equiv -\xi$. Note, since $G$ is mean zero and symmetric, both $\xi^A$ and $\xi^B$ are distributed according to $G$.

Candidate $c$ is defined as the *preferred candidate* of $(\theta, \xi)$-citizen if and only if

$$\Delta u_\theta^c + \xi^c > 0.$$

Thus, the probability that some type $\theta$ citizen prefers candidate $c$ is:[6]

$$\begin{aligned}\Pr\left\{\Delta u_\theta^c + \xi^c > 0\right\} &= 1 - G\left(-\Delta u_\theta^c\right)\\ &= G\left(\Delta u_\theta^c\right).\end{aligned}$$

I will often refer to the set of citizens that prefer candidate $c$ as candidate $c$'s *supporters*.

**Voting Decision** If a citizen votes, they vote for their preferred candidate. However, voting is costly, so a citizen will not necessarily vote for their preferred candidate. In particular, suppose that a type $\theta$ supporter of candidate $c$ will cast a vote for candidate $c$ with probability $\eta_\theta\left(D^c\right)$, where $D^c$ is the total donations candidate $c$ receives in the election. For all $\theta$, let $\eta_\theta : \mathbb{R}_+ \to [0, 1]$ be a strictly increasing and strictly concave

[5] In ongoing work, I am exploring the implications of relaxing this assumption.

[6] The second equality follows from symmetry of $G$.

function.[7] Notice, I allow this turnout probability function $\eta_\theta$ to vary by type. This reflects the possibility that individual policy preferences may be correlated with other factors that influence the baseline probability of voting and the effect of campaign spending on the probability of voting.

**Donation Decisions** In addition to making voting decisions, citizens decide whether to make monetary contributions to candidates' campaigns, and if so, how much to contribute. Citizens receive some warm-glow utility from donating to their preferred candidate. I postpone a full discussion of the microfoundations of citizen donation behavior to section 3. For the moment, I simply assume that some $(\theta, \xi)$-citizen who prefers candidate $c$ donates $\delta_\theta\left(\Delta u_\theta^c + \xi^c\right)$ to candidate $c$ and nothing to candidate $-c$, where $\delta_\theta : \mathbb{R}_+ \to \mathbb{R}_+$ is weakly increasing with $\delta_\theta(0) = 0$. Like the turnout probability functions, donation functions vary by type, reflecting the possibility that policy preferences may be correlated with other factors that influence an individual's willingness to donate money to political campaigns.

The expected donation a type $\theta$ citizen makes to candidate $c$ when $\Delta u_\theta^c = \Delta u$ is

$$\bar{\delta}_\theta(\Delta u) \equiv \int_{-\Delta u}^{\infty} \delta_\theta(\Delta u + \xi)\, \mathrm{d}G(\xi).$$

**Aggregate Behavior** If the candidates adopt policy positions $\left(x^A, x^B\right)$, then the total donations to candidate $c$ are

$$D^c\left(x^A, x^B\right) \equiv \int \bar{\delta}_\theta(\Delta u)\, \mu(\mathrm{d}\theta). \tag{1}$$

Let $\eta_\theta^c\left(x^A, x^B\right) \equiv \eta_\theta\left(D^c\left(x^A, x^B\right)\right)$ be the turnout probability for candidate $c$'s type $\theta$ supporters given the policy positions $\left(x^A, x^B\right)$. The probability that a given citizen of type $\theta$ votes for candidate $c$ is[8]

$$P_\theta^c\left(x^A, x^B\right) \equiv \eta_\theta^c\left(x^A, x^B\right) \cdot G\left(\Delta u_\theta^c\left(x^A, x^B\right)\right) \tag{2}$$

and thus the share of citizens voting for candidate $c$ is

$$P^c\left(x^A, x^B\right) \equiv \int P_\theta^c\left(x^A, x^B\right) \mu(\mathrm{d}\theta). \tag{3}$$

[7] I am adopting this approach to modeling costly voting for simplicity. In ongoing work I am exploring alternative assumptions about the use of campaign funds and the voter turnout decision. This includes, for example, the case where voter turnout decisions depend on $\Delta u_\theta^c$ and cases where donation can directly influence the choice of preferred candidate and citizen donation behavior.

[8] The probability a type $\theta$ citizen prefers candidate $c$ is

$$\begin{aligned}\text{Prob}\left\{\Delta u_\theta^c + \xi^c > 0\right\} &= 1 - G\left(-\Delta u_\theta^c\right) \\ &= G\left(\Delta u_\theta^c\right)\end{aligned}$$

where the second equality follows from symmetry of $G$. Multiplying this by candidate $c$'s turnout probability gives $P_\theta^c$.

Note that $P^c$ and $D^c$ are deterministic because there is a continuum of citizens each independently drawing a preference shock from $G$.

**Candidates' Problem** Candidate $c$ chooses their policy position to maximize their plurality of votes in the election:

$$\max_{x \in X} \left\{ P^c \left( x^c, x^{-c} \right) - P^{-c} \left( x^{-c}, x^c \right) \right\}. \tag{4}$$

Let $P\ell^c \left( x^c, x^{-c} \right) \equiv P^c \left( x^c, x^{-c} \right) - P^{-c} \left( x^{-c}, x^c \right)$ denote candidate $c$'s objective function.

Note, candidates do not seek to maximize their probability of winning the election but rather the difference between the votes they receive and the votes their opponent receives. This type of objective function is common in the probabilistic voting literature [Banks and Duggan, 2005]. However, because I have assumed a continuum of citizens with independently distributed partisan biases, any equilibrium under this objective function will also be an equilibrium under a probability of winning objective, but not vice versa.[9]

For notational convenience, in the remainder of the paper I will often suppress the dependence of the functions defined above on candidate policy choices (e.g. writing $\eta_\theta^c$ instead of $\eta_\theta^c \left( x^A, x^B \right)$).

## 1.2 Equilibrium Policy and Implicit Welfare Functions

The election game is a static game where the two candidates simultaneously choose policy positions. A pair of policy positions $\left( x^{\star A}, x^{\star B} \right)$ is an *electoral equilibrium* of this game if

$$x^{\star A} = \arg\max_{x \in X} \left\{ P^A \left( x, x^{\star B} \right) - P^B \left( x, x^{\star B} \right) \right\} \tag{5}$$

and

$$x^{\star B} = \arg\max_{x \in X} \left\{ P^B \left( x^{\star A}, x \right) - P^A \left( x^{\star A}, x \right) \right\} \tag{6}$$

for all $c \in \{A, B\}$.[10] A *convergent equilibrium* is an electoral equilibrium in which $x^{\star A} = x^{\star B} = x^\star$. This is the standard equilibrium definition used in the probabilistic voting literature [Banks and Duggan, 2005].

Candidate policy choices in an electoral equilibrium can be characterized using a simple first-order condition

[9] This is because the vote shares in this model are deterministic. Suppose that in the case of a tie each candidate has a $\frac{1}{2}$ probability of winning and otherwise the candidate with the highest vote share wins. If there exists some policy position pair where one candidate has a higher vote share and the losing candidate can deviate to an alternate policy position which would increase their vote share but not by enough to change the election outcome. This would be a profitable deviation under the vote share objective but not under the probability of winning objective.

[10] Note, this is simply a generalized Nash equilibrium. See Facchinei and Kanzow [2007] for a review of generalized Nash games.

approach.[11] Let $\mathbf{x}^\star \equiv \left(x^{\star A}, x^{\star B}\right)$ be an electoral equilibrium and $g(\xi) \equiv G'(\xi)$ be the preference shock probability distribution function. The first-order condition characterizing candidate $c$'s choice is

$$\underbrace{\int \left[g\left(\Delta u_\theta^c\right)\eta_\theta^c + \psi^c \bar{\delta}'_\theta\left(\Delta u_\theta^c\right)\right]\frac{\partial u_\theta^c}{\partial x^c}\mu(\mathrm{d}\theta)}_{=\partial P^c/\partial x^c} + \underbrace{\int \left[g\left(\Delta u_\theta^{-c}\right)\eta_\theta^{-c} + \psi^{-c}\bar{\delta}'_\theta\left(\Delta u_\theta^{-c}\right)\right]\frac{\partial u_\theta^c}{\partial x^c}\mu(\mathrm{d}\theta)}_{=-\partial P^{-c}/\partial x^c} = 0 \tag{7}$$

where

$$\psi^c \equiv \int \eta'_\theta\left(D^c\right)G\left(\Delta u_\theta^c\right)\mu(\mathrm{d}\theta)$$

is the *marginal effect of campaign spending* for candidate $c$. This parameter measures the marginal impact of an additional dollar of donations (and resulting expenditures) on candidate $c$'s objective function.[12]

Equation (7) provides a simple, intuitive description of how campaign finance issues can influence policy outcomes. Political candidates in this model choose a location in the policy space taking into account not only how changes in their policy influence their plurality, but also how such changes influence the donations that they will receive. They value these marginal donations in proportion to the marginal effect of campaign spending.

To be more explicit about how these issues translate into equilibrium policy positions, note that equation (7) implies for any electoral equilibrium $\mathbf{x}^\star$ candidate $c$'s policy choice will also satisfy

$$x^{\star c} = \arg\max_{x\in\mathbb{R}}\left\{\int \gamma_\theta^c\left(\mathbf{x}^\star\right)u_\theta(x)\,\mu(\mathrm{d}\theta)\right\} \tag{8}$$

where

$$\gamma_\theta^c\left(\mathbf{x}^\star\right) \equiv \underbrace{\sum_{k\in\{c,-c\}}\eta_\theta^k\left(\mathbf{x}^\star\right)g\left(\Delta u_\theta\left(x^{\star k}, x^{\star -k}\right)\right)}_{\text{value as marginal source of votes}} + \underbrace{\sum_{k\in\{c,-c\}}\psi^k\left(\mathbf{x}^\star\right)\bar{\delta}'_\theta\left(\Delta u_\theta\left(x^{\star k}, x^{\star -k}\right)\right)}_{\text{value as marginal source of donations}} \tag{9}$$

is candidate $c$'s *implicit welfare weight* for type $\theta$ citizens at $\mathbf{x}^\star$. That is to say, each candidate's policy choice in an electoral equilibrium can be described as the solution to some weighted utilitarian social welfare maximization problem; as the maximizer an *implicit welfare function*. To be clear, the maximization problem in equation (8) is not equivalent to the actual candidate's problem presented in equation (5). As equation (9)

[11] Here and for the remainder of the paper, I ignore the assumptions required to enable this, as exploring the technical properties of this model is not my primary objective here.

[12] This description of the marginal effect of campaign spending is very specific to this variant of my elections model. In ongoing work, I am exploring the nature of this parameter in a more general version of the model and in alternative special cases of the general model. The precise form of this parameter varies substantially depending on the assumptions made about campaigning technology.For example, when campaign spending can influence donation behavior the marginal effect of campaign spending includes feedback loops between donation behavior and spending (i.e. a dollar of spending of additional spending may induce additional donations to the candidate, which induce more spending, which induce more donations, etc) and effects of campaign spending on the behavior on donations to the candidate's opponent (i.e. a dollar of spending of additional spending may reduce donations to the opponent, which reduces their spending, which further reduces donations, etc).

makes clear, each equilibrium of the election game has its own corresponding sets of implicit welfare weights for the candidates. Nonetheless, this presentation is useful for building intuition about how campaign finance issues influence equilibrium policy outcomes.

Equations (8) and (9) imply that in any electoral equilibrium, candidate $c$ weights the policy preferences of type $\theta$ citizens in proportion to their importance as a marginal source of votes and as a *marginal source of donations*. As well, it is important to note that the candidate not only cares about how their policy position affects the votes and donations they receive, but also how it affects the votes and donations their opponent receives. The multipliers $\psi^c(\mathbf{x}^\star)$ and $\psi^{-c}(\mathbf{x}^\star)$ reflect the marginal value of donations in terms of "buying" votes, and thus determine the relative importance of the marginal vote and marginal donation incentives for the candidate's policy position.

This simple result already highlights an important issue which has received little attention in the public and academic discourse on political finance issues. Notice, the model does not imply that citizens who donate more money will have disproportionate political influence: rather it suggests that citizens who will increase/decrease their donations by relatively large amounts in response to marginal policy changes are relatively influential. Thus, it is possible that large donor who strongly favors one candidate's policy position could actually have relatively little influence on the candidate's policy choice at the margin. In the language of the model, equilibrium policies favor citizen types with large values of $\bar{\delta}'_\theta\left(\Delta u^A_\theta\right)$ and $\bar{\delta}'_\theta\left(\Delta u^B_\theta\right)$, not necessarily those with large values of $\bar{\delta}_\theta\left(\Delta u^A_\theta\right)$ and $\bar{\delta}_\theta\left(\Delta u^B_\theta\right)$.[13]

This fact will prove important to my later results so it will be helpful to define some terminology. For a $(\theta,\xi)$-citizen who prefers candidate $c$ I call $\delta'_\theta\left(\Delta u^c_\theta+\xi^c\right)$ their *donation sensitivity*. Type $\theta$ citizens as a group also have an *aggregate donation sensitivity* for candidate $c$: $\bar{\delta}'_\theta\left(\Delta u^c_\theta\right)$.

**Convergence** Note that equation (9) implies that the candidates will have identical sets of implicit welfare weights. This results from the concern each candidate has for the impact of their policy choice on their opponent's vote share and total donations. Symmetry of welfare weights implies that if there exists some electoral equilibrium, there exists a convergent equilibrium.[14] implies that there exists a convergent equilibrium policy profile $\left(x^{\star A},x^{\star B}\right)$ where $x^{\star A}=x^{\star B}=x^\star$. This equilibrium can be described as the maximizer of a simplified implicit social welfare function:

$$x^\star=\arg\max_{x\in\mathbb{R}}\left\{\int\gamma_\theta\cdot u_\theta(x)\,\mu(\mathrm{d}\theta)\right\}$$

[13] In making the case for the importance of small donors in American politics Ansolabehere et al. [2003] note the

[14] To see this, suppose that $(x,x')$ is an electoral equilibrium. This implies that $x$ is the maximizer of candidate $A$'s implicit social welfare function. By symmetry of welfare weights and because welfare weights are independent of candidate policy choices, $x$ is also a maximizer of candidate $B$'s implicit social welfare function. Hence, $(x,x)$ is an electoral equilibrium.

where

$$\gamma_\theta \equiv 2\eta_\theta (D) g(0) + \left[ \int \eta'_\theta (D) \mu (\mathrm{d}\theta) \right] \bar{\delta}'_\theta (0),$$

and

$$D = \int \bar{\delta}_\theta (0) \mu (\mathrm{d}\theta).$$

**Equilibrium with Quadratic Utility** For simplicity, let us suppose that for each type $\theta$ their policy utility function is

$$u_\theta (x) \equiv -\frac{1}{2} (x_\theta - x)^2 \tag{10}$$

so that $x_\theta$ is the *ideal policy* for type $\theta$ citizens. This implies that $\frac{\partial u_\theta (x^c)}{\partial x^c} = x_\theta - x^c$. In this case there is a unique convergent equilibrium. Letting $\bar{\gamma} \equiv \int \gamma_\theta \mu (\mathrm{d}\theta)$ the equilibrium policy is

$$\begin{aligned} x^\star &= \mathbb{E}\left[ \frac{\gamma_\theta}{\bar{\gamma}} x_\theta \right] \\ &= \bar{x} + \mathrm{Cov}\left( \frac{\gamma_\theta}{\bar{\gamma}}, x_\theta \right) \end{aligned}$$

where $\bar{x} \equiv \mathbb{E}[x_\theta]$.

# 2 Frameworks for Normative Analysis

How can the model developed above be used to answer normative questions about campaign finance regulations? In this section, I consider two different normative approaches.

## 2.1 A Welfarist Approach

A natural starting point for a welfarist analysis is to pick some normatively-relevant welfare function and use it to evaluate policy outcomes. I define the *political welfare* associated with an equilibrium policy $x^\star$ as

$$PW(x^\star) \equiv \int \omega_\theta u_\theta (x^\star) \mu (\mathrm{d}\theta).$$

where $(\omega_\theta)_{\theta \in \mathrm{supp}(\mu)}$ is some set of normatively relevant welfare weights. Since the implicit welfare weights $(\gamma_\theta)_{\theta \in \mathrm{supp}(\mu)}$ differ from the normatively relevant weights, the equilibrium policy might differ from the welfare-maximizing policy $x^M = \arg\max_x \left\{ \int \omega_\theta u_\theta (x) \mu (\mathrm{d}\theta) \right\}$.

A natural choice for the normatively-relevant welfare weights might be one which weights all individuals' policy preferences equally: $\omega_\theta = 1$ for all $\theta$. In the case of quadratic utility functions this implies that $x^M = \bar{x}$. This standard is consistent with strands of the political finance literature which highlight inequality of political influence as an important possible normative cost associated with political finance institutions.

However, even when $\omega_\theta$ are equalized, note that $PW$ can be increased without necessarily reducing inequality of political influence (as measured by inequality of implicit welfare weights). In fact, $PW$ can, in principle, be increased by regulatory changes which exacerbate inequality of political influence. In the next subsection I discuss an alternative normative criterion which reflects a direct valuation of equality of political influence.

## 2.2 Political Inequality: A Non-Welfarist Approach

As discussed above, political welfare ($PW$) is an outcome-based measure. There is not universal support in political finance literature for using outcome-based normative metrics. Notice, under the political welfare criteria, if wealthy citizens have disproportionate influence in the political process, but coincidentally have policy preferences such that the equilibrium policy outcome is the $PW$-maximizing policy, there is no normative cost associated with such disproportionate influence. Overton [2004] labels this situation "virtual representation by the wealthy" and suggests that it is objectionable on the grounds that the less wealthy citizens' are having their "individual autonomy" undermined. More generally, some scholars consider the pursuit of a democratic ideal of "political equality" to be a good in and of itself [Dahl, 2006]. I attempt formalize such process-based objectives by applying measures of the inequality of the equilibrium implicit welfare weights in my elections model. Later, in section 2.2, I develop a potential welfarist justification for the use of this criteria in evaluating the effects of political finance reforms when there is uncertainty about the joint distribution of political influence and policy preferences.

I define the *political inequality* associated with a given electoral equilibrium as measured inequality in the distribution of implicit welfare weights which characterize that equilibrium. Specifically, for the remainder of the paper I define the *political inequality* of an electoral equilibrium as

$$PI = -\int \log\left(\frac{\gamma_\theta}{\bar{\gamma}}\right) \mu\left(\mathrm{d}\theta\right).$$

This measure of inequality—called mean-log deviation—has been employed in the literature on income inequality.[15]

[15]Note that many other measures of inequality might also be appropriate for quantifying political inequality, though the use of alternate measures should yield qualitatively similar results.

Notice, since a citizens' implicit welfare weights reflects their relative influence on policy outcomes due to *all factors*, such a definition reflects not only how donation behavior affects the distribution of political influence, but also how differential turnout probabilities do so. This seems to be most consistent with the broad definitions of political (in)equality most commonly found in the literature [Dahl, 2006, Rosset, 2016, Shore, 2016].

By construction, the use of $PI$ as normative criterion avoids the concern with $PW$ I discussed above: a regulatory change which increases $PW$ can, in principle, simultaneously increase $PI$. Of course, $PI$ suffers from the opposite issue: it is possible for a regulatory change which decreases $PI$ will to simultaneously decrease $PW$. This is an unsurprising fact, familiar to the literature on non-welfarist normative criteria. It is noteworthy, however, that $PI$ is minimized when $\gamma_\theta = \gamma \in \mathbb{R}_+$ for all $\theta$ (i.e. when there is perfect equality of political influence), and that under such circumstances $PW$ would also be maximized if the normatively relevant welfare weights were equal across citizen types.

The relationship between my formalization of political equality as normative objective and the ideas that inspire it is perhaps analogous to that between a maximin social welfare function and Rawlsian notions of distributive justice. Dahl [2006] defines political equality in terms of a hypothetical ideal democracy and my measure does the same, defining the "ideal democracy" as one which yields an electoral equilibrium where implicit welfare weights are perfectly equal. My criteria is more precise than Dahl's, providing clear answers to questions about how bad different deviations from the ideal are relative to one another. At the same time, my definition of an "ideal democracy" is defined in terms of a highly simplified model and clearly does not fully capture the nuances of Dahl's definition. For example, in Dahl's ideal democracy citizens must have sufficient opportunities to engage in information acquisition and communicating with one another about their policy preferences. My model abstracts completely from any such behavior.

# 3 Linear Taxes and Subsidies on Political Donations

In this section I discuss how adding a concern for "political externalities" to the social planner's objective function influences the optimal rate of a linear tax on political contributions. Before beginning this discussion I introduce an alternative way to characterize citizens. Let

$$Q \equiv |\Delta u_\theta^c + \xi^c|$$

be the *quality* of a $(\theta, \xi)$-citizen's preferred candidate. Quality $Q \in \mathbb{R}_+$ is a measure of the intensity of a citizen's preference for their preferred candidate. Notice that a $(\theta, \xi)$-citizen can alternatively be characterized by a triplet $(\theta, Q, c)$ where $c \in \{A, B\}$ is the citizen's preferred candidate. The joint distribution of $(\theta, Q, c)$ is induced by the measure $\mu$ and the distribution $G$ together with the set of type-specific policy utility functions $u_\theta$ and the candidate policy positions $\left(x^A, x^B\right)$.

In the discussion that follows, I will refer to a type $\theta$ citizen with preferred candidate $c$ and candidate quality $Q$ as a $(\theta, Q, c)$-citizen.

Note as well that for the remainder of the paper I adopt the assumption of quadratic ideal point utility functions (see equation 10).

## 3.1 Microfoundations of Donation Behavior

To conduct an optimal tax analysis it is necessary to provide some microfoundations that rationalize my assumed donation behavior and which specify the mechanism through which taxation can influence donation behavior. In particular, suppose that a $(\theta, Q, c)$-citizen donates

$$\delta_\theta (Q) \equiv \arg\max_\delta \left\{\phi_\theta (\delta; Q) - (1+t)\,\delta\right\}$$

to their preferred candidate $c$. I assume that:

1. $\phi_\theta (\delta; 0) = \phi_\theta (0; Q) = 0$;
2. $\frac{\partial \phi_\theta}{\partial \delta} > 0$, $\frac{\partial^2 \phi_\theta}{\partial \delta^2} < 0$, and $\frac{\partial^3 \phi_\theta}{\partial \delta^3} \geq 0$ for all $Q > 0$; and,
3. $\frac{\partial \phi_\theta}{\partial Q} > 0$, $\frac{\partial^2 \phi_\theta}{\partial \delta \partial Q} > 0$, and $\frac{\partial^3 \phi_\theta}{\partial \delta^2 \partial Q} > 0$ for all $\delta > 0$.

Under these assumptions a citizen's donation choice must satisfy the first-order condition $\frac{\partial \phi_\theta}{\partial \delta} = 1 + t$ and therefore, their (compensated) response to a marginal increase in the contribution tax is $\frac{\partial \delta_\theta}{\partial t} = \frac{1}{\partial^2 \phi_\theta / \partial \delta^2} < 0$. Let $\varepsilon_\theta (Q) \equiv -\frac{1+t}{\delta_\theta(Q)} \frac{\partial \delta_\theta(Q)}{\partial t}$ be the compensated elasticity of political donations for a $(\theta, Q, c)$-citizen.

Recall that the *donation sensitivity* of citizens is a key parameter in my elections model. The donation sensitivity of a $(\theta, Q, c)$-citizen is

$$\delta'_\theta (Q) = \frac{\partial \delta_\theta (Q)}{\partial Q} = -\frac{\partial^2 \phi_\theta / \partial \delta \partial Q}{\partial^2 \phi_\theta / \partial \delta^2} < 0.$$

Importantly, this parameter is itself influenced by taxation:

$$\frac{\partial^2 \delta_\theta(Q)}{\partial Q \partial t} = -\left[\frac{\left(\partial^3 \phi_\theta / \partial^2 \delta \partial Q\right) + \frac{\partial \delta_\theta}{\partial Q}\left(\partial^3 \phi_\theta / \partial \delta^3\right)}{\left(\partial^2 \phi_\theta / \partial \delta^2\right)}\right] \frac{\partial \delta_\theta}{\partial t} < 0.$$

The sign of this parameter follows from the assumed non-negativity of $\frac{\partial^3 \phi_\theta}{\partial \delta^3}$ but also holds if $\frac{\partial^3 \phi_\theta}{\partial \delta^3}$ is negative but sufficiently low magnitude.

For convenience, let

$$\sigma_\theta(Q) \equiv -\frac{\partial^2 \delta_\theta(Q)}{\partial Q \partial t} > 0$$

denote the magnitude of the effect of taxation on the donation sensitivity of a $(\theta, Q, c)$-citizen. This donation sensitivity effect is a key sufficient statistic in the optimal tax formulae presented below.

Finally, let $V_\theta(Q) \equiv \max_\delta \{\phi_\theta(\delta; Q) - (1+t)\delta\}$ be the indirect utility function for a $(\theta, Q, c)$-citizen.

## 3.2 The Effect of Taxation on "Political Externalities"

Suppose that two candidates are competing in an election of the kind described in section 1. The donation behavior of the citizens in the election follows the model set out in the previous sub-section. For simplicity, I will rely on the example introduced in section 1.2.

Recall, the implicit welfare weights in this case are

$$\gamma_\theta \equiv 2\eta_\theta(D)\, g(0) + \left[\int \eta'_\theta(D)\, \mu(\mathrm{d}\theta)\right] \bar{\delta}'_\theta(0),$$

where $D = \int \bar{\delta}_\theta(0)\, \mu(\mathrm{d}\theta)$. As well, recall that the equilibrium policy which is adopted will be

$$x^\star = \bar{x} + \mathrm{Cov}\left(\frac{\gamma_\theta}{\bar{\gamma}}, x_\theta\right). \tag{11}$$

### Political Welfare

Suppose that the normatively relevant welfare weights are $\omega_\theta = 1$. Then the political welfare ($PW$) maximizing policy would be $x^M = \mathbb{E}[x_\theta]$. Thus, the equilibrium policy differs from the $PW$ maximizing policy if $\mathrm{Cov}\left(\frac{\gamma_\theta}{\bar{\gamma}}, x_\theta\right) \neq 0$. Whenever this holds, there may be scope for political welfare-improving interventions.

When using this outcome-based normative metric, the normative effect of a marginal increase in the political contributions tax rate is characterized in terms of the comparative statics of candidate policy choices:

$$\frac{\mathrm{d}PW}{\mathrm{d}t} = \underbrace{-\mathrm{Cov}\left(\frac{\gamma_\theta}{\bar{\gamma}}, x_\theta\right)}_{=x^M - x^\star} \frac{\mathrm{d}x^\star}{\mathrm{d}t}. \tag{12}$$

Thus, increasing the tax rate will be $PW$-improving if it moves the equilibrium policy in the direction of the $PW$-maximizing policy.

Notice that the implicit welfare weights in our example are independent of the equilibrium policy. Hence, differentiating equation 11 gives us

$$\frac{\mathrm{d}x^\star}{\mathrm{d}t} = \mathrm{Cov}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\gamma_\theta}{\bar{\gamma}}\right), x_\theta\right). \tag{13}$$

Intuitively, this equation simply says that a tax reform will alter the equilibrium policy outcome if it results in a redistribution of relative political influence (as measured by $\frac{\gamma_\theta}{\bar{\gamma}}$) towards citizen types with systematically higher/lower ideal points than the average citizen, thus moving equilibrium policy in the direction systematically preferred by those who have gained in relative influence.

A useful alternate expression for the effect of taxation on political welfare is

$$\frac{\mathrm{d}PW}{\mathrm{d}t} = \int \left\{ \left(\frac{\bar{x} - x^\star}{\bar{\gamma}}\right) (x_\theta - \bar{x}) \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}t} \right\} \mu\left(\mathrm{d}\theta\right). \tag{14}$$

**Political Inequality**

Characterizing normative effects of taxation according to our process-based measure is somewhat simpler. Because $\int \frac{\gamma_\theta}{\bar{\gamma}} \mu\left(\mathrm{d}\theta\right) = 1$, it follows that $\int \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\gamma_\theta}{\bar{\gamma}}\right) \mu\left(\mathrm{d}\theta\right) = 0$ and therefore

$$\begin{aligned} \frac{\mathrm{d}PI}{\mathrm{d}t} &= -\int \left(\frac{\gamma_\theta}{\bar{\gamma}}\right)^{-1} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\gamma_\theta}{\bar{\gamma}}\right) \mu\left(\mathrm{d}\theta\right) \\ &= -\mathrm{Cov}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\gamma_\theta}{\bar{\gamma}}\right), \frac{1}{\gamma_\theta/\bar{\gamma}}\right) \end{aligned} \tag{15}$$

Notice, equation (15) suggests a certain symmetry between first-order political welfare and political inequality effects of taxation. Consider some initial equilibrium where it happens to be the case that each type $\theta$ citizen's ideal point coincides with their inverse (normalized) implicit welfare weight: $x_\theta = \frac{1}{\gamma_\theta/\bar{\gamma}}$. This implies that the support of the distribution of citizen ideal points is a subset of $(0, \infty)$ and that citizens with ideal points that are closer to 0 have higher implicit welfare weights. In such an initial equilibrium, $x^\star$ is below the

$PW$-maximizing ideal point ($x^\star = 1 < \mathbb{E}\left[\frac{1}{\gamma_\theta/\bar{\gamma}}\right] = x^M$) and therefore[16]

$$\frac{\mathrm{d}PW}{\mathrm{d}t} > 0 \iff \frac{\mathrm{d}PI}{\mathrm{d}t} < 0.$$

Thus, the use of political inequality as a normative standard can be given a welfarist interpretation under a particular assumption about the relationship between citizen preferences and implicit welfare weights. This suggests there may be a plausible welfarist rationale for relying on political inequality as a normative metric when there is uncertainty about relevant features of the joint distribution of implicit welfare weights and policy preferences.

A useful alternate expression for the effect of taxation on political inequality is

$$\frac{\mathrm{d}PI}{\mathrm{d}t} = -\int \left\{ \left[ \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right] \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}t} \right\} \mu\left(\mathrm{d}\theta\right). \tag{16}$$

**Political Externalities**

Equations 14 and 16 show that the marginal effect of taxation on political welfare and political inequality takes a similar form. In each case, the tax-induced change in the implicit welfare weights of each type is multiplied by a type-specific factor which determines what effect this change has on the political externality in question. For the remainder of the paper I shall therefore refer in the abstract to the effect of taxation on the *political externality* $PE$ and define $\tau_\theta^{PE}$ as the *political externality wedge* for type $\theta$ citizens, reflecting marginal effect on the political externality from increasing the implicit welfare weight of type $\theta$ citizens.

When using political welfare as our normative metric,

$$PE\left(x^\star\right) \equiv PW\left(x^M\right) - PW\left(x^\star\right),$$

the difference between political welfare at the $PW$-maximizing policy and the equilibrium policy. The political externality wedges are

$$\tau_\theta^{PE} \equiv -\left(\frac{\bar{x} - x^\star}{\bar{\gamma}}\right)\left(x_\theta - \bar{x}\right).$$

When using political inequality as our normative metric,

$$PE\left(x^\star\right) \equiv PI\left(\left(\gamma_\theta\right)_{\theta\in\Theta}\right).$$

[16]This follows from the fact that $\frac{\mathrm{d}PW}{\mathrm{d}\kappa} = \left[x^M - x^\star\right] \mathrm{Cov}\left(\frac{\mathrm{d}}{\mathrm{d}\kappa}\left(\frac{\gamma_\theta}{\bar{\gamma}}\right), x_\theta\right)$, as shown in the prior subsection.

The political externality wedges are

$$\tau_\theta^{PE} \equiv \frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}.$$

Given these definitions, the marginal effect of taxation on political externalities can always be expressed as

$$\frac{\mathrm{d}PE}{\mathrm{d}t} = \int \tau_\theta^{PE} \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}t} \mu\left(\mathrm{d}\theta\right). \tag{17}$$

**The Effect of Taxation on Implicit Welfare Weights**

Equation 17 formalizes the intuitive idea that the political externality effects of taxation depend on its effect on implicit welfare weights.[17]

Even in this simple example there are several mechanisms through which taxation can alter implicit welfare weights:

$$\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}t} = \underbrace{\psi \frac{\mathrm{d}\bar{\delta}_\theta'\left(0\right)}{\mathrm{d}t}}_{\text{direct effect}} + \underbrace{\frac{\mathrm{d}\psi}{\mathrm{d}t} \bar{\delta}_\theta'\left(0\right)}_{\text{marginal value of funds effect}} + \underbrace{2g\left(0\right) \eta_\theta'\left(D\right) \frac{\mathrm{d}D}{\mathrm{d}t}}_{\text{voter turnout effect}}. \tag{18}$$

The most intuitively obvious mechanism is the *direct effect*: all else equal, reducing the sensitivity of citizen's political donations to the perceived "quality" of a candidate reduces their welfare weight. Notice, such a reduction in their welfare weight does not imply a reduction in political influence. A donation tax increase will decrease the welfare weights of all citizens. How it affects political influence depends on how it changes a particular citizen's weight relative to that of other citizens.

The *marginal value of funds effect* in equation 18 is proportional to the effect of regulation on the marginal effect of campaign expenditure,

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \int \eta_\theta''\left(D\right) \mu\left(\mathrm{d}\theta\right) \frac{\mathrm{d}D}{\mathrm{d}t}. \tag{19}$$

Because $\delta_\theta'\left(0\right) \geq 0$, $\eta_\theta' > 0$ and $\eta_\theta'' < 0$, this implies that

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} > 0 \iff \frac{\mathrm{d}D}{\mathrm{d}t} = \int \frac{\mathrm{d}\bar{\delta}_\theta\left(0\right)}{\mathrm{d}t} \mu\left(\mathrm{d}\theta\right) < 0. \tag{20}$$

This result is quite intuitive: diminishing marginal returns to campaign expenditure means that taxation can increase the marginal effect of funds by decreasing campaign expenditures.

Thus, the marginal value of funds effect captures a potential unintended consequence of donation taxation

[17] This is true more generally within my model, though there are additional complications to consider outside this simplified example.

that aims to reduce the potentially disproportionate influence of the "donor class". In this model, such disproportionate influence stems from politicians' incentive to acquire additional campaign fund: marginal value of campaign funds. All else equal, any regulatory change which reduces total donations to the candidates will drive up marginal value of campaign funds. Thus, a policy which reduces $\delta_\theta$ and simultaneously decreases $\bar{\delta}$ can actually increase the implicit welfare weight of type $\theta$ citizens by increasing $\psi^\star$. Intuitively, this implies that a reform which seeks to reduce inequality of political influence by reducing inequality in donation responsiveness could perversely have the effect of *increasing political inequality.*

The *voter turnout effect* in equation 18 is proportional to the effect of regulation on $\eta_\theta$:

$$\frac{\mathrm{d}\eta_\theta\left(D\right)}{\mathrm{d}t} = \eta'_\theta\left(D\right)\frac{\mathrm{d}D}{\mathrm{d}t}.$$

This effect captures yet another potential unintended consequence of regulatory reforms that alter total campaign expenditures. If such expenditures are used to increase voter turnout, then regulations which change total campaign expenditures can also alter citizen's relative political influence by changing their turnout probability. Even in this simple model, general statements cannot be made about whether this effect suggests increases in total campaign expenditures should be expected to increase or decrease inequality of political influence. The answer depends on how $\eta'_\theta\left(D\right)$ varies with the other components of the type $\theta$ implicit welfare weight.

An alternate presentation of the impact of taxation on implicit welfare weights splits it into two sub-components: a *sensitivity effect* and a *campaign spending effect.*

$$\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}t} = \underbrace{\psi\frac{\mathrm{d}\bar{\delta}'_\theta\left(0\right)}{\mathrm{d}t}}_{\text{sensitivity effect}} + \underbrace{\left(2g\left(0\right)\eta'_\theta\left(D\right) + \bar{\delta}'_\theta\left(0\right)\int\eta''_\theta\left(D\right)\mu\left(\mathrm{d}\theta\right)\right)\frac{\mathrm{d}D}{\mathrm{d}t}}_{\text{campaign expenditure effect}}$$

Further note that

$$\frac{\mathrm{d}\bar{\delta}'_\theta\left(0\right)}{\mathrm{d}t} = -\int_0^\infty \sigma_\theta\left(\xi\right)\mathrm{d}G\left(\xi\right)$$

and

$$\frac{\mathrm{d}D}{\mathrm{d}t} = -\int\int_0^\infty \frac{\varepsilon_\theta\left(\xi\right)\delta_\theta\left(\xi\right)}{1+t}\mathrm{d}G\left(\xi\right)\mu\left(\mathrm{d}\theta\right).$$

### 3.3 Optimal Linear Tax/Subsidy

The social planner's problem is

$$\max_{t}\left\{2\int\int_{0}^{\infty}V_{\theta}\left(t,\xi\right)\mathrm{d}G\left(\xi\right)\mu\left(\mathrm{d}\theta\right)+2tD-\rho PE\right\}$$

Note that because I have assumed that individual welfare is linear in consumption there is no redistributive motive for taxation and thus, the planner's first-order condition is

$$2t\frac{\mathrm{d}D}{\mathrm{d}t}-\rho\frac{\mathrm{d}PE}{\mathrm{d}t}=0$$

or, in it's expanded form

$$-\frac{t}{1+t}\mathbb{E}\left[\varepsilon_{\theta}\delta_{\theta}\right]-\rho\left\{\psi\int\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\bar{\delta}_{\theta}'\left(0\right)}{\mathrm{d}t}\right]\mu\left(\mathrm{d}\theta\right)+\frac{1}{2}\left(\frac{1}{1+t}\mathbb{E}\left[\varepsilon_{\theta}\delta_{\theta}\right]\right)\int\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]\mu\left(\mathrm{d}\theta\right)\right\}=0$$

where $\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}=2g\left(0\right)\eta_{\theta}'\left(D\right)+\bar{\delta}_{\theta}'\left(0\right)\int\eta_{\theta}''\left(D\right)\mu\left(\mathrm{d}\theta\right)$. After some algebra, this gives us

$$\frac{t-\frac{\rho}{2}\mathbb{E}\left[\tau_{\theta}^{PE}\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]}{1+t}=\rho\psi\cdot\frac{\mathbb{E}\left[\tau_{\theta}^{PE}\bar{\sigma}_{\theta}\right]}{\mathbb{E}\left[\varepsilon_{\theta}\delta_{\theta}\right]}$$

where $\bar{\sigma}_{\theta}\equiv\int_{0}^{\infty}\sigma_{\theta}\left(\xi\right)\mathrm{d}G\left(\xi\right)$ is the magnitude of the effect of taxation on donation sensitivity.

For simplicity, suppose that the source of political externalities is political inequality, and thus $\tau_{\theta}^{PE}=\frac{1}{\bar{\gamma}}-\frac{1}{\gamma_{\theta}}$. Three main factors determine the optimal tax rate here:

1. Does taxation disproportionately reduce the sensitivity of donations amongst relatively influential citizens? If so $\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}}-\frac{1}{\gamma_{\theta}}\right)\bar{\sigma}_{\theta}\right]>0$ and, all else equal, the optimal tax is larger.

2. Do marginal campaign expenditures increase the influence of relatively influential citizens? If so, $\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}}-\frac{1}{\gamma_{\theta}}\right)\frac{\mathrm{d}\gamma_{\theta}}{\mathrm{d}D}\right]>0$ and, all else equal, the optimal tax is larger.

3. How responsive are donations to taxation, as measured by the aggregate elasticity $\mathbb{E}\left[\varepsilon^{c}\delta\right]$? The magnitude of the tax/subsidy is increasing in this elasticity.

Notice, the second factor is a traditional atmospheric externality. In the absence of the sensitivity issue, the standard Pigouvian prescription would apply and the planner would simply levy a tax/subsidy equal to the marginal social cost/benefit of campaign donations:

$$t^{\star} = \frac{\rho}{2}\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right].$$

But the sensitivity issue generates a non-standard externality which leads creates an alternate motivation for taxation/subsidy of campaign contributions.

If taxation reduces political inequality through both mechanisms the optimal tax will be positive.[18] However, suppose that although taxation does disproportionately reduce the sensitivity of donations amongst relatively influential citizens ($\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}\right)\bar{\sigma}_\theta\right] > 0$), marginal campaign spending actually increases the influence of relatively low influence citizens ($\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] < 0$).[19] In this case, the planner faces a conundrum. They would like to subsidize donations in order to increase campaign expenditures that will drive down political inequality. However, they would also like to tax donations to reduce political inequality via the sensitivity mechanism. Simultaneously addressing these causes of "political externalities" is not possible with just a linear tax rate.

# 4 Nonlinear Taxation and Subsidization Schedules for Donations

In the previous section, I show that a social planner with a motive to address political inequality through taxation will attempt to use it to reduce political inequality by simultaneously altering the distribution of donation sensitivity and influencing the total amount of donations flowing to political candidates. However, I also demonstrated that a planner who relies on a linear tax may be substantially constrained in their ability to address political inequality, as it is possible that leveraging one of these mechanisms will requires increasing the tax while leveraging the other requires decreasing the tax. In this section, I show a nonlinear donation tax can allow a planner to overcome this problem by allowing them more precise control over the distribution of donation sensitivity.

## 4.1 Microfoundations Revisited

Here, I briefly extend the discussion of microfoundations from section 3.1 to the case of nonlinear taxation.

[18] That is to say, if $\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}\right)\hat{\sigma}_\theta\right] < 0$ and $\mathbb{E}\left[\left(\frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right] > 0$.

[19] Recall, the marginal value of funds effect discussed in section 3.1 pushes in this direction: as campaign expenditures increase, the marginal effect of campaign spending falls and the importance of donation sensitivity inequality is reduced, possibly reducing overall political inequality. This situation might also arise if marginal campaign spending has the largest effect on low turnout voters.

Suppose that for type $\theta$ citizens

$$\delta_\theta(Q;\kappa) \equiv \arg\max_\delta \left\{\phi_\theta(\delta;Q) - \delta - \mathcal{T}(\delta) - \kappa\tau(\delta)\right\}.$$

The (compensated) response to a marginal tax reform $\tau(\delta)$ is

$$\left.\frac{\partial \delta_\theta(Q)}{\partial \kappa}\right|_{\kappa=0} = \frac{\tau'(\delta_\theta(Q))}{\partial^2\phi_\theta/\partial\delta^2 - \mathcal{T}''(\delta_\theta(Q))} < 0.$$

Let $\varepsilon_\theta(Q) \equiv -\frac{1+\mathcal{T}'(\delta_\theta(Q))}{\delta_\theta(Q)\tau'(\delta_\theta(Q))}\frac{\partial\delta_\theta(Q)}{\partial\kappa}$ be the compensated elasticity of political donations for a $(\theta, Q, c)$-citizen.

The sensitivity of donations is similar to the case of linear taxation:

$$\left.\frac{\partial \delta_\theta(Q)}{\partial Q}\right|_{\kappa=0} = -\frac{\partial^2\phi_i/\partial\delta\partial Q}{\partial^2\phi_i/\partial\delta^2 - \mathcal{T}''(\delta_i)} > 0.$$

However, the effect of a tax reform on the sensitivity of donations is now somewhat modified from the linear tax case:

$$\left.\frac{\partial^2 \delta_\theta(Q)}{\partial Q\partial \kappa}\right|_{\kappa=0} = -\sigma_\theta^1(Q)\,\tau'(\delta_\theta(Q)) - \sigma_\theta^2(Q)\,\tau''(\delta_\theta(Q)), \tag{21}$$

where

$$\sigma_\theta^1(Q) \equiv \frac{\left(\partial^3\phi_\theta/\partial^2\delta\partial Q\right) + \left.\frac{\partial\delta_\theta(Q)}{\partial Q}\right|_{\kappa=0}\left(\partial^3\phi_\theta/\partial\delta^3 - \mathcal{T}'''(\delta_\theta(Q))\right)}{\left(\partial^2\phi_\theta/\partial\delta^2 - \mathcal{T}''(\delta_\theta(Q))\right)^2},$$

and

$$\sigma_\theta^2(Q) \equiv -\frac{\left.\frac{\partial\delta_\theta(Q)}{\partial Q}\right|_{\kappa=0}}{\partial^2\phi_i/\partial\delta^2 - \mathcal{T}''(\delta_i)}.$$

The magnitude of the effect of increasing the marginal tax rate on donation sensitivity is $\sigma_\theta^1(Q)$. This effect is analogous to the effect of linear taxation on donation sensitivity as discussed in section 3.1. Equation (21) also includes a new effect captured by $\sigma_\theta^2(Q)$, the magnitude of the effect of increasing the convexity of the tax schedule on donation sensitivity.

Finally, let $V_\theta(\kappa, Q) \equiv \max_\delta\{\phi_\theta(\delta;Q) - \delta - \mathcal{T}(\delta) - \kappa\tau(\delta)\}$ be the indirect utility function of a type $\theta$ citizen.

## 4.2 The Effect of Taxation on "Political Externalities"

Here again I split the impact of taxation on implicit welfare weights splits it into two sub-components: a *sensitivity effect* and a *campaign expenditure effect*.

$$\left.\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}\kappa}\right|_{\kappa=0} = \underbrace{\psi \left.\frac{\mathrm{d}}{\mathrm{d}\kappa}\bar{\delta}'_\theta(0)\right|_{\kappa=0}}_{\text{sensitivity effect}} + \underbrace{\left(\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right)\left.\frac{\mathrm{d}D}{\mathrm{d}\kappa}\right|_{\kappa=0}}_{\text{campaign expenditure effect}}$$

where

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\kappa}\bar{\delta}'_\theta(0) &= \int_0^\infty \left.\frac{\partial^2\delta_\theta(\xi)}{\partial\xi\partial\kappa}\right|_{\kappa=0}\mathrm{d}G(\xi)\\ &= -\int_0^\infty \left[\sigma^1_\theta(\xi)\,\tau'(\delta_\theta(\xi)) + \sigma^2_\theta(\xi)\,\tau''(\delta_\theta(\xi))\right]\mathrm{d}G(\xi)\,,\end{aligned}$$

$$\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D} = 2g(0)\,\eta'_\theta(D) + \bar{\delta}'_\theta(0)\int \eta''_\theta(D)\,\mu(\mathrm{d}\theta)\,,$$

and

$$\left.\frac{\mathrm{d}D}{\mathrm{d}\kappa}\right|_{\kappa=0} = -\int\int_0^\infty \frac{\varepsilon_\theta(\xi)\,\delta_\theta(\xi)\,\tau'(\delta_\theta(\xi))}{1+\mathcal{T}'(\delta_\theta(\xi))}\mathrm{d}G(\xi)\,\mu(\mathrm{d}\theta)\,.$$

## 4.3 Optimal Marginal Tax/Subsidy Rates

The social planner's problem is

$$\max_t \left\{2\int\int_0^\infty \left[V_\theta(t,\xi) + \mathcal{T}(\delta_\theta(\xi))\right]\mathrm{d}G(\xi)\,\mu(\mathrm{d}\theta) - \rho PI\right\}$$

As in the case with linear taxes, I have assumed away any redistributive motive for taxation. Hence, the corresponding first-order condition is

$$\begin{aligned}&\int\int_0^\infty \left\{\frac{2\mathcal{T}'(\delta_\theta(\xi)) - \rho\left[\int \tau^{PE}_\theta \frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\mu(\mathrm{d}\theta)\right]}{1+\mathcal{T}'(\delta_\theta(\xi))}\varepsilon_\theta(\xi)\,\delta_\theta(\xi)\right\}\tau'(\delta_\theta(\xi))\,\mathrm{d}G(\xi)\,\mu(\mathrm{d}\theta)\ldots\\ &\qquad\cdots = \rho\psi\cdot\int\left\{\tau^{PE}_\theta\int_0^\infty\left[\sigma^1_\theta(\xi)\,\tau'(\delta_\theta(\xi)) + \sigma^2_\theta(\xi)\,\tau''(\delta_\theta(\xi))\right]\mathrm{d}G(\xi)\right\}\mu(\mathrm{d}\theta)\,.\end{aligned}$$

A change of variables allows us to integrating over the type distribution to get:[20]

$$\int \left\{ \frac{\mathcal{T}'(\delta) - \frac{\rho}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]}{1+\mathcal{T}'(\delta)} \mathbb{E}\left[\varepsilon_\theta(\xi)\,|\delta_\theta(\xi)=\delta\right]\delta - \rho\psi\cdot\mathbb{E}\left[\tau_\theta^{PE}\sigma_\theta^1(\xi)\,|\delta_\theta(\xi)=\delta\right]\right\}\tau'(\delta)\,h(\delta)\,\mathrm{d}\delta\ldots$$

$$\cdots = \rho\psi\cdot\int \mathbb{E}\left[\tau_\theta^{PE}\sigma_\theta^2(\xi)\,|\delta_\theta(\xi)=\delta\right]\tau''(\delta)\,h(\delta)\,\mathrm{d}\delta$$

Applying the fundamental lemma of the calculus of variations, I obtain the following characterization of the optimal marginal tax rate on political donations:

$$\frac{\mathcal{T}'(\delta) - \frac{\rho}{2}\mathbb{E}\left[\tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}|\delta_\theta(\xi)=\delta\right]}{1+\mathcal{T}'(\delta)} = \ldots$$

$$\cdots \frac{\rho\psi}{\mathbb{E}\left[\varepsilon_\theta(\xi)\,|\delta_\theta(\xi)=\delta\right]\delta}\left(\mathbb{E}\left[\tau_\theta^{PE}\sigma_\theta^1(\xi)\,|\delta_\theta(\xi)=\delta\right] - \frac{1}{h(\delta)}\frac{\mathrm{d}}{\mathrm{d}\delta}\left\{\mathbb{E}\left[\tau_\theta^{PE}\sigma_\theta^2(\xi)\,|\delta_\theta(\xi)=\delta\right]h(\delta)\right\}\right) \quad (22)$$

for all $\delta \geq 0$.[21]

Gaining an intuitive understanding of equation (22) is tricky but feasible. It is composed of four first-order welfare effects associated with an increase in the marginal rate at $\delta$. For simplicity, I again consider the case where the political externality stems from political inequality (and thus $\tau_\theta^{PE} = \frac{1}{\bar{\gamma}} - \frac{1}{\gamma_\theta}$).

**Direct Effect** Increasing the marginal rate at $\delta$ has a direct effect on political inequality:

$$\psi\mathbb{E}\left[\left(\frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}}\right)\sigma_\theta^1(\xi)\,|\delta_\theta(\xi)=\delta\right]h(\delta).$$

Holding $\sigma_\theta^1(\xi)$ constant across the citizens located at $\delta$ this effect would be negative—putting upward pressure on the marginal rate at $\delta$—if relatively influential citizen types are disproportionately located at $\delta$. Conversely, if relatively low influence citizen types are disproportionately located at $\delta$ the effect to be positive, putting downward pressure on the marginal rate at $\delta$ as a way to boost the influence of these citizens.

[20] Let $H(\delta|\theta)$ be the conditional distribution of donations induced by $G$ and $\delta_\theta(\cdot)$. Let $h(\delta|\theta)$ be the corresponding probability density function. Then this condition can be written as

$$\int\left[\int_0^\infty \left\{\frac{2\mathcal{T}'(\delta) - \rho\left[\int \tau_\theta^{PE}\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\mu(\mathrm{d}\theta)\right]}{1+\mathcal{T}'(\delta)}\tilde{\varepsilon}_\theta(\delta)\,\delta - \rho\psi\tau_\theta^{PE}\tilde{\sigma}_\theta^1(\delta)\right\}\tau'(\delta)\,h(\delta|\theta)\,\mathrm{d}\delta\right]\mu(\mathrm{d}\theta)\ldots$$

$$\cdots = \rho\psi\cdot\int\left[\int_0^\infty\left\{\tau_\theta^{PE}\tilde{\sigma}_\theta^2(\delta)\right\}\tau''(\delta)\,h(\delta|\theta)\,\mathrm{d}\delta\right]\mu(\mathrm{d}\theta)$$

where for any function $f_\theta \in \left\{\varepsilon_\theta, \sigma_\theta^1, \sigma_\theta^2\right\}$ define $\tilde{f}_\theta(\delta) = \{f_\theta(\xi) : \delta_\theta(\xi) = \delta\}$. Let $h(\delta) = \int h(\delta|\theta)\,\mu(\mathrm{d}\theta)$. Integrating over the type distribution gives the result below.

[21] And also $\lim_{\delta\to 0}\left\{\mathbb{E}\left[\left(\frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}}\right)\sigma_\theta^2(\xi)\,|\delta_\theta(\xi)=\delta\right]h(\delta)\right\} = \lim_{\delta\to\infty}\left\{\mathbb{E}\left[\left(\frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}}\right)\sigma_\theta^2(\xi)\,|\delta_\theta(\xi)=\delta\right]h(\delta)\right\} = 0$. The latter limit should hold as long as the density of donations decreases rapidly enough. Since $h(0) > 0$, the first limit will only hold if at the optimal tax schedule there is no marginal political inequality effect of a change in the second derivative of the donation tax schedule at $\delta = 0$.

**Convexity Effect** Increasing the marginal rate at $\delta$ increases the convexity of the tax schedule just below $\delta$ and decreases it just above $\delta$, which has an additional effect on political inequality:

$$-\psi \frac{\mathrm{d}}{\mathrm{d}\delta} \left\{ \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h(\delta) \right\}.$$

Note, this term can be expanded

$$\begin{aligned} & -\psi \frac{\mathrm{d}}{\mathrm{d}\delta} \left\{ \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h(\delta) \right\} = \dots \\ & \qquad \dots - \frac{\mathrm{d}}{\mathrm{d}\delta} \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h(\delta) - \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h'(\delta). \end{aligned}$$

Holding $\sigma_\theta^2 (\xi)$ constant across the citizens located at $\delta$ and assuming that relatively influential citizen types are increasingly over-represented among large donors might imply that $\frac{\mathrm{d}}{\mathrm{d}\delta} \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] < 0$, thus suggesting a positive convexity effect on political inequality which puts downward pressure on the marginal rate at $\delta$. The rationale for this is simple. Recall, tax schedule convexity reduces donation sensitivity. Thus, when the citizens just above $\delta$ are relatively more influential than the citizens just below $\delta$, the planner prefers to increase the convexity for those just above and decrease it for those just below. This can be accomplished by decreasing the marginal rate at $\delta$.

However, the convexity effect does not only depend on $\frac{\mathrm{d}}{\mathrm{d}\delta} \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h(\delta)$ but also on a second term $\mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h'(\delta)$. To understand this second term, suppose that $h'(\delta) < 0$ so that the density of donation is decreasing in $\delta$ and the second derivative effect $\mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right]$ is negative and constant across $\delta$. In that case, it would follow that $\frac{\mathrm{d}}{\mathrm{d}\delta} \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] = 0$ and the convexity effect would put upward pressure on the marginal rate at $\delta$. The reason for this is that although increasing the convexity of the tax schedule will reduce political inequality equally for citizens just above and just below $\delta$ the population just below $\delta$ is larger than the population just above $\delta$ so the planner would prefer to increase convexity for the individuals just below $\delta$. They can accomplish this by increasing the marginal rate at $\delta$.[22]

**Campaign Spending Effect and Fiscal Externality** As in the case of linear taxation, the effect of taxation on campaign spending manifests in the form of a standard atmospheric externality. Via its impact

[22] Note however, that for any valid probability distribution it should be the case that $h'(\delta) \to 0$, and thus

$$\psi \frac{\mathrm{d}}{\mathrm{d}\delta} \left\{ \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right] h(\delta) \right\} \to \frac{\mathrm{d}}{\mathrm{d}\delta} \mathbb{E}\left[ \left( \frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}} \right) \sigma_\theta^2 (\xi) \,| \delta_\theta (\xi) = \delta \right]$$

as $\delta \to \infty$.

on campaign spending, increasing the marginal rate at $\delta$ thus has an additional effect on political inequality of

$$\frac{1}{2}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]\left(\frac{\mathbb{E}\left[\varepsilon_\theta\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]\delta}{1+\mathcal{T}'\left(\delta\right)}\right)h\left(\delta\right).$$

Finally, increasing the marginal rate at $\delta$ imposes a standard fiscal externality of

$$-\frac{\mathcal{T}'\left(\delta\right)}{1+\mathcal{T}'\left(\delta\right)}\mathbb{E}\left[\varepsilon_\theta\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]\delta h\left(\delta\right).$$

Adding multiplying the political inequality effects by $-\rho$ and adding them together with the fiscal externality produces equation (22).

## 4.4 Discussion

The optimal donation tax formula presented in equation (22) offers a way to rationalize the kinds of non-linear subsidy schedules observed in real tax policy in countries like Canada. For example, suppose that marginal campaign spending disproportionately increases the influence on low influence citizens, thereby decreasing political inequality: $\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\frac{\mathrm{d}\gamma_\theta}{\mathrm{d}D}\right]<0$. This puts downward pressure on marginal rates at all levels of donations. Further suppose that for each $j\in\{1,2\}$ $\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]<0$, $\frac{\mathrm{d}^2}{\mathrm{d}\delta^2}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]>0$, and $\lim_{\delta\to\infty}\left\{\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]\right\}=0$. This might occur if, for example, the sensitivity effects $\sigma_\theta^1\left(\xi\right)$ and $\sigma_\theta^1\left(\xi\right)$ are roughly constant with respect to $\xi$ but are higher for relatively high influence citizen types, and relatively high influence citizen types are disproportionately located at high donation amounts but type composition becomes constant becomes constant at large donation amounts.

The fact that $\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^1\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]<0$ provides a rationale for increasing marginal rates since it implies that the planner is able to use donation amount as a screening device, targeting higher marginal rates to citizens who are disproportionately influential. The assumption that $\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]<0$ and $\frac{\mathrm{d}^2}{\mathrm{d}\delta^2}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]>0$ provides an additional justification for a progressive schedule: this implies that convexity effects are strongest lower in the donation distribution, so the planner has a strong incentive to keep marginal rates low in that region but this attenuates at higher points in the distribution. The fact that $\lim_{\delta\to\infty}\left\{\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta}-\frac{1}{\bar{\gamma}}\right)\sigma_\theta^j\left(\xi\right)|\delta_\theta\left(\xi\right)=\delta\right]\right\}=0$ for $j\in\{1,2\}$ implies that both of these force for progressivity disappear in the tail of the donation distribution, where the marginal rate will eventually converge.

Finally, notice that this example provides several possible justifications for the marginal rate schedule to begin with a negative marginal rate: a subsidy for small donors. First, I have assumed donations are associated with a positive Pigouvian externality through campaign spending effects, thus pushing all marginal rates downwards. In the absence of any sensitivity effects, this would result in a linear subsidy as in section . Next, note that if relatively high influence citizen types are more concentrated in the top of the donation distribution then relatively low influence types should also be more concentrated at the bottom, and therefore $\mathbb{E}\left[\left(\frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}}\right)\sigma_\theta^1(\xi)\,|\delta_\theta(\xi) = \delta\right] > 0$ at sufficiently low values of $\delta$. This too provides a rational for subsidizing small donors as a way to increase their donation sensitivity and, consequently, their political influence. Finally, as noted above, if $\frac{\mathrm{d}}{\mathrm{d}\delta}\mathbb{E}\left[\left(\frac{1}{\gamma_\theta} - \frac{1}{\bar{\gamma}}\right)\sigma_\theta^2(\xi)\,|\delta_\theta(\xi) = \delta\right] < 0$, then this puts downward pressure on all marginal rates, but given our assumptions above it this pressure would be largest at low values of $\delta$.

# 5 Conclusion

Can tax policy be used to facilitate the proper functioning of a modern democracy? At present, a tax economist faced with this question cannot presented a complete answer. To do so requires not only a commitment to a positive theory democratic governance which captures the most important institutional features at stake, but also a normative framework for interpreting the behavior of such a model as well as an appropriate normative thought experiment within which the question can be posed and answered. Here, I have attempted such an endeavor as a way to demonstrate the various difficulties associated with this task. At each stage, I might have made different choices in developing the theory, which might yield distinct policy conclusions. My theory rests on simplifying assumptions that seem difficult to defend conceptually, particularly with respect to the normative thought experiment employed. My optimal tax formulae depend on sufficient statistics which impose an incredibly high informational burden on the social planner. The reader could be forgiven for concluding that this work primarily serves to demonstrate that addressing this topic is more trouble than it's worth.

And yet, if the goal of optimal tax theory is to provide a guide to policy action, the omission of the topic of political externalities seems untenable. Tax policies ostensibly designed to address such externalities already exist. Where they do not yet existence, some campaign finance reformers, political finance scholars, and tax economists have called for their introduction. Some have claimed that political externalities are of central importance in addressing standard tax policy questions like income and capital gains. Even if one is to reject these claims, this should ideally be done in the context a clear normative framework.

This paper necessarily leaves a great many questions about optimal taxation with political externalities unanswered. My work here is only a proof-of-concept, but it does suggest an extensive agenda for future research. For example, although I have shown that a reasonably defined political externality will not necessarily be of the standard atmospheric kind, I have not resolved the question of whether some version of the principle of targeting applied to political finance transactions. If donation taxes are set optimally, is there still scope for the social planner to adjust income or wealth taxes to achieve further corrective benefits? This is but one of many promising theoretical research questions I leave to future work, including:

- What characterizes the jointly optimal choice of donation taxes and other campaign finance instruments like spending caps or public subsidies?
- How does evasion and avoidance behavior alter optimal policy prescriptions?
- How sensitive are optimal policy prescriptions to the particulars of the elections model?
- When might limits on individual contributions be preferable to taxation of large contributors?[23]
- What alternative normative thought experiments are worth consideration and how do they alter policy prescriptions?

I also leave the even more daunting task of applied welfare analysis for a separate paper. The optimal tax formulae presented in sections 3 and 4 contain novel sufficient statistics some of which appear—at least superficially—to require an amount of information well beyond what is available to policy makers. On the one hand, prior empirical work has certainly investigated questions about how voter turnout rates differ across groups and what the marginal effect of campaign spending is. On the other hand, the effect of taxation of donation sensitivity is an empirical parameter which, to my knowledge, has not been estimated in any prior literature. In the case of the optimal nonlinear tax these sensitivity effects include the effect of the second derivative of the tax schedule on donation sensitivity. And note that even knowledge of these effects of taxation are insufficient by themselves to conduct applied welfare analysis: the planner must have knowledge of either the joint distribution of political influence and sensitivity effects (in the case of the political inequality normative criteria) or the joint distribution of policy preferences and sensitivity effects (in the case of the political welfare criteria). Ultimately, a theory of optimal taxation with political externalities will be most valuable if it is amenable to some kind of applied welfare analysis exercise and the framework developed here does not easily lend itself to such analysis.

[23]Notice, this inquiry is in the spirit of Weitzman [1974].

# References

Stephen Ansolabehere, John M De Figueiredo, and James M Snyder Jr. Why is there so little money in us politics? *Journal of Economic perspectives*, 17(1):105–130, 2003.

Jeffrey S Banks and John Duggan. Probabilistic voting in the spatial model of elections: The theory of office-motivated candidates. In *Social choice and strategic decisions*, pages 15–56. Springer, 2005.

Duncan Black. The theory of committees and elections. 1958.

Peter J Coughlin. *Probabilistic voting theory*. Cambridge University Press, 1992.

Robert A Dahl. *On political equality*. Yale University Press, 2006.

Anthony Downs. An economic theory of democracy. 1957.

Elections Canada. Compendium of election administration in canada: A comparative overview. Technical report, Government of Canada Publications, 2017.

Francisco Facchinei and Christian Kanzow. Generalized nash equilibrium problems. *4or*, 5(3):173–210, 2007.

Walter Hettich, Stanley L Winer, et al. Democratic choice and taxation. *Cambridge Books*, 1999.

Melvin Hinich. The mean versus the median in spatial voting games. *Game Theory and Political Science New York. NYU Press: New York*, 1978.

Melvin J Hinich. Equilibrium in spatial voting: The median voter result is an artifact. 1976.

Melvin J Hinich, John O Ledyard, and Peter C Ordeshook. Nonvoting and the existence of equilibrium under majority rule. *Journal of Economic Theory*, 4(2):144–153, 1972.

Melvin J Hinich, John O Ledyard, and Peter C Ordeshook. A theory of electoral equilibrium: A spatial analysis based on the theory of games. *The Journal of Politics*, 35(1):154–193, 1973.

Harold Hotelling. Stability in competition. *The Economic Journal*, 39(153):41–57, 1929.

Wojciech Kopczuk. Comment on âprogressive wealth taxationâby saez and zucman. *Unpublished manuscript, Columbia University, New York, NY*, 2019.

Assar Lindbeck and Jörgen W Weibull. Balanced-budget redistribution as the outcome of political competition. *Public choice*, 52(3):273–297, 1987.

Assar Lindbeck and Jörgen W Weibull. A model of political equilibrium in a representative democracy. *Journal of public Economics*, 51(2):195–209, 1993.

Luca Micheletto. Redistribution and optimal mixed taxation in the presence of consumption externalities. *Journal of Public Economics*, 92(10-11):2262–2274, 2008.

Spencer Overton. The donor class: campaign finance, democracy, and participation. *U. Pa. L. Rev.*, 153:73, 2004.

Jan Rosset. Economically based inequalities in political representation: Where do they come from? In *Understanding Inequality: Social Costs and Benefits*, pages 241–256. Springer, 2016.

Emmanuel Saez and Gabriel Zucman. *The triumph of injustice: How the rich dodge taxes and how to make them pay.* WW Norton & Company, 2019.

Susan E Scarrow. Political finance regulation and equality: comparing strategies and impact. *Handbook of Political Party Funding*, page 103, 2018.

Florian Scheuer and Joel Slemrod. Taxing our wealth. Technical report, Working Paper, 2020.

Jennifer Shore. Political inequality: Origins, consequences, and ways ahead. In *Understanding inequality: Social costs and benefits*, pages 225–240. Springer, 2016.

Martin L Weitzman. Prices vs. quantities. *The review of economic studies*, 41(4):477–491, 1974.