Taxing Paradise: Optimal Commodity Taxation with Tourists

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#### Abstract

In this paper, I develop a theory of the jointly optimal choice of nonlinear income taxes and commodity taxes which accounts for the presence of tourists. Standard results are augmented in two key ways. First, if commodity taxes are uniform, I show that the Pareto efficient rate is the one that maximizes revenue from tourists. Impacts of a uniform commodity tax on residents are irrelevant, as they can always be compensated through income tax changes. Second, I show that the presence of tourists in the tax base overturns the classic Atkinson-Stiglitz result justifying uniform commodity taxation. Efficient rate differentiation is characterized by a Corlett-Hague-style rule which requires placing lower taxes on goods which tourists can more easily substitute for expenditures outside of the destination economy. Notably, the impact of commodity taxes on residents do not influence the pattern of efficient differentiation nor of the overall level of commodity taxation: rather, resident consumption behavior only influencing the magnitude of efficient rate differentiation. By contrast, the endogeneity of tourist arrivals may also influence the efficient degree of rate differentiation: taxes should also be lower on those goods whose consumption most strongly predicts a larger extensive margin response. That is to say, rate differentiation may be efficient if it can achieve a tagging objective, targeting the greatest tax burden to those tourists who are relatively more price sensitive at the extensive margin or whose visits generate the relatively more tax revenue. My results can rationalize some commonly observed tax policies in tourist destinations, such as VAT refunds to tourist on large purchases, or reduced VAT rates on hotel accommodation. On the other hand, these results do not support other common practices, such as levying higher rates on goods that are disproportionately consumed by tourists.

It is a common view in tourism-dependent economies that as much of the tax burden as possible should be shifted on to tourists. In the United States, this often manifests in tax policy that levies higher tax rates on goods that are more heavily consumed by tourists: twenty-five state levy tax hotel rooms and other visitor accommodations at higher rates than other goods.<sup>1</sup> On the other hand, in some jurisdictions, concerns about competition appear to have been used to justify levying lower rates on goods that tourists buy. According to a 2014 OECD report, hotel accommodations are one of the most common goods benefiting from reduced VAT rates: twenty-two OECD countries did so at the time [OECD and of Public Finance, 2014].

Both these policy responses suggest that tourist demand provides a rationale for departing from the implications of the Atkinson-Stigliz theorem [Atkinson and Stiglitz, 1976], which implies that as long as the government can also levy nonlinear income taxes, a Pareto efficient tax system is one with uniform commodity tax rate (at least, under certain restrictions on preferences). Several justifications for departures from uniform commodity taxation have been proposed in prior literature, including preference heterogeneity [Saez, 2002, Ferey et al., 2022], and credit constraints [Boadway et al., 2019]. This paper contributes this literature by providing a rationale for rate differentiation in the presence of nonlinear income taxation that is based on tourism demand.

In so doing, I also expand on the results of two prior papers [Hämäläinen, 2004, Gooroochurn, 2009] that explore optimal commodity taxation in the presence of tourists when the planner does not have access to a nonlinear income tax. Both these papers present results showing that taxes should be higher on goods that are more heavily consumed by tourists. My findings show that these papers' results are driven by the restriction on set of tax instruments available to the planner. As I shall show, optimal tax rates are not increasing tourists' share of demand for a good when a nonlinear income tax may be levied on residents.

First, I show that in the case of a uniform commodity tax regime, the Pareto efficient rate is one which maximizes the revenue obtained from tourists. This result is quite straightforward: from the perspective of residents, there exists a continuum of different tax systems consisting of a nonlinear income tax schedule and a uniform commodity tax rate which generate equivalent budget constraints (and are consequently welfare and revenue-equivalent for residents). In the presence of tourists, Pareto efficient taxation calls for choosing from amongst these tax systems the one that levies the commodity tax that maximizes revenue extracted from tourists. Resident demand is thus rendered irrelevant to the determination of the optimal uniform tax rate.

Turning to the case of non-uniform taxation, I first consider the case of exogenous tourist arrivals, and assume resident preferences follow a weakly separable structure that yields the standard Atkinson-Stigliz result of

<sup>&</sup>lt;sup>1</sup>https://www.hvs.com/article/9469-2022-hvs-lodging-tax-report-usa

uniform commodity taxation in the absence of tourist demand. In the presence of tourist demand, I show that optimally differentiated rates follow a Corlett-Hague-style rule. Just as Corlett and Hague [1953] found that—in the absence of nonlinear income taxes—optimal commodity tax rates should be lower on goods which are more substitutable with (untaxed) leisure, my results show that rates should be lower on goods which—for tourists—are more substitutable with spending outside the tourist destination (which the tourist destination cannot tax). The magnitude of optimal tax rate differentiation is attenuated to account for the efficiency costs that differentiation creates through resident demand responses. However, resident demand shares and resident responses to taxation do not determine whether rate differentiation is optimal, nor do they influence pattern of differentiation across goods.

I extend my results to account for extensive margin responses by tourists, who may choose to skip a visit if a destination becomes too costly. I show that these responses create a further rationale for rate differentiation: all else equal, taxes should be lower on those goods which are more heavily demanded by tourists with high extensive margin elasticities, and on those goods which are more heavily consumed by tourists whose visits generate more tax revenue.

I also show that the efficient average commodity tax rate paid by tourists during their visit is characterized by a kind of inverse elasticity rule that only accounts for tourist responses to commodity taxation. Thus, although resident demand patterns play a role in determining the magnitude of optimal rate differentiation, they play no direct role in determining the optimal level of commodity taxation.

The paper concludes with a qualitative discussion of the potential policy implications of these results, and also suggests gaps in these results which could be filled in future work.

# 1 Optimal Uniform Commodity Tax

Before developing the results mentioned above regarding optimal rate differentiation in Section 2 of the paper, I will first present a simple result on the optimal uniform commodity tax rate in the presence of tourists. This result will serve as a key benchmark in the discussion the follows.

Consider an economy with a continuum of residents each indexed by some  $i \in [0, 1]$ . Each resident i chooses taxable income  $z_i$  to maximize their utility

$$u_i\left(c,z\right)$$

subject to the budget constraint

$$(1+t) c = z - T(z)$$

where  $T(\cdot)$  is a nonlinear income tax and t is a linear (uniform) commodity tax rate. Here,  $c_i$  is taken to represent the residents' total consumption across many goods. With a uniform commodity tax, this is without loss of generality.

Now suppose that commodity demand in this economy stems not only from residents, but also from tourists (or other non-residents). Let  $C_V$  be the aggregate spending of these *visitor* taxpayers on commodities in the destination economy. Further, let  $C_R \equiv \int_0^1 c_i \mathrm{d}i$  be the aggregate spending of resident taxpayers on commodities.<sup>2</sup> Total tax revenue in this economy is then

$$R \equiv \int_0^1 T(z_i) di + t (C_R + C_V).$$

Here and throughout this paper, I make the assumption that tourists are not normatively relevant the social planner. Thus, when I say that a tax system is "Pareto efficient" I mean that it cannot be changed in such a way as to make some residents better off and without making other residents worse off. I do not account for how tax changes impact tourist welfare in this definition. Given this definition, the Pareto-efficient uniform commodity tax rate in this economy follows a simple inverse elasticity rule:

$$\frac{t}{1+t} = \frac{1}{\mathcal{E}^V} \tag{1}$$

where  $\mathcal{E}^V \equiv \frac{1+t}{C_V} \frac{\partial C_V}{\partial t}$  is the elasticity of aggregate visitor expenditure in the economy. This elasticity includes both intensive and extensive margin responses by visitors. Note, without adopting this normative distinction between residents and visitors, there would be nothing particularly noteworthy about the problem of optimal taxation in the presence of non-residents.

The derivation of this result follows from the basic insights of Feldstein [1999]: in this simple static model, for any change in the commodity tax rate there exists a corresponding non-linear income tax reform which leaves the budget constraints of all residents unchanged. In particular, suppose we start with some initial tax system  $(T_0(\cdot), t_0)$  and then change the commodity tax rate to  $t_1$ . Note that, for any uniform commodity tax rate t, nonlinear income tax schedule  $T(\cdot)$ , the budget constraint of a resident can be written as

$$c = \frac{z - T(z)}{1 + t}.$$

Thus, we can completely offset the impact of the commodity tax rate change on residents if we can find a

<sup>&</sup>lt;sup>2</sup>For simplicity, I've assumed that residents do not themselves spend some of their income as tourists in other destinations. Relaxing this assumption would imply a somewhat lower optimal rate.

corresponding new income tax schedule  $T_1(\cdot)$  such that

$$\frac{z - T_0(z)}{1 + t_0} = \frac{z - T_1(z)}{1 + t_1}.$$

That is, the reformed income tax schedule that offsets the commodity tax change is

$$T_{1}(z) = z - \frac{1+t_{1}}{1+t_{0}}(z-T_{0}(z)).$$

Since the new tax system  $(T_0(\cdot), t_0)$  leaves resident budget constraints unchanged, it also leaves residents taxable income and consumption choices unchanged. This implies it also leaves total tax revenue obtained from residents unchanged (though the share derived from income vs commodity taxes has changed).

The argument above implies that for any given commodity tax rate change, the income tax schedule can be adjusted to leave all residents just as well off as they were before without impacting the revenue derived from residents. Consequently, such changes can be evaluated based on their impact on revenue derived from tourists alone. This leads to the Pareto efficient rate being the rate that maximizes revenue obtained from tourists (as per equation 1). Notice, this result holds true under very general preference assumptions. We have imposed no restrictions on the utility functions of residents or the demand of tourists.

# 2 Optimal Differentiated Commodity Tax Rates

To discuss differentiated commodity taxation we must impose more structure on the behavior of agents. The economy consists of a set of residents with types indexed by  $w \in W$  and a set of visitors with types indexed by  $\theta \in \Theta$ . The distribution of resident types is described by F and the distribution of nonresident types is described by F. Finally, let the total number of residents be F0. Initially, we will consider the case where the total number of visitors is fixed. As well, for simplicity of exposition, we will first discuss the two-good case.

Each resident type w chooses taxable income z(w) and consumption of two commodities  $x_1(w)$  and  $x_2(w)$  to solve

$$\max_{z,x_1,x_2} \left\{ u\left(\phi\left(x_1,x_2\right),z;w\right) : \left(1+t_1\right)x_1 + \left(1+t_2\right)x_2 = z - T\left(z\right) \right\},\tag{2}$$

where  $t_i$  is the commodity tax rate applied to good  $i \in \{1, 2\}$  and  $T(\cdot)$  is a nonlinear income tax schedule. In what follows, it will often be helpful to reframe the resident's problem as a two-stage problem. In the second stage, the resident chooses commodity consumption conditional on their choice of taxable income z. Let  $\Phi(z)$  represent the indirect utility the agent gets from this second stage problem for a given value of taxable income:

$$\Phi(z) \equiv \max_{x_1, x_2} \left\{ \phi(x_1, x_2) : (1 + t_1) x_1 + (1 + t_2) x_2 = z - T(z) \right\}.$$
(3)

In the first stage, the a type w resident chooses a value of taxable income z(w) which maximizes their utility, anticipating how this choice impacts the value of the second stage problem:

$$\max_{z} \left\{ u \left( \Phi \left( z \right), z; w \right) \right\}. \tag{4}$$

Notice, the fact that the second stage problem does not depend directly on a resident's type (w) implies that consumption of the two commodities is homogeneous across resident taxpayers conditional on their taxable income z. For the remainder of the paper, we will therefore let  $x_i(z)$  be the demand for good i of any resident with taxable income z. Consequently, the demand for good i of a type w resident is  $x_i(z(w))$ .

As per the classic result of Atkinson and Stiglitz [1976], these preferences imply that if the tax base consisted of only residents, the optimal commodity tax would be uniform. However, in this economy, we must also consider the presence of visitors. Each visitor type  $\theta$  chooses their consumption of the two commodities  $x_1^V(\theta)$ , and  $x_2^V(\theta)$  in the destination economy, as well as total spending on goods outside the destination economy  $x_o^V(\theta)$  to solve

$$\max_{x_1, x_2, x_o} \left\{ u\left(x_1, x_2, x_o; \theta\right) : \left(1 + t_1\right) x_1 + \left(1 + t_2\right) x_2 + x_o = m\left(\theta\right) \right\},\tag{5}$$

where  $m(\theta)$  is the income of a type  $\theta$  visitor.

In this economy, tax revenue is

$$R \equiv \int T(z(w)) dF(w) + t_1(X_1^R + X_1^V) + t_2(X_2^R + X_2^V).$$

where  $X_i^R \equiv N_R \int x_i(z(w)) dF(w)$  is total resident consumption of good i and  $X_i^V \equiv N_V \int x_i^V(\theta) dG(\theta)$  is total visitor consumption of good i.

#### 2.1 Distribution Neutral Tax Reforms

To obtain a characterization of the Pareto efficient tax structure, we will consider tax reforms that jointly change these parameters in a manner that is distribution neutral for residents, in the spirit of Kaplow [2006]. Suppose that the economy begins with the tax system  $(t_1, t_2, T(\cdot))$ . Now, consider a joint tax reform which

marginally increases tax rate on good i by  $dt_i > 0$  while also marginally changing income tax liability at every income level z by some amount dT(z). The effect of this joint reform on the value of the residents' second stage problem (equation 3) is:

$$d\Phi(z) = \frac{\partial \Phi(z)}{\partial I} \left[ x_i(z) dt_i + dT(z) \right]$$

Notice, by setting  $dT(z) = -x_i(z) dt_i$  for all z, we obtain a joint reform for which  $d\Phi(z) = 0$  for all z.

This type of joint reform is distribution neutral from the perspective of residents, as the income tax change is design to compensate every agent for the loss in purchasing power caused by the increase in the commodity tax rate on good i. Because it leaves the value the second stage problem unchanged, it also leaves the private welfare of all residents unchanged. Consequently, the welfare effect of this joint reform can be evaluated solely in terms of it's revenue implications.

Turning to revenue, we should first note an additional convenient property of a distribution neutral reform: it leaves all residents' choices of taxable income unchanged. This is a direct implication of the fact that the value function of the second stage problem is unaffected by the reform  $(d\Phi(z) = 0$  for all z). Thus, the marginal revenue effect of a distribution neutral reform which increases the tax rate on good i is

$$\frac{\mathrm{d}R}{\mathrm{d}t_{i}}\Big|_{\mathrm{d}T(z)=-x_{i}(z)\mathrm{d}t_{i}} = X_{i}^{V} + N_{V}t_{i} \int \frac{\partial x_{i}^{V}(\theta)}{\partial t_{i}}\Big|_{u} \mathrm{d}G(\theta) + t_{-i}N_{V} \int \frac{\partial x_{-i}^{V}(\theta)}{\partial t_{i}}\Big|_{u} \mathrm{d}G(\theta) 
- N_{V} \int x_{i}^{V}(\theta) \left[t_{i} \frac{\partial x_{i}^{V}(\theta)}{\partial I} + t_{-i} \frac{\partial x_{-i}^{V}(\theta)}{\partial I}\right] \mathrm{d}G(\theta) 
+ t_{i}N_{R} \int \frac{\partial x_{i}(z)}{\partial t_{i}}\Big|_{u,z=z(w)} \mathrm{d}F(w) + t_{-i}N_{R} \int \frac{\partial x_{-i}(z)}{\partial t_{i}}\Big|_{u,z=z(w)} \mathrm{d}F(w) \quad (6)$$

where  $-i = \{1,2\} \setminus i$ ,  $\frac{\partial x_j(z)}{\partial t_k}\Big|_{u,z=z(w)}$  is the compensated response of resident demand for good j to the tax rate on good k (holding taxable income constant),  $\frac{\partial x_j^V(\theta)}{\partial t_k}\Big|_u$  is the compensated response of visitor demand for good j to the tax rate on good k, and  $\frac{\partial x_j^V(\theta)}{\partial I}$  is the visitor income effect of good j. Notice, the revenue effect of this joint reform does not include income effects for residents, as—by construction—the reform holds resident income constant.

### 2.2 Efficiency Conditions

The fact that distribution neutral reforms do not impact resident welfare implies that any distribution neutral reform which increases tax revenue is Pareto-improving. Thus, we can characterize Pareto efficient commodity

taxation in this economy using the following two equations:

$$\frac{\mathrm{d}R}{\mathrm{d}t_1}\Big|_{\mathrm{d}T(z)=-x_1(z)\mathrm{d}t_1} = 0 \quad \text{and} \quad \frac{\mathrm{d}R}{\mathrm{d}t_2}\Big|_{\mathrm{d}T(z)=-x_2(z)\mathrm{d}t_2} = 0 \tag{7}$$

Derivation of the central result closely follows the standard derivation of the Corlette-Hague rule. Applying basic results from demand theory (see Appendix A), we obtain the following presentation of these efficiency conditions:

$$C = \left(\frac{t_{2}}{1+t_{2}} - \frac{t_{1}}{1+t_{1}}\right) \mathcal{E}_{1}^{V} + \frac{t_{2}}{1+t_{2}} \mathcal{E}_{1,o}^{V} + \frac{X_{1}^{R}}{X_{1}^{V}} \left(\frac{t_{2}}{1+t_{2}} - \frac{t_{1}}{1+t_{1}}\right) \mathcal{E}_{1|z}^{R} - \operatorname{Cov}\left(\frac{x_{1}^{V}(\theta)}{X_{1}^{V}}, \ t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I}\right)$$

$$(8)$$

$$C = \left(\frac{t_{1}}{1+t_{1}} - \frac{t_{2}}{1+t_{2}}\right) \mathcal{E}_{2}^{V} + \frac{t_{1}}{1+t_{1}} \mathcal{E}_{2,o}^{V} + \frac{X_{2}^{R}}{X_{2}^{V}} \left(\frac{t_{1}}{1+t_{1}} - \frac{t_{2}}{1+t_{2}}\right) \mathcal{E}_{2|z}^{R} - \operatorname{Cov}\left(\frac{x_{2}^{V}(\theta)}{X_{2}^{V}}, \ t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I}\right)$$

$$(9)$$

where  $C \equiv \mathbb{E}\left[t_1 \frac{\partial x_1^V(\theta)}{\partial I} + t_2 \frac{\partial x_2^V(\theta)}{\partial I}\right] - 1$ ,

$$\mathcal{E}_{i}^{V} \equiv \int \frac{x_{i}^{V}\left(\theta\right)}{X_{i}^{V}} \left(-\frac{1+t_{i}}{x_{i}^{V}\left(\theta\right)} \frac{\partial x_{i}^{V}\left(\theta\right)}{\partial t_{i}}\Big|_{u}\right) dG\left(\theta\right)$$

is the aggregate (compensated) elasticity of visitor demand for good i,

$$\mathcal{E}_{i|z}^{R} \equiv \int \frac{x_{i}\left(z\left(w\right)\right)}{X_{i}^{R}} \left(-\frac{1+t_{i}}{x_{i}\left(z\left(w\right)\right)} \left.\frac{\partial x_{i}\left(z\left(w\right)\right)}{\partial t_{i}}\right|_{u}\right) dG\left(\theta\right)$$

is the aggregate (compensated) elasticity of resident demand for good i, and

$$\mathcal{E}_{i,o}^{V} \equiv \int \frac{x_{i}^{V}\left(\theta\right)}{X_{i}^{V}} \left(-\frac{1}{x_{i}^{V}\left(\theta\right)} \left.\frac{\partial x_{i}^{V}\left(\theta\right)}{\partial p_{o}}\right|_{u}\right) dG\left(\theta\right)$$

is the aggregate (compensated) cross-elasticity of visitor demand for good i with respect to the price of the outside option (i.e. consumption outside the destination economy.

Combining equations (8) and (9) we can eliminate the C term. After some algebra, we obtain the following characterization of Pareto efficient differentiation of commodity taxation:

$$\frac{\frac{t_1}{1+t_1}}{\frac{t_2}{1+t_2}} = \underbrace{\frac{\mathcal{E}^{V}_{1,o} + \mathcal{E}^{V}_{1} + \mathcal{E}^{V}_{2} + \left(\frac{X^{R}_{1}}{X^{V}_{1}}\right)\mathcal{E}^{R}_{1|z} + \left(\frac{X^{R}_{2}}{X^{V}_{2}}\right)\mathcal{E}^{R}_{2|z}}_{\text{substitution effect component}} + \underbrace{\frac{\text{Cov}\left(\frac{x^{V}_{2}(\theta)}{X^{V}_{2}} - \frac{x^{V}_{1}(\theta)}{X^{V}_{1}}, \ t_{1}\frac{\partial x^{V}_{1}(\theta)}{\partial I} + t_{2}\frac{\partial x^{V}_{2}(\theta)}{\partial I}\right)}_{\text{income effect component}}$$

Notice, this characterization should appear is reminscent of the classic Corlette-Hague rule. To obtain a precise understand of the result, let us first consider the case where the *income effect component* is zero, leaving only the *substitution effect component* so that Pareto efficient commodity tax rates must satisfy

$$\frac{\frac{t_1}{1+t_1}}{\frac{t_2}{1+t_2}} = \frac{\mathcal{E}_{1,o}^V + K}{\mathcal{E}_{2,o}^V + K}$$

where  $K \equiv \mathcal{E}_1^V + \mathcal{E}_2^V + \left(\frac{X_1^R}{X_1^V}\right)\mathcal{E}_{1|z}^R + \left(\frac{X_2^R}{X_2^V}\right)\mathcal{E}_{2|z}^R$ . In this scenario, differential commodity taxation is only justified if aggregate demand for one of the two commodities is more responsive to the price of consumption outside the destination economy. That is to say, the tax rate on good 1 should be higher than the rate of good 2 if and only if good 2 is more substitutable for outside consumption than good 1:  $\mathcal{E}_{1,o}^V > \mathcal{E}_{2,o}^V$ . While other behavior responses do affect the efficient level of differentiation (through K), they do not determine whether differentiation occurs, nor its direction. Rather, these responses only influence the magnitude of the efficient level of differentiation. The more responsive visitor and resident demand for a good is to the own-price of consumption, the more attenuated the differentiation of the commodity taxes will be. Note as well, that the relative size of resident and visitor consumption also does not play a role in determining the direction of efficient rate differentiation. Rather, when resident consumption is high relative to visitor consumption for some good, this serves to increase the attenuation of efficient tax rate differentiation.

These results are fairly intuitive. Because it can neutralize the effects of commodity tax changes on resident welfare through offsetting income tax reforms, the destination economy ought to pursue any such reform that increases tax revenue. If some goods are easier for visitors to substitute for purchases outside the destination economy, the demand for these goods will be more responsive to taxation. Consequently, lower tax rates should be placed on such goods. However, this differentiation generates some distortions in consumption patterns of both residents and visitors. The term K includes the own-price elasticities of demand for both visitors and residents as a measure of the fiscal externalities these distortions generate. The larger these terms are, the greater the revenue loss caused by the distortionary effects of differential commodity taxation. If too much rate differentiation occurs, these effects will come to dominate the revenue gains generated by targeting commodity taxes towards less elastically consumed goods. And, of course, the large the resident share of consumption is, the greater the revenue loss caused by resident behavioral responses.

Recall, in the case of the Corlette-Hague rule, tax rate differentiation in a standard economy without tourists may be optimal if the social planner cannot levy a non-linear income tax. In such a setting, rate differentiation is optimal only when the untaxed good (leisure) is more substitutable for certain commodities than for others. Own-price behavioral responses to commodity taxation only serve to determine the optimal degree of rate differentiation. Similarly, in this paper, rate differentiation may be optimal if the the social planner's ability to tax visitors is restricted, as visitors cannot be subjected to a nonlinear income tax by the destination economy. Consequently, rate differentiation is optimal only when the untaxed good (outside consumption) is more substitutable for certain commodities than for others.

Having thoroughly explored the substitution effect component, let us turn to the income effect component. When visitor demand for destination commodities is influences by income effects, efficient rate differentiation is further augmented. Simply put, this component accounts for the fact that tax rate differentiation may also be desirable on tagging grounds if the size of fiscal externalities generated by visitor income effects are systematically correlated with patterns of visitor commodity demand. For example, even if  $\mathcal{E}_{1,o}^V = \mathcal{E}_{2,o}^V$ , so that the substitution effect component is equal to one, efficient taxation will require a higher tax rate to be placed on good 1 if

$$\operatorname{Cov}\left(\frac{x_{2}^{V}\left(\theta\right)}{X_{2}^{V}},\ t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I}+t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right)>\operatorname{Cov}\left(\frac{x_{1}^{V}\left(\theta\right)}{X_{1}^{V}},\ t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I}+t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right).$$

These covariances terms measure the strength of the association between visitor income effect fiscal externalities,  $t_1 \frac{\partial x_1^V(\theta)}{\partial I} + t_2 \frac{\partial x_2^V(\theta)}{\partial I}$ , and the relative consumption of each good. Therefore, the expression above can be interpreted as saying that having above average consumption of good 2 is a better predictor of having a high income effect fiscal externality than having above average consumption of good 1. That is to say, high consumption of good 2 is a useful tag, indicating that a visitors is part of a group whose demand is relatively more response to commodity taxation, and on whom a revenue-maximizing planner would want to place a lower tax burden. The planner can achieve this goal (imperfectly) by levying a higher tax rate on good 1 than good 2.

It is not immediately obvious whether we should expect the income effect component to be of much importance in practice. Purchases in the destination economy may make up a relatively small fraction of visitor income, and so the effect may be small in magnitude. Moreover, it is not immediately obvious that we should expect high consumption of some particular commodities to be indicative of high income effect fiscal externalities.

### 2.3 Extensive Margin Responses

Any analysis of commodity taxation in the presence of tourists would be incomplete without account for the possibility of extensive margin responses. Commodity taxes increase the total cost of a tourist's visit to the destination economy, and thus should be deter some from visiting.

To capture these responses, let us extend our model of visitor behavior so that each potential visitor has a

type  $(\theta, \xi)$ , and each such visitor chooses to visit the destination economy if and only if

$$v(\theta) > \xi$$

where  $v(\theta) \equiv u\left(x_1^V(\theta), x_2^V(\theta), x_o^V(\theta); \theta\right)$  is the value of a visit for a potential visitor of type  $\theta$ , and  $\xi$  is the value of the outside option (not visiting the destination economy). Note,  $v(\theta)$  is simply the indirect utility function associated with (5).

Let the conditional distribution of  $\xi$  be  $\Pi(\cdot|\theta)$ ,<sup>3</sup> so that the probability a type  $\theta$  visitor chooses to visitor the destination economy is

$$\Pr \left\{ \xi < v \left( \theta \right) | \theta \right\} = \Pi \left( v \left( \theta \right) | \theta \right)$$

In this new version of the model, the number of visitors to the destination economy is endogenous to commodity taxation. Let  $N_{PV}$  be the total number of prospective visitors to the destination economy. The total number of actual visitors is

$$N_{V} = N_{PV} \int \Pi(v(\theta)|\theta) dG(\theta).$$

Similarly, tax revenue from the visitors is now

$$t_1 X_1^V + t_2 X_2^V = N_{PV} \int \left[ t_1 x_1^V \left( \theta \right) + t_2 x_2^V \left( \theta \right) \right] \Pi \left( v \left( \theta \right) | \theta \right) dG \left( \theta \right).$$

From here, characterizing efficient commodity tax differentiation follows a similar approach to that taken for the model with extensive margin responses. The difference is that now the visitor revenue effect of slightly increasing the tax rate on good i includes an additional term to account for these extensive margin effects:

$$-N_{PV} \int \left[t_{1} x_{1}^{V}\left(\theta\right)+t_{2} x_{2}^{V}\left(\theta\right)\right] x_{i}^{V}\left(\theta\right) \epsilon_{ext}^{V}\left(\theta\right) \mathrm{d}G\left(\theta\right)$$

where the semi-elasticity of the probability of visiting with respect to the total cost of a visit is

$$\epsilon_{ext}^{V}\left(\theta\right) \equiv \frac{1}{v\left(\theta\right)} \frac{\partial v\left(\theta\right)}{\partial m\left(\theta\right)} \frac{v\left(\theta\right) \pi\left(v\left(\theta\right) | \theta\right)}{\Pi\left(v\left(\theta\right) | \theta\right)}$$

with  $\pi(\cdot|\theta)$  denoting the conditional PDF of  $\xi$ .

$$u\left(x_{1}^{V}\left(\theta\right),x_{2}^{V}\left(\theta\right),x_{o}^{V}\left(\theta\right);\theta\right)>v_{o}\left(\theta,\xi\right)$$

where  $v_o(\theta, \xi)$  is the indirect utility of not visiting the destination economy.

<sup>&</sup>lt;sup>3</sup>Allowing the distribution of  $\xi$  to vary with  $\theta$  means the model is equivalent to one where a visitor chooses to visit the destination economy if and only if

Incorporating extensive margin responses, efficient commodity tax differentiation must satisfy

$$\frac{\frac{t_{1}}{1+t_{1}}}{\frac{t_{2}}{1+t_{2}}} = \underbrace{\frac{\mathcal{E}_{1,o}^{V} + \mathcal{E}_{1}^{V} + \mathcal{E}_{2}^{V} + \left(\frac{X_{1}^{R}}{X_{1}^{V}}\right)\mathcal{E}_{1|z}^{R} + \left(\frac{X_{2}^{R}}{X_{2}^{V}}\right)\mathcal{E}_{2|z}^{R}}_{\text{substitution effect component}} + \underbrace{\frac{\text{Cov}\left(\frac{x_{2}^{V}(\theta)}{X_{2}^{V}} - \frac{x_{1}^{V}(\theta)}{X_{1}^{V}}, \ t_{1}\frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2}\frac{\partial x_{2}^{V}(\theta)}{\partial I}\right)}_{\text{substitution effect component}} \\ + \underbrace{\frac{\text{Cov}\left(\frac{x_{2}^{V}(\theta)}{X_{2}^{V}} - \frac{x_{1}^{V}(\theta)}{X_{1}^{V}}, \ \left(t_{1}x_{1}^{V}(\theta) + t_{2}x_{2}^{V}(\theta)\right)\mathcal{E}_{1|z}^{R} + \left(\frac{X_{2}^{R}}{X_{2}^{V}}\right)\mathcal{E}_{2|z}^{R}\right)}_{\text{extensive margin component}}.$$

Notice, the way that extensive margin responses enter this condition closely parallels the way income effects do. Unsurprisingly then, the intuition behind the extensive margin component is closely related. Even if  $\mathcal{E}_{1,o}^V = \mathcal{E}_{2,o}^V$ , so that the substitution effect component is equal to one, and if the income effect component were zero, efficient taxation will require placing a higher tax rate on good 1 if

$$\operatorname{Cov}\left(\frac{x_{2}^{V}\left(\theta\right)}{X_{2}^{V}},\ \left(t_{1}x_{1}^{V}\left(\theta\right)+t_{2}x_{2}^{V}\left(\theta\right)\right)\epsilon_{ext}^{V}\left(\theta\right)\right) > \operatorname{Cov}\left(\frac{x_{1}^{V}\left(\theta\right)}{X_{1}^{V}},\ \left(t_{1}x_{1}^{V}\left(\theta\right)+t_{2}x_{2}^{V}\left(\theta\right)\right)\epsilon_{ext}^{V}\left(\theta\right)\right).$$

Notice, each of these covariance terms includes  $(t_1x_1^V(\theta) + t_2x_2^V(\theta)) \epsilon_{ext}^V(\theta)$ , which is a measure of the fiscal externality associated with the extensive margin responses of type  $\theta$  visitors. As with the income effect component then, the extensive margin component is incorporates the possibility that differential commodity taxation can be used to achieve a tagging objective: to target higher tax burdens to those visitors whose extensive margin responses generate a lower fiscal externality. Note, a visitor may have a high extensive margin fiscal externality due to either having a high participation semielasticity  $(\epsilon_{ext}^V(\theta))$  or being a large source of revenue condition on visiting (a high  $t_1x_1^V(\theta) + t_2x_2^V(\theta)$ ). That is to say, it is sensible to place a higher tax rate on good 1 if relatively high consumption of good 1 is a better predictor of being the type of prospective visitor who is highly responsive at the extensive margin and/or who generates a large amount of tax revenue conditional on visiting.

### 2.4 The Efficient Level of Taxation

Combining the efficiency conditions in equation (7), we have that efficient commodity taxation must satisfy

$$(1+t_1) \frac{dR}{dt_1} \Big|_{dT(z)=-x_1(z)dt_1} + (1+t_2) \frac{dR}{dt_2} \Big|_{dT(z)=-x_2(z)dt_2} = 0.$$

This condition can be re-arranged to obtain the following, more intuitive presentation:

$$\frac{t_1 X_1^V + t_2 X_2^V}{(1 + t_1) X_1^V + (1 + t_2) X_2^V} = \frac{1 - IE^V - EM^V}{-\frac{t_1 X_1^V}{t_1 X_1^V + t_2 X_2^V} \mathcal{E}_{1,o}^V - \frac{t_2 X_2^V}{t_1 X_1^V + t_2 X_2^V} \mathcal{E}_{2,o}^V}$$
(10)

where

$$\begin{split} IE^{V} &= \mathbb{E}\left[t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right] + \frac{\left(1+t_{1}\right)X_{1}^{V}}{\left(1+t_{1}\right)X_{1}^{V} + \left(1+t_{2}\right)X_{2}^{V}} \text{Cov}\left(\frac{x_{1}^{V}\left(\theta\right)}{X_{1}^{V}},\ t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right) \\ &+ \frac{\left(1+t_{2}\right)X_{2}^{V}}{\left(1+t_{1}\right)X_{1}^{V} + \left(1+t_{2}\right)X_{2}^{V}} \text{Cov}\left(\frac{x_{2}^{V}\left(\theta\right)}{X_{2}^{V}},\ t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right) \end{split}$$

is a term capturing income effects responses of visitors and

$$\begin{split} EM^{V} &= \mathbb{E}\left[t_{1}\frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2}\frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right] + \frac{\left(1+t_{1}\right)X_{1}^{V}}{\left(1+t_{1}\right)X_{1}^{V} + \left(1+t_{2}\right)X_{2}^{V}} \text{Cov}\left(\frac{x_{1}^{V}\left(\theta\right)}{X_{1}^{V}}, \; \left(t_{1}x_{1}^{V}\left(\theta\right) + t_{2}x_{2}^{V}\left(\theta\right)\right)\epsilon_{ext}^{V}\left(\theta\right)\right) \\ &+ \frac{\left(1+t_{2}\right)X_{2}^{V}}{\left(1+t_{1}\right)X_{1}^{V} + \left(1+t_{2}\right)X_{2}^{V}} \text{Cov}\left(\frac{x_{2}^{V}\left(\theta\right)}{X_{2}^{V}}, \; \left(t_{1}x_{1}^{V}\left(\theta\right) + t_{2}x_{2}^{V}\left(\theta\right)\right)\epsilon_{ext}^{V}\left(\theta\right)\right) \end{split}$$

is a term capturing extensive margin responses of visitors.

Note, the left hand side of equation (10) is commodity taxes paid by visitors as a share of total visitor spending in the destination economy. Thus, this equation as characterizes the efficient average commodity tax rate visitors face in the destination economy. As we can see, as in the case of uniform taxation, the efficient level of taxation follows a type inverse elasticity rule, where the elasticity in question is a revenue-weighted average of cross-substitution elasticities of aggregate tourist demand, scaled up to account for the fiscal externalities generated by income effects and the endogeneity of arrivals:

$$\frac{\frac{t_{1}X_{1}^{V}}{t_{1}X_{1}^{V}+t_{2}X_{2}^{V}}\left(-\mathcal{E}_{1,o}^{V}\right)+\frac{t_{2}X_{2}^{V}}{t_{1}X_{1}^{V}+t_{2}X_{2}^{V}}\left(-\mathcal{E}_{2,o}^{V}\right)}{1-IEV-EMV}.$$

Note as well, just as in the case of uniform taxation, equation (1) characterizes the efficient level of visitor taxation only in terms of sufficient statistics that pertain to visitor commodity demand. This generalizes a key result from the uniform tax case: the efficient level of commodity taxation is one which maximizes revenue extracted from tourists.

## 3 Discussion and Conclusion

This paper fills a critical gap in the niche subject of optimal commodity taxation in the presence of tourists by addressing the important case where nonlinear income taxes are part of the social planner's toolkit. This is a highly policy relevant case, as in the case where the destination economy in question is a national or regional government, nonlinear income taxation is likely an available source of revenue. Notably, my results show that in this case, efficient rate differentiation is at odds with the intuitive idea that higher tax rates should be levied on commodities disproportionately demanded by tourists. Rather, I show that tax rates should be higher on goods which visitors can less readily substitute for outside consumption and those goods which act as tags for visitors with low income and extensive margin responses.

What might the concrete implications of these results be? Suppose policymakers in a destination economy are considering placing a higher tax rate on luxury goods, on the basis of the fact that tourists account for a disproportionate share of demand for these goods. The results of this paper suggest the opposite policy might be desirable. In most case, these purchases are readily substitutable for luxury goods purchased other jurisdictions, suggesting it may be optimal to levy a lower tax on such goods. This type of differentiation might be reinforced if visitors who purchase luxury goods during their trip are big spenders more generally, which might suggest luxury goods are a tag for those with high extensive margin fiscal externalities. On the other hand, we might also think that such big spenders are also not very price sensitive at the extensive margin, so it is not immediately clear how extensive margin responses influence optimal rate differentiation in the case of luxury goods.

Alternatively, consider the possibility of placing a relatively low tax rate on groceries. The results of this paper do not support doing so on the basis of the the fact that tourists make up a relatively small share of grocery demand. In the case of many destination economies in fact, the opposite likely holds true: in general, visitors cannot substitute food purchases in the destination economy for food purchases at home, suggesting a higher tax rate on food purchases is warranted. On the other hand, suppose that the type of tourists who spend large amounts on groceries (relative to other tourists) are budget conscious consumers whose decision to visit the destination economy at all is quite sensitive to the overall cost of the visit. In this case, grocery purchases may act as a tag for visitors with high extensive margin fiscal externalities and so it may be desirable to levy a lower tax rate on groceries. But here again, the effect of extensive margin responses is not immediately clear, since we might also expect that such budget conscious visitors also make a relatively small contribution to commodity tax revenue through their visits.

By contrast, existing results on optimal taxation with tourists do provide some support for the idea of taxing

those goods which tourists demand more highly [Hämäläinen, 2004, Gooroochurn, 2009]. This paper thus provides an important clarification by showing that these prior results are pertinent only in the case where levying a nonlinear income tax on residents is not possible. I have also provided a link between tourist taxation and recent developments in the theory of commodity taxation which characterize circumstances under which rate differentiation is Pareto efficient. As in this paper, such results often rely on the construction of distribution neutral tax reforms in the spirit of Kaplow [2006].

This connection suggests several directions for future extension of the results contained in this paper. How are these results modified if we allow for richer forms of resident preference heterogeneity [Saez, 2002, Ferey et al., 2022]? Or if residents wages are endogenous to tourist demand? What if the destination economy levy charge entry fees on non-residents? What if visitors' presence imposes externalities on residents of the destination economy? While the project of characterizing optimal taxation in a tourism-dependent economy is incomplete without addressing such topics, this paper provides a clear foundation upon which a more complete guide to taxation for tourism dependent economies may be constructed.

The paper also presents characterizations of efficient commodity taxation in terms of sufficient statistics which are in principle estimable. In so doing, it provides clear guidance on how existing empirical results on tourist demand can (or cannot) be applied to inform the analysis of tax policy in tourism dependent economies. Furthermore, the results presented here suggest directions for novel, policy-relevant empirical work. For example, quantification of the role of extensive margin responses may prove critical in determining the real world implications of the results presented in this paper.

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# A Derivation of Corlett-Hague-Style Condition

As per equation 6, the optimality condition  $\frac{dR}{dt_i}\Big|_{dT(z)=-x_i(z)dt_i} = 0$  can be written as

$$0 = X_{i}^{V} + t_{i} N_{V} \int \frac{\partial x_{i}^{V}(\theta)}{\partial t_{i}} \Big|_{u} dG(\theta) + t_{-i} N_{V} \int \frac{\partial x_{-i}^{V}(\theta)}{\partial t_{i}} \Big|_{u} dG(\theta) - N_{V} \int x_{i}^{V}(\theta) \left( t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I} \right) dG(\theta) + t_{i} N_{R} \int \frac{\partial x_{i}(z)}{\partial t_{i}} \Big|_{u,z=z(w)} dF(w) + t_{-i} N_{R} \int \frac{\partial x_{-i}(z)}{\partial t_{i}} \Big|_{u,z=z(w)} dF(w).$$

$$(11)$$

Applying standard properties of Hicksian demand functions to the second stage resident compensated responses, we have

$$(1+t_i) \left. \frac{\partial x_i(z)}{\partial t_i} \right|_{u,z} + (1+t_{-i}) \left. \frac{\partial x_i(z)}{\partial t_{-i}} \right|_{u,z} = 0.$$

Combined with Slutsky symmetry, this implies that

$$\left. \frac{\partial x_{-i}\left(z\right)}{\partial t_{i}}\right|_{u,z} = -\frac{1+t_{i}}{1+t_{-i}} \left. \frac{\partial x_{i}\left(z\right)}{\partial t_{i}}\right|_{u,z}.$$

Similarly, visitor compensated responses satisfy

$$(1+t_i) \left. \frac{\partial x_i^V(\theta)}{\partial t_i} \right|_{u} + (1+t_{-i}) \left. \frac{\partial x_i^V(\theta)}{\partial t_{-i}} \right|_{u} + \left. \frac{\partial x_i^V(\theta)}{\partial p_o} \right|_{u} = 0$$

where  $p_o$  is the price of (untaxed) consumption outside the tourist destination. Together with Slutsky symmetry, this gives us

$$\left. \frac{\partial x_{-i}^{V}\left(\theta\right)}{\partial t_{i}}\right|_{u} = -\frac{1+t_{i}}{1+t_{-i}} \left. \frac{\partial x_{i}^{V}\left(\theta\right)}{\partial t_{i}}\right|_{u} - \frac{1}{1+t_{-i}} \left. \frac{\partial x_{i}^{V}\left(\theta\right)}{\partial p_{o}}\right|_{u}.$$

Using these equalities, equation ?? can be re-written as

$$0 = X_{i}^{V} + \left(\frac{t_{-i}}{1 + t_{-i}} - \frac{t_{i}}{1 + t_{i}}\right) N_{V} \int x_{i}^{V}(\theta) \, \varepsilon_{i}^{V}(\theta) \, dG(\theta) + \frac{t_{-i}}{1 + t_{-i}} N_{V} \int x_{i}^{V}(\theta) \, \varepsilon_{i,o}^{V}(\theta) \, dG(\theta)$$
$$- N_{V} \int x_{i}^{V}(\theta) \left(t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I}\right) dG(\theta) + \left(\frac{t_{-i}}{1 + t_{-i}} - \frac{t_{i}}{1 + t_{i}}\right) N_{R} \int x_{i} \left(z\left(w\right)\right) \varepsilon_{i|z}^{R}\left(z\left(w\right)\right) dF(w)$$

where

$$\varepsilon_{i|z}^{R}\left(z\right) \equiv -\frac{1+t_{i}}{x_{i}\left(z\right)} \left.\frac{\partial x_{i}\left(z\right)}{\partial t_{i}}\right|_{u,z}$$

is the compensated elasticity of resident demand for good i (holding taxable income constant),

$$\varepsilon_{i}^{V}\left(\theta\right) \equiv -\frac{1+t_{i}}{x_{i}^{V}\left(\theta\right)} \left. \frac{\partial x_{i}^{V}\left(\theta\right)}{\partial t_{i}} \right|_{u}$$

is the compensated elasticity of visitor demand for good i,

$$\varepsilon_{i,o}^{V}\left(\theta\right) \equiv -\frac{1}{x_{i}^{V}\left(\theta\right)} \left. \frac{\partial x_{i}^{V}\left(\theta\right)}{\partial p_{o}} \right|_{u}$$

and  $\varepsilon_{i,o}^V(\theta)$  is the cross-elasticity of visitor demand for good i with respect to the price of consumption outside the destination economy.

Alternatively, we can write this efficiency condition in terms of the elasticities of aggregate demand:

$$0 = X_{i}^{V} + \left(\frac{t_{-i}}{1 + t_{-i}} - \frac{t_{i}}{1 + t_{i}}\right) X_{i}^{V} \mathcal{E}_{i}^{V} + \frac{t_{-i}}{1 + t_{-i}} X_{i}^{V} \mathcal{E}_{i,o}^{V}$$

$$- X_{i}^{V} \mathbb{E} \left[ t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I} \right] - X_{i}^{V} \operatorname{Cov} \left( \frac{x_{i}^{V}(\theta)}{X_{i}^{V}}, \ t_{1} \frac{\partial x_{1}^{V}(\theta)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}(\theta)}{\partial I} \right)$$

$$+ \left( \frac{t_{-i}}{1 + t_{-i}} - \frac{t_{i}}{1 + t_{i}} \right) X_{i}^{R} \mathcal{E}_{i|z}^{R}$$

$$(12)$$

where the income effect component has been re-written, using the fact that

$$\begin{split} N_{V} \int x_{i}^{V}\left(\theta\right) \left(t_{1} \frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right) \mathrm{d}G\left(\theta\right) &= X_{i}^{V} \mathbb{E}\left[t_{1} \frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right] \\ &+ \mathrm{Cov}\left(x_{i}^{V}\left(\theta\right), \ t_{1} \frac{\partial x_{1}^{V}\left(\theta\right)}{\partial I} + t_{2} \frac{\partial x_{2}^{V}\left(\theta\right)}{\partial I}\right). \end{split}$$

Re-arranging equation (12), we can easily obtain equations (8) and (9).