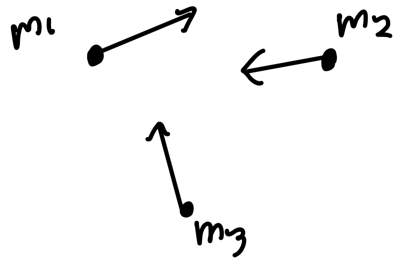
3 body problem



$$U = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_2 m_3}{r_{23}} + \frac{m_3 m_1}{r_{31}}\right)$$

$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$$

$$\mathcal{L} = T - U = \frac{1}{2}\left(m_1(\dot{x}_1^2 + \dot{y}_1^2) + m_2(\dot{x}_2^2 + \dot{y}_2^2) + m_3(\dot{x}_3^2 + \dot{y}_3^2)\right)$$
$$+ G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_2 m_3}{r_{23}} + \frac{m_3 m_1}{r_{31}}\right)$$

$$= \frac{1}{2}\left(m_1(\dot{x}_1^2 + \dot{y}_1^2) + m_2(\dot{x}_2^2 + \dot{y}_2^2) + m_3(\dot{x}_3^2 + \dot{y}_3^2)\right)$$
$$+ G\left(\frac{m_1 m_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} + \frac{m_2 m_3}{\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}} + \frac{m_3 m_1}{\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}}\right)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_1} \Rightarrow G m_1 m_2 \left[\frac{(x_1 - x_2)}{((x_1 - x_2)^2 + (y_1 - y_2)^2)^{3/2}}\right]$$
$$+ G m_3 m_1 \left[\frac{-(x_3 - x_1)}{((x_3 - x_1)^2 + (y_3 - y_1)^2)^{3/2}}\right] = m_1 \ddot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_2} = G m_2 m_3 \left[\frac{(x_2 - x_3)}{((x_2 - x_3)^2 + (y_2 - y_3)^2)^{3/2}}\right]$$

$$+ G m_1 m_2 \left[\frac{-(x_1 - x_2)}{\left((x_1 - x_2)^2 + (y_1 - y_2)^2\right)^{3/2}}\right] = m_2 \ddot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x_3} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_3} = G m_3 m_1 \left[\frac{(x_3 - x_1)}{\left((x_3 - x_1)^2 + (y_3 - y_1)^2\right)^{3/2}}\right]$$

$$+ G m_2 m_3 \left[\frac{-(x_2 - x_3)}{\left((x_2 - x_3)^2 + (y_2 - y_3)^2\right)^{3/2}}\right] = m_3 \ddot{x}$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = \frac{\partial H}{\partial q_i}$$

$$\mathcal{H} = \sum p_i \dot{q}_i - \mathcal{L}$$

$$p_{x_1} = \frac{\partial \mathcal{L}}{\partial \dot{x}_1} = m_1 \dot{x}_1 \qquad p_{y_1} = m_1 \dot{y}_1 \quad \ldots$$

$$\mathcal{H} = m_1(\dot{x}_1^2 + \dot{y}_1^2) + m_2(\dot{x}_2^2 + \dot{y}_2^2) + m_3(\dot{x}_3^2 + \dot{y}_3^2) - \mathcal{L}$$

$$= \frac{1}{2}\left(m_1(\dot{x}_1^2 + \dot{y}_1^2) + m_2(\dot{x}_2^2 + \dot{y}_2^2) + m_3(\dot{x}_3^2 + \dot{y}_3^2)\right)$$

$$- G\left(\frac{m_1 m_2}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}} + \frac{m_2 m_3}{\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}} + \frac{m_3\, m}{\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}}\right.$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$H = \frac{p_{x_1}^2 + p_{y_1}^2}{2m_1} + \frac{p_{x_2}^2 + p_{y_2}^2}{2m_2} + \frac{p_{x_3}^2 + p_{y_3}^2}{2m_3}$$

$$- G\left(\frac{m_1 m_2}{\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}} + \frac{m_2 m_3}{\sqrt{(x_2-x_3)^2 + (y_2-y_3)^2}} + \frac{m_3 m_1}{\sqrt{(x_3-x_1)^2 + (y_3-y_1)^2}}\right.$$

$$\dot{p}_{x_1} = G m_1 m_2 \left[\frac{(x_1-x_2)}{((x_1-x_2)^2 + (y_1-y_2)^2)^{3/2}}\right] + G m_3 m_1 \left[\frac{-(x_3-x_1)}{((x_3-x_1)^2 + (y_3-y_1)^2)^{3/2}}\right]$$

$$\dot{x}_1 = \frac{p_{x_1}}{m}$$

base — 125 +

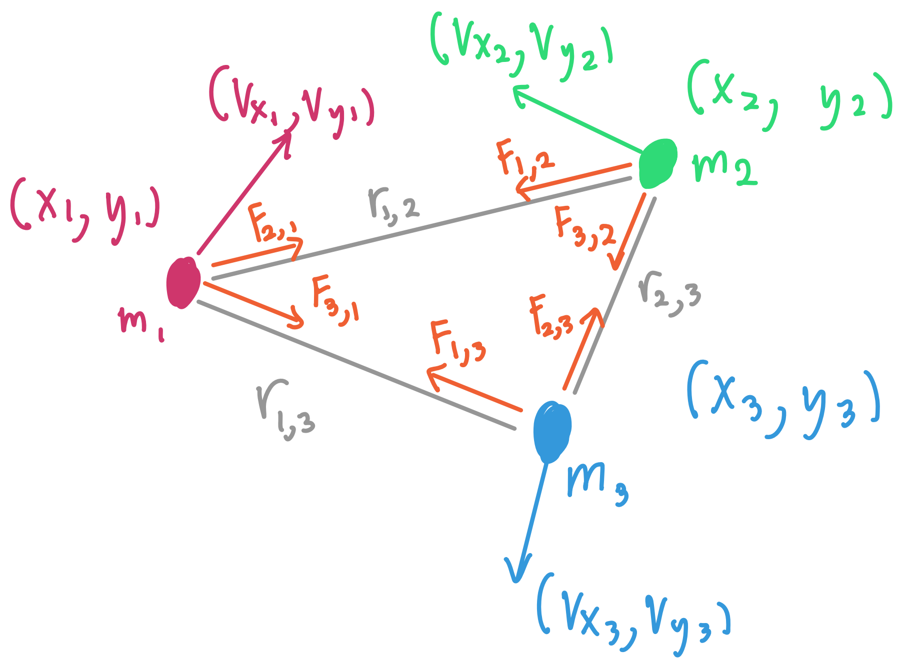
hamiltonian — 25

animation — 50

interactivity — 25

markdown — 25

symmetries / stable orbit — rest



(x_1, y_1)
(v_{x_1}, v_{y_1})
m_1
(v_{x_2}, v_{y_2})
(x_2, y_2)
m_2
$F_{1,2}$
$F_{2,1}$
$r_{1,2}$
$F_{3,1}$
$F_{3,2}$
$r_{2,3}$
$F_{2,3}$
$F_{1,3}$
$r_{1,3}$
(x_3, y_3)
m_3
(v_{x_3}, v_{y_3})