

#2

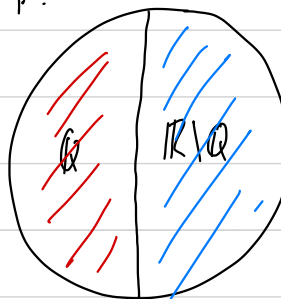
$\mathbb{R} \setminus \mathbb{Q}$

\mathbb{R} :

Hint: Proceed by contradiction,

- assume the statement holds, and

show that irrational = rational



Claim: There is no smallest positive rational number.

Proof: Proceed by contradiction. Assume N is the smallest positive rational.

We know that $N \in \mathbb{Q}$, $N > 0$, $N < r \quad \forall r \in \mathbb{Q}, r \neq N$. [1] (definitions)

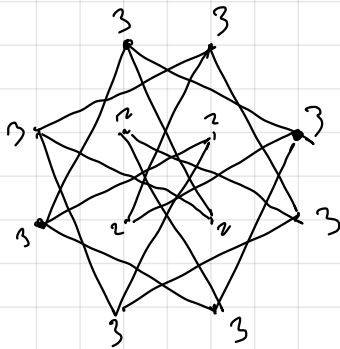
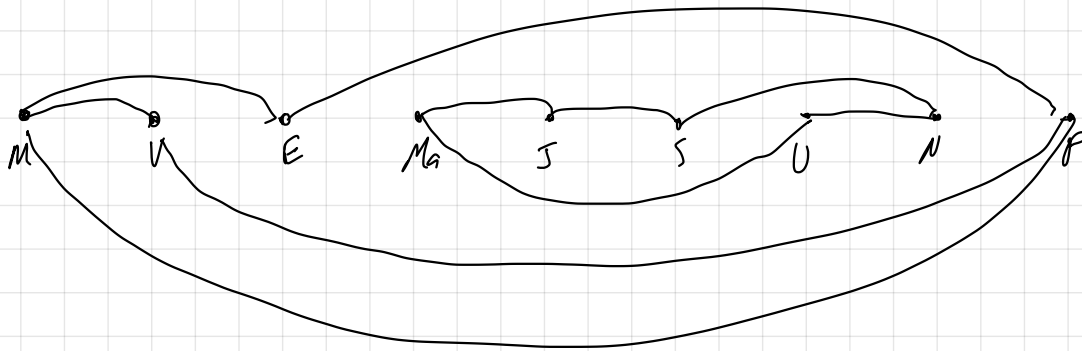
However, $\frac{1}{2} \cdot N < N$, and $\frac{1}{2} \cdot N \in \mathbb{Q}$. \S [2] (contradiction)

Claim: The sum of a rational and an irrational cannot be rational. (is irrational)

Let $x \in \mathbb{Q}$, $y \in \mathbb{R} \setminus \mathbb{Q}$. Assume $x + y = z$, where $z \in \mathbb{Q}$. [1] (definitions)

$\Rightarrow y = z - x$. Since $z \in \mathbb{Q}$, $x \in \mathbb{Q}$ [2] (manipulation)

But, $z - x \in \mathbb{Q}$, a contradiction \S . [3] (contradiction)



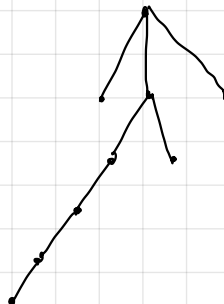
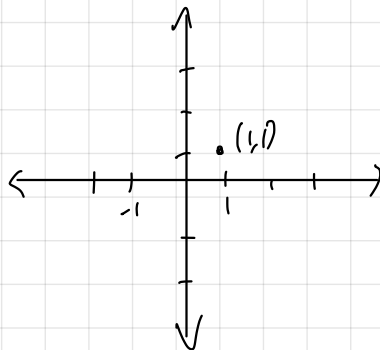
$$2E = D$$

$$V = \{A, B, C, \dots\}$$

$$V \times V = (A, B)$$

$$\mathbb{N} \times \mathbb{N} \quad \boxed{\mathbb{R} \times \mathbb{R}} \quad (x, y), x \in \mathbb{R}, y \in \mathbb{R}$$

$$(1, 1) \in \mathbb{R} \times \mathbb{R} \\ \in \mathbb{N} \times \mathbb{N}$$



$$\begin{array}{ccccccc}
 & & & & 1 & & \\
 & & & 1 & & 1 & \\
 & & 1 & & 2 & & 1 \\
 & 1 & & 3 & & 3 & & 1 \\
 1 & & 4 & & 6 & & 4 & & 1 \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1
 \end{array}$$

$$\begin{array}{cccc}
 \binom{3}{0} & \binom{3}{1} & \binom{3}{2} & \binom{3}{3} \\
 \binom{4}{0} & \binom{4}{1} & \binom{4}{2} & \binom{4}{3} & \binom{4}{4}
 \end{array}$$

$$(x+y)^3 = (x^2+2xy+y^2)(x+y)$$

$$(x^3+2x^2y+y^2x+x^2y+2xy^2+y^3)$$

$$(x^3+3x^2y+3xy^2+y^3)(x+y)$$

$$\rightarrow \frac{n!}{(n-k)!k!}$$

N

$$\binom{n}{k}$$

$$nCr(n,k)$$

$$\binom{n}{k}$$

$$\binom{n}{0} \rightarrow 0 \quad \binom{n}{n}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

what is the coefficient of x^3y^6 of

$$2^2 \cdot 3 \cdot 5 \quad (2x+6y)^{27}$$

$$\frac{n!}{(n-(n-k))! (n-k)!} = \frac{n!}{k! (n-k)!}$$