Claim: $\sqrt{3}+\sqrt{2}$ is irrational

Lemma: $\sqrt{n}$ is irrational for $n \in \mathbb{Z}^+$

Suppose $\sqrt{n}$ is rational. Then, $\sqrt{n} = \frac{a}{b}$, $a,b \in \mathbb{Z}$, $\gcd(a,b)=1$

$\Rightarrow n = \frac{a^2}{b^2} \Rightarrow nb^2 = a^2 \Rightarrow a = nk$ for some $k \in \mathbb{Z}$

$\Rightarrow nb^2 = n^2k^2 \Rightarrow b^2 = nk^2 \Rightarrow b = nl$ for some $l \in \mathbb{Z}$

$\Rightarrow \frac{a}{b} = \frac{nk}{nl} \Rightarrow$ not fully simplified $\rightarrow$ contradiction $\lightning$

Proof:

Suppose $\sqrt{3}+\sqrt{2}$ is rational. Then,

$\sqrt{3}+\sqrt{2} = \frac{a}{b}$, $a, b \in \mathbb{Z}$, fully simplified

$\Rightarrow (\sqrt{3}+\sqrt{2})^2 = \frac{a^2}{b^2} = 3+2+2\sqrt{6} = 5+2\sqrt{6}$

$\Rightarrow 2\sqrt{6} = \frac{a^2}{b^2} - 5$

$\Rightarrow \sqrt{6} = \frac{a^2-5b^2}{2b^2} \Rightarrow \sqrt{6}$ is rational, a contradiction by the lemma $\blacksquare$.