Linear Regression

Intro to Machine Learning: Beginner Track #3

Feedback form: forms.gle/SrnCkwjSiNR4ox7s5
Attendance code: **Serbia**
Discord: bit.ly/ACMdiscord

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Today's Content

- Motivating Example
- Hypothesis Functions
- Some Useful Math
- Loss Function

A Motivating Example

How well will Joe Bruin do on his midterm?

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Problem: Predicting Your Midterm Performance

- What information might be useful?
  - Time spent studying, Lecture hours attended, etc.
- We call this information about the student: **features**
- The **midterm score** of the student **depends** on these **features.** But what is this relation?
  - Are they positively/negatively correlated?
  - How would you model it?

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Some terms

- **Feature** - some **property** of the object that impacts the **target**

  Eg: For predicting midterms, time spent procrastinating is a feature.

- **Target/Label** - the **true** value of what we are trying to predict.

  Eg: For predicting midterms, the target would be how the student scores.

How do we represent our data?

- Each **row** represents the information for **one student** in our data set
- Each **column** represents one **feature**: Like number of hours spent studying
- The **target** is the list of **midterm scores**. This is what we want to predict. We call this vector **y**

`y :target`

HW Avg	Study Hrs	Lec. Hrs	Midterm %
70%	10	18	85
95%	15	19	90
64%	5	10	60
77%	13	16	76

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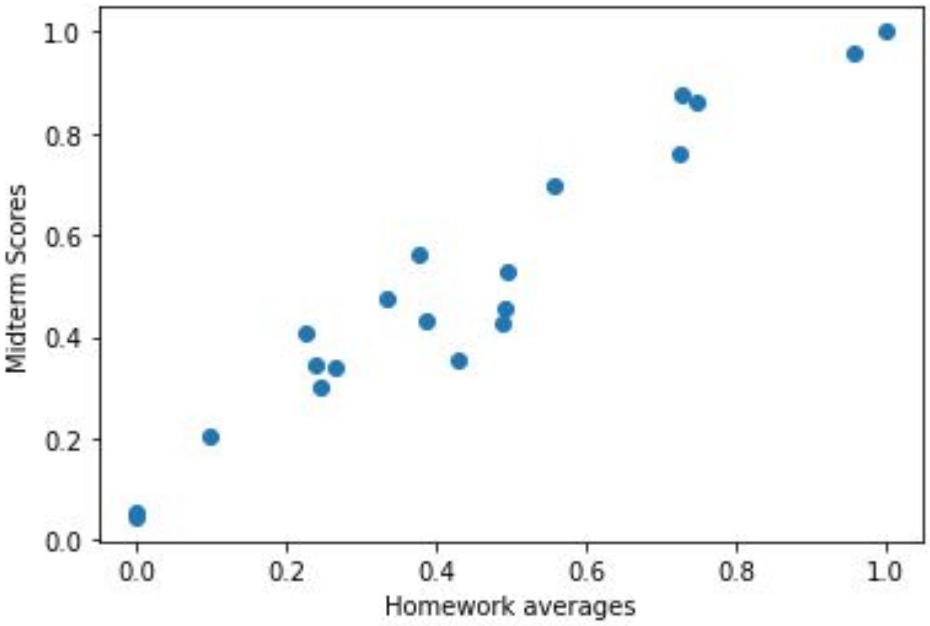
How would you model this relationship?

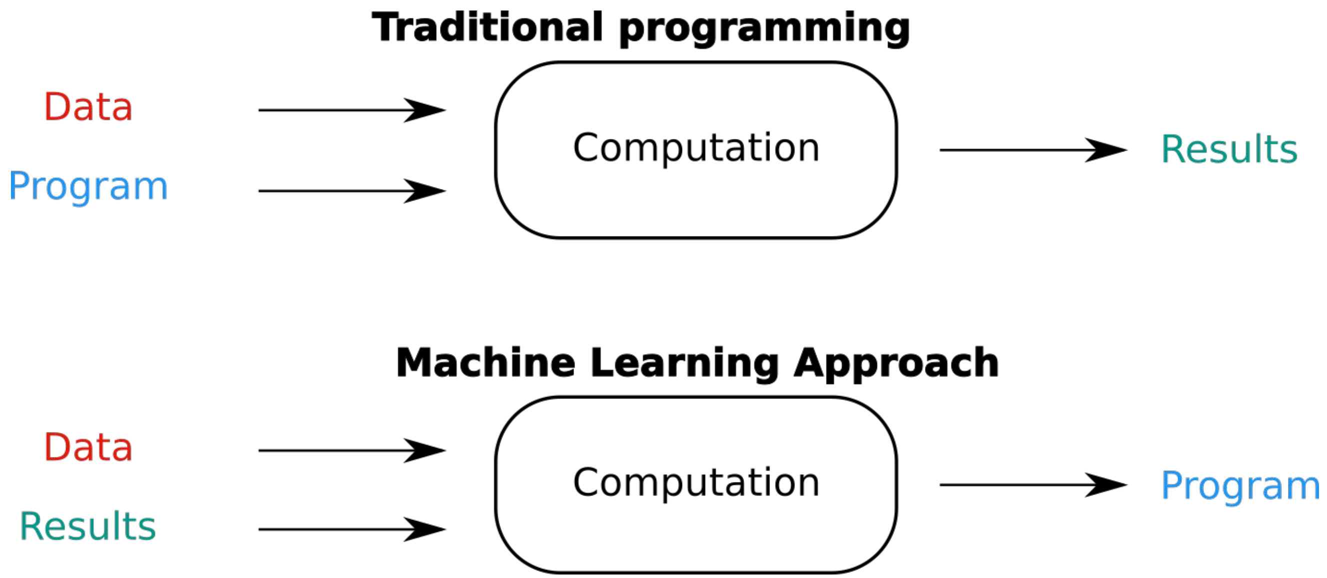
- One way is to use a linear model
- How do we determine it's parameters?



The ML way - Pattern Recognition

Through machine learning, we aim to predict the midterm score of a student by determining the **pattern** between certain features of students and the target: midterm score

The ML way vs The Old Fashioned way



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ML pipeline

Input data → **Preprocessing** → **Training** → **Testing** → **Interpretation**

Input data

Training and testing data
The more data we have, the better

Preprocessing

Scaling data, removing outliers, etc.

Training

Choosing the **function** and representing the data in that form; Training data is used to develop model

Testing

Evaluating the model using testing data

Interpretation

Choosing a new model, introducing more variables, etc.

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Formalizing what we've learned

Hypothesis

- We want to learn what **function** will **output the score**, given some **features as input.**
- This function is called the **hypothesis**. It models the **pattern** we are interested in.
- Let's call our hypothesis **y-hat**

$$\hat{y}(x_1, x_2 \ldots x_n)$$

and the features would be the inputs: x_1, x_2, … , x_n

- Our goal now is to determine y-hat

What model should we use?

- First, what is a model?
- The word *model* is thrown around a lot in ML, and there doesn't seem to be one rigorous definition.
- Think of the model as the machine's interpretation of the problem, how it "models" the situation provided.

What model should we use? (contd.)

- Now, for midterm scores , features like hours spent studying and scores on homework are proportional to the result
- number of overall classes taken may negatively correlate to the score
- For all these features, there seems to be a direct relationship: the feature either directly increases or directly decreases the price.
- A **linear model** might be a good choice: i.e. something like y = mx+b

$$\hat{y}(x) = b + w_1 x_1 + w_2 x_2 + \ldots\ldots w_n x_n$$

- This is our **hypothesis**

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Model

$$\hat{y}(x) = b + w_1 x_1 + w_2 x_2 + \ldots\ldots w_n x_n$$

So our model, **yhat(x)** can also be represented in the following manner :

$$\begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} + b \qquad X \cdot W + b \qquad XW + b$$

Takes the dot product of X and W vectors, then adds b

The parameters

$$\hat{y}(x_1, x_2 \ldots x_n) = b + w_1 x_1 + w_2 x \ldots + w_n x_n$$

An input **X** is an n-dimensional **row vector** for the n features in the example

$$[x_1 \; x_2 \; x_3 \; \ldots \; x_n]$$

The weight **W** is also an n-dimensional **column vector**.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix}$$

The bias **b** is a real number.

$$b$$

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Weights

$$\hat{y}(x) = b + w_1x_1 + w_2x_2 + \ldots\ldots w_nx_n$$

- The **weights** are $\mathbf{w_1, w_2 \ldots. w_n}$ and $\mathbf{b}$.
- They are the **learnable parameters** of our model.
- In the hypothesis above, we can change the function by changing the values of $\mathbf{b}$ and $\mathbf{w_1, w_2, \ldots. w_n}$
- We need to find the *best* possible weights for our model.

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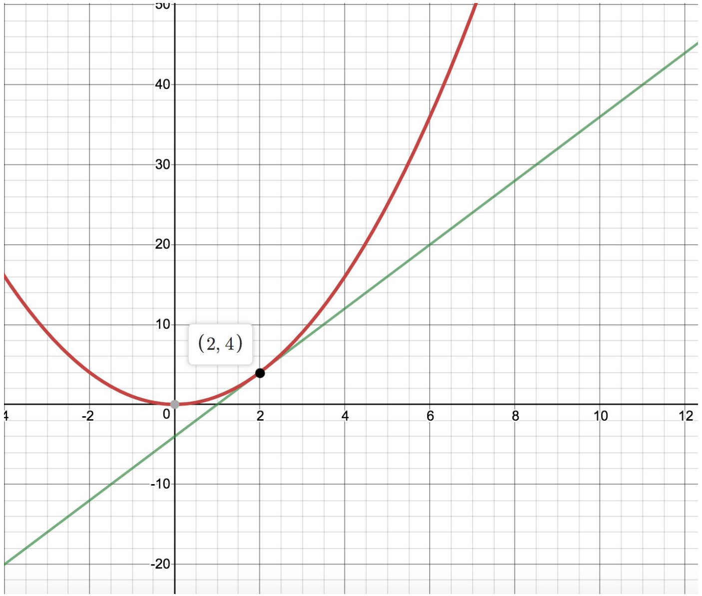
Weights - Quick Question

- Rank these four features on how much they affect midterm scores.
    - Hours spent studying for midterm
    - Number of classes being taken
    - Number of pets student has
    - Hours of sleep the night before

Let’s talk math

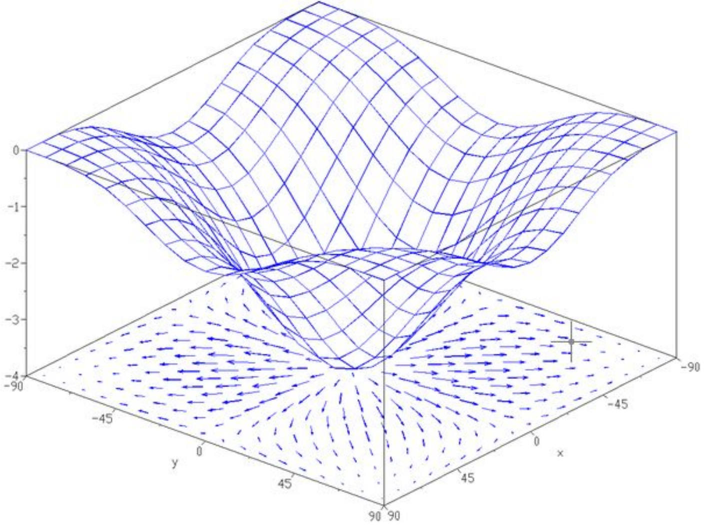
Derivatives



- The derivative of a function is the **rate of change** of the function
- If the derivative is **positive**, the function is **increasing**
- If the derivative is **negative,** the function is **decreasing**

Partial Derivatives

- To take the partial derivative of **f(x,y)** with respect to **x**, we assume **y** to be **constant** and take the derivative as you would for a single variable function
- To take the partial derivative of **f(x_1,x_2,…,x_n)** with respect to some **x_i** we **take every other variable to be constant**, and continue.



$$f(x, y) = 2x + 3y^2$$

$$\frac{\partial f}{\partial x} = 2$$

$$\frac{\partial f}{\partial y} = 6y$$

Calculating a gradient

A gradient of an **n-dimensional function** is an **n-dimensional vector** of the partial derivatives of the function with respect to each variable

$$\nabla f = < \frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz} >$$

$$f(x, y, z) = xsin(y) + 2z^3$$

$$\nabla f(x, y, z) = < sin(y), xcos(y), 6z^2 >$$

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A Quick Quiz

What is the gradient of $f(x,y,z) = x^2 + y^2 + z^2$?

a. $\nabla f = \; < 2x, \; 2y, \; 2z >$

b. $\nabla f = \; 2x + 2y + 2z$

c. $\nabla f = \; 1/3 < x^3, \; y^3, \; z^3 >$

d. $\nabla f = \; < x^2, \; y^2, z^2 >$

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Loss Function: a measure of error

- To update W, we need to first talk about something called the loss/cost function
- It measures the **error** in your predictions compared to the target values.
- There are many choices of function to measure the error. We will go with the **Mean Squared Error (MSE)**

$$L(\hat{y_1}, \hat{y_2}, \dots \hat{y_m}) = \frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y_i})^2$$

y: the actual **target** value

yhat: the output **predicted value**

i: the ith **training sample**

Loss Function as a function of weights and bias

- The loss function depends on your predictions (**yhat**)
- Your predictions depend on the weights and bias of your model
- The loss function can also be thought as a **function** of the **weights** and **bias** of your model!

$$L(\hat{y_1}, \hat{y_2}, \ldots \hat{y_m}) = \frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y_i})^2$$

$$\hat{y}(x) = b + w_1x_1 + w_2x_2 + \ldots\ldots w_nx_n$$

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Loss Function - Quick Poll

- What is the loss function (MSE) measuring?
  - Error of the predictions that your model makes (compared to true value)
  - The inherent error in your dataset (such as when the dataset is not properly cleaned)
  - Weight(s) of your model
  - Your predictions

Learning Weights: Minimize Loss

- Our loss is a function of the **weights** and **bias** of our model
- Our model learns by updating its **weights** and **bias**
- If loss function outputs a small value → our model is **accurate**
- So, we need to find those **weights** for which the **loss is minimized**
- How do we do that?

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Single Variable Gradient Descent

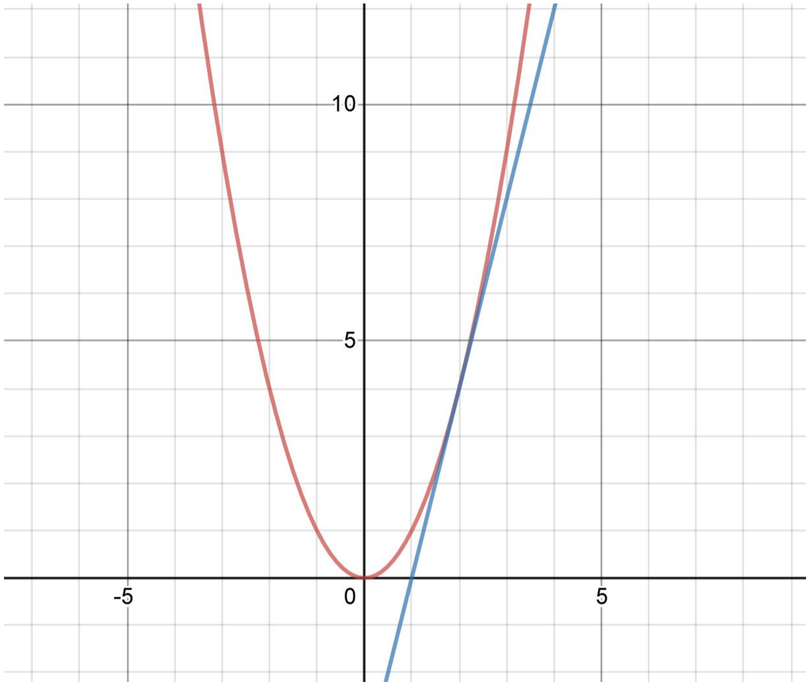
f(x) is a function of one variable: **x**

f′(x), the derivative, indicates whether the function is increasing or decreasing

If **f′(x)** is positive, the function is increasing. So if **x** increases, **f(x)** increases.

We want to decrease **f(x)** so we **decrease** x. i.e. we **subtract** *something* from x

Similarly if **f′(x)** is negative, if we want to decrease f(x) we **increase** x



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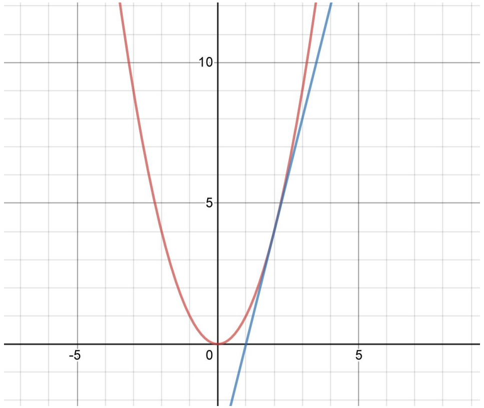
Single Variable Gradient Descent

To summarize: We want to **minimize** f(x), so
if **f'(x)** is **positive**, we want to **subtract** *something* from x
if **f'(x)** is **negative**, we want to **add** *something* to x

How do we do that?
Use **f'(x)** itself!
But careful! We want to do the **opposite** of what f'(x) tells us to

So we can **update** x like this $$x = x - \alpha f'(x)$$

alpha is just a constant we choose to scale f'(x). We call it the **learning rate.**



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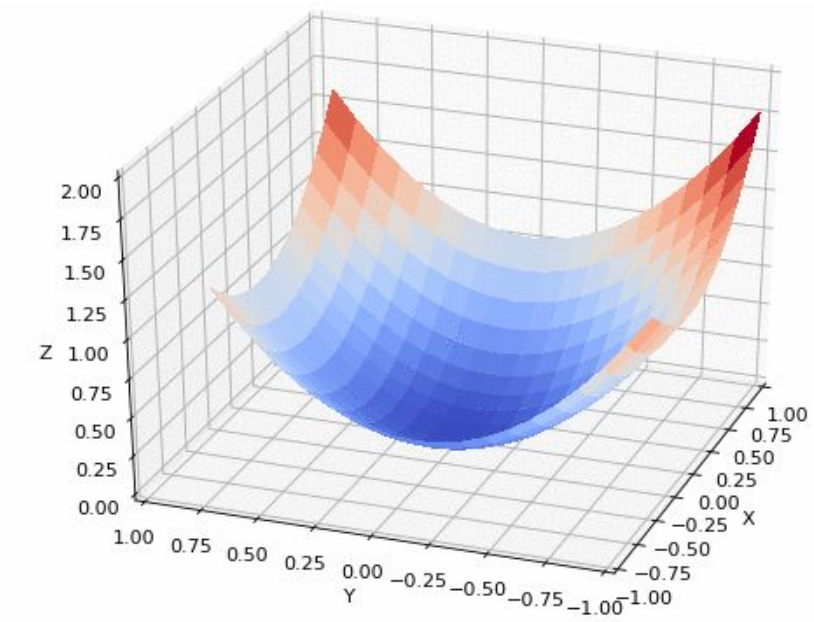
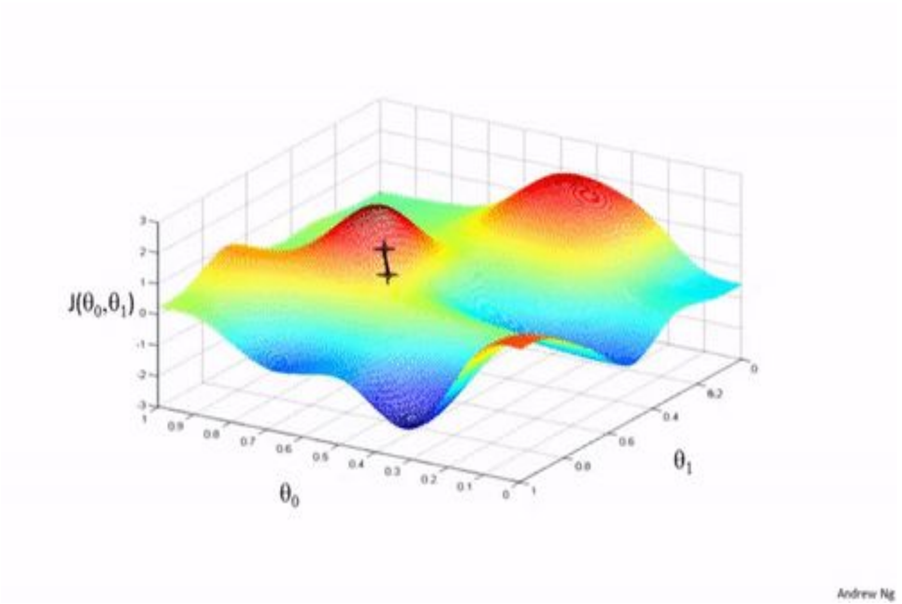
Here's a Cool Gradient Descent Visualizer

https://uclaacm.github.io/gradient-descent-visualiser/

Gradient Descent: How we minimize the value of a function





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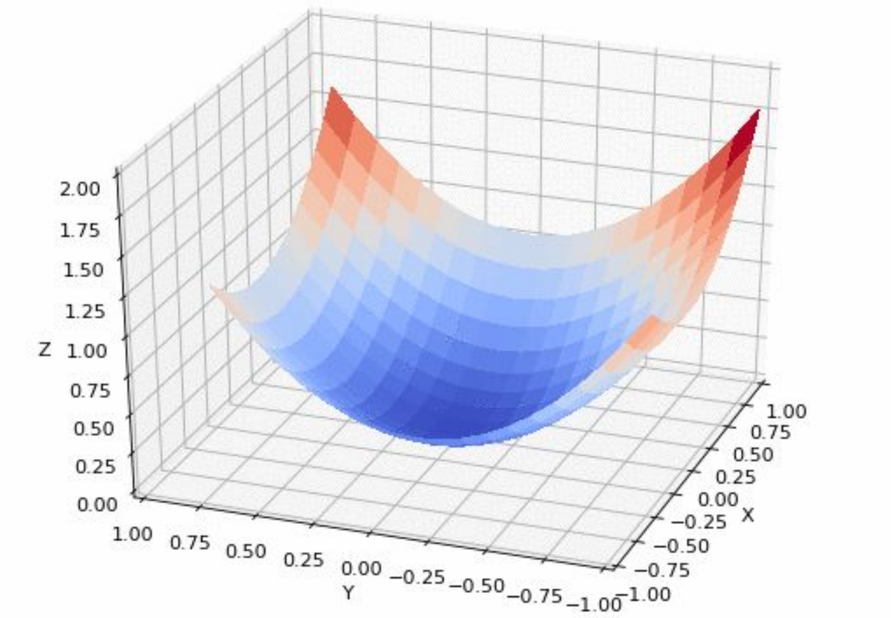
Gradient Descent



$$\vec{x} = [x_1, x_2, \ldots . x_n]$$

$$\nabla f(\vec{x}) = [\frac{\delta f}{\delta x_1}, \frac{\delta f}{\delta x_2} \ldots . . \frac{\delta f}{\delta x_n}]$$

$$\vec{x} = \vec{x} - \alpha \nabla f(\vec{x})$$

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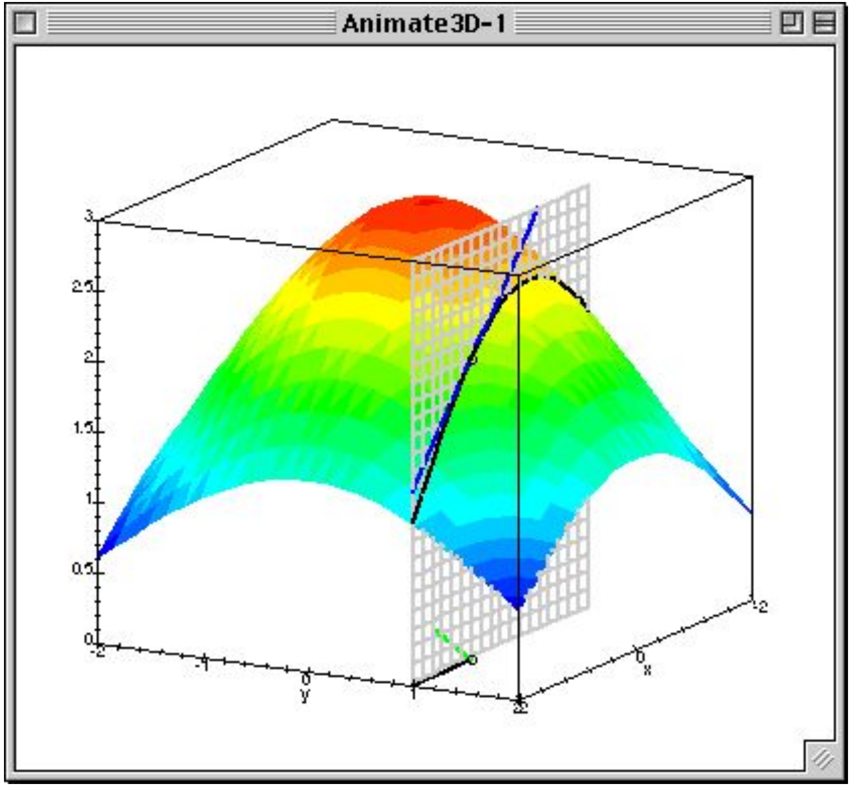
Multivariable Gradient Descent

- The gradient is the direction of **steepest ascent**
- Meaning that if we go in the same direction as the gradient we **increase** the value of the function
- But we want to **decrease** the value of the function
- So we go in the **opposite** direction as the gradient i.e. **gradient descent!**



Multivariable Gradient Descent

x is now a **vector**

$$\vec{x} = [x_1, x_2, \ldots . x_n]$$

The **gradient** is also a **vector**

$$\nabla f(\vec{x}) = [\frac{\delta f}{\delta x_1}, \frac{\delta f}{\delta x_2} \ldots . . \frac{\delta f}{\delta x_n}]$$

So we **update** the x vector using the gradient vector

$$\vec{x} = \vec{x} - \alpha \nabla f(\vec{x})$$

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Minimize loss using gradient descent!

Taking the **gradient** of our MSE loss function

$$\frac{\partial L}{\partial w_j} = \frac{2}{m}\sum_{i=1}^{m}(\hat{y}_i - y_i)x_{ij}$$

$$w_j = w_j - \alpha\frac{\partial L}{\partial w_j}$$

$$\frac{\partial L}{\partial b} = \frac{2}{m}\sum_{i=1}^{m}(\hat{y}_i - y_i)$$

$$b = b - \alpha\frac{\partial L}{\partial b}$$

i refers to the **i**th training sample, **j** refers to the **j**th feature
Here's the full derivation of the gradients of MSE

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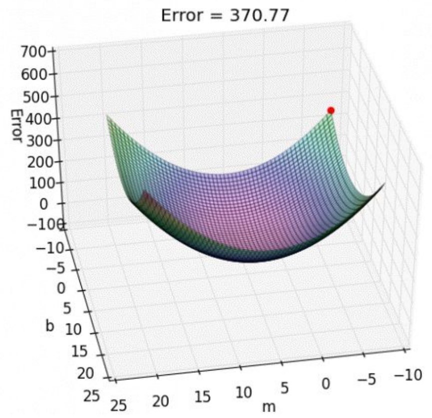
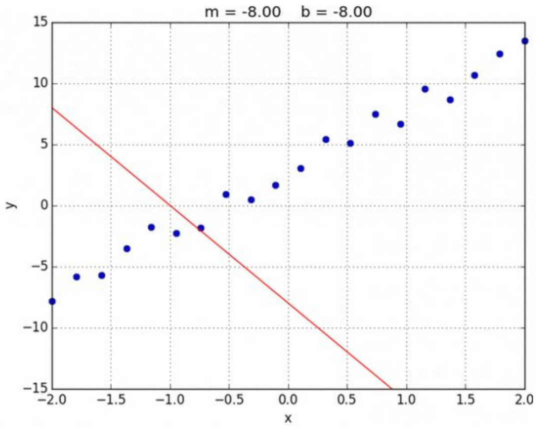
Best Fit

- Minimizing the loss function can be thought of as finding the **best fit “curve”** for your data. The best fit curve is the optimal solution.





- This is linear regression with one feature. We are trying to fit a line with the given data.
- You will implement this from scratch in a project later in the quarter!

After we learn weights: Testing

- To test our model, we first select some of the data points we have (test set)
- We then feed the **input features** of those data points into our model and keep aside the true **y** values
- Our model generates **predictions** using the input features
- We calculate the **loss** between our predictions and the true values
- And that loss tells us how well our model has performed!

Linear Regression Coding Exercise

Code Along at:
https://colab.research.google.com/drive/1i_YZOeKPcw6s4_pOwdp6Jb8NzFD3FfMi?usp=sharing

What we just did: Supervised learning

- In very simple terms, we told our model what the right answer was
- Types of supervised learning
  - Classification: output labels
  - Regression: map input to continuous output
- Classification or regression?
  - Cat vs dog?
  - Number of fish in certain reef?
  - Normal mail or spam?

So there you have it

- What we did today was a form of Supervised Learning
- We'll be concentrating on Supervised Learning in this series.
- The next topic to learn is Logistic Regression, where we'll be classifying objects, instead of predicting values.

Thank you! We'll see you next week!

Please fill out our feedback form: [forms.gle/SrnCkwjSiNR4ox7s5]

Next week: Logistic Regression

How does a computer recognise cats and dogs?

Today's event code: **Serbia**

FB group: facebook.com/groups/uclaacmai

Github: github.com/uclaacmai/beginner-track-spring-2021

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