Glossary: Theorems and Definitions From Higher Theory and Application of Differential Equations

Dylan Wang (z5422214)

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Definition. First-order ODE.

A differential equation of the form $y'(t) = f(t, y(t))$.

*It is called a 'first-order' ODE because the degree of the derivative is only 1.

Definition. Linear first-order ODE.

An ODE of the form

$$a(t)y'(t) + b(t)y(t) = f(t)$$

with $a(t) \neq 0$ on some interval $I \subseteq \mathrm{R}$.

*It is 'linear' because the 'variables' (i.e. y, y', y'', etc.) are linear.

Definition. Separable first-order ODE.

An ODE of the form

$$y'(t) = f(t)g(y).$$

The solution is (implicitly) found by writing

$$\int \frac{1}{g(y)} \, \mathrm{d}y = \int f(t) \, \mathrm{d}t.$$

*These equations are called separable differential equations because we can separate everything involving t from everything involving y.

Definition. Integrating factor method.

For a linear first-order ODE, we generally cannot separate the variables. However, suppose there is a function $\mu(t)$, called an **integrating factor** such that

$$[\mu y](t)' = \mu(t)(y' + a(t)y) = \mu(t)f(t),$$

for if this happens, then the general solution of the ODE should be

$$y(t) = \frac{1}{\mu(t)} \int \mu(t)f(t) \, \mathrm{d}t + \frac{C}{\mu(t)}.$$

We find that $\mu(t) = e^{\int a(t) \, \mathrm{d}t}$.