Quantum Computing

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Introduction

Quantum computing is a method of computing that uses the quantum states of subatomic particles to store information. Quantum computing is used for solving complex computational problems faster than normal computers. The way quantum computing works is that it uses qubits rather than the conventional bits used in normal computers. Quantum computers are used to solve problems that are impossible to solve on regular computers.

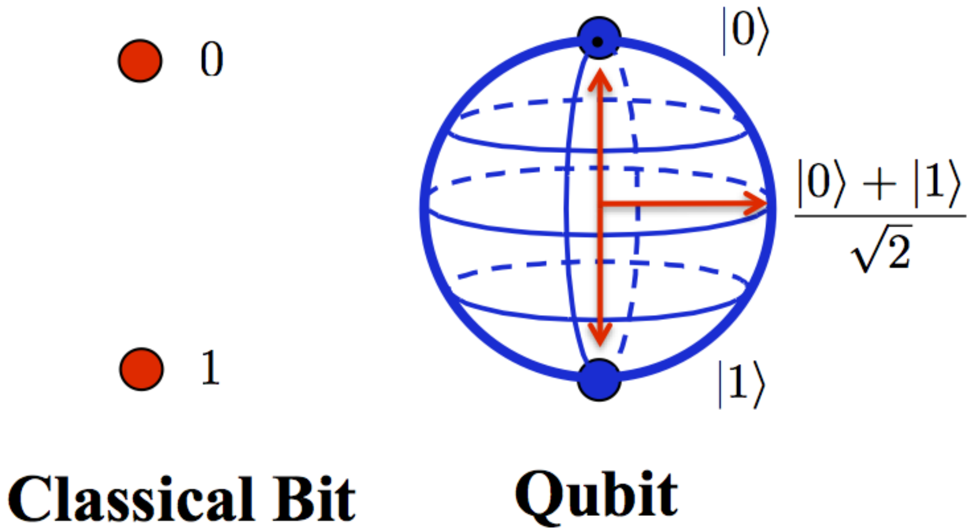


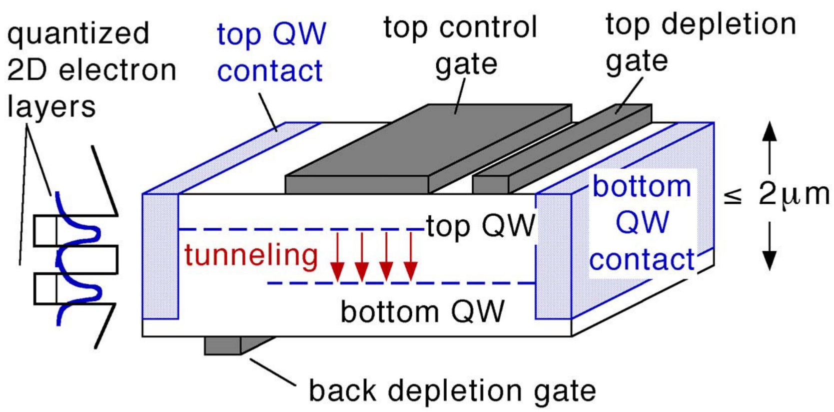
Figure 1
Comparison between the classical bits from normal computer and quantum bits (qubits) from a quantum computer. A normal bit can be represented as either 1 or 0 whereas a qubit can be both 1 and zero using linear superposition.

Key Concepts

The basis for quantum computing is to break the limiting barriers that classical computers today contain by harnessing quantum physics and subatomic particles into storing information into qubits that efficiently produce better algorithms that classical computers struggle with. For example, "a 64-qubit quantum computer processes 36 billion bytes of information in each computation step while a 64-bit computer processes 8 bytes in each step" (Foy). The size in processing power is trillion of times more efficient and quicker than classical computers and is used for the most complex of computations of algorithms because classical computers would not be able to handle even the easiest quantum based algorithms. Modern technology today does revolve using classical computing bits of only 0 and 1 but there are various calculations too difficult to process even for the most advanced computing designs on the planet and that is the stepping stone for quantum computers in place.

Transistors

The ideal proposition on superposition and probability elevates the terms of real life physics into countless different variables of information that qubit and quantum logic gates store and process information through where instead of relying on two outputs for bits, different values of numbers stored in qubits can unleash unlimited potential in speed and processing algorithms in seconds. Quantum tunneling through transistors allows for higher speeds due to instantaneous switching while normal transistors have a sparking delay. Also, quantum transistors are very tiny and compact allowing for easier circuits in quantum computers to allow more space to interfere with, and more of these transistors to be added into.



Instead of classical physics where the electrons follow the rules of direct current and redefining that current to flow into another path, quantum transistors here follow quantum mechanics and their principles using quantum dots to direct that current out from a switch and into another junction in which can all interlock as a single qubit. The precision control on the number of quantum dots is interlocked through the current's control as shown above through the use of quantum tunneling of those quantum dots. Quantum computers use these transistors in their circuits to allow qubits to store information and follow their functionalities.

Qubits

Quantum computers use a qubit (quantum bit) and is the basic unit of quantum information. Similarly to how a normal computer computes using a binary bit with a value of either 1 or 0, a quantum computer computes using qubits with a value than can be 1 and 0 simultaneously by coherent superposition. Superposition is the ability of a quantum system to be in multiple states at the same time.

Notation

The way qubits are represented is by a linear superposition of two states using the Dirac notation. The Dirac notation uses the ket symbol (|>) to represent the quantum state vector. For example, the qubit states would be represented as $|\psi\rangle = |0\rangle + |1\rangle$. When a qubit is in a superposition of states, their linear combination would be $\alpha|0\rangle + \beta|1\rangle$ where α and β are complex numbers. Qubits can also be combined to create a quantum register, which is just a system comprised of multiple qubits and these registers can manipulate each individual qubit within the register.

Superposition

Superposition gives quantum computers the ability to process information much faster than a normal system because the amount of information that a qubit can hold is increased exponentially instead of linearly like in classical computers. In quantum mechanics, particles such as atoms and electrons are represented as waves but the waves do not represent the movement of the particles but their probability of having a certain value of position or energy. The shape of the wave changes based on the energy level of the electrons.

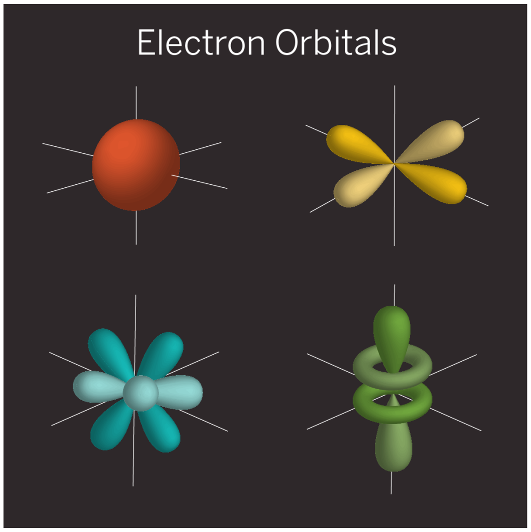
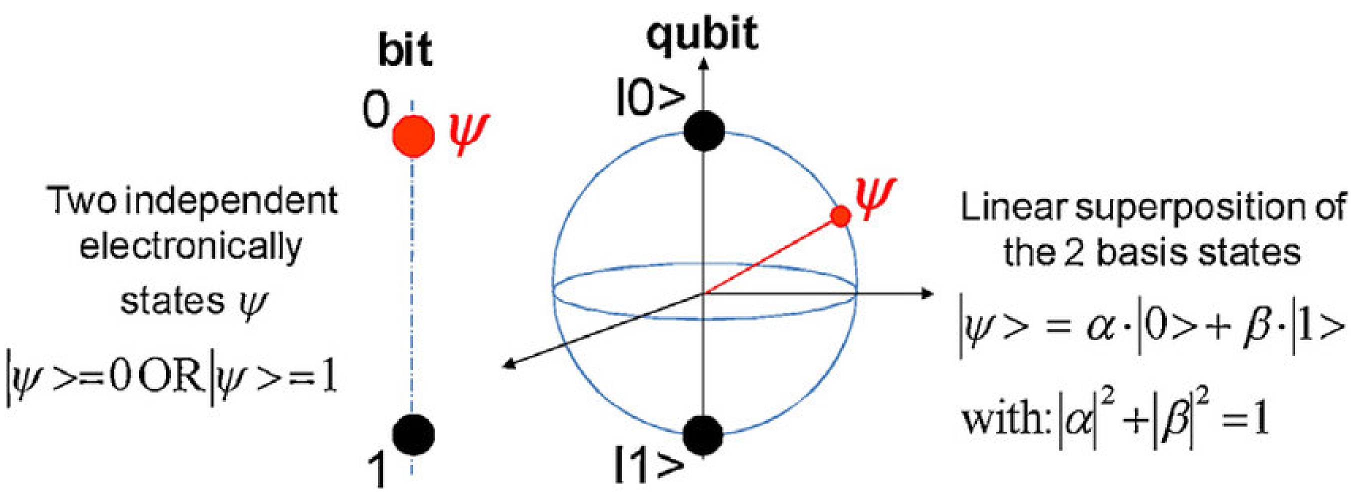


Figure 2
An example of an Electron Orbital, an electron orbital is a function that describes the location and wave-like behavior of the electron.

Bloch Sphere

Bloch Sphere is a graphical representation of the quantum state of a two-level quantum system like qubits which uses three dimensions of its locations in vectors, and the north and south poles representing ket 0 and ket 1. However, the equation of the sphere represents all of the possible superpositions of the two states and their values and are crucial in labeling out qubits. Any quantum state can be drawn by a point on the sphere's surface and the point corresponds to the direction of the quantum state's state vector in the three-dimensional space and the state vector is a complex number with two components with one for each qubit's possible state. This diagram is an easier way to illustrate quantum gates built on quantum circuits and their rotations on their values are easily depicted on the Bloch Sphere by rotating the qubit's state vector on the sphere.

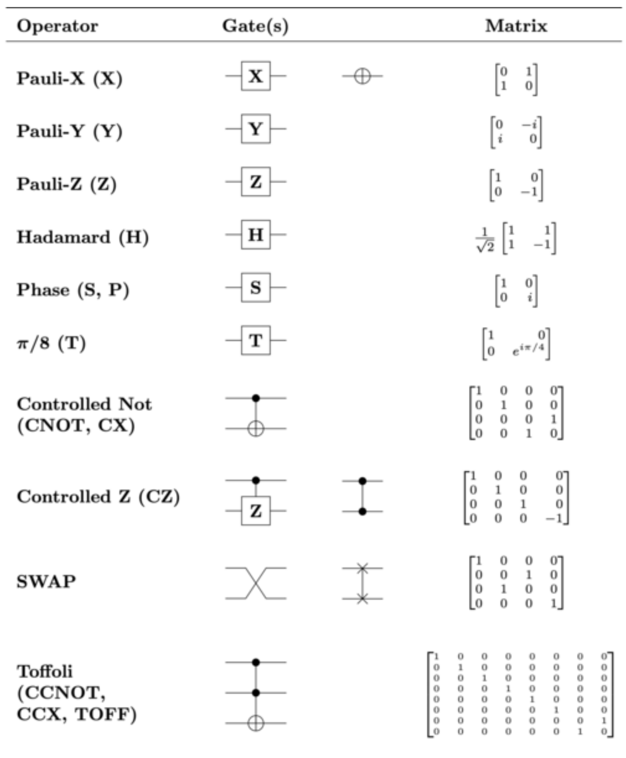


Bloch Sphere is a visual diagram of the state of one qubit with a direction of an x,y, or z axis using 0 or 1 (ket 0 or ket 1), or superposition value for these appending states. The Bloch sphere is a unit sphere with radius 1, where each point on the sphere represents a different quantum state of the qubit. The north and south poles of the sphere represent the two basis states of the qubit, conventionally labeled as $|0\rangle$ and $|1\rangle$ and the rotational depiction of the point is represented through the angle theta of Rx, Ry, Rz.

Formulas:
*Rx(θ) = cos(θ/2) * I - i sin(θ/2) * X,*
*Ry(θ) = cos(θ/2) * I - i sin(θ/2) * Y,*
*Rz(θ) = cos(θ/2) * I - i sin(θ/2) * Z,*

Quantum Logic Gates

Quantum logic gates perform a specific set of logic operations similar to logic gates used for classical computers except their functions are dependent on the qubits initialized and set. Similarly to classical logic gates, quantum logic gates are used as building blocks for quantum circuits. However, quantum gates are reversible unlike classical logic gates. Being reversible allows the operations that are performed in a way that can be reversed, meaning the output can be used to determine the input without any loss of information in a computation. Quantum logic gates are classified as unitary operators and use unitary matrices as shown in figure 3.



Operator	Gate(s)		Matrix
Pauli-X (X)	$\boxed{X}$	$\oplus$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	$\boxed{Y}$		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	$\boxed{Z}$		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	$\boxed{H}$		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	$\boxed{S}$		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	$\boxed{T}$		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	$\boxed{Z}$		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Figure 3
A schematic of the most commonly used quantum logic gates along with their matrix values that build the components for quantum computers. The Pauli XYZ logic gates are commonly used for rotational purposes and the Hadamard Gate is commonly used for huge quantum algorithms.

Quantum Density Matrices

Quantum density matrices describe the states of the quantum system and there are various different density matrices represented by both real and imaginary numbers for their precise location and such. They play a fundamental role in describing qubits and their states all around pertaining to quantum logic gates and operations for quantum systems and are vital to the building blocks for quantum computing. The wave function is used to describe pure states and the density matrix is a generalization of that wave function with the purpose of describing mixed states, when a quantum system is not in a pure state but in a statistical combination of multiple pure states.

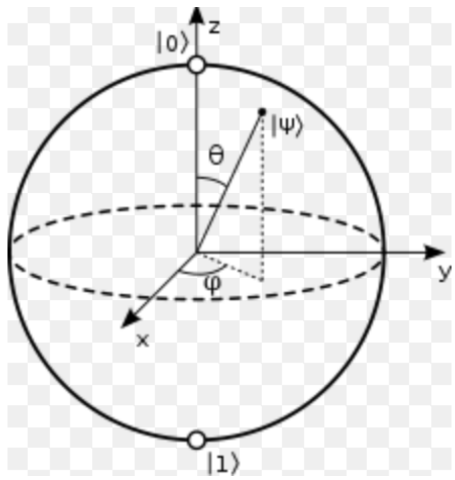


Figure 5 is a quantum density matrix diagram.

$\rho = |\Psi\rangle\langle\Psi|$ is the formula for density matrices and if the quantum system is not known exactly, then the matrix is a ***mixed state***, meaning it is a combination of ***pure states*** **($\langle\Psi|\Psi\rangle = 1$)**. Pure states often are used as the initial states in quantum computation because they are easier to analyze and manipulate at first and quantum algorithms generally rely on manipulating entangled pure states which describe correlations between two or more quantum systems.

Quantum Entanglement with Qubits

Quantum entanglement is the proposition of two or more particles in the quantum realm are correlated as the same object such as photons of light despite how large the distance, the quantum paradox for quantum entanglement is that the probability of a particle being an absolute value only one part of the information is detected. Using probability, if one qubit is 1 and is confirmed to be 1, then it is absolutely certain that the other qubit entangled is 0 and vice versa if the original qubit is 0, then the other qubit is one using probability.

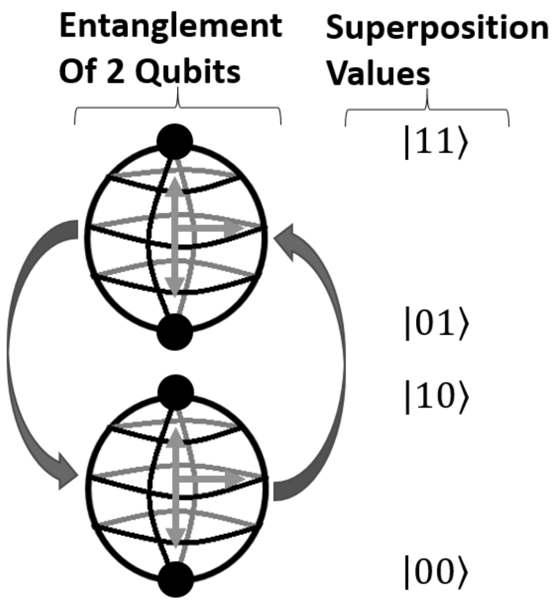


Figure 6 shows an example of two qubits entangling with one another along with their values.

Quantum Teleportation

Quantum entanglement with qubits is important because it allows for the creation of complex quantum states that cannot be described as a simple combination for other qubits meaning that entanglement can be used for quantum algorithms that solve certain problems much faster than classical algorithms. Quantum teleportation is a technique for transmitting quantum information between different qubits without physically transmitting the qubit itself to the other so a basis of interlocking positions without directly touching or moving the quantum particles is the basis of quantum entanglement as a whole and is a tool for qubits and quantum computers.

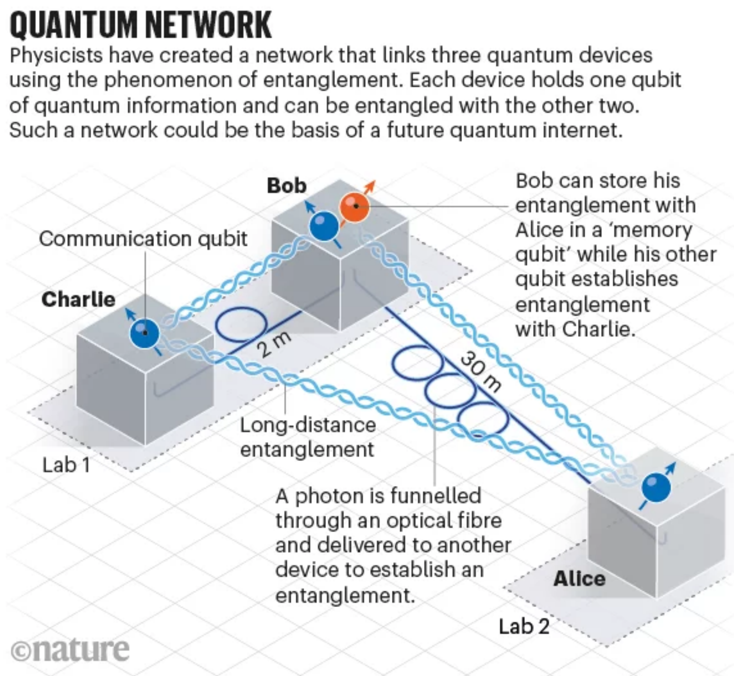


Figure 7
Example of quantum entanglement with information over different spans of distance.

Qubit Entanglement

Qubits can also be entangled with one another and this information actually is used to speed up quantum algorithms in terms of processing and efficiency that normal computers do and it is due to the fact that quantum entanglement exists for these qubits of information. Bits in classical computers only output values of 0 or 1 but are never entangled much as qubits because of the laws of quantum mechanics.

Benefits of Qubit Entanglement

Entangled qubits are essential in quantum computers because of how more efficient these computing systems are from this phenomenon in which a regular computer calculates linearly, quantum computers use quantum entanglement to calculate algorithms exponentially due to the overlapping of information that they usually pertain to at times.

Figure 8
Real Life Quantum Computer that has to be kept at a certain temperature to reduce environmental noise impacts.

Qubit entanglement is very sensitive so the conditions for a quantum computer must be met to limit and reduce noise as much as possible especially qubits entangling with one another because they must only entangle with each other or the whole system becomes corrupted. Cold to freezing conditions in a sealed chamber is the current stabilization for quantum computers with higher processing power. Quantum computers have the potential to keep private data safe from hackers despite where it is stored, processed, and run through.

Material Calculations

A quantum computer mimics the computing style of nature, allowing the process to understand the complex basis of micro-material and even the largest material in the universe to depict size and volumes of enormous calculations of objects in space.

Machine Learning/ Artificial Intelligence

A.I. is cognitive data that holds unlimited potential to algorithms and in recognizing thousands of algorithms to process at high speed. Quantum computing accelerates machine learning through these quantum based advantages in their qubits because of speed and higher ends of memory space to store all of these pieces of information.

Handling Enormous Data and Calculations

Quantum computing can ultimately root out the biggest of calculations at an incredible speed and locate connections to all of its data that has a great enough impact across many industries.

Simulation Tutorials

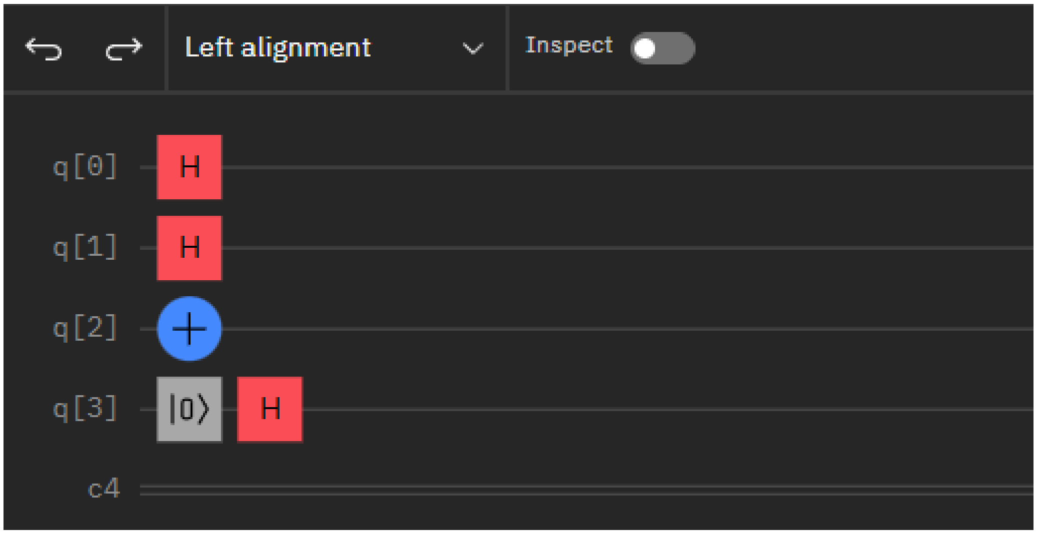
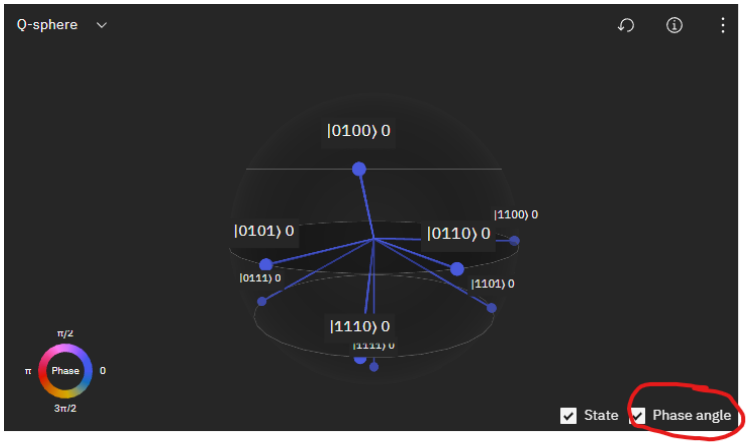


Figure 9
Set the qubits to connect using the different quantum gates to see how the output values are affected.

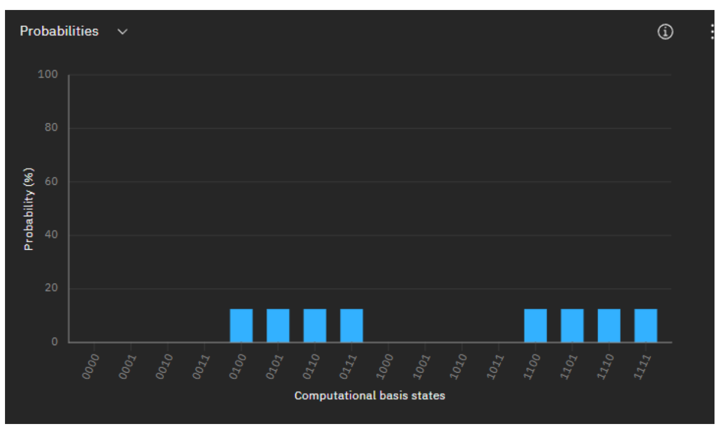
Setting Quantum Gates

There is info on the gates to set for the qubits assigned in the Q-sphere, H gate applies rotates the states of |0> and |1> to |+> and |-> respectively; ket 1 and 0 are similar to classical 0 and 1 states for classical bits but all these probabilities complicate it along with these gates. Qubit operations set aside the state transformation for each qubit to set a probable value like setting a bit in classical computing, quantum computing bits are set a specific initial state and gate operation and plotted on the probability of that value's superposition (weighted sum of two or more quantum states where n qubits exist in a superposition of 2^n states). Right clicking the gates will display all of the gate operations for the basis of quantum logic gates that are similar fundamentals such as AND, NOT, OR, XOR classical gates for classical computer bits.

IBM Quantum Simulation

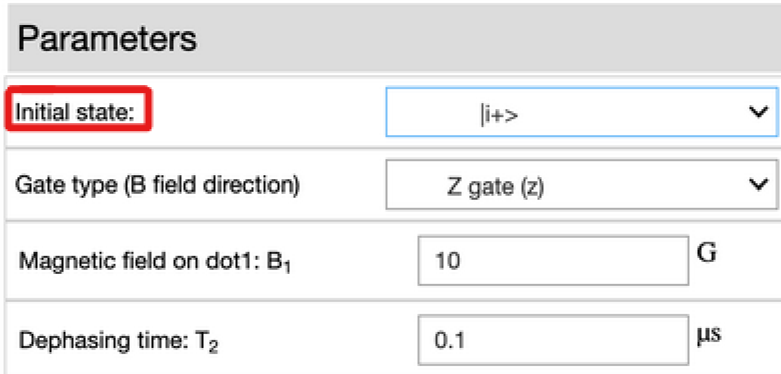


Setting the phase angle on will display the complex number that describes the relative phase between the two quantum states EACH QUBIT from ket 0 and ket (|0> & |1>). Quantum algorithms usually rely on changing this phase angle for quantum computing to be efficient or noticeable for a change. Setting the phase angle on will display the complex number that describes the relative phase between the two quantum states EACH QUBIT from ket 0 and ket (|0> & |1>). Quantum algorithms usually rely on changing this phase angle for quantum computing to be efficient or noticeable for a change.

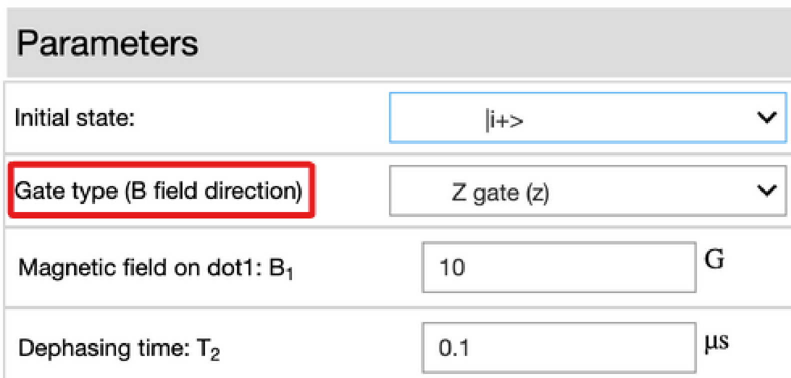


The probabilities graph shows the ranging outputs for 8 qubits as shown since the qubits range from q[3:0] and the probabilities for each value qubit is important to simulate because neither value will ever be absolutely determined until each selected value is viewed by the user one by one, increasing the chances of that accuracy. The value can be a vector of 0111 or 1101 for any of the qubits and this probability graph showcases the likelihood of that potential.

Nanohub Spin Coupled Quantum Dots Simulation



Initial state refers to the state at the beginning of the computation either |0⟩ or state |1⟩ or it can be set to a superposition calculation such as $(|0\rangle + |1\rangle)/\sqrt{2}$ or the equal superposition state where the Qubit can collapse into one of those values at an instance of time, and it is the probability that determines it.



Logic gate operating on one or more qubits performs a quantum operation on the qubits:

- Hadamard Gate: puts the value of |0> or |1> of the qubit in a superposition state
- The Pauli Gates (X, Y, and Z) gate performs a rotation shift on the qubit around the respective axis
- S Gate combines the Hadamard and Z gate for a quarter-phase shift
- The S gate performs a $\pi/2$ phase shift around the Bloch sphere
- The T Gate in a state of |0> leaves the qubit unchanged but a state of |1> rotates the qubit about $\pi/4$
  Also, if combined with the H and CNOT gate, the classical AND logic gate can be made with these gates together

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