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APPLIED NUMERICAL METHODS II: HOMEWORK 1

Q1) See other document [MATH-428.Hw-1-sub.pdf]

Q2) See other document [MATH-428.Hw-1-sub.pdf]

The derivative of f(x)=ln(x2+1) at x=1.3 was approximated using forward and central difference formulas with step sizes h=0.01and h=0.001. The exact derivative was computed analytically and used to evaluate the absolute error. The results show that the central difference method produces significantly smaller errors than the forward difference method, and that decreasing h reduces the error for both methods, consistent with their theoretical orders of accuracy.

```
(SCRIPT)[hw01_p02.py]
```

```
(OUTPUT)[python3 scripts/hw01_p02.py]
```

```
x = 1.300000
Exact derivative = 0.966543

h = 0.01
  Forward Difference  = 0.965583
  Forward Error       = 9.593253e-04
  Central Difference  = 0.966537
  Central Error       = 5.832992e-06

h = 0.001
  Forward Difference  = 0.966447
  Forward Error       = 9.541350e-05
  Central Difference  = 0.966543
  Central Error       = 5.832671e-08
```

Q3) See other document [MATH-428.Hw-1-sub.pdf]

The initial value problem,

(x^3)(y') + 20(x^2)(y) = x, y(2)=0, x∈[2,10]

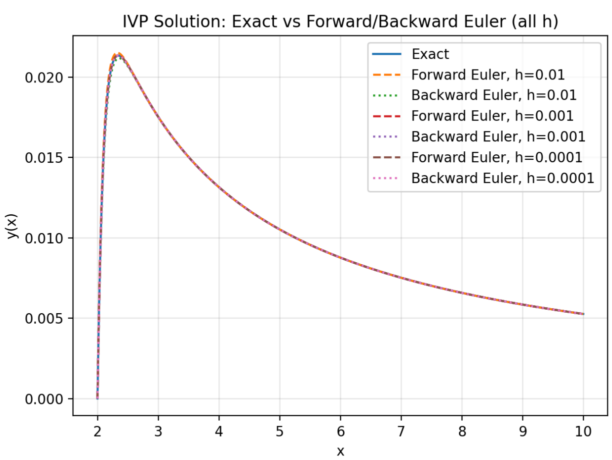
, was solved numerically using Forward Euler and Backward Euler methods with step sizes h=0.01, 0.001, and 0.0001. The numerical solutions were compared with the given exact solution. Absolute errors were computed point-wise to study convergence behavior and to compare the two methods.

(SCRIPT)[hw01_p03.py]

(OUTPUT)[python3 scripts/hw01_p03.py]

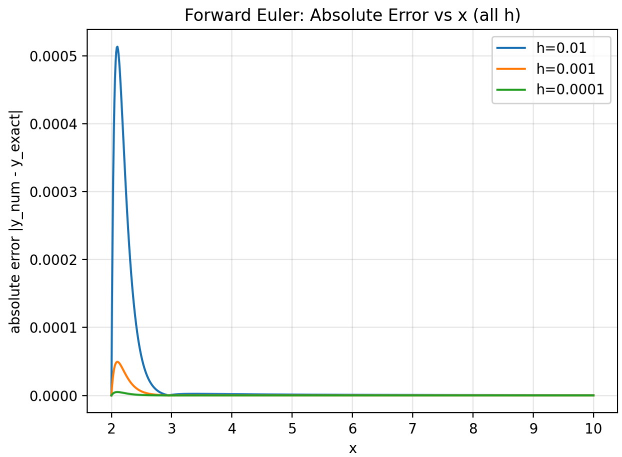
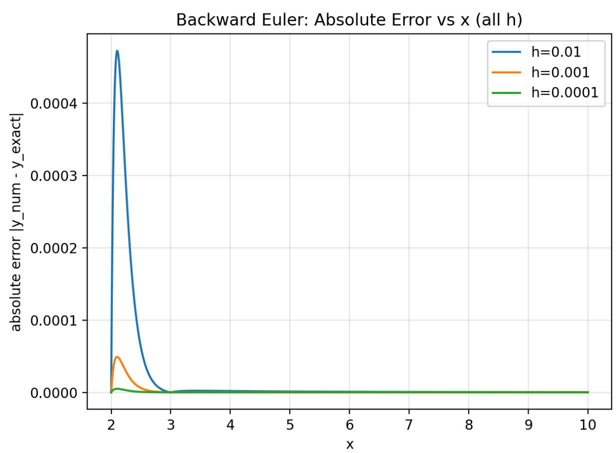
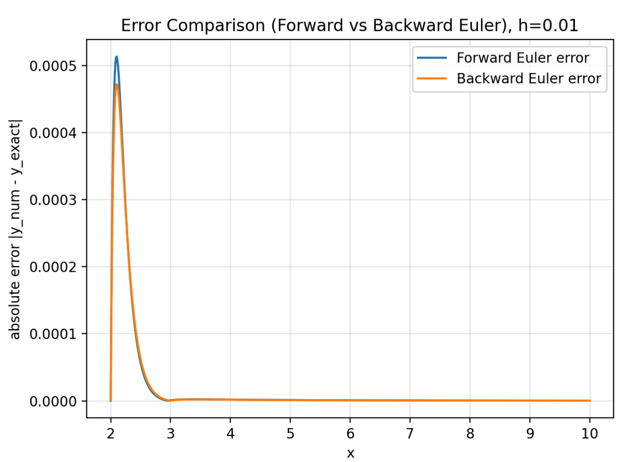
```
IVP on x ∈ [2.000000, 10.000000], y(2.000000) = 0.000000
Exact solution: y(x) = 1/(19x) - 524288/(19x^20)

h = 0.01
  x_end = 10.000000 (target 10.000000)
  y_exact(x_end) = 0.005263
  Forward Euler:  y(x_end) = 0.005263,   max|error| = 5.136893e-04
  Backward Euler: y(x_end) = 0.005263,   max|error| = 4.724398e-04

h = 0.001
  x_end = 10.000000 (target 10.000000)
  y_exact(x_end) = 0.005263
  Forward Euler:  y(x_end) = 0.005263,   max|error| = 4.941078e-05
  Backward Euler: y(x_end) = 0.005263,   max|error| = 4.899855e-05

h = 0.0001
  x_end = 10.000000 (target 10.000000)
  y_exact(x_end) = 0.005263
  Forward Euler:  y(x_end) = 0.005263,   max|error| = 4.922424e-06
  Backward Euler: y(x_end) = 0.005263,   max|error| = 4.918301e-06
```

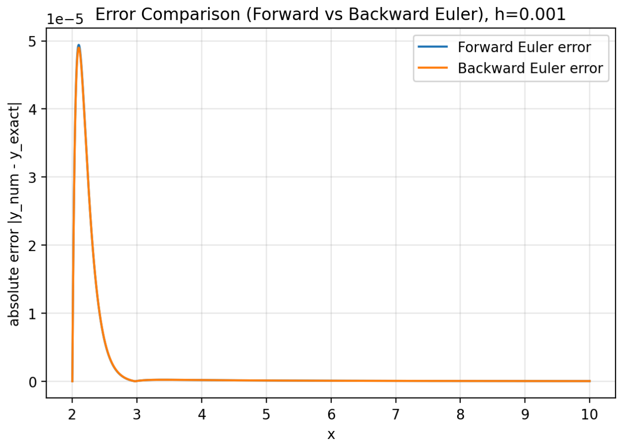
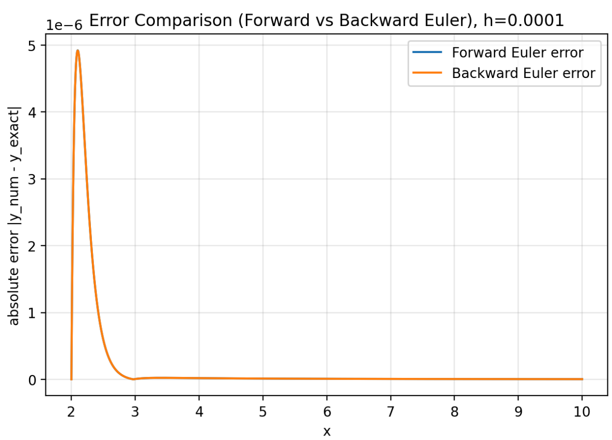
Figures:



Exact vs Forward/Backward Euler (all h)

Exact solution and numerical solutions computed using Forward Euler and Backward Euler for h=0.01, 0.001, and 0.0001. As the step size decreases, both numerical methods converge to the exact solution.

 **Forward Euler: absolute error vs x (all h)**	Absolute error of the Forward Euler method as a function of x for three step sizes. The error decreases as h becomes smaller, indicating first-order convergence.
 **Backward Euler: absolute error vs x (all h)**	Absolute error of the Backward Euler method as a function of x for three step sizes. The magnitude of the error decreases with smaller h, and the method remains stable over the entire interval.
 	Comparison of absolute errors for Forward Euler and Backward Euler with h=0.01. Both methods exhibit similar error behavior, with Backward Euler producing slightly smaller errors near the initial point.

Error comparison **(Forward vs Backward Euler), h=0.01**	
 **Error comparison** **(Forward vs Backward Euler), h=0.001**	Comparison of absolute errors for Forward Euler and Backward Euler with h=0.001. The errors for both methods decrease by approximately one order of magnitude compared to h=0.01.
 **Error comparison** **(Forward vs Backward Euler), h=0.0001**	Comparison of absolute errors for Forward Euler and Backward Euler with h=0.0001. Both methods show further reduction in error, confirming first-order convergence.

Overall, both Forward Euler and Backward Euler methods converge as the step size decreases. The Backward Euler method produces slightly smaller errors and exhibits greater stability near the initial point, consistent with its implicit formulation and class discussion.

Q4) [Extra Credit]