3450:428/528 Applied Numerical Methods II, HW1, Due Jan ??, 2026

Instructions:

- **Show your work**, in a reasonably neat and coherent way. All answers must be justified by valid mathematical reasoning.
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written or typed. Work scattered all over the page without a clear ordering will receive from little to no credit.
- You must also **submit all your executable Matlab codes** along with your write-up, when a question asks for programming. Please comment codes for detailed explanations.
- To show your results, you have to summarize about what you do and what you find, and may also discuss the reason about your observation.
- Your figures should include at least a caption, labels of axis, and legends of the plots. To compare different results, different colors or styles should be used. You can use Matlab's publish functionality to store your results.
- Code for each problem should be in separate script.

Q1) We know function f has 5 bounded derivatives in a neighborhood of point x_0 that includes $[x_0 - 2h, x_0 + 2h]$, and we are given

$$g = \frac{1}{h^2}(f(x_0 - h) - 2f(x_0) + f(x_0 + h)), \ \hat{g} = \frac{1}{4h^2}(f(x_0 - 2h) - 2f(x_0) + f(x_0 + 2h)).$$

i) Explain that the expression

$$\hat{c} = \frac{g - \hat{g}}{3}$$

is an estimation for the expression of the 2nd derivative of f subtract g at x_0, i.e., $f''(x_0) - g$; and ii) also explain that the g is an approximation to $f''(x_0)$ of error order 2. iii) For graduate students, show the error term in $(f''(x_0) - g) - \hat{c} = O(h^3))$

Q2) Write a MATLAB code to approximate the derivative of $f(x) = \ln(x^2 + 1)$ (natural logarithmic function use 'log' in MATLAB) at $x = 1.3$ using central difference and forward difference with $h = 0.01$ and $h = 0.001$. Print the derivative approximations and errors with at least 6 digits after the decimal.

Q3) Solve the initial value problem $x^3y' + 20x^2y = x$ for $x \in [2, 10]$ with $y(2) = 0$, using forward Euler and backward Euler method. Make 3 runs for each method, with $h = .01$, $h = .001$ and $h = .0001$. Plot the numerical results and the exact solution in a single figure for each problem, and plot the errors comparison for i) each method with different h ii) different method with same h. Please explain your finding and relate the conclusions with class discussions. Note the analytical solution is $y(x) = 1/(19x) - 524288/(19x^{20})$

Q4) Extra credit (+10 or make 100) Consider the boundary value problem $y''(x)+5y'(x)-(2+x)y(x)=e^x$ on $x\in(0,2)$ with boundary conditions $3y(0)+y'(0)=5$ and $y'(2)=7$. This is a theoretical question, no coding needed. Given the points $x_i, i=0,1,\ldots,n$, with $x_0=0$ and $x_n=2$. We assume the numerical approximation of $y(x_i)$ are denoted by ϕ_i

(a) Discretize the differential equation at a generic interior point x_i using the central difference formulas for both first and second derivative (i.e., for $y'(x_i)$ and $y''(x_i)$ we use central difference). The discretized equation should be about ϕ_i, ϕ_{i+1} and ϕ_{i-1}.

(b) Discretize the left boundary condition $3y(0)+y'(0)=5$ using forward difference formula on the standard grid (x_0 and x_1).

(c) Discretize the right boundary condition $y'(2)=7$ using backward difference on the standard grid.

To simplify formula, you can assume $x_{i+1}-x_i=h$.