Problem 12

Given the grouped frequencies (nm):

Class	[3.0, 3.5)	[3.5, 4.0)	[4.0, 4.5)	[4.5, 5.0)	[5.0, 5.5)	[5.5, 6.0)	[6.0, 6.5)	[6.5, 7.0)	[7.0, 7.5)	[7.5, 8.0)
f	5	15	27	34	22	14	7	2	4	1

$$n = \sum f = 5 + 15 + 27 + 34 + 22 + 14 + 7 + 2 + 4 + 1 = 131.$$

(a) Proportion < 5:

$$f_{<5} = 5 + 15 + 27 + 34 = 81, \qquad \hat{p}(<5) = \frac{81}{131} \approx 0.6183.$$

(b) Proportion ≥ 6:

$$f_{\geq 6} = 7 + 2 + 4 + 1 = 14, \qquad \hat{p}(\geq 6) = \frac{14}{131} \approx 0.1069.$$

(c) Relative-frequency histogram (heights $r_i = f_i/n$):

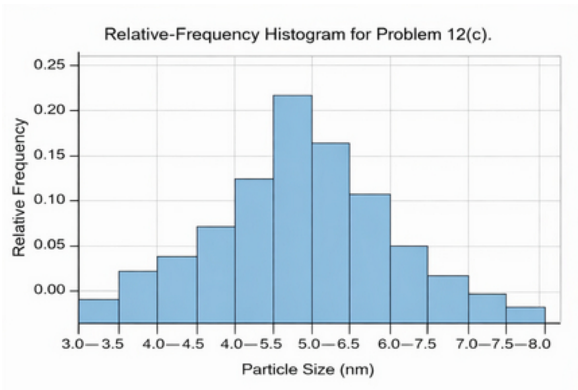

Figure 1: Relative-frequency histogram

(d) Density histogram (heights $h_i = r_i/0.5 = 2f_i/131$):

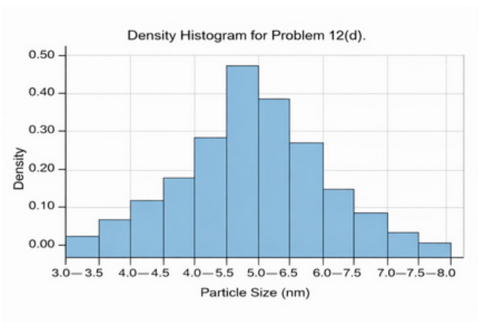

Figure 2: Density histogram (heights

Problem 16

Cylinder strengths:

6.1, 5.8, 7.8, 7.1, 7.2, 9.2, 6.6, 8.3, 7.0, 8.3, 7.8, 8.1, 7.4, 8.5, 8.9, 9.8, 9.7, 14.1, 12.6, 11.2.

(a) Stem-and-leaf (stem = integer part, leaf = tenths):

5	8
6	1 6
7	0 1 2 4 8 8
8	1 3 3 5 9
9	2 7 8
11	2
12	6
14	1

(b) Comparison to beam data: cannot do without the beam observations (not provided).

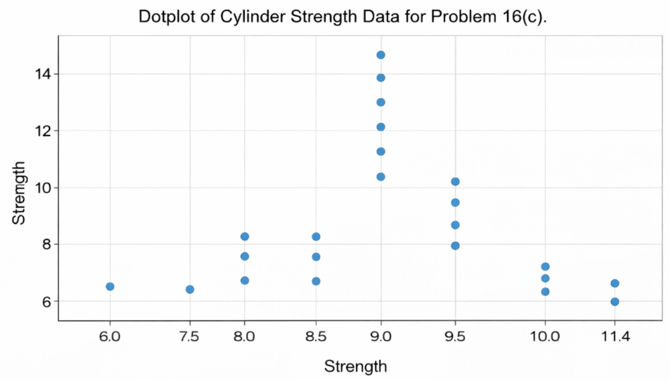

Figure 3: Dotplot

(c)

Problem 18

Frequencies for number of directors ($n = 204$):

x	4	5	6	7	8	9	10	11	12	13	14	15	16	17	21
24	32														
f	3	12	13	25	24	42	23	19	16	11	5	4	1	3	1
1	1														

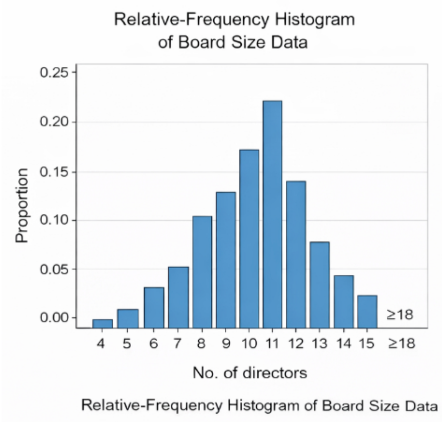

Figure 4: Relative-frequency histogram:

(a)

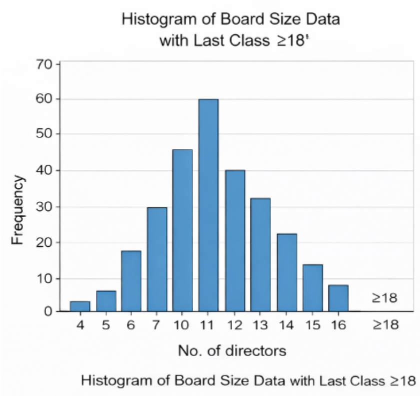

Figure 5: Part B: Relative-frequency histogram:

(b)

(c) Proportion with at most 10 directors:

$$f(\leq 10) = 3+12+13+25+24+42+23 = 142, \qquad \hat{p}(\leq 10) = \frac{142}{204} \approx 0.696.$$

Problem 30

Frequencies:

$incorrectcomponent$	210
$failedcomponent$	126
$missingcomponent$	131
$insufficientsolder$	67
$excesssolder$	54

$n = 210 + 131 + 126 + 67 + 54 = 588.$

$Category$	f	$f/588$	$Cum.\%$
$incorrectcomponent$	210	0.3571	35.71
$missingcomponent$	131	0.2228	58.00
$failedcomponent$	126	0.2143	79.43
$insufficientsolder$	67	0.1140	90.83
$excesssolder$	54	0.0918	100.00

Problem 34

Urban (U), $n_U = 9$:

$$6, 5, 11, 33, 4, 5, 80, 18, 35 \quad (\sum = 197).$$

Farm (F), $n_F = 13$:

$$4, 14, 11, 9, 9, 8, 4, 20, 5, 9.2, 3, 2, 0.3 \quad (\sum = 98.5).$$

(a) Means:

$$\bar{x}_U = \frac{197}{9} = 21.8889 \approx 21.89, \qquad \bar{x}_F = \frac{98.5}{13} = 7.5769 \approx 7.58.$$

(b) Medians (sort each):

U sorted:

$$4, 5, 5, 6, 11, 18, 33, 35, 80 \Rightarrow \tilde{x}_U = x_{(5)} = 11.$$

F sorted:

$$0.3, 2, 3, 4, 4, 5, 8, 9, 9, 9.2, 11, 14, 20 \Rightarrow \tilde{x}_F = x_{(7)} = 8.$$

Urban mean vs median: right-skew due to large values (80, 35, 33) pulls mean up.

(c) Trimmed mean (drop min and max):

Urban:

$$\bar{x}_{U,trim} = \frac{197 - 4 - 80}{7} = \frac{113}{7} = 16.1429 \approx 16.14, \quad trim\frac{1}{9} = 11.11\% each tail.$$

Farm:

$$\bar{x}_{F,trim} = \frac{98.5 - 0.3 - 20}{11} = \frac{78.2}{11} = 7.1091 \approx 7.11, \quad trim\frac{1}{13} = 7.69\% each tail.$$

Problem 36

Data ($n = 26$) and given $\sum x_i = 9638$.

(a) Stem-and-leaf (stem=tens, leaf=ones):

32	5 5
33	4 9
35	6 6 9 9
36	3 4 4 6 9
37	0 3 3 4 5
38	9
39	2 3 4 7
40	2 3
42	4

Right tail suggests $\bar{x} > \tilde{x}$.

(b) Mean:

$$\bar{x} = \frac{9638}{26} = 370.6923 \approx 370.69.$$

Ordered data (given):

$$325, 325, 334, 339, 356, 356, 359, 359, 363, 364, 364, 366, 369,$$

$$370, 373, 373, 374, 375, 389, 392, 393, 394, 397, 402, 403, 424.$$

Median (average of 13th and 14th):

$$\tilde{x} = \frac{369 + 370}{2} = 369.5.$$

(c) Largest value 424 and median: median depends on 13th and 14th values (369, 370). Changing the largest value does not affect the median as long as it remains ≥ 370. Largest possible decrease:

$$424 - 370 = 54.$$

Increase: no bound.

(d) Convert to minutes ($y = x/60$):

$$\bar{y} = \frac{370.6923}{60} = 6.1782 \approx 6.18, \qquad \tilde{y} = \frac{369.5}{60} = 6.1583 \approx 6.16.$$

Problem 40

Ordered lifetimes ($n = 50$).

1. Median:
$$\tilde{x} = \frac{x_{(25)} + x_{(26)}}{2} = \frac{91 + 93}{2} = 92.$$

2. 25% trimmed mean: remove $\lfloor 0.25(50) \rfloor = 12$ from each end $\Rightarrow$ 26 left. Given remaining sum $= 2480$:
$$\bar{x}_{0.25} = \frac{2480}{26} = 95.3846.$$

3. 10% trimmed mean: remove $0.10(50) = 5$ from each end $\Rightarrow$ 40 left. Given remaining sum $= 4089$:
$$\bar{x}_{0.10} = \frac{4089}{40} = 102.225.$$

4. Mean: total sum 5963:
$$\bar{x} = \frac{5963}{50} = 119.26.$$

5. Comparison:
$$\tilde{x} = 92 < \bar{x}_{0.25} \approx 95.38 < \bar{x}_{0.10} = 102.225 < \bar{x} = 119.26$$
(right-skew; trimming reduces influence of large upper-end values).

Problem 44

Data ($n = 12$):

$$180.5, 181.7, 180.9, 181.6, 182.6, 181.6, 181.3, 182.1, 182.1, 180.3, 181.7, 180.5.$$

(a) Range:
$$x_{\max} - x_{\min} = 182.6 - 180.3 = 2.3.$$

(b) Variance from definition (using $y_i = x_i - 180$):

$$\bar{y} = \frac{\sum y_i}{12} = \frac{16.9}{12} = 1.408333\ldots$$

$$\sum (y_i - \bar{y})^2 = 5.769166\ldots$$

$$s^2 = \frac{5.769166\ldots}{11} = 0.5244697\ldots \approx 0.5245.$$

(c) Standard deviation:

$$s = \sqrt{0.5244697\ldots} = 0.7242028\ldots \approx 0.7242.$$

(d) Shortcut method:

$$S = \sum x_i = 2176.9, \qquad SS = \sum x_i^2 = 394913.57,$$

$$s^2 = \frac{SS - S^2 n}{n-1} = \frac{394913.57 - (2176.9)^2 12}{11} \approx 0.5245.$$

Problem 46

Cooler ($n = 15$):

$$1.59, 1.43, 1.88, 1.26, 1.91, 1.86, 1.90, 1.57, 1.79, 1.72, 2.41, 2.34, 0.83, 1.34, 1.76$$

Control ($n = 11$):

$$1.92, 2.00, 2.19, 1.12, 1.78, 1.84, 2.45, 2.03, 1.52, 0.53, 1.90$$

Warmer ($n = 14$):

$$2.57, 2.60, 1.93, 1.58, 2.30, 0.84, 2.65, 0.12, 2.74, 2.53, 2.13, 2.86, 2.31, 1.91$$

(a) Centers (computed from samples):

$$\bar{x}_{Cooler} \approx 1.73, \quad \tilde{x}_{Cooler} \approx 1.76;$$

$$[\ \bar{x}_{Control} \approx 1.75, \quad \tilde{x}_{Control} \approx 1.84; [\ \bar{x}_{Warmer} \approx 2.07, \quad \tilde{x}_{Warmer} \approx 2.30.$$

(b) Standard deviations (computed from samples):

$$s_{Cooler} \approx 0.40, \qquad s_{Control} \approx 0.54, \qquad s_{Warmer} \approx 0.80.$$

(c) Fourth spreads (IQR) vs s: ordering matches

$$Cooler < Control < Warmer.$$

(d)

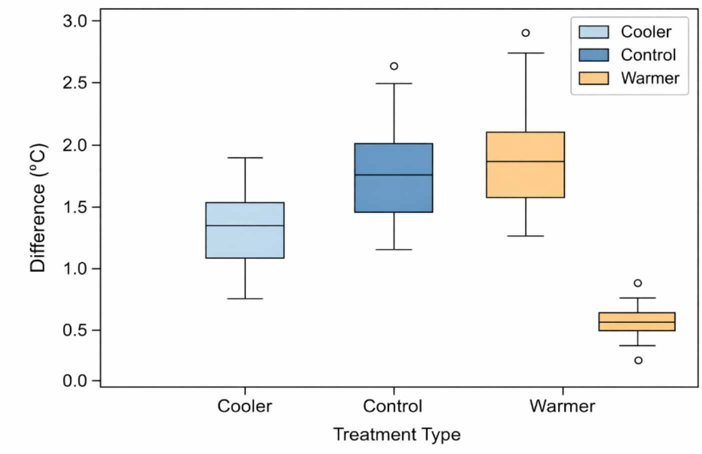

Figure 6: Comparative boxplot

Problem 48

Settled dust:

$U:\ 6, 5, 11, 33, 4, 5, 80, 18, 35\ (n = 9)$, $\qquad F:\ 4, 14, 11, 9, 9, 8, 4, 20, 5, 9.2, 3, 2, 0.3\ (n = 13)$.

Given:

$\sum x = 237.0,\ \sum x^2 = 10079\ (urban)$, $\qquad \sum x = 128.4,\ \sum x^2 = 1617.94\ (farm)$.

(a) Standard deviations (shortcut):

$$s^2 = \frac{\sum x^2 - (\sum x)^2 n}{n-1}.$$

Urban: $s_U^2 = \frac{10079-(237)^2 9}{8}$
$= \frac{10079-561699}{8}$
$= \frac{10079-6241}{8}$
$= \frac{3838}{8}$
$= 479.75,$

$$s_U = \sqrt{479.75} \approx 21.90.$$

Farm: $s_F^2 = \frac{1617.94-(128.4)^2 13}{12}$
$= \frac{1617.94-16486.5613}{12}$
$= \frac{1617.94-1268.1969...}{12}$
$= 29.1453\ldots,$

$$s_F = \sqrt{29.1453\ldots} \approx 5.40.$$

(c) :

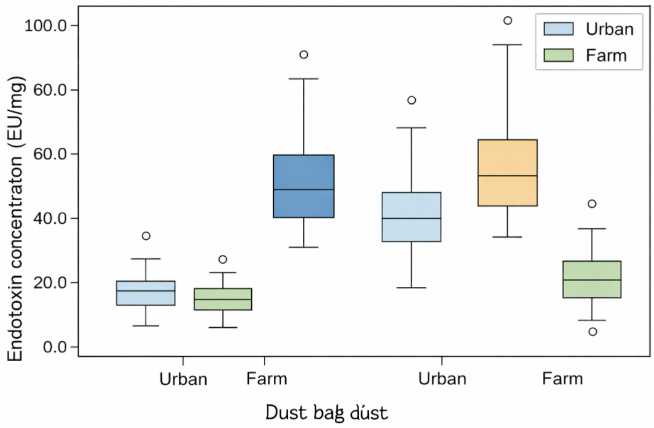

Figure 7: Dust bag Box Plot

Problem 52

Deviations from mean for $n = 5$:

$$0.3,\ 0.9,\ 1.0,\ 1.3,\ d_5.$$

1. Fifth deviation:

$$\sum(x_i - \bar{x}) = 0 \Rightarrow d_5 = -(0.3 + 0.9 + 1.0 + 1.3) = -3.5.$$

2. One possible sample (pick $\bar{x} = 10$):

$$10.3,\ 10.9,\ 11.0,\ 11.3,\ 6.5.$$

Problem 58

Comparative boxplot (two machines, $n = 20$ each):
Center: Machine 2 has larger median than Machine 1.
Spread: Machine 2 has larger IQR (more variability).
Outliers/shape: Machine 1 shows a possible high outlier; Machine 2 no clear outliers.
Conclusion: Machine 2 higher typical dimension but less consistent; Machine 1 more consistent.

Problem 76

Resistance to outliers:

- Mean: not resistant.
- Median: resistant.
- Trimmed mean: resistant (depends on trimming).
- Midrange $(x_{\min} + x_{\max})/2$: not resistant.
- Midfourth $(Q_1 + Q_3)/2$: resistant.

Problem 78

Let $x_1, \ldots, x_n$ have mean $\bar{x}$ and sample sd s.

(a) $y_i = x_i - \bar{x}$:

$$\bar{y} = \frac{1}{n} \sum (x_i - \bar{x}) = \frac{1}{n} \left(\sum x_i - n\bar{x} \right) = 0,$$

$$s_y^2 = \frac{1}{n-1} \sum (y_i - \bar{y})^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = s^2.$$

(b) $z_i = x_i - \bar{x}s$:

$$\bar{z} = \frac{1}{n} \sum \frac{x_i - \bar{x}}{s} = \frac{1}{s} \cdot \frac{1}{n} \sum (x_i - \bar{x}) = 0,$$

$$s_z^2 = \frac{1}{n-1} \sum (z_i - \bar{z})^2 = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s} \right)^2 = \frac{1}{s^2} \cdot \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{s^2}{s^2} = 1.$$