

Math Models, Spring 2026: HW 2

Note that some problems or parts of problems may not be graded.

1. In class we derived the IVP

$$(1) \quad \frac{d^2x}{dt^2} = -\frac{gR^2}{(x+R)^2}, \quad x(0) = 0, \quad \frac{dx}{dt}(0) = V,$$

for the projectile problem.

Instead of rescaling as we did in class, in this problem rescale using $y = x/V^2g^{-1}$ and $\tau = t/Vg^{-1}$.

(a) Check that the rescaled variables are dimensionless.

(b) Give a physical interpretation of the intrinsic reference time Vg^{-1} . (Saying that “ Vg^{-1} is the initial velocity divided by gravity” is NOT an acceptable answer. Rather, your answer should look something like “ Vg^{-1} is the time it takes the projectile to ... do something ... under ... certain conditions,” where you need to supply the something and the conditions.)

(c) Show that the rescaled IVP is

$$(2) \quad \frac{d^2y}{d\tau^2} = -\frac{1}{(\varepsilon y + 1)^2}, \quad y(0) = 0, \quad \frac{dy}{d\tau}(0) = 1,$$

where $\varepsilon = V^2/gR$.

(d) Check that ε is dimensionless.

(e) Recall that our first model for projectile motion, in which we assume gravity is constant, is

$$(3) \quad \frac{d^2x}{dt^2} = -g, \quad x(0) = 0, \quad \frac{dx}{dt}(0) = V.$$

Rescale (3) using the same rescalings you just used to rewrite (1) as (2). Show that this yields

$$(4) \quad \frac{d^2y}{d\tau^2} = -1, \quad y(0) = 0, \quad \frac{dy}{d\tau}(0) = 1.$$

(f) Note that the right-hand side of the ODE in (2) goes to the right-hand side of the ODE in (4) as $\varepsilon \rightarrow 0$. Hence mathematically we expect the solution to the IVP (2) to be close to the solution to the IVP (4) for ε small. Using the definition of ε in terms of the original parameters, explain based on the physics of the problem why we expect the solution to the IVP (2) to be close to the solution to the IVP (4) if ε is small.

If you are taking the class at the 500 level, please read the material in Groesen-Molenaar-Continuum-Modeling-pp4-11.pdf, which is also posted with the announcement. I will assign problems based on this reading in the next HW.

Additional Instructions

To facilitate the HW grading, please do the following.

- Write your HW solutions double spaced.
- Write your HW solutions in the same order in which the problems are assigned above.
- Clearly label each problem. Draw a dark box or circle around the label—the point is that I should be able to quickly flip through your HW solution and find a given problem.