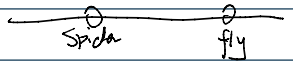
Question 7 ecen 689

Friday, February 12, 2021 10:55 PM



(I hand write the question after got some hint from class, but run out of time, so I can not put it in latex)

consider one unit away from the spider and fly will move to same direction which is $X_1 \to X_1$

$P\{$ the spider will move $\} = 2p$, since the spider will move left or right, that is $p+p$

$P\{$ the spider will stay $\} = 1-2p$

consider spider will catch the fly, which is $X_1 \to X_0$

$P\{$ spider will move $\} = 1-2p$

$P\{$ spider will stay $\} = p$

Consider if the spider and fly will move to two unit, which is spider will stay

$P\{$ spider will stay $\} = p$

So the bellman optimization function is

$$V^*(x) = 1 + pV^*(x) + (1-2p)V^*(x-1) + pV^*(x-2), \text{ for } x \geq 2$$

but for the special cases $V^*(1)$, consider the above statement

$$V^*(1) = 1 + \min\{2pV^*(1),\ pV^*(0) + (1-2p)V^*(1)\}$$

$$V^*(0) = 1 + pV^*(0) + (1-2p)V^*(1)$$

$$\Rightarrow$$

$$V^*(1) = 1 + \min\{2p \cdot V^*(1),\ (p + p(1-2p)V^*(1) + (1-p)(1-2p)V^*(1))/(1-p)\}$$

consider the one unit away bellman function of $V^*(1)$

$$V^*(1) = \begin{cases} 1 + 2pV^*(1) & \text{if } 2pV^*(1) < pV^*(2) + (1-2p)V^*(1) \Rightarrow V^*(1) = 1/1-2p \\ 1 + pV^*(0) + (1-2p)V^*(1) & \text{if } 2pV^*(1) > pV^*(1) + (1-2p)V^*(1) \end{cases}$$

$$\Rightarrow 2pV^*(1) < (p + (1-2p)V^*(1))(1-p)$$

$$\Rightarrow 2p(1-p) < (1-p)(1-2p) \Rightarrow 2p - 2p^2 \leq 1 - 2p - p + p^2 \Rightarrow p \leq 1/3$$

$$\Rightarrow V^*(1) = 1/p$$

$$\Rightarrow 2pV^*(1) \geq p/(1-p) + (1-2p)V^*(1)/(1-p)$$

$$\Rightarrow (1-2p)V^*(1) \leq 2(1-p)$$

$$\Rightarrow p \geq 1/3$$

$$\Rightarrow \begin{cases} V^*(1) = 1/1-2p & \text{if } p \leq \frac{1}{3} \\ V^*(1) = \frac{1}{p} & \text{if } p \geq \frac{1}{3} \end{cases}$$

then for the two unit away from the spider to catche the fly

we have $V^*(2) = \frac{1}{1-p}(1 + V^*(1) + 2pV^*(1))$

$= \frac{1}{1-p}(1 + \min\{2pV^*(1),\ p + p(1-2p)V^*(1) + (1-p)(1-2p)V^*(1)\})/(1-p)$

we have $V^*(x) = \frac{1}{1-p}(1+V^*(1) + 2pV'(1))$

$= \frac{1}{1-p}(1+(1-2p)(1+\min\{2p, V^*(1)\}) - p + p(1-p)V^*(1) + (1-p)((d-1)V^*(1) + (d-1)))/(1-p)]$

similar to the closed form of $V^*(x)$

$$V^*(x) = \begin{cases} \frac{1}{p} & \text{if } p \geq \frac{1}{3} \\ 2/(1-p) & \text{if } p \leq \frac{1}{3} \end{cases}$$