Polymorphic distributivity

Dylan Bumford, *UCLA*

Abstract This article describes a novel pattern of interpretations associated with universal determiners like 'each' and 'every'. It is demonstrated that these canonically distributive quantifiers can give rise to surprising collective readings when they quantify into sub-clausal constituents, especially other Determiner Phrases. For instance, 'two cards from each player' can be understood to pick out a single assorted deck of cards, one whose contents co-vary with the players. Yet this deck as a whole may be said to participate in a range of collective activities (being shuffled together, being traded en masse, not fitting into a standard pack, etc.). Such examples are shown to differ from more familiar *cumulative* readings of the same quantifiers. A compositional analysis is offered that generalizes Krifka's (2001) method of quantification into speech acts in order to accommodate quantification into a larger class of non-truth-denoting semantic objects, including in these cases, entities.

1 Introduction

The 17th Amendment to the U.S. Constitution, ratified in 1913, begins with the sentence in (1). The register is appropriately formal, but the line is still perfectly interpretable in contemporary standard English.

(1) The Senate of the United States shall be composed of two Senators from each State, elected by the people thereof, for six years; and each Senator shall have one vote.

The sentence establishes several rules at once. In particular, it guarantees that at any given time:

(i) each state will have two elected senators

(ii) each of these senator will serve a six-year term

(iii) the collection of these 100 elected officials will make up the governing body called the Senate

These truth conditions would suggest a kind of inverse-linking semantics. Since senators vary with states, the universal quantifier 'each state' should scope over the object DP that contains it, 'two senators [elected…] for six years'. But the truth conditions also suggest a kind of collective predication. No pair of senators can be said to compose the Senate, so certainly the amendment does not require that for each state, *its* two senators compose the Senate.

It is easy enough to characterize informally how the intended meaning might arise. The direct object asks us to find, for each state, two individuals who have been elected by its constituents to the appropriate office. Once we have accomplished this, we set the states aside. The rest of the sentence is concerned only with the officials we have found. If those individuals collectively perform the actions of the Senate, then the law is satisfied.

But what theory of universal quantification predicts this? Whence the "finding" and "collecting"? Canonical analyses of distributive DPs like 'each state' assume that such a DP quantifies over a property, or a set, or something that one way or another would distinguish abiding from non-abiding states. There are only two candidate properties in (1): the property of being a place that a senator is from, and the property of being a place that the senate is composed of two senators from. Neither of these is a property that (1) requires each state to have.

More sophisticated compositional theories like those of Moltmann & Szabolcsi 1994, Heim & Kratzer 1998 (Ch. 8.6), or Büring 2004 would allow the universal to quantify directly into its host DP, creating a complex quantifier 'λQ. each state [λx [two senators from x]] Q'. But this merely kicks the can down the distributive road. The composite DP still demands a property Q so as to assert what it is that each state state's senators must do. In this case, Q can only be the property of composing the Senate, but this again is not the right reading of the law; no particular state is responsible for fielding the entire Senate, much less each of them.

The same is true of yet more systematic mechanisms for recursive scope-taking, like that of Barker & Shan 2014. The higher-order function $\lambda k.\, [\![\text{each state}]\!]\, (\lambda x.\, k\, (\lambda Q.\, [\![\text{two}]\!]\, ([\![\text{senators}]\!]\, ([\![\text{from}]\!]\, x))\, Q))$ leaves two continuations open for further computation, k and Q, one of which will eventually soak up the denotation of 'compose the Senate'. But both occur in the distributive scope of the universal, and so both will again erroneously require at least one Senate per state.

Indeed, the challenge that (1) poses is not one of scope, *per se*, but of the lexical semantics of words like 'each'. To arrive at the intended meaning, 'each state' should distribute over its host DP (so that the senators vary with states) without thereby distributing over the entire rest of the sentence (so that the composition of the Senate varies with states). Its argument therefore should be a function from entities x to whatever sort of thing 'two senators from x' denotes, and its result should be whatever sort of thing '100 senators' denotes.

There is no difficulty in concocting such a function. Ideally though, it should in some way follow from the canonical denotation for 'each', the one that contributes to the truth conditions of the final provision 'each Senator shall have one vote'. In other words, (1) does not equivocate between its first and second uses of the word 'each'.

This manuscript describes one way in which a universal denotation might be constructed to derive both clauses of (1), while hewing as close as possible to standard theories of quantification, scope, and distributivity. The strategy will be to generalize Krifka's (2001) polymorphic theory of quantification-into-speech-acts, largely following the compositional lead of Bumford 2015. Consider, for example, a pair-list question like (2).

(2) Which book did each student read?

a. John read *Structures*; Mary read *Aspects*; Fred read *SPE*

Like (1), this sentence seems to require its 'each'-DP to scope over something that is not a property, and to return something that is not a truth value. In this case it turns a simple question — one whose answers specify for some student which book the student has read — into a compound question — one whose answers specify for each student which book the student has read. Krifka (2001) argues that this is as expected if the semantics of 'each' simply "conjoins" the various meanings generated by its restrictor, where the relevant notion of conjunction is determined by the conjuncts. In (2), for instance, the conjuncts are questions (say, sets of propositions), and conjunction is refinement (say, pointwise set intersection).

Krifka formulates his analysis specifically as a theory of conjoint speech acts, dealing with pair-list interpretations of questions, commands, blessings, and the like. And thus the flexibility of conjunction is closely connected to the notion of sequential performance. To conjoin two acts is to perform one and then the other. To distribute a restrictor over an act-valued function is to perform actions for all of the elements of the restrictor. Bumford (2015) detaches this idea from speech act theory, adapting the pair-list vocabulary to other side-effects of "iterated conjunctions", including in ordinary declarative clauses, even when embedded in conditionals.

Here the idea is relaxed further to include quantification into entities: pair-list individuals, so to speak. The thesis is the same though: whatever can be conjoined can be distributed over. Take the schematic examples in (3) and (4).

(3) Who did each state elect?

a. each state [λx who did x elect]

(4) two senators from each state

a. each state [λx two senators from x]

As described above, in (3), the argument of 'each state' is a function from entities to sets of propositions. Thus for each state this function generates a question. Conjoining these questions is intersecting their respective possible answers pointwise. The resulting meaning is then a new set of propositions each of which entails for every state one of the answers to the question generated by that state (Alabama elected John and Alaska Mary and …). In (4), the argument to 'each state' is a function from entities to sets of (plural) entities. Conjoining these indeterminate individuals is joining their respective possible witnesses pointwise. The resulting meaning is then a new set of (plural) entities each of which contains for every state one of the candidate pairs in the description generated by that state (Mary of Alabama + John of Alabama + Sue of Alaska + Bill of Alaska + …).

For further lexical and compositional details, see Section 3.2. Before that, Section 2 presents a range of data to establish and circumscribe the empirical pattern that (1) exemplifies.

2 Data

The basic pattern represented in (1) is repeated in the following attested examples. In every case, some kind of existential determiner hosts an inversely linked 'each'-DP. And in every case, the whole inversely linked DP satisfies a collective predicate. For example, the first sentence in (5a) places the attendance of CCA Day, a single event, at somewhere above 60. The second sentence in (5a) clarifies that the facilities, of which there are almost certainly six, were all well represented in the set of attendees. Specifically, each facility was represented by at least 10 employees, and all 60 or so of these employees assembled for the event.

(5) a. More than 60 CCA employees recently had an opportunity to rub elbows with some of the state Capitol's most prestigious officials [...] At least 10 employees from each facility assembled for CCA Day, including all six wardens.

b. The jobs in each set are now combined into a mega-job.

c. If you don't know what an acrostic is, it's a type of poem where the first letter of each line spells out a word. Mine spells "SUPERHERO".

d. But the words of each inquiry rattled and collided in my brain and spun into an unintelligible tangle

e. School performance estimates combine [three years of data from each school] to provide an estimate of the expected percent of students proficient or advanced [...]

f. In this analysis, feature measurements of at most 100 cells from each slide were pooled to form two groups.

g. Tell you what. I'll trade you two of each kind for the King's Rock.

The other examples are similar, and present a variety of hosting determiners. In (5g), as in (1), the host is a bare numeral, while in (5a), the hosting numeral is complex. The hosts of (5b) and (5d) are definite plurals; that of (5c) a singular definite superlative. Examples (5e) and (5f) are more complicated. (5e) is plausibly syntactically ambiguous between an outer pseudo-partitive parse ('[[three years] of [data [from each school]]]') and an inner pseudo-partitive parse ('[[[three years] of data] from each school]'). In the former case, the host determiner is null, since the host NP is the mass noun 'data'; in the latter, the host determiner is itself a pseudo-partitive. There are probably other options as well. In any case, although the sentence could in principle describe a cumulative situation in which each school provides at least part of the data, and the total time period over which data is collected is three years, my own judgment is that the distributive DP *must* outscope the numeral 'three' (so each school provided three years of data, and all of the data was subsequently aggregated). (5f) presents even more difficulty. However the attachments are resolved, the sentence should end up with at least one interpretation in which the scopes are totally inverted: from each slide, at most 100 cells were measured, and then all of the measurements from all of the cells from all of the slides were pooled for analysis.

What's crucial here is that universal distributors like 'each' generally resist aggregation. This, after all, is why we call them "distributors". Simplified variants of the sentences above, with vanilla 'each'-DPs, do not make much sense, since they require a bunch of atomic entities — employees, jobs, etc. — to have a bunch of collective properties — assembling, combining, etc.

(6) a. #Each employee assembled for CCA Day, including all six wardens
  b. #Each job is now combined into a mega-job
  c. #An acrostic is a type of poem where each letter in the first column spells out a word
  d. #Each word rattled and collided in my brain and spun into an unintelligible tangle
  e. #Estimates combine each relevant data point to predict student performance
  f. #Each measurement was pooled to form two groups
  g. #I'll trade you each kind for the King's Rock

The examples above all involve distributive DPs embedded in complements or adjuncts of non-distributive DPs. Distributors embedded in possessive specifiers exhibit similar interpretive possibilities. The adaptations in (7) have the same "pair-list entity" readings as the examples above.

(7) a. ✓Each facility's ten employees assembled for CCA Day
  b. ✓Each set's jobs are now combined into a mega-job
  c. ✓An acrostic is a type of poem where each line's first letter spells out a word
  d. ✓Each inquiry's words rattled and collided in my brain and spun into an unintelligible tangle
  e. ✓Estimates combine each school's (three years of) data to predict performance
  f. ✓Each cell's few feature measurements were pooled to form two groups

Attested instances of the relevant pattern are harder to track down, but (8) may present some examples.

(8) a. Lunaris represent a new breed of metal, breaking every rule within the genre to create something new and unique. [...] Individuality is the essence, and each member's personal skill combined, evolves into the sound of Lunaris.
  b. Scouts began probing each army's areas to try to discern the commanders' intentions. A skirmish occurred at Auburn on 1 October when one of each other's patrols collided.
  c. The merger between Verint and Witness would combine each company's unique area of superiority.

Before turning to the analysis of this pattern, let me note several empirical points. First, I have been speaking generically about "distributive universals", but the only universal quantifier to appear in (5)–(8) is 'each'. To be sure, analogous sentences with 'every' or 'all' in place of 'each' are easy to find, as exemplified in (9) and (10). However, as discussed in Section 3.1, 'every' and especially 'all' are known to take on definite or cumulative interpretations in certain environments. So while I see no reason to doubt that sentences like those in (9) might be analyzed along exactly the lines laid out in what follows, I will concentrate on the examples with 'each' to avoid potential confounds.

(9) a. Our Favorite Wedge Sneakers Combine the Best of Every Trend.
  b. Our master invoice template will combine the line items of every current invoice for that company and show a combined total.
  c. During the next meeting you attend, add up the hourly cost of every person in the room.

d. Later, in the story of the flood, Genesis says that God told Noah to gather two of every animal and take them on his ark to save them.

e. As the governing body of the union, the Board of Player Representatives is comprised of one player representative from every team.

(10) a. Not all characterisation tests provided evidence of "superiority" of one mAb over another, but together the results from all characterisation studies allowed for selection of an antibody pair to be taken forward to assay development.

b. Figure 1: Hypothetical trees of a community composed of two species from all monophyletic families from APG III (APG III, 2009)

c. SAAC (Student-Athlete Advisory Committee) is comprised of two student-athletes from all our teams.

Second, almost all of the sentences in (5) contain a singular distributive quantifier like 'each set' embedded within a larger *plural* DP like 'the jobs in …'. The entire complex DP denotes a collection of entities, a model-theoretic plurality. In these cases, unsurprisingly, inflection on the collective predicate is plural, matching both the form and denotation of the argument. In (5c) and (7c), however, the head nominal 'first letter' is *singular*, though the denotation of the entire argument is still multipartite. This creates a clash between syntactic and semantic number. The author of (5c) opts for syntactic rather than semantic agreement on the predicate: 'spells' rather than 'spell'. In my own judgment, this is by far the more natural choice, though the attested (11) may give some doubts about the strength of the morphological force exerted by atomic subjects.

(11) [T]he single electron from each atom's orbital combine to form an electron pair creating a sigma bond.

On other occasions, as in (12), the multiplicity of the emergent pair-list collection seems to pull people toward pluralizing the head noun, even when the referents described by the host NP are clearly atomic: a kind of *i*-within-*i* dependent plurality.

(12) a. Just used AI to combine the faces of every single US President.

b. This is what you get when you combine the male leads from every Nicholas Sparks movie. Turns out a bunch of hot guys makes one single, also hot guy.

In yet other instances of this configuration, neither morphological choice seems especially acceptable. For example, (13a) — with a plural verb form — is all but ungrammatical, but (13b) — with a singular verb form — all but disambiguates to the nonsensical reading in which there are a number of single-jersey piles. Unfortunately I will not have anything to say about the Catch-22 of inflection and agreement here.[1]

(13) a. *A jersey from each team are piled up on the couch

b. $^{?}$A jersey from each team is piled up on the couch

Third, not all collective predicates accept these pair-list sums. Dowty (1987), among many others, distinguishes between 'gather'-type predicates and 'numerous'-type predicates, largely on the basis of their acceptability with 'all'-DPs. For instance, (14b) patterns with (14c) rather than (14a) in asserting that the campers *collectively* gathered around the fire. But (15b), like (15a) and unlike (15c), incoherently requires that the campers are *individually* numerous.

1 A reviewer notes that they are comfortable resolving (13b) to the pragmatically sensible reading — a single pile containing one jersey per team — which is in line with (5c). Again I will have to leave the interaction of number-marking with the relevant readings to future research.

(14) a. #Each camper gathered around the fire
b. ✓All the campers gathered around the fire
c. ✓The campers gathered around the fire

(15) a. #Each camper is numerous
b. #All the campers are numerous
c. ✓The campers are numerous

At present it seems there is little agreement on what general conceptual or theoretical classes these predicates are representative of, or what diagnostics distinguish them, or even what the empirical facts are regarding those diagnostics (Champollion 2020). In my own judgment, the collective predicates most willing to take 'all'-DPs (but not 'every' or 'each'-DPs) as arguments are predicates of joint action ('gather', 'elect') or joint use ('shuffle'), including acts of collaboration and cooperation ('work together', 'put on a play'), predicates of similarity or compatibility ('agree', 'make a good team'), predicates of constitution and composition ('comprise', 'form', 'make up'), and of combination ('add up', 'pool', 'pile'). The predicates most resistant to all universally quantified DPs, including those headed by 'all', are predicates of cardinality ('be numerous', 'outnumber'), distribution ('be denser in the middle', 'be homogeneous'), and to a lesser extent arrangement ('be arranged in a triangle').

And also in my judgment, it seems that these latter, 'numerous'-type predicates are as incompatible with the pair-list collections exemplified in (5) as they are with ordinary distributive arguments like (15a). For example, (16) cannot mean that because there are so many bands in the lineup, a collection consisting of two songs from each would be a very large collection.

(16) #Two songs by every band in the lineup are numerous

Even when the host DP is a simple plural definite description — the sort of phrase that should have no trouble generally satisfying 'numerous'-type predicates, as in (15c) — numerosity is unassertable. That is, (17a) can only mean that each borough is populated by more than a million people, not that the boroughs have a combined population of over a million. Likewise, (17c) cannot mean that the marching bands, taken together, were arranged so that the highest concentration of bands was in the middle of the arrangement; it means that *each* band was arranged in a middle-heavy formation.

(17) a.	The residents of each borough number in the millions	**#coll, ✓dist**
b.	The residents of the boroughs number in the millions	**✓coll, ✓dist**
c.	The formations of each marching band were densest in the middle	**#coll, ✓dist**
d.	The formations of the marching bands were densest in the middle	**✓coll, ✓dist**

This is perhaps surprising, since the agreement facts above suggest that the features of the host nominal and determiner can retain some influence over the predicate. Instead, these pair-list arguments appear to show neither the semantic properties of the embedded distributor, nor the properties of the definite host. Rather they show the properties of 'all'-DPs. That is, they readily saturate predicates of joint activity, composition, combination, etc., but stubbornly distribute over predicates of cardinality and arrangement. It would seem that for whatever reason, a sum assembled compositionally via distributive quantification is too motley to be considered as a holistic group entity, and so cannot have properties like numerosity or uniformity. Such entities, like DPs headed by 'all', apparently must be construed as loose sets of individuals, acting together to some end. Again, though, given the theoretical and empirical murkiness of the 'gather'/'numerous' split, I'm afraid I will have nothing more insightful to say about this contrast.

The final point I wish to draw attention to is that the readings of interest here depend on a fundamentally ordinary kind of inverse scope: an embedded universal takes scope over a DP that contains it. This means the phenomenon is delimited by normal constraints of scope and binding. For instance, if the universal is embedded in an island, then the pair-list reading is unavailable. For instance, the rules of the Constitution would be unsatisfiable if the 17th amendment were written as in (18), where the universal is in a relative clause, as it would require potential senators to be born in multiple states.

(18) # The Senate shall be composed of two Senators who come from each State.

As far as binding, recall that in a typical case of inverse linking, an embedded quantifier distributes over the rest of its clause in addition to its host. In (19), for instance, the cheese eating events vary with the cities, just as the cheese eaters do. And in virtue of scoping over this clausal continuation, the quantifier may also bind pronouns in it, as in May's well-rehearsed example (19b).

(19) a. Someone in every city is eating cheese right now
 b. Someone in every city hates it

But with a collective predicate and a pair-list interpretation of its complex argument, the embedded quantifier by definition does not scope over any more of the clause other than its host DP. As a result, it should not be able to bind anything outside of that host (Barker 2012). Checking this requires consideration of rather careful artificial sentences, but the following perhaps bear out the prediction.

(20) I walked back into the writers workshop and saw that someone had …
 a. ✓ piled up one book from each of my favorite authors on top of the table in front of me
 b. # piled up one book from each of my favorite authors on top of the table in front of her
 c. # placed one book from each of my favorite authors on top of the table in front of her
 d. # piled up books from all my favorite authors on top of the table in front of each of them

The sentence in (20a) is a typical example of the pattern under discussion: the object of the collective predicate 'piled up' denotes a single heap containing one book per author. The minimal variant in (20b) sounds unremarkable at first blush, but doesn't actually make any sense. For instance, it doesn't mean what (20c) means, where the the universal takes scope over the entire clause, because that would require accepting that a solitary book can constitute a pile. It also can't mean anything like what (20d) means, in which each author has their own book-per-author pile, which perhaps might be expected if the universal could take scope both inside the object DP 'one book from x' *and* over the entire clause, so as to bind the pronoun.

3 Analysis

3.1 Analytical background

The English determiners 'each', 'every', and 'all (the)' are *universal*. The truth of a sentence containing one of these determiners will generally depend on whether the entirety of the set denoted by its restrictor satisfies a certain property. The former two, 'each' and 'every', are also generally *distributive*. A sentence containing 'each', for instance, will require that the individuals in the extension of its restrictor, one by one, satisfy the property corresponding to its scope. The latter determiner, 'all (the)', appears in both distributive and collective contexts. In collective contexts, it suffices that the set of individuals described by the restrictor, as a whole, meet the condition described by the scope. A typical sort of contrast is given in (21).

(21) a. ✓ All the students collided in the middle of the field
 b. # Each student collided in the middle of the field

There are a few recognized exceptions to this generalization, at least for ‘every’. For instance, synthetic, lexicalized universal DPs like ‘everybody’ and ‘everywhere’ can satisfy collective predicates, unlike their analytic counterparts with independent restrictors, as illustrated in (22). There are no analogous words formed from ‘each’.

(22) a. ✓Everybody circled up around the coach
b. #{Every, Each} player circled up around the coach

Beghelli & Stowell (1997) point out the contrast in (23). Descriptively speaking, the DPs in (23a) and (23b) seem to be coerced into an abstract degree-of-toughness determined by calculating the combined strengths of the students. In that sense, there is something collective about the interpretations of the clauses, though not in the usual sense that the DPs successfully saturate the argument of a collective predicate. Notably, this total-strength interpretation is unavailable with ‘each’, though as far as I know, the semantics of the ‘it took’ construction is not well-studied, and there is no theory of the difference between universals here.

(23) a. ✓It took all the students to lift the piano
b. ✓It took every student to lift the piano
c. #It took each student to lift the piano

More robustly, ‘all’- and ‘every’-DPs are known to sometimes stand in what are called *cumulative* relations to other plural arguments. Kratzer (2000), following Schein (1993), discusses sentences like (24b), on the reading in which all of the mistakes were caught by (at least) one of the three editors, and all of the editors did some catching. This is quite unexpected, given that (24a), for instance, lacks an analogous reading; there is no sense in which (24a) could be true merely because three mistakes were caught in total, with some editors catching only one or two. To make matters worse, Schein observes that ‘every’ can accumulate with some of its co-arguments, yet distribute over others. For instance, (24c) has a reading on which three very influential video games can be held cumulatively responsible for every quarterback’s having learned two (possibly distinct) plays.

(24) a. Every editor caught three mistakes
b. Three copy editors caught every mistake
c. Three video games taught every quarterback two new plays

Schein and Kratzer both assess (24c) as an argument for the necessity of event variables, but Champollion (2010) and Brasoveanu (2013) have provided event-free analyses of these sorts of split-distributivity cases. In keeping with a line of research pursued by, e.g. Szabolcsi (1997) and Matthewson (2001), Brasoveanu (2013) decomposes ‘every’ into a maximal, existential component and a distributive, quantificational component. The existential component accumulates with “higher” operators, while the quantificational component distributes over “lower” properties. The effect is approximated in English by replacing ‘every NP’ with ‘the NPs each’, as in (25).

(25) a. Every girl wore a dress to the party
b. The girls each wore a dress to the party

What exactly counts as “higher” in these contexts is somewhat contentious. In the event semantics of Kratzer 2000, (obligatory) distributivity is correlated with agency, so that ‘every’-DPs playing the role of agents are necessarily distributive, while ‘every’-DPs in non-agentive roles are optionally cumulative. For Brasoveanu, it is simply semantic scope that determines the potential for cumulativity; anything within the nuclear scope of the universal will also fall within its distributive scope. Champollion (2010) criticizes both of these accounts. The former makes predictions that are at odds with the judgments of Bayer (1997), given in (26), and Zweig (2008), given in (27). And the latter cannot explain why (24a) lacks a cumulative reading, given that DPs with bare numeral determiners are generally free to take arbitrarily wide scope.

(26) a. # Every screenwriter in Hollywood wrote *Gone with the Wind*
    b. ✓ *Gone with the Wind* was written by every screenwriter in Hollywood

(27) a. ✓ The Fijians and the Peruvians won every game
    b. # Every game was won by the Fijians and the Peruvians

Zweig (2008: p. 137) conjectures that 'every' is only obligatorily distributive when it is in subject position. In other positions he expects it to exhibit cumulative 'all'-like behavior. A bit less radically, Champollion proposes that an 'every'-DP is doomed to distribute over anything it (syntactically) c-commands. This would of course largely privilege subject positions, but should extend also to lower distinctions within the thematic argument hierarchy (see Flor 2017 and Chatain 2020 for some discussion of this prediction).

Unfortunately, none of these analyses provide much help with the data in Section 2. For one, the sentences in (5) all show surprising *collective* interpretations, not cumulative ones. Brasoveanu (2013: p. 40, 48) in fact explicitly denies that 'every'-DPs can support collective interpretations, in the middle of his discussion of their cumulative potential. For another, the scope-splitting mechanisms are still too coarse. In the sentences of interest like those in (5), we do not see a universal associating distributively with one constituent and cumulatively (or collectively) with another. Instead, the universal distributes over some element, and then *that element*, or rather the sum of elements distributed over, assumes a collective role in the rest of the sentence. We would need to split the *distributive scope* of the universal in two (and then do some work to pick up the distributed pieces), which would mean actually breaking the universal into three arguments, a plurality formed from the restrictor, a distributive share, and then a collection of distributees. And then most plainly, all of the sentences in (1) and (5) contain 'each', which as far as I know has never been reported to participate in the cumulative readings just discussed.

In a different strain of research concerned with the semantic flexibility of universals, many theorists have argued that interrogatives like (28) exhibit readings best accounted for by treating the universal DP as taking scope over its containing question. On the relevant interpretation, (28) is answered by any proposition that specifies for each student a book they read.

(28) Which book did every student read?
    a. John read *Structures*; Mary read *Aspects*; Fred read *SPE*

Indeed, the closest example that I have been able to find in the literature to the Constitutional excerpt in (1) comes from Belnap's (1982) early study of question composition, given in (29).

(29) What the average grade is depends (only) on what each student receives

In Belnap's words: "Obviously the scope of 'each' cannot be the whole declarative; for it is not true that for each student, the average grade depends (only) on what grade that student receives. Nor can the scope of 'each' be inside the what, for that would presuppose that each student received the same grade. Instead, one has to look at what grade each student received as a closed unit — an indirect question which is a complement of depend — in which 'each' is wide with respect to 'what'." Of course for this to be possible, for 'each' to scope over just the question that embeds it, its denotation would have to be capable of accepting questions as arguments and returning questions as results.

Taking this assumption as a starting point, Krifka (2001) suggests that the sentence in (28) ought to have exactly the same meaning as the sequence of sentences in (30). They are both "conjunctions" of question acts. Since a question act is not a proposition, 'conjunction' here should not be understood in the Boolean, intersective sense, but rather as a sequencing operator in some more dynamic denotational algebra. Krifka sketches what this might look like in a semantics of acts oriented around *commitment states* (see, e.g., Krifka 2015 for subsequent elaboration of this idea).

(30) Which book did John read? And which book did Mary read? And which book did Fred read?

a. John read *Structures*; Mary read *Aspects*; Fred read *SPE*

Bumford (2015) argues that reconfiguring the denotation of 'every' in terms of generalized dynamic conjunction has benefits below the level of speech acts. He demonstrates that ordinary matters of intra-sentential scope and binding can be explained in terms of "iterated conjunction". For instance, consider the so-called *internal* reading of (31), on which it asserts the existence of an injection from students to books within the read relation.

(31) Every student read a different book

(32) John read a different book; And Mary read a different book; And Fred read a different book

a. (John read *Structures*; Mary read *Aspects*; Fred read *SPE*) or
(John read *Aspects*; Mary read *Structures*; Fred read *SPE*) or
(John read *SPE*; Mary read *Aspects*; Fred read *Structures*) or
(…)

Bumford (2015) models this reading by assuming that 'different' means "distinct from the relevant previously mentioned entities", and that (31) has the Krifka-esque structure of (32). Crucially by dynamicizing the conjunction that the universal uses to glue together the results of evaluation, the extension of 'different' evolves as the semantic computation unfolds. Thus students evaluated later will have stricter standards for different-book-reading than the students evaluated earlier.

In the next section, the non-Boolean conjunction at the heart of Krifka's and Bumford's universals is further generalized to include non-deterministic *entities* in addition to non-deterministic propositions generated by *wh*-words and indefinites.

3.2 A Fragment

3.2.1 Composition

The following formal fragment caches the previous ideas out. The metalanguage is simply-typed with ground types for entities ($\mathtt{e}$) and truth values ($\mathtt{t}$), and constructors for functions ($\sigma \to \tau$) and sets ($\mathbf{S}_\alpha$). Semantic combination is function application. Object-language variables and abstractions run through an assignment function, as usual. The notation $v :: \alpha$ indicates that the variable v has type α.

(33) a. **Function Application** (FA)

$$\left[\!\!\left[\begin{array}{ccc} & \tau & \\ & \text{FA} & \\ \sigma & & \sigma \to \tau \\ \varphi & & \psi \end{array}\right]\!\!\right]^g = [\![\varphi]\!]^g\, [\![\psi]\!]^g$$

b. **Function Abstraction** (ABS)

$$\left[\!\!\left[\begin{array}{ccc} & \sigma \to \tau & \\ & \text{ABS} & \\ \Lambda_{v :: \sigma} & & \tau \\ & & \varphi \end{array}\right]\!\!\right]^g = \lambda d.\, [\![\varphi]\!]^{g^{v \mapsto d}}$$

To fix terms, I will say that in a configuration of the form $[\varphi\ [\Lambda_v\ \psi]]$, the expression φ "scopes over" the expression ψ. When φ is of type $\mathbf{S}_\alpha$, I will sometimes say that its denotation is "indeterminate", roughly following the terminology in Coppock & Beaver 2014. For instance, where a definite description like 'the student', of type $\mathtt{e}$, might refer *determinately* to a particular student — say, $[\![\text{the student}]\!] = \mathbf{john}$ — an indefinite description like 'a student', of type $\mathbf{S}_\mathtt{e}$, will be said to refer *indeterminately* to the set of potential student referents — $[\![\text{a student}]\!] = \{d \mid \textbf{student}\, d\}$. But to be clear, I do not mean to attach any technical significance to this way of talking about sets. Nor is this particular set-theoretic analysis of indefinite DPs — of which there are many versions going back at least to Milsark 1974 — in any way essential to the analysis of distributivity on offer (see Section 3.2.3 for discussion). I invoke these notions only for concreteness.

Following Charlow 2014, I will assume ordinary values may be coerced to indeterminate values with an IDENT-like type-shifter $(\cdot)^\eta$, and indeterminate objects may be combined with their scopes by way of a LIFT-like operator $(\cdot)^\star$ (cf. Partee 1986).[2] For both of these operations, α and β may be instantiated at any type; anything can be made trivially indeterminate, and any indeterminate expression can be given scope over any other.[3]

(34)

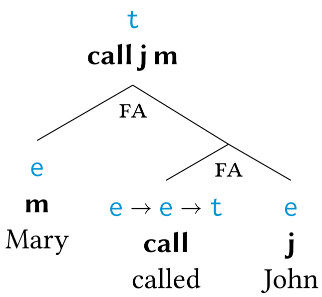
Operator	Type	Denotation
$(\cdot)^\eta$	$\alpha \to \mathsf{S}_\alpha$	$\lambda d.\ \{d\}$
$(\cdot)^\star$	$\mathsf{S}_\alpha \to (\alpha \to \mathsf{S}_\beta) \to \mathsf{S}_\beta$	$\lambda m \lambda k.\ \bigcup\{k\, d \mid d \in m\}$

Examples of derivations with basic, quantificational, and indeterminate DPs are given in (35). The point here is just to illustrate how the compositional components (application, abstraction, $(\cdot)^\eta$, and $(\cdot)^\star$) work together. Lexical denotations will be discussed in Section 3.2.3.

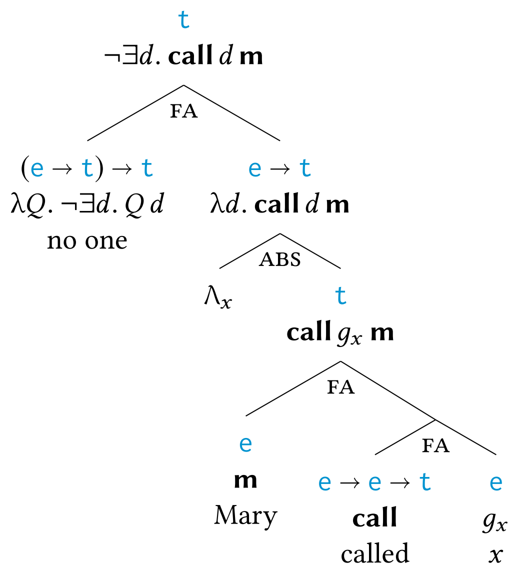
2 The notation $(\cdot)^o$ is intended to signify that I will generally represent the application of o to an argument by attaching o as a superscript to that argument; e.g., the squaring function might be defined by $(\cdot)^2 \coloneqq \lambda n.\ n \times n$.

3 For those keeping score at home, $(\cdot)^\eta$ and $(\cdot)^\star$ are the two algebraic components of the (power)set monad (Wadler 1992, Shan 2001).

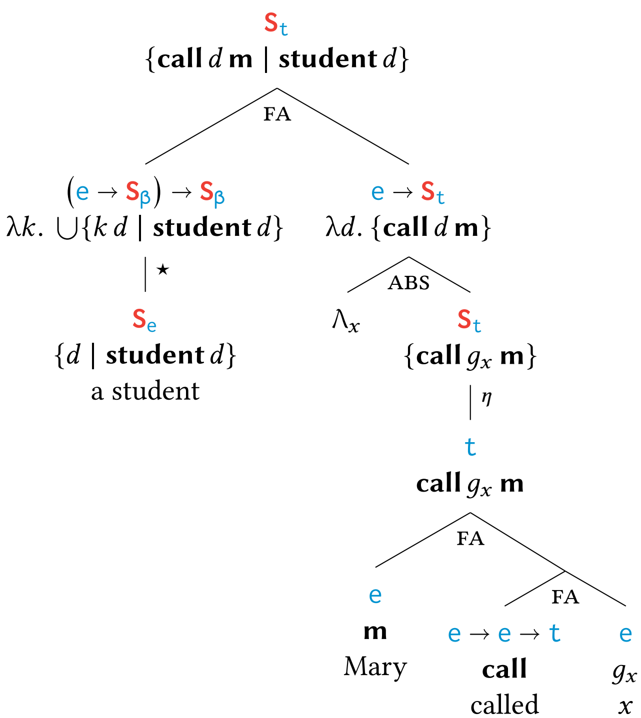
(35) a.



b.



c.



3.2.2 Conjunction across types

With this grammar established, the only crucial theoretical notion is that of what I'llc all a *multiplicative type*. This determines the range of semantic objects over which distributors like ‘every’ can scope. As a hypothesis, I assume that any sort of expression that can be conjoined by the word ‘and’ is also the sort of expression that can act as the nuclear scope of a distributive universal, and any operation that ⟦and⟧ can compute with two arguments is an operation that ⟦every⟧ generalizes to a set of such arguments. Since such expressions come in a variety of types, I will say that conjunctions and universals are *polymorphic*.[4]

Let Π^{α} represent the family of type-indexed actions associated with conjunction. At what types should Π^{α} be defined? For starters, we might include the fundamental, bitwise multiplication of (36a), the classic

4 Note that since my aims are semantic and not computational, I will not distinguish between the *parametric* polymorphism enjoyed by $(\cdot)^{\star}$, which “does the same thing” at every type, and the so-called *ad-hoc* polymorphism of ⟦and⟧, which might be better thought of as a kind of type-governed ambiguity.

Boolean generalizations of this that Partee & Rooth (1983) called *Generalized Conjunction* (36b, 36c), as well as the collectivizing actions associated with plurality symbolized in (36d).

(36) a. $\prod^{\mathsf{t}} := \lambda T.\ \bigwedge T$

b. $\prod^{\sigma\to\mathsf{t}} := \lambda T \lambda d.\ \bigwedge\{P\,d \mid P \in T\}$

c. $\prod^{\sigma_1\to\cdots\to\sigma_n\to\mathsf{t}} := \lambda T \lambda d_1 \cdots \lambda d_n.\ \bigwedge\{R\,d_1\ \cdots\ d_n \mid R \in T\}$

d. $\prod^{\mathsf{e}} := \lambda X.\ \oplus X$

We might also assume, again following Bumford 2015, that for any type α such that $\prod^{\alpha}$ is defined, there is a valid instantiation of $\prod^{\mathsf{S}_\alpha}$ for indeterminate analogs of α, as in (37). This operation takes a set M of α-valued sets m_1 through m_n, and returns the pointwise product of these sets, where the product operation is appropriate to the type α. Just for illustration, if $\prod^{\alpha}$ did nothing more than place its arguments in a tuple, then $\left(\prod^{\mathsf{S}_\alpha} M\right)$ would yield a Cartesian product of M.[5]

(37) $$\prod^{\mathsf{S}_\alpha} := \lambda M.\ m_1{}^\star\left(\lambda a_1.\ m_2{}^\star\left(\lambda a_2.\ \cdots m_n{}^\star\left(\lambda a_n.\ (\textstyle\prod^{\alpha}\{a_1, a_2, \ldots, a_n\})^\eta\right)\right)\right)$$
$$\textbf{where}\ M = \{m_1, m_2, \ldots, m_n\}$$

For instance, let M be the set of singleton truth-valued sets recording for each dog whether or not it barked. That is, for each dog d, let M contain $\{\mathbf{T}\}$ if d barked and $\{\mathbf{F}\}$ if d didn't (one should feel free to switch to propositions under intersection if finer semantic resolution is desired):

$$\begin{aligned} M &= \{(\mathbf{bark}\,d)^\eta \mid \mathbf{dog}\,d\} \\ &= \{\{\mathbf{bark}\,\mathbf{d}_1\},\ \{\mathbf{bark}\,\mathbf{d}_2\},\ \ldots,\ \{\mathbf{bark}\,\mathbf{d}_n\}\} \end{aligned}$$

Then $\prod^{\mathsf{S}_\mathsf{t}} M$ is just the singleton set that contains $\mathbf{T}$ if every dog barked and $\mathbf{F}$ if any dog didn't:

$$\begin{aligned} \textstyle\prod^{\mathsf{S}_\mathsf{t}} M &= \left\{ \textstyle\prod^{\mathsf{t}}\{p_1, \ldots, p_n\} \;\middle|\; \begin{array}{l} p_1 \in \{\mathbf{bark}\,\mathbf{d}_1\}, \\ \ldots, \\ p_n \in \{\mathbf{bark}\,\mathbf{d}_n\} \end{array} \right\} \\ &= \{\mathbf{bark}\,\mathbf{d}_1 \wedge \cdots \wedge \mathbf{bark}\,\mathbf{d}_n\} \\ &= \left(\textstyle\prod^{\mathsf{t}}\{\mathbf{bark}\,d \mid \mathbf{dog}\,d\}\right)^\eta \end{aligned}$$

In other words, the product of a bunch of coerced values is exactly the coercion of these values' product.

If M contains non-trivial indeterminate truth values, then their product is crossed. For example, let M contain for each dog d the indeterminate truth value that d chased a cat, where truth varies with cats:

$$\begin{aligned} M &= \{\{\mathbf{chase}\,c\,d \mid \mathbf{cat}\,c\} \mid \mathbf{dog}\,d\} \\ &= \{\{\mathbf{chase}\,c\,\mathbf{d}_1 \mid \mathbf{cat}\,c\},\ \{\mathbf{chase}\,c\,\mathbf{d}_2 \mid \mathbf{cat}\,c\},\ \ldots,\ \{\mathbf{chase}\,c\,\mathbf{d}_n \mid \mathbf{cat}\,c\}\} \end{aligned}$$

5 This presentation is made slightly awkward by the fact that M is a set, but the definition of $\prod^{\mathsf{S}_\alpha}$ requires that its constituent sets $m_1 \ldots m_n$ be evaluated in some arbitrary order. Bumford argues that this emergent order-sensitivity is a good thing in the context of *dynamic* conjunctions. But all of the $\prod^{\alpha}$ operations considered in this paper are commutative, as is the lifting operator $(\cdot)^\star$:

$$m_1{}^\star\,(\lambda a_1.\ m_2{}^\star\,(\lambda a_2.\ f\,a_1\,a_2)) = m_2{}^\star\,(\lambda a_2.\ m_1{}^\star\,(\lambda a_1.\ f\,a_1\,a_2)) = \bigcup\{f\,a_1\,a_2 \mid a_1 \in m_1,\ a_2 \in m_2\}$$

So in this context, $\prod^{\mathsf{S}_\alpha}$ in (37) defines a perfectly valid function (setting aside infinite sets and the Axiom of Choice).

Then $\prod^{\mathsf{S}_\mathsf{t}} M$ is itself an indeterminate truth value; for every way f of choosing a cat per dog, $\prod^{\mathsf{S}_\mathsf{t}} M$ contains **T** if every dog d chased $f\,d$ and **F** if any didn't:

$$\begin{aligned} M &= \left\{ \prod^{\mathsf{t}}\{p_1,\dots,p_n\} \;\middle|\; \begin{array}{l} p_1 \in \{\mathbf{chase}\,c\,\mathbf{d}_1 \mid \mathbf{cat}\,c\}, \\ \dots, \\ p_n \in \{\mathbf{chase}\,c\,\mathbf{d}_n \mid \mathbf{cat}\,c\} \end{array} \right\} \\ &= \{\mathbf{chase}\,(f\,\mathbf{d}_1)\,\mathbf{d}_1 \wedge \cdots \wedge \mathbf{chase}\,(f\,\mathbf{d}_n)\,\mathbf{d}_n \mid f : \mathbf{dog} \to \mathbf{cat}\} \end{aligned}$$

Assuming indeterminate denotations correspond to judgments of truth when they contain **T** among their alternatives, $\prod^{\mathsf{S}_\mathsf{t}} M$ will be judged true whenever every dog chased at least one cat.

Most relevant to the topic at hand, consider the case when M consists of a variety of indeterminate entities, type S_e. For example, let M contain for each dog the set of cats it chased:

$$\begin{aligned} M &= \{\{c \mid \mathbf{chase}\,c\,d\} \mid \mathbf{dog}\,d\} \\ &= \{\{c \mid \mathbf{chase}\,c\,\mathbf{d}_1\},\ \{c \mid \mathbf{chase}\,c\,\mathbf{d}_2\},\ \dots,\ \{c \mid \mathbf{chase}\,c\,\mathbf{d}_n\}\} \end{aligned}$$

Then $\prod^{\mathsf{S}_\mathsf{e}} M$ is itself an indeterminate (plural) entity; for every way f of choosing a cat per dog, $\prod^{\mathsf{S}_\mathsf{e}} M$ contains the cat sum corresponding to the range of f:

$$\begin{aligned} \prod^{\mathsf{S}_\mathsf{e}} M &= \left\{ \prod^{\mathsf{e}}\{c_1,\dots,c_n\} \;\middle|\; \begin{array}{l} c_1 \in \{c \mid \mathbf{chase}\,c\,\mathbf{d}_1\}, \\ \dots, \\ c_n \in \{c \mid \mathbf{chase}\,c\,\mathbf{d}_n\}, \end{array} \right\} \\ &= \left\{ \bigoplus\{f\,d \mid \mathbf{dog}\,d\} \;\middle|\; f : \mathbf{dog} \to \mathbf{cat} \right\} \end{aligned}$$

That is, $\prod^{\mathsf{S}_\mathsf{e}} M$ is a set of cat clusters, each cluster containing for each dog one of the cats it chased. As a technicality, note that if any dog left well alone of the cats, then $\prod^{\mathsf{S}_\mathsf{e}} M = \emptyset$; in this case there are no cat pluralities that can bear witness to the respective pursuits of the dogs. As expected from a product, if one of its factors is zero, the entire product is zero as well.

3.2.3 Denotations

With this in hand, (38) clarifies the lexical semantic hypothesis advanced in the introductory sections. Unsurprisingly, $[\![\text{every}]\!]$ is a binary wrapper for the $\prod$ operation, and is therefore a straightforward generalization of the traditional Boolean Generalized Quantifier.[6] The rest of the table spells out standard non-quantificational denotations for indefinites (e.g., Bittner 1994, Krifka 1999, Kratzer & Shimoyama 2002, Landman 2004, Charlow 2019) to complete the fragment.

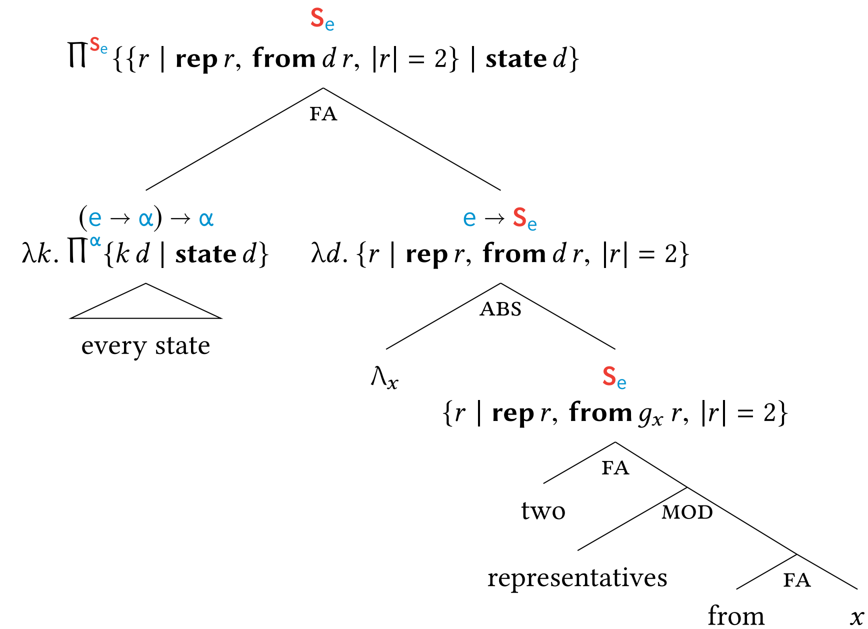
(38)

Word	Type	Denotation
every	$(\mathsf{e} \to \mathsf{t}) \to (\mathsf{e} \to \alpha) \to \alpha$	$\lambda P \lambda k.\, \prod^{\alpha}\{k\,d \mid d :: \mathsf{e},\, P\,d\}$
a	$(\mathsf{e} \to \mathsf{t}) \to \mathsf{S}_\mathsf{e}$	$\lambda P.\, \{d \mid d :: \mathsf{e},\, P\,d\}$
two	$(\mathsf{e} \to \mathsf{t}) \to \mathsf{S}_\mathsf{e}$	$\lambda P.\, \{d \mid d :: \mathsf{e},\, P\,d,\, \lvert d\rvert = 2\}$

In garden-variety cases, 'every' will scope over a truth-denoting constituent, and the lexical entry in (38) will reduce to Boolean infinitary conjunction. In the cases of interest, 'every' will scope over an entity-denoting constituent, and the semantics will behave as in the final cat example above.

6 "Generalization" here only in the technical sense that the polymorphic 'every' defined in (38) includes the traditional GQ denotation as a special case (when $\alpha = \mathsf{t}$). So in any derivation where the traditional 'every' could appear, the polymorphic 'every' may appear as well, and the derivations will have the same denotation. I make no claim as to whether the non-Boolean instances of conjunction should be thought of as generalizations of Boolean conjunction in any natural, conceptual, or psychological sense (see Schmitt 2020 for a recent overview of this widely pondered question).

(39) two representatives from every state



The denotation of (39) is a set of sum entities.[7] Each sum contains for every state exactly two representatives. If any state happens to have more than two representatives, then the denotation here is properly indeterminate, containing all the ways of choosing respective pairs out of the states' representative pools. If all the states have exactly two representatives, then the denotation is singleton, containing just the one collection of pairs; in this case we would be justified in talking about *the* two representatives of every state. If any state has fewer than two representatives, then the denotation above will be empty, as no collection of representatives could meet the constraint that it contain two from each state.

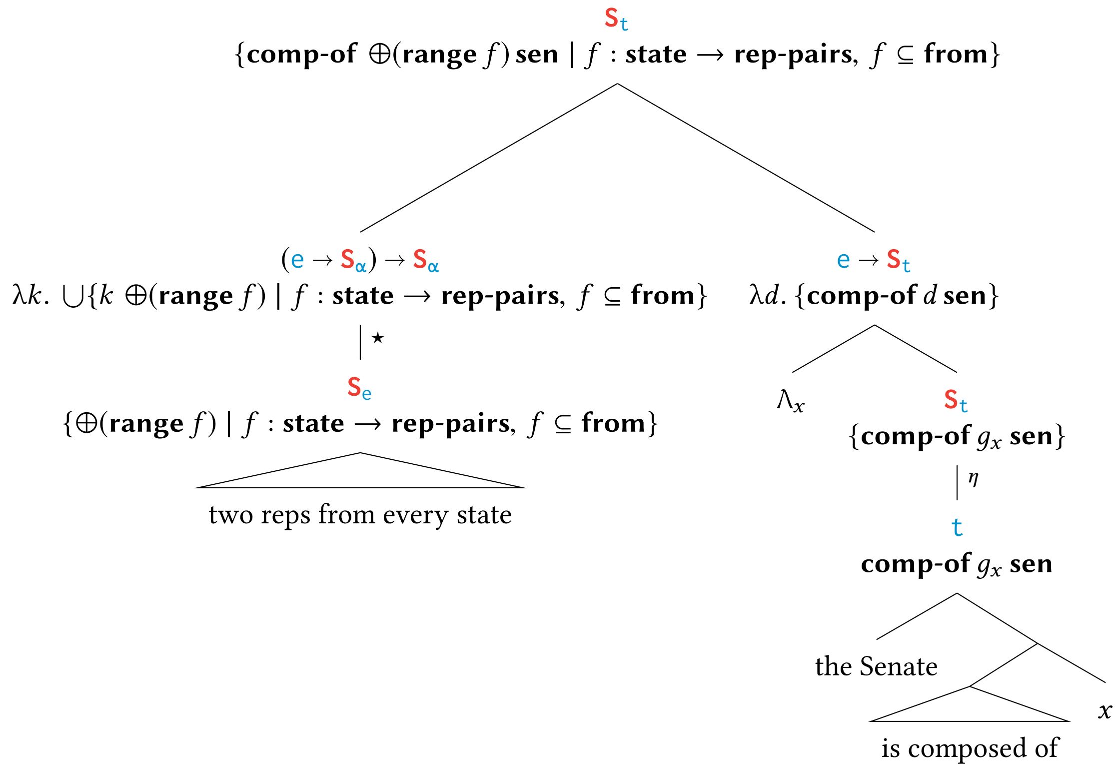
Putting this denotation together with a collective predicate poses no subsequent challenge. The object of (40) denotes an indeterminate diversity of representatives. It takes scope over the sentence just as any ordinary indefinite like 'two cats' would. The whole sentence is predicted to be judged true if any alternative in the resulting indeterminate set of truth values is **T**. That is, if the Senate really does comprise 100 representatives, two per state.

7 MOD stands for the usual intersective mode of combination used in modification structures:

$$\left[\!\!\left[\begin{array}{ccc} & \sigma \to t & \\ & \text{MOD} & \\ \sigma \to t & & \sigma \to t \\ \varphi & & \psi \end{array} \right]\!\!\right]^g = \lambda v.\, [\![\varphi]\!]^g\, v \wedge [\![\psi]\!]^g\, v$$

This could certainly be generalized to support any multiplicative return type α, replacing $\wedge$ with $\prod^{\alpha}$, but the semantics of adjunction is entirely outside the scope of present concerns.

(40) The Senate is composed of two representatives from every state



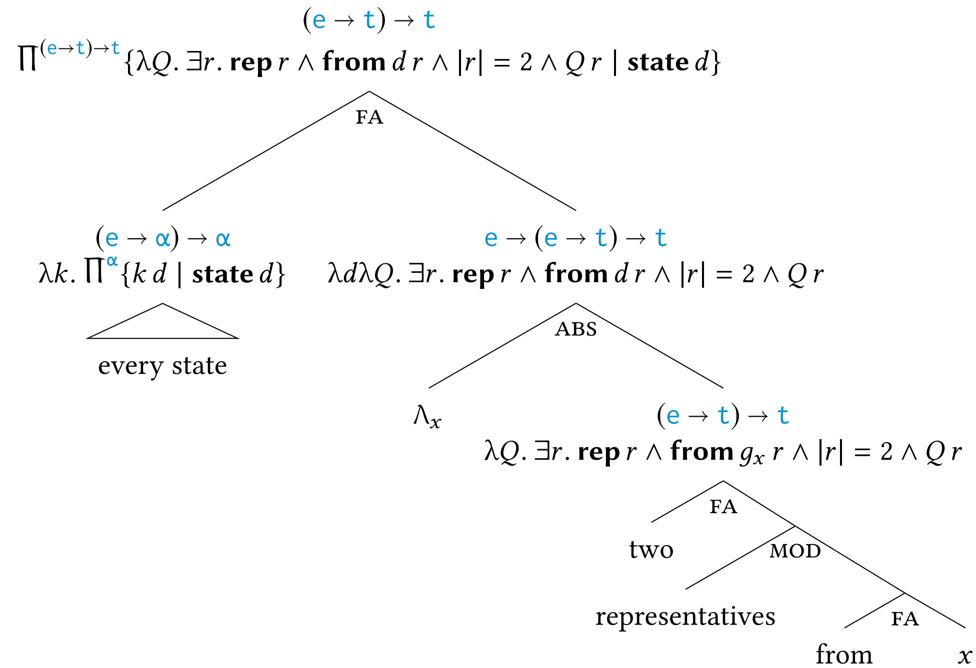
Note that, as promised in Section 1, the distributive universal in the example takes scope only over its own host DP. This much is similar to analyses in the tradition of May 1985, like Heim & Kratzer's (1998) and Büring's (2004). But unlike those accounts, the universal quantifier here is not type-shifted in any way, the mode of combination that combines the object with its scope is simply Function Application, and the result is simply a kind of entity, with no arguments left to saturate. This is a consequence of taking the host itself to denote a kind of entity, so that distributive conjunction reduces to entity-agglomeration. Similar LFs will deliver the appropriate readings for all of the sentences in (5). In each case, a universal takes a (possibly indeterminate) entity-valued function as argument and returns a (possibly indeterminate) plurality as a result.

Still, because the universal must take scope over its host in order for the nominal witnesses to co-vary with the universal's restrictor, we predict the scope and binding effects introduced at the end of Section 2. If the universal is trapped inside an island within the host, a collective pair-list interpretation will be impossible. And whenever a collective pair-list interpretation does arise, the universal will not be able to bind a pronoun outside of the DP it lives in.

Before moving on to discuss possible extensions of the technique, let me try and be clear about the interplay of polymorphism and the universal's host. The denotation of (39), and the resulting truth conditions of (40), depend on both the *type* α of the universal's scope and the *instance of conjunction* $\sqcap^\alpha$ made available by the lexicon for that type. For instance, if the indefinite subject in (39) were analyzed as a Generalized Quantifier, with GQ conjunction defined as intersection, as in (36c), then the derivation would be effectively equivalent to the type-shifting analyses of Heim & Kratzer 1998 and Büring 2004. This is shown in (41). As expected, the resulting truth conditions are fully distributive; presumably not what the framers intended, but certainly a possible reading of the sentence.

(41) two representatives from every state

a.



b. $\prod^{(e\to t)\to t}\{\lambda Q.\, \exists r.\, \mathbf{rep}\, r \wedge \mathbf{from}\, d\, r \wedge |r| = 2 \wedge Q\, r \mid \mathbf{state}\, d\}$
$= \lambda Q.\, \bigwedge\{\exists r.\, \mathbf{rep}\, r \wedge \mathbf{from}\, d\, r \wedge |r| = 2 \wedge Q\, r \mid \mathbf{state}\, d\}$
$= \lambda Q.\, \forall d.\, \mathbf{state}\, d \rightarrow \exists r.\, \mathbf{rep}\, r \wedge \mathbf{from}\, d\, r \wedge |r| = 2 \wedge Q\, r$
$= \lambda Q.\, [\![\text{every state}]\!]^g \left(\lambda d.\, [\![\text{two states from } x]\!]^{g^{x\mapsto d}}\, Q\right)$

But collective notions of Generalized Quantifier conjunction have also been entertained (see Winter 2001 for a careful study of some options). For instance, Hoeksema (1983, 1988) suggests that GQs might be conjoined by "lifting" them over collective arguments, as in (42). Leaving the LF in (41a) exactly as it is, but using the notion of conjunction in (42) instead of the intersective notion from (36c), would again yield the (intended) collective truth conditions of (40).

(42) $$\prod^{(e\to t)\to t} := \lambda M.\, m_1 \left(\lambda a_1.\, m_2 \left(\lambda a_2.\, \cdots m_n \left(\lambda a_n.\, \prod^{e}\{a_1, a_2, \ldots, a_n\}\right)\right)\right)$$
$$\textbf{where } M = \{m_1, m_2, \ldots, m_n\}$$

Yet another route to the collective reading arises in the context of choice-functional indefinites (see again Winter 2001 for an extensive study). For instance, let 'two' be a pronoun over pair-valued choice functions, such that for any property Q, $([\![\text{two}]\!]\, Q) \in Q \times Q$. Then using the ordinary (determinate) conjunction for entities $\prod^{e}$, the LF in (39) would denote an ordinary (determinate) sum of representative pairs, two per state. Which sum it represents would of course depend on which choice function $[\![\text{two}]\!]$ is resolved to, and whether the entire sentence in (40) is true would presumably depend on existentially closing over this choice.

The point here is that it doesn't much matter what particular analysis of (in)definite semantics one ascribes to the host DPs in (1) and (5), and to some extent it doesn't even matter what types one assigns them. As long as the grammar provides a collective way to combine objects of that type, the polymorphic lexical entry for universals will derive the correct collective truth conditions. That is, as long as there is some notion of conjunction under which (43a) can be made sense of, then a polymorphic ‘every’ will make sense of (43b) as well.

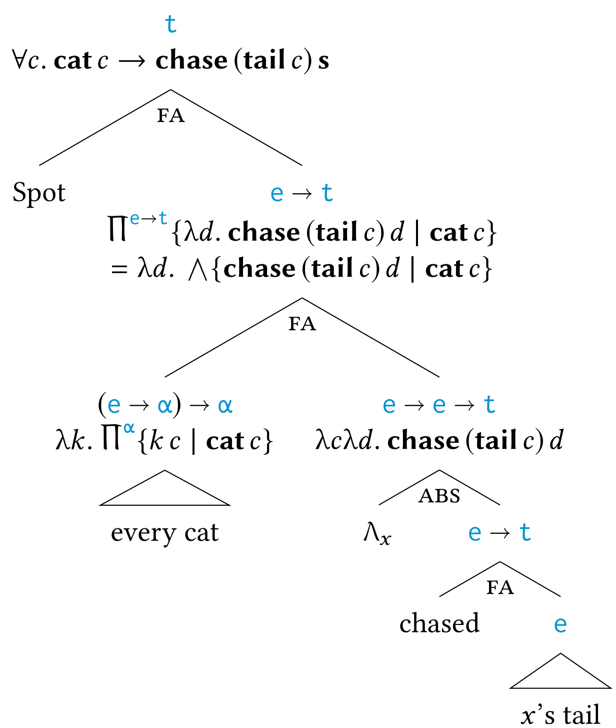
(43) a. Two representatives from Indiana and two representatives from California comprise the House Subcommittee on Technology Modernization
b. Two representatives from every state comprise the Senate

In contrast, any grammar in which the denotational options for a universal DP like ‘every state’ are the Generalized Quantifier ($\lambda Q.\, \forall x.\, \textbf{state}\, x \to Q\, x$) and perhaps the maximal collection of states $\oplus\{d \mid \textbf{state}\, d\}$ will continue to undergenerate readings for (43b).

4 Extensions

A natural question to ask then is what other kinds of constituents such a polymorphic distributor can scope over. That in turn depends entirely on how many types α the conjunction operation $\prod^{\alpha}$ is defined for. Since the Partee & Rooth 1983 instances in (36c) define conjunction for any “type ending in $\mathtt{t}$”, distributors should have no trouble scoping over a predicative or relational constituent, as in (44). Because of the recursive nature of the Partee & Rooth definition, this reduces to familiar recursive generalizations of Boolean GQs as in, e.g., Keenan & Moss 2016.

(44) Spot chased every cat’s tail



But just as there are reasonable non-Boolean products of entities, there could in principle be non-Boolean products of relations. Link (1983) and Krifka (1990) offer such operations based on sentences like (45), where the individual predicates are understood to describe different parts of the subject. Lasersohn (1995) and Winter (2001) find these cases unconvincing, arguing that they are better analyzed as instances of entity conjunction with mass terms, along the lines of (46). I will set these sorts of examples aside, and leave it to scholars of mereology to decide whether it is worth hunting for similar instances with universal distributors instead of conjunctions.

(45) a. The flag is green and white
b. This is beer and lemonade

(46) a. Green and white is my favorite combination of colors
b. Beer and lemonade is my favorite combination of drinks

More recently, some researchers have set sights on the surprising *cumulative* readings of distributors that were mentioned earlier, due to Schein (1993) and Kratzer (2000). Recall that the relevant reading of (47) is one on which the number of games played across quarterbacks is three, but the number of plays learned might be as many as twice the number of players.

(47) Three video games taught every quarterback two new plays

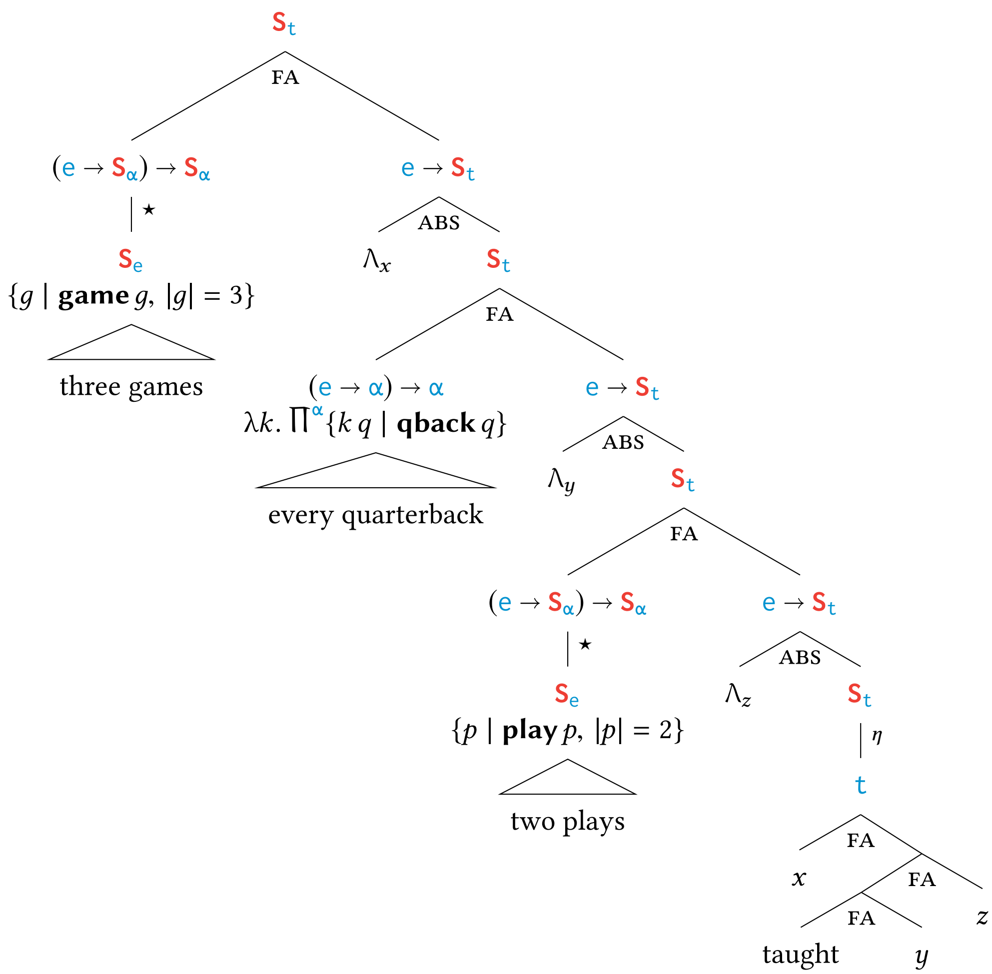
The Schein sentence *seems* tantalizingly similar to the Constitution sentence that kicked things off in (1). In both cases, a distributive universal induces co-variation with a nearby indefinite, yet does not distribute over anything else in the sentence. But as discussed in Section 3.1, existing analyses of the former do not explain the latter. And in fact nothing said here about the Constitution predicts the relevant reading of the Schein sentence. Just as in (44), with *Generalized Conjunction* as the multiplicative mode for property conjunction, the two LFs in (48) are denotationally equivalent. When $\prod^{\mathsf{e}\to\mathsf{t}}$ simply passes the eventual subject, the games, in to each quarterback's nuclear scope, it doesn't matter whether the universal scopes above or below the trace of the subject — that is, over the whole sentence, or over just the VP.[8,9] The resulting truth conditions for both (48a) and (48b) are met whenever some trio of games is such that every quarterback played all three games and acquired a couple of (possibly distinctive) plays cumulatively from the three games. But the reading of interest allows quarterbacks to vary in which game(s) they learn their plays from, so long as the total number of games across quarterbacks is three.

8 I am assuming predicate denotations are lexically cumulative: $\mathbf{R}\,X\,Y$ holds iff every atomic $y \ll Y$ stands in the $\mathbf{R}$ relation to some atomic $x \ll X$, and every atomic $x \ll X$ is stood in the $\mathbf{R}$ relation to by some atomic $y \ll Y$.

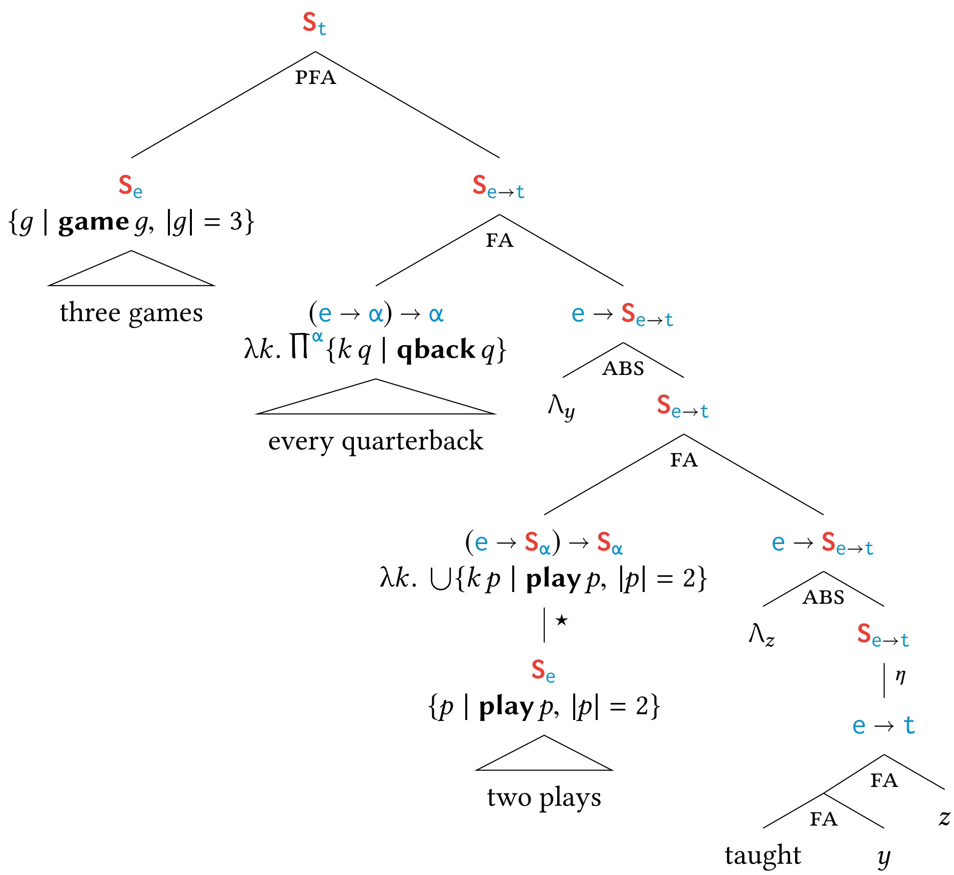
9 The PFA mode of combination at the root of (48b) stands for Pointwise Function Application. It is an LF abbreviation:

$$\left[\!\!\left[\begin{array}{ccc} & \mathsf{S}_{\tau} & \\ & \text{PFA} & \\ \mathsf{S}_{\sigma} & & \mathsf{S}_{\sigma\to\tau} \\ \varphi & & \psi \end{array}\right]\!\!\right]^g := \left[\!\!\left[\left[\varphi^\star \,\Lambda_x\, \left[\psi^\star \,\Lambda_f\, [f\ x]^\eta\right]\right] \right]\!\!\right]^g = \left\{h\,d \;\middle|\; d \in [\![\varphi]\!]^g,\ h \in [\![\psi]\!]^g\right\}$$

(48) a.


S_t
FA
(e → S_α) → S_α
⋆
S_e
{g | **game** g, |g| = 3}
three games
e → S_t
ABS
Λ_x
S_t
FA
(e → α) → α
λk. ∏^α {k q | **qback** q}
every quarterback
e → S_t
ABS
Λ_y
S_t
FA
(e → S_α) → S_α
⋆
S_e
{p | **play** p, |p| = 2}
two plays
e → S_t
ABS
Λ_z
S_t
η
t
FA
x
FA
FA
taught
y
z

b.



Whether or not the relevant reading can be derived from this semantics depends on whether there is another instance of $\prod^{\mathsf{e}\to\mathsf{t}}$, one based on a cumulative notion of property conjunction rather than the distributive (intersective) notion underlying (36c). In fact, such an operation has recently been proposed by Haslinger & Schmitt (2018). Their analysis is ultimately aimed at just the example at hand, but builds on the interesting and extensive compositional framework for plurality in Schmitt 2017. Here, I just want to point out the minimum assumption about conjunction needed to bag the Schein sentence, given the polymorphism inherent in 'every'.

Starting with the LF in (48b), the denotation after 'every quarterback' has composed with its scope is given schematically in (49), assuming the relevant set of quarterbacks is $\{\mathbf{q}_1, \ldots, \mathbf{q}_n\}$.

(49) $\{(\textbf{teach}\, \mathbf{q}_1\, Z_1) \sqcap (\textbf{teach}\, \mathbf{q}_2\, Z_2) \sqcap \cdots \sqcap (\textbf{teach}\, \mathbf{q}_n\, Z_n) \mid Z_1, \ldots, Z_n \text{ are play-pairs}\}$

What this denotation actually amounts to depends on how property combination $\sqcap$ is understood. Let us say that a sum of properties P applies (cumulatively) to a sum of entities X whenever each atomic property $p \ll P$ holds of some collection of the entities $X' < X$, and every entity (atomic or otherwise) $x < X$ participates in at least one of the properties $p \ll P$. This is very close to what Haslinger & Schmitt call *cumulative composition*. We can in fact define cumulative property combination from this:

(50) For any possibly complex P and Q,

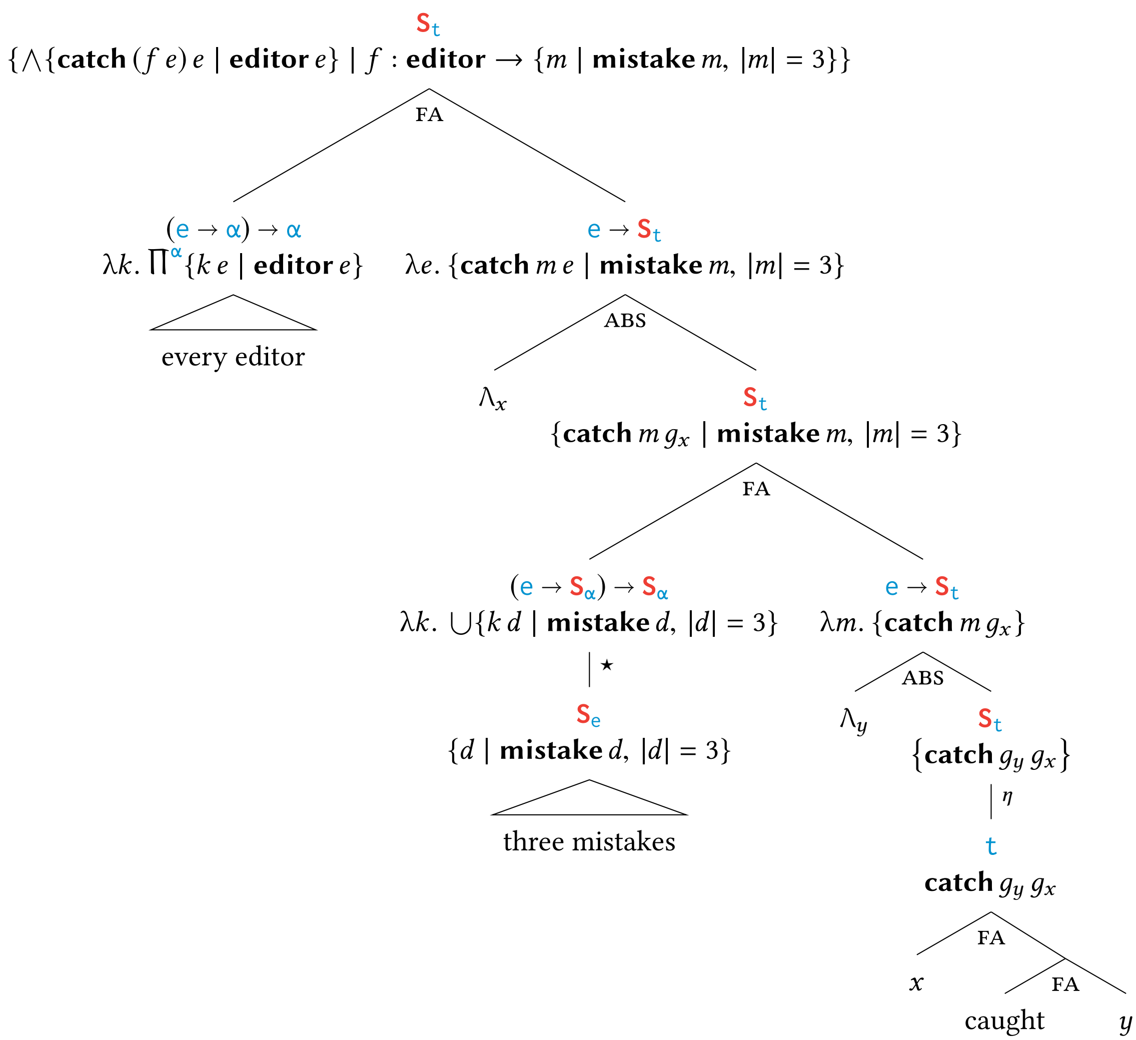
$$P \sqcap Q := \lambda X.\, \mathbf{T} \text{ iff both } \begin{array}{ll} \text{(i)} & \forall p \in (\textbf{atoms}\, P \cup \textbf{atoms}\, Q).\, \exists X' < X.\, p\, X' \\ \text{(ii)} & \forall X' < X.\, \exists p \in (\textbf{atoms}\, P \cup \textbf{atoms}\, Q).\, p\, X' \end{array}$$

(51) $\prod^{\mathsf{e}\to\mathsf{t}} := \lambda P.\, \bigsqcap P$

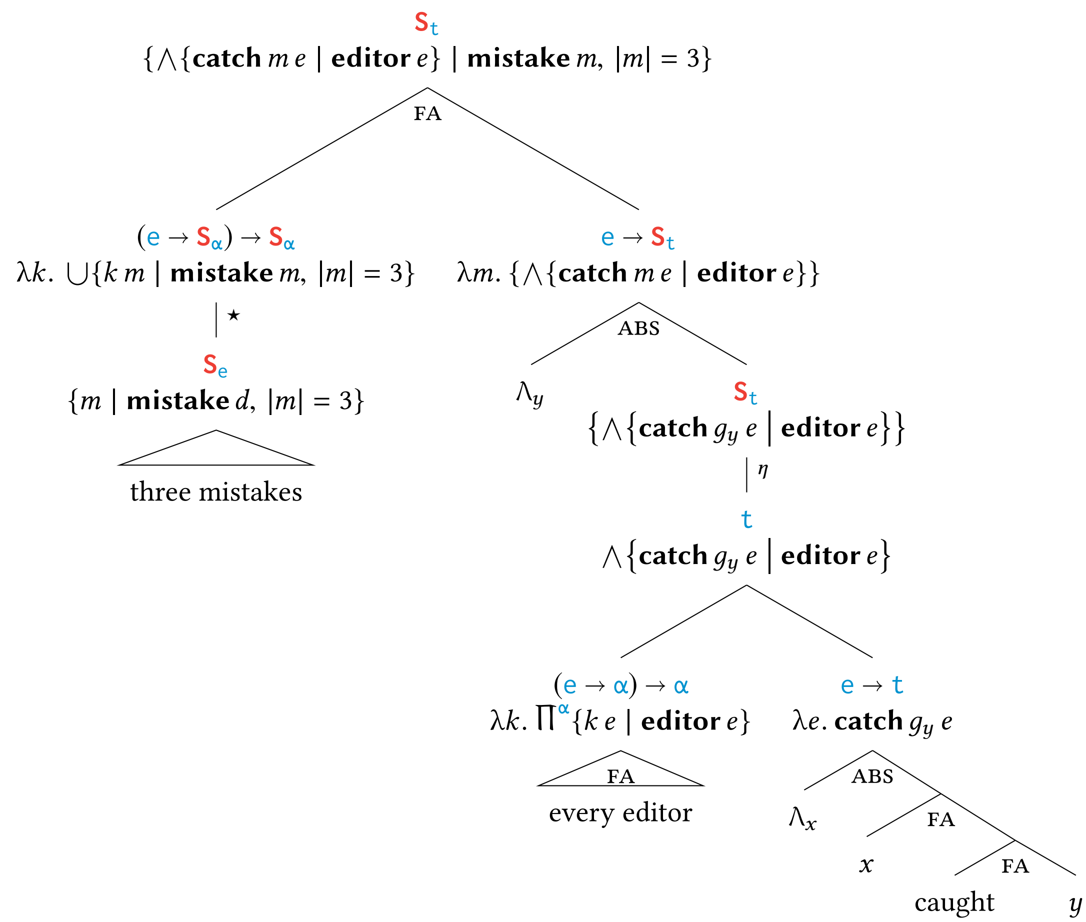
The parametric operation in (37) then determines a lifted instance of $\prod^{\mathsf{S}_{e\to t}}$ to indeterminate sets of such properties. And this instance yields the denotation in (49), fixed with the definition of $\sqcap$ in (50). The sentence is expected to be true on this cumulative reading iff there's a game triple Z such that for each quarterback q, some collection of games $Z' < Z$ is responsible for teaching q a couple of plays, so long as all of the games in Z were useful to at least one quarterback.

So this resolves the *undergeneration* issue posed by the sentences in (24b) and (24c). That is, the Haslinger & Schmitt-adjacent definition of property conjunction is all that's needed to derive the appropriate cumulative truth conditions for the Schein sentence. The other half of the empirical puzzle is preventing the *overgeneration* of such readings for, e.g., distributors in subject position. I will not pursue this challenge in any depth, except to say that it is in fact not obvious to me how the offending reading of (24a) *could* be generated, given what has been said so far. Consider the two conceivable LFs for (24a) depicted in (52).

(52) a.



b.



Even in (52b), where the indefinite object takes scope over the distributive subject, there is no cumulativity between the two DPs. The reason is twofold. On the one hand, in both LFs, the nuclear scope of the universal traffics in truth values, be they ordinary (52b) or indeterminate (52a), and the only instance of $\prod^{\mathsf{t}}$ is Boolean conjunction.

But it also wouldn't change matters here if one were to define a type-t algebra to model proposition-collections or sum such. The wide scope of the indefinite merely guarantees that the editors were uniform in their mistake-catching. That is, for some triple of errors X, Editor 1 caught X "and" Editor 2 caught X "and" Editor 3 caught X, etc. Each "conjunct" relates an individual editor to X, a sum of mistakes. It is irrelevant how the truth values (or propositions) are combined, as long as the lexical semantics of 'catch' guarantees that for an *individual* to catch a plurality of errors, that individual has gotta catch 'em all. If so, the sentence will entail that every editor individually caught all three mistakes, not that the three were caught between them. It remains to be seen whether this line of analysis predicts, or is even consistent with, the full range of asymmetries in cumulativity, whatever the facts turn out to be.[10]

Finally, I'd like to return to the fundamental asymmetry between distributive and plural universals, repeated in (53).

(53) a. ✓The students collided in the middle of the field
b. #Each student collided in the middle of the field

10 It should be said that Haslinger & Schmitt (2018) also derive the asymmetry in cumulative potential between subject- and object-universals. However, like Brasoveanu (2013), they attribute the asymmetry to the universal's differing semantic scope in the two syntactic configurations, but do not discuss the possibility of scope inversion. It is moreover not immediately clear how to incorporate typical scope-shifting mechanisms, given the novel mode of plural composition that they pursue.

The denotation of 'each' proposed here is still distributive, in the sense that it maps a property across the elements of its restrictor, one by one, and then multiplies the results. So it would seem that the basic infelicity in (53b) is still predicted. But technically — now that entities, indeterminate or otherwise, can be multiplied — it is possible for the universal to scope over *nothing*. In terms of LFs, this would amount to maximally "short" movement of the universal. In semantic terms, this would correspond to a trivial delimitation of the universal's continuation. The result is the total collection of campers, exactly the sort of thing that 'the campers' should denote.

(54)

$$\begin{array}{c} \mathsf{e} \\ \bigoplus\{d \mid \textbf{camper}\, d\} \end{array}$$

FA

$$\begin{array}{c} (\mathsf{e} \to \alpha) \to \alpha \\ \lambda k.\, \prod^{\alpha}\{k\, d \mid \textbf{camper}\, d\} \end{array} \qquad \begin{array}{c} \mathsf{e} \to \mathsf{e} \\ \lambda d.\, d \end{array}$$

each camper $\qquad \Lambda_x$ ABS x

So for the analysis here to have any purchase on the fundamental contrast in (53), constituents or derivations like (54) need to be ruled out. The idea is simple enough: a distributive quantifier should distribute over *something* (cf. Thomas & Sudo 2016). I leave it to future work to determine the precise nature that such a constraint ought to take, be it formal — perhaps something in the *antilocality* family — or pragmatic — perhaps rooted in competition with 'the'.

5 Conclusion

Sentences like (1) from the U.S. Constitution provide evidence that distributive universals do not always quantify over truth-denoting constituents. In a range of similar cases, I have argued the quantifiers are best thought of as using their restrictors to assemble a collection of associated individuals, and that these collections behave for linguistic purposes like ordinary plural expressions. Consequently, the lexical semantics of the distributor should be polymorphic enough to accommodate such uses, but unified enough to explain what the various uses have in common. I have provided one analysis, rooted initially in dynamic semantics and speech act theory, but generalized to include a greater variety of non-Boolean conjunctions, most pressingly those of entities.

References

Barker, Chris. 2012. Quantificational binding does not require c-command. *Linguistic Inquiry* 43(4). 614–633.

Barker, Chris & Chung-chieh Shan. 2014. *Continuations and natural language*. Vol. 53 (Oxford Studies in Theoretical Linguistics). Oxford University Press.

Bayer, Samuel Louis. 1997. *Confessions of a lapsed Neo-Davidsonian: Events and arguments in compositional semantics*. Taylor & Francis.

Beghelli, Filippo & Tim Stowell. 1997. Distributivity and negation: The syntax of *each* and *every*. English. In Anna Szabolcsi (ed.), *Ways of scope taking*, vol. 65 (Studies in Linguistics and Philosophy), 71–107. Dordrecht, The Netherlands: Kluwer Academic.

Belnap, Jr., Nuel D. 1982. Questions and answers in Montague Grammar. In Stanley Peters & Esa Saarinen (eds.), *Processes, beliefs, and questions*, vol. 16 (Synthese Language Library), 165–198. Springer Netherlands.

Bittner, Maria. 1994. Cross-linguistic semantics. *Linguistics and Philosophy* 17(1). 53–108.
Brasoveanu, Adrian. 2013. Modified numerals as post-suppositions. *Journal of Semantics* 30(2). 155–209. https://doi.org/10.1093/jos/ffs003.
Bumford, Dylan. 2015. Incremental quantification and the dynamics of pair-list phenomena. *Semantics and Pragmatics* 8(9). 1–70.
Büring, Daniel. 2004. Crossover situations. *Natural Language Semantics* 12(1). 23–62.
Champollion, Lucas. 2010. *Parts of a whole: Distributivity as a bridge between aspect and measurement*. University of Pennsylvania PhD Dissertation.
Champollion, Lucas. 2020. Distributivity, collectivity, and cumulativity. *The Wiley Blackwell Companion to Semantics*. 1–38.
Charlow, Simon. 2014. *On the semantics of exceptional scope*. New York, NY: New York University PhD Dissertation.
Charlow, Simon. 2019. The scope of alternatives: Indefiniteness and islands. *Linguistics and Philosophy*. 1–46.
Chatain, Keny. 2020. Cumulative readings of "every" and leaks. In Joseph Rhyne, Kaelyn Lamp, Nicole Dreier & Chloe Kwon (eds.), *Semantics and linguistic theory (SALT) 30*. Cornell University.
Coppock, Elizabeth & David Beaver. 2014. A superlative argument for a minimal theory of definiteness. In *Semantics and linguistic theory (SALT) 24*, 177–196. https://doi.org/10.3765/salt.v24i0.2432.
Dowty, David. 1987. Collective predicates, distributive predicates and *all*. In Fred Marshall, Ann Miller & Zheng-sheng Zhang (eds.), *3rd Eastern states conference on linguistics*, 97–115.
Flor, Enrico. 2017. Cumulative readings of italian *ogni*. Unpublished manuscript.
Haslinger, Nina & Viola Schmitt. 2018. Scope-related cumulativity asymmetries and cumulative composition. In *Semantics and linguistic theory (SALT) 28*, 197–216.
Heim, Irene & Angelika Kratzer. 1998. *Semantics in Generative Grammar*. Oxford: Blackwell.
Hoeksema, Jack. 1983. Plurality and conjunction. In Alice ter Meulen (ed.), *Studies in modeltheoretic semantics*, 63–83. Dordrecht: Reidel.
Hoeksema, Jack. 1988. The semantics of non-Boolean "and". *Journal of Semantics* 6(1). 19–40.
Keenan, Edward & Lawrence Moss. 2016. *Mathematical structures in language* (CSLI Lecture Notes). CSLI Publications.
Kratzer, Angelika. 2000. The event argument and the semantics of verbs. Chapter 2.
Kratzer, Angelika & Junko Shimoyama. 2002. Indeterminate pronouns: The view from Japanese. In Yukio Otsu (ed.), *Third Tokyo conference on psycholinguistics*, 1–25. Tokyo. https://doi.org/10.1007/978-3-319-10106-4_7.
Krifka, Manfred. 1990. Boolean and non-Boolean 'and'. In *Papers from the second symposium on logic and language*, 161–188.
Krifka, Manfred. 1999. At least some determiners aren't determiners. *The semantics/pragmatics interface from different points of view* 1. 257–291.
Krifka, Manfred. 2001. Quantifying into question acts. *Natural Language Semantics* 9(1). 1–40.
Krifka, Manfred. 2015. Bias in commitment space semantics: Declarative questions, negated quetions, and question tags. In *Semantics and linguistic theory (SALT) 25*, 328–345.
Landman, Fred. 2004. *Indefinites and the type of sets*. Oxford: Blackwell.
Lasersohn, Peter. 1995. *Plurality, conjunction, and events*. Vol. 55 (Studies in Linguistics and Philosophy). Springer Dordrecht.
Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In Rainer Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, use, and interpretation of language* (Foundation of Communication), 302–323. New York: Walter de Gruyter.
Matthewson, Lisa. 2001. Quantification and the nature of crosslinguistic variation. *Natural Language Semantics* 9(2). 145–189.

May, Robert. 1985. *Logical form: Its structure and derivation*. Cambridge, MA: MIT Press.

Milsark, Gary. 1974. *Existential sentences in English*. Massachusetts Institute of Technology PhD Dissertation.

Moltmann, Friederike & Anna Szabolcsi. 1994. Scope interactions with pair-list quantifiers. In *North east linguistic society (NELS) 24*, vol. 24, 381–395.

Partee, Barbara. 1986. Noun phrase interpretation and type-shifting principles. In Jeroen Groenendijk, Dick de Jongh & Martin Stokhof (eds.), *Studies in Discourse Representation Theory and the theory of generalized quantifiers*, 115–144. Dordrecht: Foris. https://doi.org/10.1002/9780470751305.ch10.

Partee, Barbara & Mats Rooth. 1983. Generalized conjunction and type ambiguity. In Rainer Bäuerle, Christoph Schwarze & Arnim von Stechow (eds.), *Meaning, use, and interpretation of language*, 361–384. Berlin: Walter de Gruyter.

Schein, Barry. 1993. *Plurals and events*. Cambridge, MA: MIT Press.

Schmitt, Viola. 2017. Cross-categorial plurality and plural composition. Unpublished ms, University of Vienna.

Schmitt, Viola. 2020. Boolean and non-Boolean conjunction. *The Wiley Blackwell Companion to Semantics*. 1–32.

Shan, Chung-chieh. 2001. Monads for natural language semantics. In Kristina Striegnitz (ed.), *ESSLLI 2001 student session*, 285–298.

Szabolcsi, Anna. 1997. Quantifiers in pair-list readings. In Anna Szabolcsi (ed.), *Ways of scope taking* (Studies in Linguistics and Philosophy), chap. 9, 311–347. Springer Netherlands.

Thomas, Guillaume & Yasutada Sudo. 2016. Cumulative readings of *each*. Presentation at the Workshop on (Co-)Distributivity. Paris.

Wadler, Philip. 1992. The essence of functional programming. In *19th ACM SIGPLAN-SIGACT symposium on principles of programming languages* (POPL '92), 1–14. Albuquerque, NM.

Winter, Yoad. 2001. *Flexibility principles in Boolean semantics*. Vol. 37 (Current Studies in Linguistics). Cambridge, MA: MIT Press.

Zweig, Eytan. 2008. *Dependent plurals and plural meaning*. New York University PhD Dissertation.