$$m \,/\, n := \begin{cases} m\, n & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k .\, m\left(\lambda f .\, n\left(\lambda x .\, k\,(f \,/\, x)\right)\right) & \textbf{otherwise} \end{cases}$$

$$m \setminus n := \begin{cases} n\, m & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k .\, m\left(\lambda f .\, n\left(\lambda x .\, k\,(f \setminus x)\right)\right) & \textbf{otherwise} \end{cases}$$

$$\begin{aligned} \eta\, x &:= \lambda g .\, \left\{\langle x, g\rangle\right\} \\ m \star f &:= \lambda g .\, \bigcup\left\{ f\, x\, g' \,\middle|\, \langle x, g'\rangle \in m\, g\right\} \\ m^{\downarrow} &:= m\left(\lambda x s .\, \left\{\langle x, s\rangle \,\middle|\, x \neq \mathsf{F}\right\}\right) \\ x^{\uparrow} &:= \lambda k .\, \eta\, x \star k \end{aligned}$$

$$\begin{aligned} \textbf{the}_u &:= \lambda c k g .\, |G_u| = 1 .\, G, \\ &\quad \text{where } G = \bigcup\left\{ k\, x\, g' \,\middle|\, \langle \mathsf{T}, g'\rangle \in c\, x\, g^{u \mapsto x}\right\} \\ \textbf{circle} &:= \mathsf{circ} \\ \textbf{square} &:= \mathsf{sq} \\ \textbf{in} &:= \mathsf{in} \end{aligned}$$

- $\textbf{the}_u$ is just $\textbf{a}_u$ plus a uniqueness presupposition
- The presupposition restricts what the *set* of outputs is allowed to look like: they need to all agree on the value of u
- That means its uniqueness effect is *delayed* until some program containing it is evaluated, (i.e., until its continuation is delimited). This sort of delayed global test on the set of outputs is very much like a postsupposition (Brasoveanu 2012, Henderson 2014), but here it is regulated by continuations, rather than logical subscripts (Charlow 2014)

$$[\![\text{the square}]\!] \rightsquigarrow \left(\frac{\textbf{the}_v\,(\lambda y .\, [\,])}{y} \;\Big\backslash\; \frac{[\,]}{\textbf{square}}\right)^{\downarrow\downarrow\star} \rightsquigarrow \left(\textbf{the}_v\left(\lambda y g .\, \left\{\langle \mathsf{sq}\, y, g\rangle\right\}\right)\right)^{\downarrow\star}$$

$$\rightsquigarrow \left(\frac{\lambda g .\, \bigcup\left\{[\,]\, g^{v \mapsto y} \,\middle|\, \mathsf{sq}\, y\right\}}{y}\right)^{\downarrow\star} \rightsquigarrow \frac{\lambda g \colon |G_v| = 1 .\, \bigcup\left\{[\,]\, g^{v \mapsto y} \,\middle|\, \mathsf{sq}\, y\right\}}{y}$$

- Note that resetting $[\![\text{the square}]\!]$ in the last reduction step here has no effect on its semantic shape, because it's essentially $[\![\text{a square}]\!]$
- But it does fix the presupposition; For any input g, $G = \left\{\langle y, g^{v \mapsto y}\rangle \,\middle|\, \mathsf{sq}\, y\right\}$, and the presup will require all those g's to map v to the same square (which is only possible if there's exactly one square available to assign v to in the first place)

$[\![\text{the circle in the square}]\!]$

$$\left(\frac{\textbf{the}_u\,(\lambda x .\, [\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Big|\; \frac{[\,]}{\textbf{in}} \;\Big/\; \left(\frac{\textbf{the}_v\,(\lambda y .\, [\,])}{y} \;\Big\backslash\; \frac{[\,]}{\textbf{square}}\right)^{\downarrow\downarrow\star}\right)\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\textbf{the}_u\,(\lambda x .\, [\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Big|\; \frac{[\,]}{\textbf{in}} \;\Big/\; \frac{\lambda g \colon |G_v| = 1 .\, \bigcup\left\{[\,]\, g^{v \mapsto y} \,\middle|\, \mathsf{sq}\, y\right\}}{y}\right)\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\textbf{the}_u\left(\lambda x g \colon |G_v| = 1 .\, \bigcup\left\{[\,]\, g^{v \mapsto y} \,\middle|\, \mathsf{sq}\, y\right\}\right)}{\mathsf{circ}\, x \wedge \mathsf{in}\, y\, x}\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\lambda g .\, \bigcup\left\{[\,]\, g^{\substack{u \mapsto x \\ v \mapsto y}} \,\middle|\, \mathsf{sq}\, y,\ \mathsf{circ}\, x,\ \mathsf{in}\, y\, x\right\}}{x}\right)^{\downarrow\star}$$

$$\frac{\lambda g \colon |G'_u| = |G_v| = 1 .\, \bigcup\left\{[\,]\, g^{\substack{u \mapsto x \\ v \mapsto y}} \,\middle|\, \mathsf{sq}\, y,\ \mathsf{circ}\, x,\ \mathsf{in}\, y\, x\right\}}{x}$$

Absolute Reading

$\vdots$

- Once again, resetting the outer DP just fixes its presupposition. This time all the outputs need to agree on v, which is only possible if there's exactly one circle in the square that all the outputs now assign to u

⟦the circle in the square⟧

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Bigg\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Bigg|\; \frac{[\,]}{\textbf{in}} \;\Bigg/\; \left(\frac{\textbf{the}_v\,(\lambda y\,.\,[\,])}{y} \;\Bigg\backslash\; \frac{[\,]}{\textbf{square}}\right)^{\downarrow}\right)\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Bigg\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Bigg|\; \frac{[\,]}{\textbf{in}} \;\Bigg/\; \frac{\lambda g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\textbf{the}_u\,\left(\lambda x g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\downarrow\star}$$

$$\left(\frac{\lambda g\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\downarrow\star}$$

$$\frac{\lambda g\colon |G_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

Relative Reading (cf. Haddock, Champollion and Saurland)

- The only difference here is that we do not reset the inner DP, which staves off its presupposition until more information is accumulated in its scope
- But now when the outer DP is reset, it sets the presuppositions of *both* definites
- For any input g, $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \;\middle|\; \mathsf{sq}\,x,\ \mathsf{circ}\,y,\ \mathsf{in}\,y\,x\right\}$ is the set out outputs that map u onto a circle in some square that it maps to v.
- So requiring that there be exactly one such v is tantamount to requiring that there be exactly one square *that has a circle in it* and exactly one circle *in that square*. In other words, there should be exactly one pair $\langle x, y\rangle$ in $\mathsf{circ} \times \mathsf{sq}$ such that *in* $y\,x$.

$$\mathsf{M}_u^f \coloneqq \lambda G.\left\{\langle \cdot, g\rangle \in G \,\middle|\, \neg\exists\langle \cdot, g'\rangle \in G.\, g'\,u > g\,u\right\}$$

$$
\begin{aligned}
\mathbf{M}_u^f &= \lambda G\,.\,\left\{\langle \alpha, g\rangle \,\middle|\, \langle\langle \alpha, \cdot\rangle, g\rangle \in G,\ \ \mathsf{truthy}\,\alpha,\ \ \neg\exists\big\langle\langle \cdot, \beta\rangle, g'\big\rangle \in G\,.\,\bigvee\beta \wedge f\,(g\,u)\,(g'\,u)\right\} \\
\textbf{larger} &= \lambda x y\,.\,\mathsf{size}\,x < \mathsf{size}\,y \\
\textbf{est}_u &= \lambda f\,.\,\mathbf{M}_u^f \\
\textbf{largest}_u &= \textbf{est}_u\,\textbf{larger} = \mathbf{M}_u^{\mathsf{sz}} = \lambda G\,.\,\left\{\langle \mathsf{T}, g\rangle \,\middle|\, \langle \mathsf{T}, g\rangle \in G \wedge \neg\exists\langle \mathsf{T}, g'\rangle \in G\,.\,\mathsf{size}\,(g\,u) < \mathsf{size}\,(g'\,u)\right\} \\
\textbf{the}_u &= \lambda \mathcal{M}ckg\colon |G'_u| = 1\,.\,G',\ \text{where } G' = \mathcal{M}\bigcup\left\{k\,x\,g' \,\middle|\, x \in \mathcal{D}_e,\ \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\}
\end{aligned}
$$

This is some stuff

$$
\begin{array}{c}
[\,] \\ \hline
\mathsf{j}^{\mathsf{F}} \star (\lambda j\,.\,[\,]) \\ \hline
j^{\triangleright} \star (\lambda x\,.\,[\,]) \\ \hline
x
\end{array}
\Bigg/
\begin{array}{c}
[\,] \\ \hline
[\,] \\ \hline
[\,] \\ \hline
\mathsf{drew}
\end{array}
\Bigg\backslash
\begin{array}{c}
\lambda g\,.\,\mathbf{M}_u \bigcup\{[\,]\,g^{u\mapsto y} \mid \mathsf{sq}\,y\} \\ \hline
[\,] \\ \hline
[\,] \\ \hline
y
\end{array}
$$

stuff

$$
\begin{aligned}
\mathcal{F}\,\alpha &\coloneqq \sigma \to \{\alpha * \sigma\} * \{\alpha * \sigma\} \\
\eta\, x &\coloneqq \lambda g\,.\left\langle \left\{\langle x, g\rangle\right\}, \left\{\langle x, g\rangle\right\}\right\rangle \\
m \star f &\coloneqq \lambda g\,.\left\langle \bigcup\left\{(f\,x\,g')_1 \,\middle|\, \langle x, g'\rangle \in (m\,g)_1\right\},\ \bigcup\left\{(f\,x\,g')_2 \,\middle|\, \langle x, g'\rangle \in (m\,g)_2\right\}\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\textbf{the} &\coloneqq \lambda \mathcal{M} c k g : |G_u| = 1\,.\, G_u,\ \text{where } G = \mathcal{M} \bigcup\left\{k\,y\,g' \,\middle|\, \langle \mathsf{T}, g'\rangle \in c\,y\,g^{u\mapsto y}\right\} \\
\textbf{only} &\coloneqq \lambda G : \mathrm{true}\,G\,.\left\{\langle \mathsf{T}, g\rangle \in G \,\middle|\, \neg\exists g'\,.\,\langle \mathsf{T}, g'\rangle \in G \wedge g'\,u \sqsupset g\,u\right\}
\end{aligned}
$$