$$m \,/\, n := \begin{cases} m\,n & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k\,.\, m\left(\lambda f\,.\, n\left(\lambda x\,.\, k\,(f \,/\, x)\right)\right) & \textbf{otherwise}\end{cases}$$

$$m \setminus n := \begin{cases} n\,m & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k\,.\, m\left(\lambda x\,.\, n\left(\lambda f\,.\, k\,(x \setminus f)\right)\right) & \textbf{otherwise}\end{cases}$$

The usual applicators for left and right continuized FA

State.Set monad combinators for managing drefs

A slightly tweaked lower operator to filter out falsity (more like traditional standard dynamic semantics). This is not essential, but makes the bookkeeping a little smoother.

$$\eta\, x := \lambda g\,.\,\left\{\langle x, g\rangle\right\}$$
$$m^\star := \lambda k g\,.\,\bigcup\left\{k\,x\,g' \,\middle|\, \langle x, g'\rangle \in m\,g\right\}$$
$$m^\downarrow := m\left(\lambda x s\,.\,\{\langle x, s\rangle \mid x \neq \mathsf{F}\}\right)$$
$$x^\uparrow := (\eta\,x)^\star = \lambda k\,.\,k\,x$$
$$x^{\downharpoonleft\upharpoonright} := \left(x^\downarrow\right)^\star$$

$$\begin{aligned} \textbf{circle} &:= \mathsf{circ} \\ \textbf{square} &:= \mathsf{sq} \\ \textbf{in} &:= \mathsf{in} \\ \textbf{the}_u &:= \lambda c k g \colon |G_u| = 1\,.\,G, \\ &\qquad \text{where } G = \bigcup\left\{k\,x\,g' \,\middle|\, \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\} \end{aligned}$$

- $\textbf{the}_u$ is just $\mathbf{a}_u$ plus a uniqueness presupposition. The presupposition restricts what the *set* of outputs is allowed to look like: they need to all agree on the value of u
- That means its uniqueness effect is *delayed* until some program containing it is evaluated, (i.e., until its continuation is delimited). This sort of delayed global test on the set of outputs is very much like a postsupposition (Brasoveanu 2012, Henderson 2014), but here it is regulated by continuations, rather than logical subscripts (Charlow 2014)

$$\begin{aligned} [\![\text{the square}]\!] &\rightsquigarrow \left(\frac{\textbf{the}_v\,(\lambda y\,.\,[\,])}{y} \;\Big\backslash\; \frac{[\,]}{\textbf{square}}\right)^{\downarrow\,\downharpoonleft\upharpoonright} \rightsquigarrow \left(\textbf{the}_v\left(\lambda y g\,.\,\left\{\langle \mathsf{sq}\,y, g\rangle\right\}\right)\right)^{\downharpoonleft\upharpoonright} \\ &\rightsquigarrow \left(\frac{\lambda g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)^{\downharpoonleft\upharpoonright} \rightsquigarrow \frac{\lambda g \colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y} \end{aligned}$$

- Note that resetting $[\![\text{the square}]\!]$ in the last reduction step here has no effect on its semantic shape, because it's essentially $[\![\text{a square}]\!]$.
- But it does fix the presupposition; For any input g, G will be equal to $\left\{\langle y, g^{v\mapsto y}\rangle \,\middle|\, \mathsf{sq}\,y\right\}$, and the presup will require all those g's to map v to the same square (which will only be possible if there's exactly one square available to assign v to in the first place).

$[\![\text{the circle in the square}]\!]$

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Bigg\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Bigg|\; \frac{[\,]}{\textbf{in}} \;\Bigg/\; \left(\frac{\textbf{the}_v\,(\lambda y\,.\,[\,])}{y} \;\Big\backslash\; \frac{[\,]}{\textbf{square}}\right)^{\downarrow\,\downharpoonleft\upharpoonright}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Bigg\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\Bigg|\; \frac{[\,]}{\textbf{in}} \;\Bigg/\; \frac{\lambda g \colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\textbf{the}_u\,\left(\lambda x g \colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\lambda g \colon |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\downharpoonleft\upharpoonright}$$

$$\frac{\lambda g \colon |G'_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

Absolute Reading

- The inner definite is reset, freezing its presupposition as above. When the input assignment g is eventually inserted, we will have $G = \left\{\langle y, g^{v\mapsto y}\rangle \,\middle|\, \mathsf{sq}\,y\right\}$, and the presupposition will guarantee that $g'\,v$ is constant across the outputs.
- As with the inner DP, the host DP's presupposition is fixed when it is reset. This time, we have $G' = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}$, where g is whatever the input happens to be. In particular, all the outputs will now need to agree on the value of u in addition to v, which will only be possible if there's exactly one circle in the square that all outputs assign to v.

⟦the circle in the square⟧

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \,\middle\backslash\, \left(\frac{[\,]}{\textbf{circle}} \,\middle|\, \frac{[\,]}{\textbf{in}} \,\middle/\, \left(\frac{\textbf{the}_v\,(\lambda y\,.\,[\,])}{y} \,\middle\backslash\, \frac{[\,]}{\textbf{square}}\right)^{\downarrow}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \,\middle\backslash\, \left(\frac{[\,]}{\textbf{circle}} \,\middle|\, \frac{[\,]}{\textbf{in}} \,\middle/\, \frac{\lambda g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\textbf{the}_u\,\left(\lambda x g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\lambda g\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\downharpoonleft\!\upharpoonright}$$

$$\frac{\lambda g\colon |G_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

Relative Reading (cf. Haddock, Champollion and Saurland)

- The only difference here is that we do not reset the inner DP, which staves off its presupposition until more information is accumulated in its scope
- But now when the outer DP is reset, it sets the presuppositions of *both* definites
- For any input g, $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \,\middle|\, \mathsf{sq}\,x,\ \mathsf{circ}\,y,\ \mathsf{in}\,y\,x\right\}$ is the set out outputs that map u onto a circle in some square that it maps to v.
- So requiring that there be exactly one such v is tantamount to requiring that there be exactly one square *that has a circle in it* and exactly one circle *in that square*. In other words, there should be exactly one pair $\langle x, y\rangle$ in $\mathsf{circ} \times \mathsf{sq}$ such that $\mathsf{in}\,y\,x$.

. . . .

$$\begin{aligned}
\mathbf{M}_u &\coloneqq \lambda G\,.\,\left\{\langle\cdot, g\rangle \in G \,\middle|\, \neg\exists\langle\cdot, g'\rangle \in G\,.\,g'\,u \sqsupset g\,u\right\}\\
\textbf{the}_u &\coloneqq \lambda c k g\colon |G_u| = 1\,.\,G,\\
&\quad\ \text{where } G = \mathbf{M}_u \bigcup\left\{k\,x\,g' \,\middle|\, \langle\mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\}\\
\textbf{-s} &\coloneqq \lambda P x\,.\,x \in \left\{\bigoplus P' \,\middle|\, P' \subseteq P\right\}
\end{aligned}$$

- **-s** is a boilerplate plural morpheme that builds sums from the atoms in its complement.
- $\mathbf{M}_u$ is a kind of maximization operator on outputs (Brasoveanu 2012, Charlow 2014). It filters out those assignments in $g \in G$ that are strictly dominated, in the sense that they assign u to a value that is a proper part of something assigned to u by some other $g' \in G$.

. . . .

Derivations …

$$
\begin{aligned}
\mathbf{M}^f_u &\coloneqq \lambda G\,.\,\left\{\langle\cdot,g\rangle\in G \,\middle|\, \neg\exists\langle\cdot,g'\rangle\in G\,.\,f\,(g'\,u)\,(g\,u)\right\}\\
\textbf{older} &\coloneqq \lambda xy\,.\,\mathsf{age}\,x > \mathsf{age}\,y\\
\textbf{est}_u &\coloneqq \lambda f\,.\,\mathbf{M}^f_u\\
\textbf{oldest}_u &= \textbf{est}\,\textbf{older} = \lambda G\,.\,\left\{\langle\cdot,g\rangle\in G \,\middle|\, \neg\exists\langle\cdot,g'\rangle\in G\,.\,\mathsf{age}\,(g'\,u) > \mathsf{age}\,(g\,u)\right\}\\
\textbf{the}_u &\coloneqq \lambda\mathcal{M}ckg\,.\,|G_u| = 1\,.\,G,\\
&\quad\text{where } G = \mathcal{M}_u \bigcup\left\{k\,x\,g' \,\middle|\, \langle\top,g'\rangle\in c\,x\,g^{u\mapsto x}\right\}
\end{aligned}
$$

- **est** abstracts over the ordering function that **M** uses to compare individuals. In the case of pluralities, $\mathbf{M}_u$ filters away any output that assigns u to a *smaller sum* than it could have (ie., a smaller sum than one of the other outputs assigns to u). In the case of **oldest**, $\mathbf{M}_u$ filters outputs that assign u to a *younger* individual than they could have.
- And now here's the swim move: **the** absorbs the max operator and then carries it along for the ride. "Absolute" definite become absolute superlatives; "Haddock" definites become relative superlatives. This part is my favorite.

⟦the oldest squirrel⟧

$$
\rightsquigarrow \left(\frac{\textbf{the}_v\,\mathbf{M}^{\mathsf{ag}}_v\,(\lambda y\,.\,[\,])}{y} \,\middle\backslash\, \frac{[\,]}{\textbf{squirrel}}\right)^{\downarrow\Downarrow} \rightsquigarrow \left(\textbf{the}_v\,\mathbf{M}^{\mathsf{ag}}_v\left(\lambda yg\,.\,\left\{\langle\mathsf{sq}\,y,g\rangle\right\}\right)\right)^{\Downarrow}
$$

$$
\rightsquigarrow \left(\frac{\lambda g\,.\,\mathbf{M}^{\mathsf{ag}}_v \bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)^{\Downarrow} \rightsquigarrow \frac{\lambda g\colon |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{v\mapsto y} \,\middle|\, \begin{array}{l}\mathsf{sq}\,y,\\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{y}
$$

- Resetting the superlative DP has two effects: (1) it fixes the set of individuals that the superlative operator **M** compares, in this case squirrels with respect to age; and (2) it freezes the presupposition associated with the definite article.
- Here $G = \left\{\langle y, g^{v\mapsto y}\rangle \,\middle|\, \mathsf{sq}\,y,\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\right\}$. This set could in principle have multiple winners (multiple squirrels with the same age). The definite determiner rules that out; all the $g\in G$ need to point v to *the same* squirrel.

⟦the circus with the oldest squirrel⟧

$$
\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \,\middle\backslash\, \left(\frac{[\,]}{\textbf{circus}} \,\middle|\, \frac{[\,]}{\textbf{with}} \,\middle/\, \left(\frac{\textbf{the}_v\,\mathbf{M}^{\mathsf{ag}}_v\,(\lambda y\,.\,[\,])}{y} \,\middle\backslash\, \frac{[\,]}{\textbf{squirrel}}\right)^{\downarrow\Downarrow}\right)\right)^{\downarrow\Downarrow}
$$

$$
\left(\frac{\textbf{the}_u\,(\lambda x\,.\,[\,])}{x} \,\middle\backslash\, \left(\frac{[\,]}{\textbf{circus}} \,\middle|\, \frac{[\,]}{\textbf{with}} \,\middle/\, \frac{\lambda g\colon |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{v\mapsto y} \,\middle|\, \begin{array}{l}\mathsf{sq}\,y,\\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{y}\right)\right)^{\downarrow\Downarrow}
$$

$$
\left(\frac{\textbf{the}_u\left(\lambda xg\colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y,\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{with}\,y\,x}\right)^{\downarrow\Downarrow}
$$

$$
\left(\frac{\lambda g\colon |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \begin{array}{l}\mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x,\\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{x}\right)^{\Downarrow}
$$

$$
\frac{\lambda g\colon |G'_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \begin{array}{l}\mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x,\\ \forall z\colon\mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{x}
$$

Absolute Superlative Reading

- When the inner DP is reset, it has the effect just demonstrated: the set of outputs is restricted to those that map v to the (unique) oldest squirrel.
- The outer definite has no explicit maximization operator. For simplicity, I assume that in such cases, it defaults to the basic plural max operator introduced above. This will have no effect when the predicates are all singular, but the uniqueness presupposition it brings to bear is still forceful.
- Altogether, after the host DP has been reset, we will have an essentially nondeterministic (continuized) individual with two presuppositions: (1) there is exactly one squirrel with no elders; and (2) there is exactly one circus that he is with.

$[\![$the circus with the oldest squirrel$]\!]$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\mathbf{circus}} \;\Big|\; \frac{[\,]}{\mathbf{with}} \;\Big/\; \left(\frac{\mathbf{the}_v\,\mathbf{M}_v^{\text{ag}}\,(\lambda y\,.\,[\,])}{y} \;\Big\backslash\; \frac{[\,]}{\mathbf{squirrel}}\right)^{\downarrow}\right)\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\mathbf{circus}} \;\Big|\; \frac{[\,]}{\mathbf{with}} \;\Big/\; \frac{\lambda g\,.\,\mathbf{M}_v^{\text{ag}} \bigcup\{[\,]\,g^{v\mapsto y} \mid \text{sq}\,y\}}{y}\right)\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda xg\,.\,\mathbf{M}_v^{\text{ag}} \bigcup\{[\,]\,g^{v\mapsto y} \mid \text{sq}\,y\}\right)}{\text{circ}\,x \wedge \text{with}\,y\,x}\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\lambda g\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \text{sq}\,y,\ \text{circ}\,x,\ \text{with}\,y\,x\right\}}{x}\right)^{\Downarrow}$$

$$\frac{\lambda g\colon |G_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \text{sq}\,y,\ \text{circ}\,x,\ \text{with}\,y\,x\right\}}{x}$$

Relative Reading (cf. Haddock, Champollion and Saurland)

- The only difference here is that we do not reset the inner DP, which staves off its presupposition until more information is accumulated in its scope
- But now when the outer DP is reset, it sets the presuppositions of *both* definites
- For any input g, $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \;\middle|\; \text{sq}\,x,\ \text{circ}\,y,\ \text{in}\,y\,x\right\}$ is the set out outputs that map u onto a circle in some square that it maps to v.
- So requiring that there be exactly one such v is tantamount to requiring that there be exactly one square *that has a circle in it* and exactly one circle *in that square*. In other words, there should be exactly one pair $\langle x, y\rangle$ in circ $\times$ sq such that in $y\,x$.

Adding focus-sensitivity

$$\begin{aligned}
\mathbf{M}_u^f &= \lambda G\,.\,\left\{\langle\alpha, g\rangle \;\middle|\; \langle\langle\alpha,\cdot\rangle, g\rangle \in G,\ \text{truthy}\,\alpha,\ \neg\exists\langle\langle\cdot,\beta\rangle, g'\rangle \in G\,.\,\bigvee\beta \wedge f\,(g\,u)\,(g'\,u)\right\}\\
\mathbf{larger} &= \lambda xy\,.\,\text{size}\,x < \text{size}\,y\\
\mathbf{est}_u &= \lambda f\,.\,\mathbf{M}_u^f\\
\mathbf{largest}_u &= \mathbf{est}_u\,\mathbf{larger} = \mathbf{M}_u^{\text{sz}} = \lambda G\,.\,\left\{\langle\mathsf{T}, g\rangle \;\middle|\; \langle\mathsf{T}, g\rangle \in G \wedge \neg\exists\langle\mathsf{T}, g'\rangle \in G\,.\,\text{size}\,(g\,u) < \text{size}\,(g'\,u)\right\}\\
\mathbf{the}_u &= \lambda\mathcal{M}ckg\colon |G'_u| = 1\,.\,G', \text{ where } G' = \mathcal{M}\bigcup\left\{k\,x\,g' \;\middle|\; x \in \mathcal{D}_e,\ \langle\mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\}
\end{aligned}$$

John drew the largest square

$$\begin{array}{c|c|c}
[\,] & [\,] & \lambda g\,.\,\mathbf{M}_u \bigcup\{[\,]\,g^{u\mapsto y} \mid \text{sq}\,y\}\\ \hline
\mathbf{j}^{\mathbf{F}} \star (\lambda j\,.\,[\,]) & [\,] & [\,]\\ \hline
j^{\triangleright} \star (\lambda x\,.\,[\,]) & [\,] & [\,]\\ \hline
x & \text{drew} & y
\end{array}$$

$$\begin{aligned}
\mathcal{F}\,\alpha &\coloneqq \sigma \to \{\alpha * \sigma\} * \{\alpha * \sigma\}\\
\eta\,x &\coloneqq \lambda g\,.\,\left\langle\{\langle x, g\rangle\}, \{\langle x, g\rangle\}\right\rangle\\
m \star f &\coloneqq \lambda g\,.\,\left\langle\bigcup\left\{(f\,x\,g')_1 \;\middle|\; \langle x, g'\rangle \in (m\,g)_1\right\},\ \bigcup\left\{(f\,x\,g')_2 \;\middle|\; \langle x, g'\rangle \in (m\,g)_2\right\}\right\rangle
\end{aligned}$$

- Rolling focus and state simultaneously to grease the wheels

---

$$
\begin{aligned}
\mathsf{true} &\coloneqq \lambda G\,.\ \bigvee\left\{\alpha \,\middle|\, \langle \alpha, g\rangle \in G\right\}\\
\textbf{not} &\coloneqq \lambda m g\,.\ \eta\left(\neg(m\,g)\right) g\\
\textbf{only}_u &\coloneqq \lambda G\colon \mathsf{true}\,G\,.\ \mathbf{M}_u\,G\\
\textbf{the}_u &\coloneqq \lambda \mathcal{M} c k g\,.\ |G_u| = 1\,.\ G,\\
&\qquad \text{where } G = \mathcal{M}_u \bigcup\left\{k\,x\,g' \,\middle|\, \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\}
\end{aligned}
$$

- true and **not** are bog standard dynamic booleans: an update is "true" iff it generates at least one successful output
- Adjectival **only** is just $\mathbf{M}$ plus a presupposition (cf. Coppock and Beaver 2012, 2014)!
- So here's the swim move: feed **only** into **the**, and then let the scope of the exclusivity ride the scope of the determiner. This will predict relative readings for adj **only**.

John sold the only cars

Anna didn't give the only good talk