0.1 Grammar and combinatorics

0.1.1 Types

Type Constructors Unary types include entities e, truth values t, and discourse contexts σ (modeled here as functions from variables to entities). Constructed types take one of the following forms, where α and β are any two types:

- $\alpha \to \beta$, the type of a function from α to β.
- $\{\alpha\}$, the type of a set of α objects.
- $\alpha * \beta$, the type of an α object paired with a β object, in that order.

Type Abbreviations To keep type descriptions readable, I use the following type synonyms:

- $\mathbb{D}_\alpha \equiv \sigma \to \{\alpha * \sigma\}$, the type of updates corresponding to constituents of type α.
- $\mathbb{K}_\alpha^\rho \equiv (\alpha \to \mathbb{D}_\rho) \to \mathbb{D}_\rho$, the type of generalized quantifiers with base type α and return type $\mathbb{D}_t$.
- $\mathbb{F}_\alpha \equiv \mathbb{D}_\alpha \to \mathbb{D}_\alpha$, the type of filters on updates.

The notation $m :: \alpha$ indicates that m is of type α.

0.1.2 Scope

$$m \mathbin{/\!\!/} n := \begin{cases} m\,n \quad \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k.\, m\,(\lambda f.\, n\,(\lambda x.\, k\,(f \mathbin{/\!\!/} x))) \quad \textbf{otherwise} \end{cases}$$

$$m \mathbin{\backslash\!\!\backslash} n := \begin{cases} n\,m \quad \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k.\, m\,(\lambda x.\, n\,(\lambda f.\, k\,(x \mathbin{\backslash\!\!\backslash} f))) \quad \textbf{otherwise} \end{cases}$$

$$m \parallel n := \begin{cases} \lambda x.\, m\,x \wedge n\,x \quad \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \to \beta \\ \lambda k.\, m\,(\lambda x.\, n\,(\lambda f.\, k\,(f \parallel x))) \quad \textbf{otherwise} \end{cases}$$

0.1.3 Mereology

$$m \oplus n := \begin{cases} m \sqcup n \quad \textbf{if } m :: e,\ n :: e \\ m \star \lambda x.\, n \star \lambda y.\, \eta\,(x \oplus y) \quad \textbf{otherwise} \end{cases}$$

0.1.4 Binding

$$\eta\,x := \lambda g.\, \{\langle x, g\rangle\}$$

$$m \star k := \lambda g.\, \bigcup\{k\,x\,g' \mid \langle x, g'\rangle \in m\,g\}$$

$$m^{\triangleright u} := m \star (\lambda xg.\, \{\langle x, g^{u \mapsto x}\rangle\})$$

0.1.5 Evaluation

$$m^{\Downarrow} := \begin{cases} m\,\eta \quad \textbf{if } m :: (\alpha \to \mathbb{D}_\alpha) \to \beta \\ m\,(\lambda n.\, n^{\Downarrow}) \quad \textbf{otherwise} \end{cases} \quad \equiv m\,(\star\,(\eta\ \ldots\ (\star\,(\eta\,\eta))))$$

$$m^{\downarrow\!\uparrow} := \lambda k.\, \left(m^{\Downarrow}\right) \star k \quad \equiv \star\,(\Downarrow m)$$

$$m^{\overline{\Downarrow}} := \lambda k.\, m\,\left(\lambda n.\, k\,n^{\Downarrow}\right) \quad \equiv (\star\,(\eta\ \Downarrow)) \mathbin{/\!\!/} m$$

$$\mu\,m := m \star \lambda x.\, x \quad \equiv \Downarrow \left((\star\,m) \mathbin{/\!\!/} (\star\,(\eta\,\star))\right)$$

1 Fragment

Item	*Type*	*Denotation*	
boy	$e \to t$	boy	
girl	$e \to t$	girl	
left	$e \to t$	left	
fought	$e \to t$	fought	
some	$(e \to \mathbb{D}_t) \to \mathbb{D}_e$	$\lambda cg.\, \{\langle x, g'\rangle \mid x \in \mathcal{D}_e,\ \langle \mathbf{T}, g'\rangle \in c\,x\,g\}$	
the$_u$	$\mathbb{K}^\rho_{(e \to \mathbb{D}_t) \to \mathbb{D}_e}$	$\lambda kg.\, \mathbf{1}_u\,(k\,(\lambda c.\, (\mathbf{some}\,c)^{\triangleright u}))\,g$	
$\mathbf{1}_u$	$\mathbb{F}_\alpha$	$\lambda mg{:}\, \lvert (m\,g)_u \rvert = 1.\, m\,g$	
every$_u$	$\mathbb{K}^t_{(e \to \mathbb{D}_t) \to \mathbb{D}_e}$	$\lambda kg.\, \mathbf{A}_u\,(k\,(\lambda c.\, (\mathbf{some}\,c)^{\triangleright u}))\,g$	where $G_u = \{g\,u \mid \langle \cdot, g\rangle \in G\}$
$\mathbf{A}_u$	$\mathbb{F}_t$	$\lambda mg.\, \left\{\left\langle \forall x \in (m\,g)_u.\, \exists\langle \mathbf{T}, g'\rangle \in m\,g.\, x = g'\,u,\ \ g\right\rangle\right\}$	
who	$\mathbb{D}_t \to \mathbb{F}_\alpha$	$\lambda rng.\, \{\langle x, g''\rangle \mid \langle x, g'\rangle \in n\,g,\ \langle \mathbf{T}, g''\rangle \in r\,g\}$	
—$_u$/**pro**$_u$	$\mathbb{D}_e$	$\lambda g.\, \{\langle g\,u, g\rangle\}$	
—$_{u+v}$	$\mathbb{D}_e$	$\lambda g.\, \{\langle g\,u \oplus g\,v, g\rangle\}$ — split antecedence	

1.1 Simple Relative Clauses

$[\![$some girl who — left$]\!]$ =

$$\left(\begin{array}{c}(\mathbf{some}\,(\lambda x.\,[\,]))^{\triangleright u}\\ \hline x\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \textsf{girl}\end{array}\right)^{\Downarrow} \mathbin{\backslash\!\backslash} \mathbf{who} \mathbin{/\!/} \left(\begin{array}{c}\textbf{—}_u \star \lambda y.\,[\,]\\ \hline y\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \textsf{left}\end{array}\right)^{\Downarrow}$$

$[\![$the girl who — left$]\!]$ =

$$\left(\begin{array}{c}\mathbf{1}_u\,[\,]\\ \hline (\mathbf{some}\,(\lambda x.\,[\,]))^{\triangleright u}\\ \hline x\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{girl}\end{array}\right)^{\Downarrow} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \mathbf{who}\end{array} \mathbin{/\!/} \left(\begin{array}{c}[\,]\\ \hline \textbf{—}_u \star \lambda y.\,[\,]\\ \hline y\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{left}\end{array}\right)^{\Downarrow}$$

$[\![$every girl who — left$]\!]$ =

$$\left(\begin{array}{c}\mathbf{A}_u\,[\,]\\ \hline (\mathbf{some}\,(\lambda x.\,[\,]))^{\triangleright u}\\ \hline x\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{girl}\end{array}\right)^{\Downarrow} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \mathbf{who}\end{array} \mathbin{/\!/} \left(\begin{array}{c}[\,]\\ \hline \textbf{—}_u \star \lambda y.\,[\,]\\ \hline y\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{left}\end{array}\right)^{\Downarrow}$$

1.2 Hydrae

1.2.1 Basic Hydra

$[\![$the boy and the girl who — fought$]\!]$ =

$$\left(\begin{array}{c}\mathbf{1}_u\,[\,]\\ \hline \mathbf{sm.boy}^{\triangleright u} \star \lambda x.\,[\,]\\ \hline x\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \oplus\end{array} \mathbin{/\!/} \begin{array}{c}\mathbf{1}_v\,[\,]\\ \hline \mathbf{sm.girl}^{\triangleright v} \star \lambda y.\,[\,]\\ \hline y\end{array}\right)^{\Downarrow} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \mathbf{who}\end{array} \mathbin{/\!/} \left(\begin{array}{c}[\,]\\ \hline \textbf{—}_{u+v} \star \lambda z.\,[\,]\\ \hline z\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{fought}\end{array}\right)^{\Downarrow}$$

- entails a unique boy-girl pair that fought

1.2.2 Nonconstituent Hydra

$[\![$John punished the boy and Mary punished the girl who — fought$]\!]$ =

$$\left(\begin{array}{c}\mathbf{1}_u\,[\,]\\ \hline \mathbf{sm.boy}^{\triangleright u} \star \lambda x.\,[\,]\\ \hline \textsf{punish}\,x\,\textsf{j}\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \wedge\end{array} \mathbin{/\!/} \begin{array}{c}\mathbf{1}_v\,[\,]\\ \hline \mathbf{sm.girl}^{\triangleright v} \star \lambda y.\,[\,]\\ \hline \textsf{punish}\,y\,\textsf{m}\end{array}\right)^{\Downarrow} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline \mathbf{who}\end{array} \mathbin{/\!/} \left(\begin{array}{c}[\,]\\ \hline \textbf{—}_{u+v} \star \lambda z.\,[\,]\\ \hline z\end{array} \mathbin{\backslash\!\backslash} \begin{array}{c}[\,]\\ \hline [\,]\\ \hline \textsf{fought}\end{array}\right)^{\Downarrow}$$

- entails a unique boy-girl pair that fought
- crucially, does not entail a unique boy-girl pair that were punished by john and mary, respectively, that fought; even though the uniqueness scopes over the VPs
- this is because the RC actually filters out alternatives (its content is effected on the right side of the set comprehension), but the VPs only modulate the contents of the alternatives (their contents are effected on the left side of the comprehension, which **1** doesn't look at)

1.3 Reconstructed Relative Clauses

1.3.1 Basic Reconstruction

⟦the relative of his⟧ =

$$\begin{array}{c}[\,]\\\hline\triangleright\nu\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\eta\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,[\,]\\\hline(\textbf{some}\,(\lambda x.\,[\,]))^{\triangleright u}\\\hline x\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline\mathsf{rel}\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\textbf{pro}_k\star\lambda z.\,[\,]\\\hline z\end{array}\right)^{\Downarrow} \rightsquigarrow \begin{array}{c}[\,]\\\hline\triangleright\nu\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\eta\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,[\,]\\\hline\lambda g.\,\{\langle x,\, g^{u\mapsto x}\rangle \mid \mathsf{rel}\,(g\,k)\,x\}\end{array}\right)$$

$$\rightsquigarrow \begin{array}{c}\mathbf{1}_u\,[\,]\\\hline(\eta\,\textbf{sm.rel.pro}_k^{\triangleright u})^{\triangleright\nu}\end{array}$$

⟦the relative of his that John hates **—**⟧ =

$$\begin{array}{c}[\,]\\\hline\mu\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,[\,]\\\hline(\eta\,\textbf{sm.rel.pro}_k^{\triangleright u})^{\triangleright\nu}\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline\textbf{that}\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}[\,]\\\hline(\eta\,\mathrm{j})^{\triangleright k}\star\lambda x.\,[\,]\\\hline[\,]\\\hline x\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline[\,]\\\hline\mathsf{hates}\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\textbf{---}_\nu\star\lambda m.\,[\,]\\\hline m\star\lambda y.\,[\,]\\\hline y\end{array}\right)^{\Downarrow}\right)$$

1.3.2 Reconstructed Hydra

⟦the friend of hers and the friend of John's⟧ =

$$\begin{array}{c}[\,]\\\hline\triangleright\nu\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\eta\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,[\,]\\\hline\textbf{sm.fr.pro}_k^{\triangleright u}\star\lambda x.\,[\,]\\\hline x\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline\oplus\end{array} \mathbin{/\!\!/} \begin{array}{c}\mathbf{1}_{u'}\,[\,]\\\hline\textbf{sm.fr.j}^{\triangleright u'}\star\lambda y.\,[\,]\\\hline y\end{array}\right)^{\Downarrow}$$

$$\rightsquigarrow \begin{array}{c}[\,]\\\hline\triangleright\nu\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\eta\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,(\mathbf{1}_{u'}\,[\,])\\\hline\lambda g.\,\left\{\left\langle x\oplus y,\, g^{\substack{u\mapsto x\\u'\mapsto y}}\right\rangle \,\middle|\, \mathsf{fr}\,(g\,k)\,x,\ \mathsf{fr}\,\mathrm{j}\,y\right\}\end{array}\right)$$

$$\rightsquigarrow \begin{array}{c}\mathbf{1}_u\,(\mathbf{1}_{u'}\,[\,])\\\hline\left(\eta\,\textbf{sm.fr.pro}_k\textbf{.fr.j}^{\triangleright u,u'}\right)^{\triangleright\nu}\end{array}$$

⟦the friend of hers and the friend of John's that Mary thinks **—** would make a good couple⟧ =

$$\begin{array}{c}[\,]\\\hline\mu\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}\mathbf{1}_u\,(\mathbf{1}_{u'}\,[\,])\\\hline\left(\eta\,\textbf{sm.fr.pro}_k\textbf{.fr.j}^{\triangleright u,u'}\right)^{\triangleright\nu}\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline\textbf{that}\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}[\,]\\\hline(\eta\,\mathrm{m})^{\triangleright k}\star\lambda x.\,[\,]\\\hline[\,]\\\hline x\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline[\,]\\\hline\mathsf{thinks.couple}\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\textbf{---}_\nu\star\lambda m.\,[\,]\\\hline m\star\lambda w.\,[\,]\\\hline w\end{array}\right)^{\Downarrow}\right)$$

1.3.3 Nonconstituent Reconstructed Hydra, or Reconstitution

⟦Bill brought the friend of hers and Sam the friend of John's that Mary thinks **—** would make a good couple⟧ =

$$\left(\begin{array}{c}[\,]\\\hline[\,]\\\hline\Downarrow\end{array} \mathbin{/\!\!/} \left(\left(\begin{array}{c}\mathbf{1}_u\,[\,]\\\hline(\eta\,\textbf{sm.fr.pro}_k^{\triangleright u})^{\triangleright\nu}\star\lambda m.\,[\,]\\\hline m\star\lambda x.\,[\,]\\\hline \mathsf{brought}\,x\,\mathrm{b}\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline[\,]\\\hline\wedge\end{array} \mathbin{/\!\!/} \begin{array}{c}\mathbf{1}_{u'}\,[\,]\\\hline\left(\eta\,\textbf{sm.fr.j}^{\triangleright u'}\right)^{\triangleright\nu'}\star\lambda n.\,[\,]\\\hline n\star\lambda y.\,[\,]\\\hline \mathsf{brought}\,y\,\mathrm{s}\end{array}\right)\right)^{\Downarrow} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline\textbf{that}\end{array} \mathbin{/\!\!/} \left(\begin{array}{c}[\,]\\\hline(\eta\,\mathrm{m})^{\triangleright k}\star\lambda z.\,[\,]\\\hline[\,]\\\hline z\end{array} \mathbin{\backslash\!\!\backslash} \begin{array}{c}[\,]\\\hline[\,]\\\hline[\,]\\\hline\mathsf{cpls}\end{array} \mathbin{/\!\!/} \begin{array}{c}[\,]\\\hline\textbf{---}_{\nu+\nu'}\star\lambda m.\,[\,]\\\hline m\star\lambda w.\,[\,]\\\hline w\end{array}\right)^{\Downarrow}$$