Brasoveanu's σ operators don't nest innocently

(40) $[\![\sigma x\,.\,\phi]\!]^{g\,h}$ iff

a. $[\![[x] \wedge \phi]\!]^{g\,h}$

b. $\neg\exists h'\,.\,h\,x < h'\,x \wedge [\![[x] \wedge \phi]\!]^{g\,h'}$

Brasoveanu 2012, p.12

$$[\![\sigma x\,.\,\sigma y\,.\,\text{see}\;y\;x]\!]^{g\,h}$$

$$= \begin{cases} [\![[x] \wedge \sigma y\,.\,\text{see}\;y\;x]\!]^{g\,h} \\ \neg\exists j\,.\,h\,x < j\,x \wedge [\![[x] \wedge \sigma y\,.\,\text{see}\;y\;x]\!]^{g\,j} \end{cases}$$

$$= \begin{cases} \exists k\,.\,g[x]k \wedge [\![\sigma y\,.\,\text{see}\;y\;x]\!]^{k\,h} \\ \neg\exists j\,.\,h\,x < j\,x \wedge \exists k\,.\,g[x]k \wedge [\![\sigma y\,.\,\text{see}\;y\;x]\!]^{k\,j} \end{cases}$$

$$= \begin{cases} \exists k\,.\,g[x]k \wedge \begin{cases} [\![[y] \wedge \text{see}\;y\;x]\!]^{k\,h} \\ \neg\exists h'\,.\,h\,y < h'\,y \wedge [\![[y] \wedge \text{see}\;y\;x]\!]^{k\,h'} \end{cases} \\ \\ \neg\exists j\,.\,h\,x < j\,x \wedge \exists k\,.\,g[x]k \wedge \begin{cases} [\![[y] \wedge \text{see}\;y\;x]\!]^{k\,j} \\ \neg\exists j'\,.\,j\,y < j'\,y \wedge [\![[y] \wedge \text{see}\;y\;x]\!]^{k\,j'} \end{cases} \end{cases}$$

$$= \begin{cases} \exists k\,.\,g[x]k \wedge \begin{cases} k[y]h \wedge [\![\text{see}\;y\;x]\!]^{h\,h} \\ \neg\exists h'\,.\,h\,y < h'\,y \wedge k[y]h' \wedge [\![\text{see}\;y\;x]\!]^{h'\,h'} \end{cases} \\ \\ \neg\exists j\,.\,h\,x < j\,x \wedge \exists k\,.\,g[x]k \wedge \begin{cases} k[y]j \wedge [\![\text{see}\;y\;x]\!]^{j\,j} \\ \neg\exists j'\,.\,j\,y < j'\,y \wedge k[y]j' \wedge [\![\text{see}\;y\;x]\!]^{j'\,j'} \end{cases} \end{cases}$$

$$= \begin{cases} \exists k\,.\,g[x]k \wedge k[y]h \wedge [\![\text{see}\;y\;x]\!]^{h\,h} \wedge \neg\exists h'\,.\,h\,y < h'\,y \wedge k[y]h' \wedge [\![\text{see}\;y\;x]\!]^{h'\,h'} \\ \neg\exists j\,.\,h\,x < j\,x \wedge \exists k\,.\,g[x]k \wedge k[y]j \wedge [\![\text{see}\;y\;x]\!]^{j\,j} \wedge \neg\exists j'\,.\,j\,y < j'\,y \wedge k[y]j' \wedge [\![\text{see}\;y\;x]\!]^{j'\,j'} \end{cases}$$

$$= \begin{cases} \exists k\,.\,g[x]k \wedge k[y]h \wedge [\![\text{see}\;y\;x]\!]^{h\,h} \wedge \neg\exists h'\,.\,h\,y < h'\,y \wedge k[y]h' \wedge [\![\text{see}\;y\;x]\!]^{h'\,h'} & (\approx \text{for given } x,\, y \text{ is all the things that } x \text{ saw}) \\ \forall j\,.\,\forall k\,.\,h\,x < j\,x \wedge g[x]k \wedge k[y]j \wedge [\![\text{see}\;y\;x]\!]^{j\,j} \Rightarrow \exists j'\,.\,j\,y < j'\,y \wedge k[y]j' \wedge [\![\text{see}\;y\;x]\!]^{j'\,j'} & (\approx \text{updates with more } x\text{s have more } y\text{s}) \end{cases}$$

No guarantee that $h\,x$ contains everybody! Only that if it does, then $h\,y$ contains everything seen, and if it doesn't, then anyone outside of $h\,x$ saw something outside of $h\,y$ (actually, the postsup is even weaker: if $h\,x$ were bigger, then there'd be *some* bigger $h\,y$ that preserves viewership, though the "new" member of $h\,x$ doesn't need to be the one that sees the "new" member of $h\,y$).

Modified numerals without postsuppositions

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Main pieces:

i. Normal DPL-style dynamic interpretation scheme (Muskens 1996). Dynamic propositions are relations on assignments, or, equivalently, functions from assignments into sets of assignments. Propositions are *not* relations on *sets* of assignments (cf. Brasoveanu 2007,2013). Notion of "random re-assignment"—i.e. mapping input assignments into more than one output.

ii. Add domain-level pluralities in the usual way, e.g. Schwarzschild 1996. For cumulativity, assume *saw* denotes (or shifts into) a dynamic relation on pluralities (i.e. a function from two plural variables into a dynamic proposition), see (2), where saw does cumulative predication ($\forall x \exists y, \forall y \exists x$). Add selective maximization, see (3).

iii. Continuations for quantification/scope (Barker & Shan 2008).

iv. For modified numerals: a third level of the tower with a cardinality test $\mathbf{N}_{m,v}$ (just checks that the plurality in register v has size m), see (4). The cardinality test scopes at a distinct from the level at which maximization occurs.

v. A distinguished category 'Assn' (for 'assertion'); a syntactic requirement that the cardinality test scopes over an Assn (i.e. is a speech act modifier, à la Krifka 1999); and a Force head for making an Assn out of an S (with a trivial semantics), see (5). Upshot: both cardinality tests must out-scope both maximizations.

Derivation:[1]

(1)

$$\begin{array}{ccc}
\begin{array}{c} \mathsf{Assn} \mid \mathsf{Assn} \\ \hline \mathsf{S} \mid \mathsf{S} \\ \hline \mathsf{DP} \end{array}
&
\left(\begin{array}{c} \mathsf{Assn} \mid \mathsf{Assn} \\ \hline \mathsf{S} \mid \mathsf{S} \\ \hline (\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP} \end{array}\right.
&
\left.\begin{array}{c} \mathsf{Assn} \mid \mathsf{Assn} \\ \hline \mathsf{S} \mid \mathsf{S} \\ \hline \mathsf{DP} \end{array}\right)
\\
\text{exactly three}^{v}\text{ boys} & \text{saw} & \text{exactly five}^{u}\text{ movies}
\\
\begin{array}{c} \mathbf{N}_{3,v}\,[\,] \\ \hline \lambda g.\,\mathbf{M}_v \bigcup\{[\,]\,g[X/v] : \mathsf{boys}\,X\} \\ \hline v \end{array}
&
\begin{array}{c} [\,] \\ \hline [\,] \\ \hline \mathbf{saw} \end{array}
&
\begin{array}{c} \mathbf{N}_{5,u}\,[\,] \\ \hline \lambda g.\,\mathbf{M}_u \bigcup\{[\,]\,g[Y/u] : \mathsf{movs}\,Y\} \\ \hline u \end{array}
\end{array}$$

$$\overset{\text{Combine}}{\rightsquigarrow}\quad
\begin{array}{c} \mathsf{Assn} \mid \mathsf{Assn} \\ \hline \mathsf{S} \mid \mathsf{S} \\ \hline \mathsf{S} \\ \text{exactly three}^{v}\text{ boys saw exactly five}^{u}\text{ movies} \\ \mathbf{N}_{3,v}\,\mathbf{N}_{5,u}\,[\,] \\ \hline \lambda g.\,\mathbf{M}_v \mathbf{M}_u \bigcup\{[\,]\,g[X/v, Y/u] : \mathsf{boys}\,X \wedge \mathsf{movs}\,Y\} \\ \hline \mathbf{saw}\,u\,v \end{array}$$

$$\overset{\text{Lower, Force}}{\rightsquigarrow}\quad
\begin{array}{c} \mathsf{Assn} \mid \mathsf{Assn} \\ \hline \mathsf{Assn} \\ \text{exactly three}^{v}\text{ boys saw exactly five}^{u}\text{ movies} \\ \mathbf{N}_{3,v}\,\mathbf{N}_{5,u}\,[\,] \\ \hline \lambda g.\,\mathbf{M}_v \mathbf{M}_u \{g[X/v, Y/u] : \mathsf{boys}\,X \wedge \mathsf{movs}\,Y \wedge \mathsf{saw}\,Y\,X\} \end{array}$$

Ancillary definitions:

(2) $$\mathbf{saw}\,u\,v = \lambda g.\,\{g\}\ \textbf{if}\ \mathsf{saw}\,(g\,u)\,(g\,v)\ \textbf{else}\ \{\,\}$$

(3) $$\mathbf{M}_v\,G = \{g : g \in G \wedge \neg\exists g' \in G.\, g\,v < g'\,v\}$$

(4) $$\mathbf{N}_{m,v}\,R = \lambda g.\,\{g' : g' \in R\,g \wedge |g'\,v| = m\}$$

(5) $$\begin{array}{c} \mathsf{Assn/S} \\ \mathsf{Force} \\ \lambda p.\,p \end{array}$$

One more Lower gets the right truth conditions: find all the boys who saw movies and all the movies seen by boys. You end up with three such boys and five such movies. Important bits: the cardinality update can only take scope over an assertion. This forces both cardinality updates to take scope over both maximality operators. Avoids over-generation (the maximal number of boys that between them saw exactly 5 movies is 3).

Standard Barker & Shan (2008) continuations semantics for scope. Combination is scope-y functional application. Lowering is application to the identity function.

Combines Kubota & Uegaki's (2009) treatment of benefactive inferences in Japanese, Krifka's (1999) idea that the cardinality requirement in these constructions is a speech act modifier, and domain-level maximization (cf. Brasoveanu 2007,2013).

"Post-suppositions" purely a matter of scope. No representational architecture, no non-standard definitions of truth (standard dynamic definition as existential closure over outputs). Replaces Brasoveanu's (2013) "pseudo wide scope" with bona fide wide scope. Conservative in the sense that this is just the continuations+DPL version of a scope splitting account (widely used for modified numerals), with a Krifka-esque stipulation about *what* sorts of things the split-scoped bit can scope over.

Handled fine with domain-level plurality and simple maximization over registers in the output assignments (i.e. '$<$' has a standard mereological interpretation). No need here for discourse-level plurality.

To do: generate the distributive reading (not hard). Look at scope splitting, interaction with intensional operators. Think about other embedded contexts. (I.e. do embedding operators embed "assertions"? See Krifka.) Think about whether there exist *empirical* reasons to prefer this to the representational post-suppositional architecture (possibly having to do with islands). Semantic effects of Force? Implemented here.

1 Note that we push $\mathbf{M}_u$ out: $\mathbf{M}_v \bigcup\{\mathbf{M}_u \bigcup\{\xi : \Psi\} : \Phi\} = \mathbf{M}_v \mathbf{M}_u \bigcup\{\xi : \Phi \wedge \Psi\}$. Two other equivalences worth mentioning: the relative scope of the maximizers doesn't matter, and likewise for the cardinality tests.

Max operators as (scopal) filters on outputs

$$\llbracket \mathbf{M}_u \rrbracket = \lambda G\,.\,\{g \mid g \in G \wedge \neg\exists g' \in G\,.\, g\,u < g'\,u\}$$

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$$\llbracket \text{circle} \rrbracket = \lambda u g\,.\begin{cases}\{g\} & \textbf{if } \text{circ}\,(g\,u) \\ \varnothing & \textbf{otherwise}\end{cases}$$

$$\llbracket \text{the}_1^u \rrbracket = \lambda c k g : |G'| = 1\,.\bigcup\{k\,u\,g' \mid g' \in G'\},\ \text{where } G' = \mathbf{M}_u \bigcup\{c\,u\,g^{u\mapsto x} \mid x \in \mathcal{D}_e\}$$

$$\llbracket \text{the}_2^u \rrbracket = \lambda c k g : |G'| = 1\,.\,G',\ \text{where } G' = \mathbf{M}_u \bigcup\left\{k\,u\,g' \;\middle|\; g' \in \bigcup\{c\,u\,g^{u\mapsto x} \mid x \in \mathcal{D}_e\}\right\}$$

. . . .

$\llbracket$the circle in the square$\rrbracket$ Bonus: Haddock Reading

$$= \frac{\llbracket \text{the}_1^u \rrbracket\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{\llbracket \text{circle} \rrbracket} \;\Bigg|\; \frac{[\,]}{\llbracket \text{in} \rrbracket} \Bigg/ \left(\frac{\llbracket \text{the}_2^v \rrbracket\,(\lambda v\,.\,[\,])}{v} \Bigg\backslash \frac{[\,]}{\llbracket \text{square} \rrbracket}\right)^{\downarrow}\right)$$

$$= \frac{\llbracket \text{the}_1^u \rrbracket\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{\llbracket \text{circle} \rrbracket} \;\Bigg|\; \frac{[\,]}{\llbracket \text{in} \rrbracket} \Bigg/ \frac{\lambda g\,.\,\mathbf{M}_v \bigcup\{[\,]\,g' \mid g' \in \bigcup\{\llbracket \text{sq} \rrbracket\,v\,g^{v\mapsto y} \mid y \in \mathcal{D}_e\}\}}{v}\right)$$

$$= \frac{\llbracket \text{the}_1^u \rrbracket\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{\llbracket \text{circle} \rrbracket} \;\Bigg|\; \frac{[\,]}{\llbracket \text{in} \rrbracket} \Bigg/ \frac{\lambda g\,.\,\mathbf{M}_v \bigcup\{[\,]\,g^{v\mapsto y} \mid \text{sq}\,y\}}{v}\right)$$

$$= \frac{\llbracket \text{the}_1^u \rrbracket\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \frac{\lambda g\,.\,\mathbf{M}_v \bigcup\{[\,]\,g^{v\mapsto y} \mid \text{sq}\,y\}}{\lambda u g\,.\,\llbracket \text{circ} \rrbracket\,u\,g \cap \llbracket \text{in} \rrbracket\,v\,u\,g}$$

$$= \left(\frac{\llbracket \text{the}_1^u \rrbracket\,\left(\lambda u g\,.\,\mathbf{M}_v \bigcup\{[\,]\,g^{v\mapsto y} \mid \text{sq}\,y\}\right)}{\lambda g\,.\,\llbracket \text{circ} \rrbracket\,u\,g \cap \llbracket \text{in} \rrbracket\,v\,u\,g}\right)^{\downarrow}$$

$$= \frac{\lambda g\,.\,\bigcup\left\{[\,]\,g' \;\middle|\; g' \in \mathbf{M}_u \bigcup\left\{\mathbf{M}_v \bigcup\left\{\llbracket \text{circ} \rrbracket\,u\,g^{\substack{u\mapsto x\\ v\mapsto y}} \cap \llbracket \text{in} \rrbracket\,v\,u\,g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \text{sq}\,y\right\} \;\middle|\; x \in \mathcal{D}_e\right\}\right\}}{u}$$

$$= \frac{\lambda g\,.\,\bigcup\left\{[\,]\,g' \;\middle|\; g' \in \mathbf{M}_u\mathbf{M}_v\left\{g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \text{circ}\,x,\ \text{sq}\,y,\ \text{in}\,y\,x\right\}\right\}}{u}$$

$G = \left\{g^{\substack{u\mapsto x\\ v\mapsto y}} \;\middle|\; \text{circ}\,x,\ \text{sq}\,y,\ \text{in}\,y\,x\right\}$ is the set of assignments that map u to a circle and v to a square that u is in. Because both nominals are singular, $\mathbf{M}_u$ and $\mathbf{M}_v$ are vacuous filters on G. But the definite determiners require that u and v, respectively, be constant across G, which guarantees that there is only one circle-square pair in the in relation. Thus $\mathbf{M}$ cumulates where σ nests.

Superlatives as Max operators

$$\begin{aligned}
\text{larger} &= \lambda xy\,.\,\text{size}\,x < \text{size}\,y \\
[\![\mathbf{M}]\!] &= \lambda fuG\colon |G'_u| = 1\,.\,G',\ \text{where } G' = \{g \mid g \in G \wedge \neg\exists g' \in G\,.\,f\,(g\,u)\,(g'\,u)\} \\
\left[\!\!\left[\mathbf{M}^{|\cdot|}\right]\!\!\right] &= [\![\mathbf{M}]\!]\,(\lambda xy\,.\,|x| < |y|) \\
[\![\text{largest}]\!] = [\![\mathbf{M}^{\text{sz}}]\!] = [\![\mathbf{M}]\!]\,\text{larger} &= \lambda uG\colon |G'_u| = 1\,.\,G',\ \text{where } G' = \{g \mid g \in G \wedge \neg\exists g' \in G\,.\,\text{size}\,(g\,u) < \text{size}\,(g'\,u)\} \\
[\![\text{the}_1^u]\!] &= \lambda\mathcal{A}\left[= \mathbf{M}^{|\cdot|}\right].\,\lambda c\,.\,\frac{\lambda g\,.\,\bigcup\{[\,]\,g' \mid g' \in \mathcal{A}\,u\,\bigcup\{c\,u\,g^{u\mapsto x} \mid x \in \mathcal{D}_e\}\}}{u} \\
[\![\text{the}_2^u]\!] &= \lambda\mathcal{A}\left[= \mathbf{M}^{|\cdot|}\right].\,\lambda c\,.\,\frac{\lambda g\,.\,\mathcal{A}\,u\,\bigcup\{[\,]\,g' \mid g' \in \bigcup\{c\,u\,g^{u\mapsto x} \mid x \in \mathcal{D}_e\}\}}{u}
\end{aligned}$$

Relative Reading

$[\![\text{the}_1^u \text{ circle in the}_2^v \text{ largest square}]\!]$

$$= \frac{[\![\text{the}_1^u]\!]\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{[\![\text{circle}]\!]} \Bigg| \frac{[\,]}{[\![\text{in}]\!]} \Bigg/ \left(\frac{[\![\text{the}_2^v]\!]\,[\![\text{largest}]\!]\,(\lambda v\,.\,[\,])}{v} \Bigg\backslash \frac{[\,]}{[\![\text{square}]\!]}\right)^{\downarrow}\right)$$

$$= \frac{[\![\text{the}_1^u]\!]\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{[\![\text{circle}]\!]} \Bigg| \frac{[\,]}{[\![\text{in}]\!]} \Bigg/ \frac{\lambda g\,.\,\mathbf{M}_v^{\text{sz}}\,\bigcup\{[\,]\,g' \mid g' \in \bigcup\{[\![\text{sq}]\!]\,v\,g^{v\mapsto y} \mid y \in \mathcal{D}_e\}\}}{v}\right)$$

$$= \frac{\lambda g\,.\,\bigcup\left\{[\,]\,g' \,\middle|\, g' \in \mathbf{M}_u\mathbf{M}_v^{\text{sz}}\left\{g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \text{circ}\,x,\ \text{sq}\,y,\ \text{in}\,y\,x\right\}\right\}}{u}$$

Absolute Reading

$[\![\text{the}_1^u \text{ circle in the}_1^v \text{ largest square}]\!]$

$$= \frac{[\![\text{the}_1^u]\!]\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{[\![\text{circle}]\!]} \Bigg| \frac{[\,]}{[\![\text{in}]\!]} \Bigg/ \left(\frac{[\![\text{the}_1^v]\!]\,[\![\text{largest}]\!]\,(\lambda v\,.\,[\,])}{v} \Bigg\backslash \frac{[\,]}{[\![\text{square}]\!]}\right)^{\downarrow}\right)$$

$$= \frac{[\![\text{the}_1^u]\!]\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{[\![\text{circle}]\!]} \Bigg| \frac{[\,]}{[\![\text{in}]\!]} \Bigg/ \frac{\lambda g\,.\,\bigcup\{[\,]\,g' \mid g' \in \mathbf{M}_v^{\text{sz}}\,\bigcup\{[\![\text{sq}]\!]\,v\,g^{v\mapsto y} \mid y \in \mathcal{D}_e\}\}}{v}\right)$$

$$= \frac{[\![\text{the}_1^u]\!]\,(\lambda u\,.\,[\,])}{u} \Bigg\backslash \left(\frac{[\,]}{[\![\text{circle}]\!]} \Bigg| \frac{[\,]}{[\![\text{in}]\!]} \Bigg/ \frac{\lambda g\,.\,\bigcup\{[\,]\,g' \mid g' \in \mathbf{M}_v^{\text{sz}}\,\{g^{v\mapsto y} \mid \text{sq}\,v\}\}}{v}\right)$$

$$= \frac{\lambda g\,.\,\bigcup\{[\,]\,g' \mid g' \in \mathbf{M}_u\,\{g'' \mid \text{circ}\,(g''\,u),\ \text{in}\,(g''\,v)\,(g''\,u),\ g'' \in \mathbf{M}_v^{\text{sz}}\,\bigcup\{g^{v\mapsto y} \mid \text{sq}\,y\}\}\}}{u}$$

Generalizing w/ Focus and State Monads

$$\llbracket \mathbf{M}^{|\cdot|} \rrbracket = \lambda u G.\, \{\langle \mathsf{T}, g\rangle \mid \langle \mathsf{T}, g\rangle \in G,\ \neg\exists \langle \mathsf{T}, g'\rangle \in G.\, |g\,u| < |g'\,u|\}$$

$$\llbracket \text{largest} \rrbracket = \llbracket \mathbf{M}^{\mathsf{sz}} \rrbracket = \lambda u G.\, \{\langle \mathsf{T}, g\rangle \mid \langle\langle \mathsf{T}, \cdot\rangle, g\rangle \in G,\ \neg\exists \langle\langle \cdot, \beta\rangle, g'\rangle \in G.\, \bigvee\beta \wedge \mathsf{size}\,(g\,u) < \mathsf{size}\,(g'\,u)\}$$

$$\llbracket \text{the}_1^u \rrbracket = \lambda \mathcal{A}\big[= \mathbf{M}^{|\cdot|}\big].\, \lambda c k g \colon |G'_u| = 1.\, \bigcup\{k\,x\,g' \mid g' \in G'\},\ \text{where } G' = \mathcal{A}\,u \bigcup\{c\,x\,g^{u\mapsto x} \mid x \in \mathcal{D}_e\}$$

$$\llbracket \text{the}_2^u \rrbracket = \lambda \mathcal{A}\big[= \mathbf{M}^{|\cdot|}\big].\, \lambda c k g \colon |G'_u| = 1.\, G',\ \text{where } G' = \mathcal{A}\,u \bigcup\{k\,x\,g' \mid \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x},\ x \in \mathcal{D}_e\}$$

Focus-Sensitive Relative Reading

$\llbracket \text{John}_{\mathsf{F}}$ drew the$_2^u$ largest circle$\rrbracket$

$$= \dfrac{[\,]}{\dfrac{\mathsf{j}^{\mathsf{F}} \multimap_{\mathsf{F}} (\lambda x.\,[\,])}{x}} \;\Big\backslash\; \dfrac{[\,]}{\dfrac{[\,]}{\mathsf{drew}}} \;\Big/\; \left(\dfrac{\llbracket \text{the}_2^u \rrbracket\, \llbracket \text{largest} \rrbracket\, (\lambda y.\,[\,])}{y} \;\Big\backslash\; \dfrac{[\,]}{\mathsf{square}}\right)^{\downarrow\Uparrow}$$

$$= \dfrac{[\,]}{\dfrac{\mathsf{j}^{\mathsf{F}} \multimap_{\mathsf{F}} (\lambda x.\,[\,])}{x}} \;\Big\backslash\; \dfrac{[\,]}{\dfrac{[\,]}{\mathsf{drew}}} \;\Big/\; \dfrac{\lambda g.\, \mathbf{M}_u^{\mathsf{sz}} \bigcup\{[\,]\, g^{u\mapsto y} \mid \mathsf{sq}\, y\}}{\dfrac{[\,]}{y}}$$

$$= \left(\dfrac{\lambda g.\, \mathbf{M}_u^{\mathsf{sz}} \bigcup\{[\,]\, g^{u\mapsto y} \mid \mathsf{sq}\, y\}}{\dfrac{\mathsf{j}^{\mathsf{F}} \multimap_{\mathsf{F}} (\lambda x.\,[\,])}{\mathsf{drew}\, y\, x}}\right)^{\Downarrow_{\mathsf{F}}}$$

$$= \lambda g.\, \mathbf{M}_u^{\mathsf{sz}}\, \{\langle\langle \mathsf{drew}\, y\, \mathsf{j}, \{\mathsf{drew}\, y\, z \mid \mathsf{Alt}\,\mathsf{j}\, z\}\rangle, g^{u\mapsto y}\rangle \mid \mathsf{sq}\, y\} \quad = \quad \lambda g.\, \{\langle \mathsf{T}, g^{u\mapsto y}\rangle \mid \mathsf{sq}\, y,\ \mathsf{drew}\, y\, \mathsf{j},\ \neg\exists x \in \mathsf{sq}.\, \exists z \in \mathsf{Alt}\,\mathsf{j}.\, \mathsf{drew}\, x\, z \wedge y < z\}$$

DP-Internal (Focus-less) Relative Reading

$\llbracket$the$_1^u$ circle in the$_2^v$ largest square$\rrbracket$

$$= \dfrac{\llbracket \text{the}_1^u \rrbracket\, (\lambda x.\,[\,])}{x} \;\Big\backslash\; \left(\dfrac{[\,]}{\llbracket \text{circle} \rrbracket} \;\Big|\; \dfrac{[\,]}{\llbracket \text{in} \rrbracket} \;\Big/\; \left(\dfrac{\llbracket \text{the}_2^v \rrbracket\, \llbracket \text{largest} \rrbracket\, (\lambda v.\,[\,])}{v} \;\Big\backslash\; \dfrac{[\,]}{\llbracket \text{square} \rrbracket}\right)^{\downarrow}\right)$$

$$= \dfrac{\llbracket \text{the}_1^u \rrbracket\, (\lambda x.\,[\,])}{x} \;\Big\backslash\; \left(\dfrac{[\,]}{\mathsf{circle}} \;\Big|\; \dfrac{[\,]}{\mathsf{in}} \;\Big/\; \dfrac{\lambda g.\, \mathbf{M}_v^{\mathsf{sz}} \bigcup\{[\,]\, g^{v\mapsto y} \mid \mathsf{sq}\, y\}}{y}\right)$$

$$= \left(\dfrac{\llbracket \text{the} \rrbracket\, (\lambda x g.\, \mathbf{M}_v^{\mathsf{sz}}\, \{[\,]\, g^{v\mapsto y} \mid \mathsf{sq}\, y\})}{\mathsf{circ}\, x \wedge \mathsf{in}\, y\, x}\right)^{\downarrow_{\mathsf{F}}}$$

$$= \dfrac{\lambda g.\, \bigcup\Big\{[\,]\, g' \;\Big|\; g' \in \mathbf{M}_u\, \mathbf{M}_v^{\mathsf{sz}}\, \Big\{\Big\langle \langle \mathsf{circ}\, x \wedge \mathsf{in}\, y\, x, \{\mathsf{circ}\, x \wedge \mathsf{in}\, y\, x\}\rangle, g^{\substack{v\mapsto y \\ u\mapsto x}} \Big\rangle \;\Big|\; \mathsf{sq}\, y\Big\}\Big\}}{x}$$

$$
\begin{aligned}
&[\![\text{Who}^{v} \text{ drew the}^{u}_{2} \text{ largest square?}]\!] \\
&= \frac{[\,]}{\dfrac{\lambda g\,.\bigcup\{[\,]\,g^{v\mapsto x} \mid \text{who}\,x\}}{x}} \;\Big\backslash\; \frac{[\,]}{\dfrac{[\,]}{\text{drew}}} \;\Big/\; \left(\frac{[\![\text{the}^{u}_{2}]\!]\,[\![\text{largest}]\!]\,(\lambda y\,.\,[\,])}{y} \;\Big\backslash\; \frac{[\,]}{\text{square}}\right)^{\downarrow\Uparrow} \\
&= \frac{[\,]}{\dfrac{\lambda g\,.\bigcup\{[\,]\,g^{v\mapsto x} \mid \text{who}\,x\}}{x}} \;\Big\backslash\; \frac{[\,]}{\dfrac{[\,]}{\text{drew}}} \;\Big/\; \frac{\lambda g\,.\,\mathbf{M}^{\text{sz}}_{u}\bigcup\{[\,]\,g^{u\mapsto y} \mid \text{sq}\,y\}}{\dfrac{[\,]}{y}} \\
&= \left(\frac{\lambda g\,.\,\mathbf{M}^{\text{sz}}_{u}\bigcup\{[\,]\,g^{u\mapsto y} \mid \text{sq}\,y\}}{\dfrac{\lambda g\,.\bigcup\{[\,]\,g^{v\mapsto x} \mid \text{who}\,x\}}{\text{drew}\,y\,x}}\right)^{\Downarrow} \\
&= \lambda g\,.\,\mathbf{M}^{\text{sz}}_{u}\left\{\left\langle \text{drew}\,y\,x,\, g^{\substack{u\mapsto y\\ v\mapsto x}}\right\rangle \,\middle|\, \text{who}\,x,\ \text{sq}\,y\right\}
\end{aligned}
$$

Note that this gives the Karttunen answers to the question rather than the Hamblin answers, since $\mathbf{M}^{\text{sz}}_{u}$ throws away any updates paired with falsity. That is, it keeps only those alternatives in which $g\,v$ really did draw at least as big a square as anyone else.

Some Thoughts

- Superlatives like ‘largest’ filter out all those updates that either fail the test contributed by the main assertion (John drew y), or fail to be maximal with respect to their alternatives (y is larger than any circle drawn by anyone else).
- This means that they are certainly sensitive to focal alternatives, but they do not depend on them. For instance, in the DP-internal example, the focal alternative is trivial, and so the superlative maximizes only over circles in squares.
- The default cardinality maximizer that both of the ‘the’s come equipped with (in case of plural complements) is not focus-sensitive; it has the wrong type to compare focus-ful updates.
- But ‘the_2’ is still scopal, so still Haddock readings. But no focus-sensitve Haddock readings.
- Also, the nominal complement of ‘the [superlative]’ is not appropriately typed to capture focal alternatives. This predicts no relative reading for things like “The largest square that JOHN bought is blue” (in contrast to other scope-based accounts of relative readings, which predict the sentence is true if John bought a larger blue square than anyone else).
- Unifies S-level and DP-level relative readings. Heim-style scope story would have to say the RC gap has alternatives, and those alternatives are precisely the extension of the raised head noun.
- Definite article is still definite, in that it still presupposes uniqueness (wrt the possible referents of a particular variable, i.e. $|G'_u| = 1$) of the set that its maximizer returns.
- Superlatives defined in terms of comparatives, rather than positive form (Bobalijk, Szabolcsi).
- No parasitic scope. Superlative outscopes the alternatives it quantifies over, like other sorts of operators that capture nondeterminism. Those alternatives may come from focus or from state; it’s all the same to $\mathbf{M}$.

Some Questions

- Why no bare instances of ‘the_2’? Weak definites (“I heard it on [the radio [that I heard it on]]”)?
- Is ‘the_2’ also useful for other “nonlocal” adjectives (‘same’, ‘average’, ‘only’, ‘usual’, ‘right’, ‘whole’, ‘possible’, ‘unknown’, etc. … Morzycki)?
- The constituent structure here is unusual: [[the largest] book]. Could it be that if ‘the’ is really what’s responsible for the scopal possibilities of superlative DPs (as the superlative-less Haddock readings suggest), that this is why no relative readings for ‘his largest book’? I.e., ‘his’ never puts its maximizer outside of its scope. Are there Haddock readings with inner possessives?
- The TCs for the S-level relative are still not quite right. One would really love for the definite to end maximizing over alternatives to John, not over squares. That is, John could have drawn two identically-sized squares that were both larger than everyone else’s. But it couldn’t be that John and Mary drew identical squares that were both larger than everyone else’s. So John needs to be unique, not the square. Can we get this by dissociating the index on the definite from the index on the superlative? In order to get John and his alternatives onto the stack, do we need to relayer the Focus and State monads, or maybe nest them recursively?
- Relatedly, upstairs de-dicto? How to keep the nominal inside the modal, while giving ‘the’ scope over it?