$$m \,/\, n := \begin{cases} m\, n & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k \,.\, m \left(\lambda f \,.\, n \left(\lambda x \,.\, k \left(f \,/\, x\right)\right)\right) & \textbf{otherwise} \end{cases}$$

$$m \setminus n := \begin{cases} n\, m & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k \,.\, m \left(\lambda f \,.\, n \left(\lambda x \,.\, k \left(f \setminus x\right)\right)\right) & \textbf{otherwise} \end{cases}$$

- The usual applicators for left and right continuized FA
- State.Set monad combinators for managing drefs
- A slightly tweaked lower operator to filter out falsity (more like traditional standard dynamic semantics). This is not essential, but makes the bookkeeping a little smoother.

$$\begin{aligned} \eta\, x &:= \lambda g \,.\, \left\{\left\langle x, g\right\rangle\right\} \\ m^{\star} &:= \lambda k g \,.\, \bigcup \left\{ k\, x\, g' \,\middle|\, \left\langle x, g'\right\rangle \in m\, g \right\} \\ m^{\downarrow} &:= m \left(\lambda x s \,.\, \{\langle x, s\rangle \mid x \neq \mathsf{F}\}\right) \\ x^{\uparrow} &:= (\eta\, x)^{\star} = \lambda k \,.\, k\, x \\ x^{\downharpoonleft\upharpoonright} &:= \left(x^{\downarrow}\right)^{\star} \end{aligned}$$

$$\begin{aligned} \textbf{the}_u &:= \lambda c k g \colon |G_u| = 1 \,.\, G, \\ &\qquad \text{where } G = \bigcup \left\{ k\, x\, g' \,\middle|\, \left\langle \mathsf{T}, g'\right\rangle \in c\, x\, g^{u \mapsto x} \right\} \\ \textbf{circle} &:= \mathsf{circ} \\ \textbf{square} &:= \mathsf{sq} \\ \textbf{in} &:= \mathsf{in} \end{aligned}$$

- $\textbf{the}_u$ is just $\mathbf{a}_u$ plus a uniqueness presupposition. The presupposition restricts what the *set* of outputs is allowed to look like: they need to all agree on the value of u
- That means its uniqueness effect is *delayed* until some program containing it is evaluated, (i.e., until its continuation is delimited). This sort of delayed global test on the set of outputs is very much like a postsupposition (Brasoveanu 2012, Henderson 2014), but here it is regulated by continuations, rather than logical subscripts (Charlow 2014)

$$[\![\text{the square}]\!] \rightsquigarrow \left(\frac{\textbf{the}_v\, (\lambda y \,.\, [\,])}{y} \;\middle\backslash\; \frac{[\,]}{\textbf{square}} \right)^{\downarrow\, \downharpoonleft\upharpoonright} \rightsquigarrow \left(\textbf{the}_v \left(\lambda y g \,.\, \left\{ \left\langle \mathsf{sq}\, y, g \right\rangle \right\} \right) \right)^{\downharpoonleft\upharpoonright}$$

$$\rightsquigarrow \left(\frac{\lambda g \,.\, \bigcup\{[\,]\, g^{v \mapsto y} \mid \mathsf{sq}\, y\}}{y} \right)^{\downharpoonleft\upharpoonright} \rightsquigarrow \frac{\lambda g \colon |G_v| = 1 \,.\, \bigcup\{[\,]\, g^{v \mapsto y} \mid \mathsf{sq}\, y\}}{y}$$

- Note that resetting $[\![\text{the square}]\!]$ in the last reduction step here has no effect on its semantic shape, because it's essentially $[\![\text{a square}]\!]$.
- But it does fix the presupposition; For any input g, G will be equal to $\left\{\left\langle y, g^{v \mapsto y}\right\rangle \,\middle|\, \mathsf{sq}\, y\right\}$, and the presup will require all those g's to map v to the same square (which will only be possible if there's exactly one square available to assign v to in the first place).

$[\![\text{the circle in the square}]\!]$

$$\left(\frac{\textbf{the}_u\, (\lambda x \,.\, [\,])}{x} \;\middle\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\middle|\; \frac{[\,]}{\textbf{in}} \;\middle/\; \left(\frac{\textbf{the}_v\, (\lambda y \,.\, [\,])}{y} \;\middle\backslash\; \frac{[\,]}{\textbf{square}} \right)^{\downarrow\, \downharpoonleft\upharpoonright} \right) \right)^{\downarrow\, \downharpoonleft\upharpoonright}$$

$$\left(\frac{\textbf{the}_u\, (\lambda x \,.\, [\,])}{x} \;\middle\backslash\; \left(\frac{[\,]}{\textbf{circle}} \;\middle|\; \frac{[\,]}{\textbf{in}} \;\middle/\; \frac{\lambda g \colon |G_v| = 1 \,.\, \bigcup\{[\,]\, g^{v \mapsto y} \mid \mathsf{sq}\, y\}}{y} \right) \right)^{\downarrow\, \downharpoonleft\upharpoonright}$$

$$\left(\frac{\textbf{the}_u \left(\lambda x g \colon |G_v| = 1 \,.\, \bigcup\{[\,]\, g^{v \mapsto y} \mid \mathsf{sq}\, y\} \right)}{\mathsf{circ}\, x \wedge \mathsf{in}\, y\, x} \right)^{\downarrow\, \downharpoonleft\upharpoonright}$$

$$\left(\frac{\lambda g \,.\, \bigcup \left\{ [\,]\, g^{\substack{u \mapsto x \\ v \mapsto y}} \,\middle|\, \mathsf{sq}\, y,\ \mathsf{circ}\, x,\ \mathsf{in}\, y\, x \right\}}{x} \right)^{\downharpoonleft\upharpoonright}$$

$$\frac{\lambda g \colon |G'_u| = |G_v| = 1 \,.\, \bigcup \left\{ [\,]\, g^{\substack{u \mapsto x \\ v \mapsto y}} \,\middle|\, \mathsf{sq}\, y,\ \mathsf{circ}\, x,\ \mathsf{in}\, y\, x \right\}}{x}$$

Absolute Reading

- The inner definite is reset, freezing its presupposition as above. When the input assignment g is eventually inserted, we will have $G = \left\{\left\langle y, g^{v \mapsto y}\right\rangle \,\middle|\, \mathsf{sq}\, y\right\}$, and the presupposition will guarantee that $g'\, v$ is constant across the outputs.
- As with the inner DP, the host DP's presupposition is fixed when it is reset. This time, we have $G' = \left\{ \left\langle x, g^{\substack{u \mapsto x \\ v \mapsto y}} \right\rangle \,\middle|\, \mathsf{sq}\, y,\ \mathsf{circ}\, x,\ \mathsf{in}\, y\, x \right\}$, where g is whatever the input happens to be. In particular, all the outputs will now need to agree on the value of u in addition to v, which will only be possible if there's exactly one circle in the square that all outputs assign to v.

⟦the circle in the square⟧

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\mathbf{circle}} \;\Big|\; \frac{[\,]}{\mathbf{in}} \;\Big/\; \left(\frac{\mathbf{the}_v\,(\lambda y\,.\,[\,])}{y} \;\Big\backslash\; \frac{[\,]}{\mathbf{square}}\right)^{\downarrow}\right)\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \;\Big\backslash\; \left(\frac{[\,]}{\mathbf{circle}} \;\Big|\; \frac{[\,]}{\mathbf{in}} \;\Big/\; \frac{\lambda g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda x g\,.\,\bigcup\{[\,]\,g^{v\mapsto y} \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\,\Downarrow}$$

$$\left(\frac{\lambda g\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\Downarrow}$$

$$\frac{\lambda g\colon |G_u| = |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}} \,\middle|\, \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

Relative Reading (cf. Haddock, Champollion and Saurland)

- The only difference here is that we do not reset the inner DP, which staves off its presupposition until more information is accumulated in its scope
- But now when the outer DP is reset, it sets the presuppositions of *both* definites
- For any input g, $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}\right\rangle \,\middle|\, \mathsf{sq}\,x,\ \mathsf{circ}\,y,\ \mathsf{in}\,y\,x\right\}$ is the set out outputs that map u onto a circle in some square that it maps to v.
- So requiring that there be exactly one such v is tantamount to requiring that there be exactly one square *that has a circle in it* and exactly one circle *in that square*. In other words, there should be exactly one pair $\langle x, y\rangle$ in $\mathsf{circ} \times \mathsf{sq}$ such that $\mathsf{in}\,y\,x$.

$$\begin{aligned}
\mathbf{M}_u &:= \lambda G\,.\,\left\{\langle\cdot, g\rangle \in G \,\middle|\, \neg\exists\langle\cdot, g'\rangle \in G\,.\,g'\,u \sqsupset g\,u\right\}\\
\mathbf{the}_u &:= \lambda c k g\colon |G_u| = 1\,.\,G,\\
&\qquad \text{where } G = \mathbf{M}_u \bigcup\left\{k\,x\,g' \,\middle|\, \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\right\}\\
\textbf{-s} &:= \lambda P x\,.\,x \in \left\{\bigoplus P' \,\middle|\, P' \subseteq P\right\}
\end{aligned}$$

- **-s** is a boilerplate plural morpheme that builds sums from the atoms in its complement.
- $\mathbf{M}_u$ is a kind of maximization operator on outputs (Brasoveanu 2012, Charlow 2014). It filters out those assignments in $g \in G$ that are strictly dominated, in the sense that they assign u to a value that is a proper part of something assigned to u by some other $g' \in G$.

Derivations …

$$\mathsf{true} \coloneqq \lambda G .\, \bigvee \left\{ \alpha \,\middle|\, \langle \alpha, g \rangle \in G \right\}$$

$$\textbf{only}_u \coloneqq \lambda G \colon \mathsf{true}\, G .\, \mathbf{M}_u\, G$$

$$\textbf{the}_u \coloneqq \lambda \mathcal{M} c k g .\, |G_u| = 1 .\, G,$$

$$\text{where } G = \mathcal{M}_u \bigcup \left\{ k\, x\, g' \,\middle|\, \langle \top, g' \rangle \in c\, x\, g^{u \mapsto x} \right\}$$

- true is a bog standard dynamic truth predicate: true iff at least one update succeeds.
- Adjectival **only** is just **M** plus a presupposition (cf. Coppock and Beaver 2012, 2014)!
- So here's the swim move: feed **only** into **the**, and then let the scope of the exclusivity ride the scope of the determiner. This will immediately predict relative readings for adj **only**.

John sold the only cars

Anna didn't give the only good talk

$$\mathbf{M}_u^f = \lambda G .\, \left\{ \langle \alpha, g \rangle \,\middle|\, \langle \langle \alpha, \cdot \rangle, g \rangle \in G,\ \mathsf{truthy}\, \alpha,\ \neg\exists \left\langle \langle \cdot, \beta \rangle, g' \right\rangle \in G .\, \bigvee \beta \wedge f\, (g\, u)\, (g'\, u) \right\}$$

$$\textbf{larger} = \lambda x y .\, \mathsf{size}\, x < \mathsf{size}\, y$$

$$\textbf{est}_u = \lambda f .\, \mathbf{M}_u^f$$

$$\textbf{largest}_u = \textbf{est}_u\, \textbf{larger} = \mathbf{M}_u^{\mathsf{sz}} = \lambda G .\, \left\{ \langle \top, g \rangle \,\middle|\, \langle \top, g \rangle \in G \wedge \neg\exists \langle \top, g' \rangle \in G .\, \mathsf{size}\, (g\, u) < \mathsf{size}\, (g'\, u) \right\}$$

$$\textbf{the}_u = \lambda \mathcal{M} c k g \colon |G'_u| = 1 .\, G', \text{ where } G' = \mathcal{M} \bigcup \left\{ k\, x\, g' \,\middle|\, x \in \mathcal{D}_e,\ \langle \top, g' \rangle \in c\, x\, g^{u \mapsto x} \right\}$$

This is some stuff

$$\left.\begin{array}{c} [\,] \\ \hline \mathsf{j}^{\mathsf{F}} \star (\lambda j .\, [\,]) \\ \hline j^{\triangleright} \star (\lambda x .\, [\,]) \\ \hline x \end{array}\right| \left.\begin{array}{c} [\,] \\ \hline [\,] \\ \hline [\,] \\ \hline \mathsf{drew} \end{array}\right| \begin{array}{c} \lambda g .\, \mathbf{M}_u \bigcup \{ [\,]\, g^{u \mapsto y} \mid \mathsf{sq}\, y \} \\ \hline [\,] \\ \hline [\,] \\ \hline y \end{array}$$

stuff

$$\begin{aligned}
\mathcal{F}\,\alpha &:= \sigma \to \{\alpha * \sigma\} * \{\alpha * \sigma\} \\
\eta\, x &:= \lambda g\,.\, \Big\langle \big\{\langle x, g\rangle\big\},\, \big\{\langle x, g\rangle\big\} \Big\rangle \\
m \star f &:= \lambda g\,.\, \Big\langle \bigcup \Big\{ (f\,x\,g')_1 \,\Big|\, \langle x, g'\rangle \in (m\,g)_1 \Big\},\ \bigcup \Big\{ (f\,x\,g')_2 \,\Big|\, \langle x, g'\rangle \in (m\,g)_2 \Big\} \Big\rangle
\end{aligned}$$

$$\mathbf{the} := \lambda \mathcal{M} c k g : |G_u| = 1\,.\, G_u,\ \text{where } G = \mathcal{M} \bigcup \Big\{ k\, y\, g' \,\Big|\, \langle \mathsf{T}, g'\rangle \in c\, y\, g^{u \mapsto y} \Big\}$$

$$\mathbf{only} := \lambda G : \mathsf{true}\, G\,.\, \Big\{ \langle \mathsf{T}, g\rangle \in G \,\Big|\, \neg \exists g'\,.\, \langle \mathsf{T}, g'\rangle \in G \wedge g'\, u \sqsupset g\, u \Big\}$$