1 The Grammar

Scope

$$m \mathbin{/\!/} n := \begin{cases} m\,n & \textbf{if } m :: \alpha \to \beta,\ n :: \alpha \\ \lambda k\,.\, m\,(\lambda f\,.\, n\,(\lambda x\,.\, k\,(f \mathbin{/\!/} x))) & \textbf{otherwise} \end{cases} \qquad m \mathbin{\backslash\!\backslash} n := \begin{cases} n\,m & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k\,.\, m\,(\lambda x\,.\, n\,(\lambda f\,.\, k\,(x \mathbin{\backslash\!\backslash} f))) & \textbf{otherwise} \end{cases} \qquad m \parallel n := \begin{cases} \lambda x\,.\, m\,x \wedge n\,x & \textbf{if } n :: \alpha \to \beta,\ m :: \alpha \\ \lambda k\,.\, m\,(\lambda x\,.\, n\,(\lambda f\,.\, k\,(f \parallel x))) & \textbf{otherwise} \end{cases}$$

Binding

$$\eta\,x := \lambda gh\,.\,\{\langle x, g, h\rangle \mid x \neq \mathbf{F}\} \qquad m^{\star} := \lambda kgh\,.\,\bigcup\{k\,x\,g'\,h' \mid \langle x, g', h'\rangle \in m\,g\,h\} \qquad m^{\downarrow} := m\,\eta \qquad x^{\uparrow} := (\eta\,x)^{\star} = \lambda k\,.\,k\,x \qquad m^{\rhd u} := m^{\star}\,(\lambda xgh\,.\,\{\langle x, g^{u\mapsto x}, h\rangle\})$$

Evaluation

$$m^{\Downarrow\!\Uparrow} := (\lambda gh\,.\,\{\langle x, g', h\rangle \mid \langle \cdot, \cdot, h'\rangle \in G,\ \ \langle x, g', \cdot\rangle \in h'\,G\})^{\star}, \quad \text{where } G = m^{\downarrow}\,g\,\mathsf{id}$$
$$\mathsf{true}_g\ m := \exists \langle \cdot, \cdot, h\rangle \in G\,.\,h\,G \neq \emptyset, \quad \text{where } G = m^{\downarrow}\,g\,\mathsf{id}$$

- Postsups are evaluated at reset boundaries and at closing time. A computation is true at an input g if it has a successful output *after its postsups have applied.* A computation is not fully reset until its postsuppositions have each had a whack at the set of outputs, at which point the survivors are collected, and the postsups flushed.

- Following Brasoveanu 2012, contexts are split into two components, an assignment function, and a "postsupposition". Postsups here have the recursive type $C \equiv \{\alpha * \gamma * C\} \to \{\alpha * \gamma * C\}$, the type of filters on sets of outputs.

- The dynamic machinery now bears an intriguing resemblance to something Wadler (1994) calls the InputOutput Monad. Assignments play the role of input, postsups that of outputs, in the sense that they are simply accumulated (composed) over the course of the computation, and used to post-process the final result.

2 Basic Definite Descriptions

$$\begin{aligned} \textbf{circle} &:= \mathsf{circ} \\ \textbf{square} &:= \mathsf{sq} \\ \textbf{in} &:= \mathsf{in} \\ \textbf{the}_u &:= \lambda ckgh\,.\,\bigcup\{k\,x\,g'\,h' \mid \langle \mathsf{T}, g', h'\rangle \in c\,x\,g^{u\mapsto x}\,h^{\mathbf{1}_u}\} \\ \mathbf{1}_u &\equiv \lambda G\,.\begin{cases} G & \textbf{if } |\{g\,u \mid \langle \cdot, g, \cdot\rangle \in G\}| = 1 \\ \emptyset & \textbf{otherwise} \end{cases} \end{aligned}$$

- $\textbf{the}_u$ is just $\mathbf{a}_u$ plus a uniqueness postsup, which restruicts what the *global set of outputs* is allowed to look like: they must all agree on the value of u.

- Thus the uniqueness effect associated with **the** is effectively *delayed* until some program containing it is evaluated, similar to the way that other cardinality impositions have been argued to operate (Brasoveanu 2012, Henderson 2014).

Reset

$$[\![\text{the square}]\!] \rightsquigarrow \left(\frac{\textbf{the}_v\,(\lambda y\,.\,[\,])}{y} \mathbin{\Big\backslash\!\!\Big\backslash} \frac{[\,]}{\textbf{square}}\right)^{\downarrow\,\Downarrow\!\Uparrow} \rightsquigarrow (\textbf{the}_v\,(\lambda ygh\,.\,\{\langle \mathsf{sq}\,y, g, h\rangle\}))^{\Downarrow\!\Uparrow}$$
$$\rightsquigarrow \left(\frac{\lambda gh\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v} \mid \mathsf{sq}\,y\}}{y}\right)^{\Downarrow\!\Uparrow} \rightsquigarrow \frac{\lambda gh\colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h \mid \mathsf{sq}\,y\}}{y}$$

- Note that resetting $[\![$the square$]\!]$ in the last reduction step here has no effect on its semantic shape, because it's essentially $[\![$a square$]\!]$.

- But it does fix its postsup (which I've switched to representing as a *pre*sup, since it now just constrains the input). For any incoming g, the G of $\mathbf{1}_u^G$ will be equal to $\{\langle y, g^{v\mapsto y}, \mathbf{1}_u\rangle \mid \mathsf{sq}\,y\}$. Nothing from this set will survive the $\mathbf{1}_u$ filter unless all the assignment funcs in G happen to map v to the same square. That is, unless there's exactly one square available to assign v to in the first place.

ABSOLUTE DEFINITE READING

$[\![\text{the circle in the square}]\!]$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x}\ \mathbin{\backslash\!\backslash}\ \left(\frac{[\,]}{\mathbf{circle}}\ \Big\|\ \frac{[\,]}{\mathbf{in}}\ \mathbin{/\!/}\ \left(\frac{\mathbf{the}_v\,(\lambda y\,.\,[\,])}{y}\ \mathbin{\backslash\!\backslash}\ \frac{[\,]}{\mathbf{square}}\right)^{\downarrow\,\downharpoonleft\upharpoonright}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x}\ \mathbin{\backslash\!\backslash}\ \left(\frac{[\,]}{\mathbf{circle}}\ \Big\|\ \frac{[\,]}{\mathbf{in}}\ \mathbin{/\!/}\ \frac{\lambda gh\colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda xgh\colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\lambda gh\colon |G_v| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h^{\mathbf{1}_u}\ \middle|\ \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\downharpoonleft\upharpoonright}$$

$$\frac{\lambda gh\colon |G_v| = \left|G'_u\right| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h\ \middle|\ \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

- The inner definite is reset, freezing its presupposition as above. When the input assignment g is eventually inserted, we will have $G = \{\langle y, g^{v\mapsto y}, \mathbf{1}_v\rangle \mid \mathsf{sq}\,y\}$, and the presupposition will guarantee that $g'\,v$ is constant across the outputs.
- As with the inner DP, the host DP's presupposition is fixed when it is reset. This time, we have $G' = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}, \mathbf{1}_u\right\rangle\ \middle|\ \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}$, where g is whatever the input happens to be. In particular, all the outputs will now need to agree on the value of u in addition to v, which will only be possible if there's exactly one circle in the square that all outputs assign to v.

RELATIVE DEFINITE (HADDOCK) READING

$[\![\text{the circle in the square}]\!]$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x}\ \mathbin{\backslash\!\backslash}\ \left(\frac{[\,]}{\mathbf{circle}}\ \Big\|\ \frac{[\,]}{\mathbf{in}}\ \mathbin{/\!/}\ \left(\frac{\mathbf{the}_v\,(\lambda y\,.\,[\,])}{y}\ \mathbin{\backslash\!\backslash}\ \frac{[\,]}{\mathbf{square}}\right)^{\downarrow}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x}\ \mathbin{\backslash\!\backslash}\ \left(\frac{[\,]}{\mathbf{circle}}\ \Big\|\ \frac{[\,]}{\mathbf{in}}\ \mathbin{/\!/}\ \frac{\lambda gh\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v} \mid \mathsf{sq}\,y\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda xgh\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v} \mid \mathsf{sq}\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{in}\,y\,x}\right)^{\downarrow\,\downharpoonleft\upharpoonright}$$

$$\left(\frac{\lambda gh\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,\left(h^{\mathbf{1}_u}\right)^{\mathbf{1}_v}\ \middle|\ \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}\right)^{\downharpoonleft\upharpoonright}$$

$$\frac{\lambda gh\colon |G_v| = |G_u| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h\ \middle|\ \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{in}\,y\,x\right\}}{x}$$

- The only difference here is that we do not reset the inner DP, which staves off its postsup until more information is accumulated in its scope.
- So when the outer DP is eventually reset, it ends up fixing the postsup of both definites to the same set of potential outputs.
- For any input g, the set of outputs G that $\mathbf{1}_u$ and $\mathbf{1}_v$ will target is given below. All of the outputs in this set will map u to some circle and v to some square containing the circle they map to u.
  $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}, \mathbf{1}_u^{\mathbf{1}_v}\right\rangle\ \middle|\ \mathsf{sq}\,x,\ \mathsf{circ}\,y,\ \mathsf{in}\,y\,x\right\}$.
- Requiring that there be exactly one such v and one such u is tantamount to requiring that there be exactly one square *that has a circle in it* and exactly one circle *in that square.* In other words, there should be exactly one pair $\langle x, y\rangle$ in $\mathsf{circ} \times \mathsf{sq}$ such that $\mathsf{in}\,y\,x$.

3 Plurals

$$\begin{aligned}
\textbf{-s} &\coloneqq \lambda P x \,.\, x \in \{\bigoplus P' \mid P' \subseteq P\} \\
\textbf{the}_u &\coloneqq \lambda ckgh \,.\, \bigcup \left\{ k\, x\, g' h' \;\middle|\; \langle \mathbf{T}, g', h' \rangle \in c\, x\, g^{u \mapsto x}\, h^{\mathbf{1}_u \circ \mathbf{M}_u} \right\} \\
\mathbf{M}_u &\coloneqq \lambda G \,.\, \{\langle \cdot, g, \cdot \rangle \in G \mid \neg \exists \langle \cdot, g', \cdot \rangle \in G \,.\, g'\, u \sqsupset g\, u\} \\
\textbf{in} &\coloneqq \lambda XY \,.\, (\forall x < X \,.\, \exists y < Y \,.\, \mathsf{in}\, x\, y) \wedge (\forall y < Y \,.\, \exists x < X \,.\, \mathsf{in}\, x\, y)
\end{aligned}$$

- **-s** is a boilerplate plural morpheme that builds sums from the atoms in its complement.
- $\mathbf{M}_u$ is a kind of maximization operator on outputs (Brasoveanu 2012, Charlow 2014). It filters out those assignments in $g \in G$ that are strictly dominated, in the sense that they assign u to a value that is a proper part of something assigned to u by some other $g' \in G$.
- Predicates and relations are extended to sums in the usual cumulative way (Link, …)

Plural Reset

$$\begin{aligned}
[\![\text{the squares}]\!] &\rightsquigarrow \left(\frac{\textbf{the}_v\, (\lambda Y \,.\, [\,])}{Y} \;\Big\backslash\!\!\Big\backslash\; \frac{[\,]}{\textbf{square}} \;\Big/\!\!\Big/\; \frac{[\,]}{\textbf{-s}} \right)^{\downarrow \Downarrow} \rightsquigarrow (\textbf{the}_v\, (\lambda Y g h \,.\, \{\langle \mathsf{sq's}\, Y, g, h\rangle\}))^{\Downarrow} \\
&\rightsquigarrow \left(\frac{\lambda g h \,.\, \bigcup \left\{ [\,]\, g^{v \mapsto Y}\, h^{\mathbf{1}_u \circ \mathbf{M}_v} \;\middle|\; \mathsf{sq's}\, Y \right\}}{Y} \right)^{\Downarrow} \\
&\rightsquigarrow \frac{\lambda g h \colon |G_v| = 1 \,.\, \bigcup \{ [\,]\, g^{v \mapsto Y}\, h \mid \mathsf{sq}\, Y,\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y \}}{Y}
\end{aligned}$$

- Once again, resetting $[\![$the square$]\!]$ in the last reduction step has no effect on its semantic shape, because it's still essentially $[\![$a square$]\!]$.
- But again, it fixes the constituent's postsup. For any incoming g, G will be equal to $\{\langle y, g^{v \mapsto Y}, \mathbf{1}_v \circ \mathbf{M}_v \rangle \mid \mathsf{sq's}\, Y\}$. The only output triples that will survive the $\mathbf{M}_v$ filter will be those that are undominated in their choice of v, i.e., those that assign v to a sum that isn't part of any sum assigned to v by any other output. The subsequent $\mathbf{1}_v$ filter will then guarantee that there is exactly one such maximal referent across outputs (this should generally by trivial, given usual mereological assumptions).

Absolute Plural Reading

$[\![$the circles in the squares$]\!]$

$$\left(\frac{\textbf{the}_u\, (\lambda X \,.\, [\,])}{X} \;\Big\backslash\!\!\Big\backslash\; \left(\frac{[\,]}{\textbf{circles}} \;\Big\|\; \frac{[\,]}{\textbf{in}} \;\Big/\!\!\Big/\; \left(\frac{\textbf{the}_v\, (\lambda Y \,.\, [\,])}{Y} \;\Big\backslash\!\!\Big\backslash\; \frac{[\,]}{\textbf{squares}} \right)^{\downarrow \Downarrow} \right) \right)^{\downarrow \Downarrow}$$

$$\left(\frac{\textbf{the}_u\, (\lambda X \,.\, [\,])}{X} \;\Big\backslash\!\!\Big\backslash\; \left(\frac{[\,]}{\textbf{circles}} \;\Big\|\; \frac{[\,]}{\textbf{in}} \;\Big/\!\!\Big/\; \frac{\lambda g h \colon |G_v| = 1 \,.\, \bigcup \left\{ [\,]\, g^{v \mapsto Y}\, h \;\middle|\; \begin{array}{l} \mathsf{sq's}\, Y, \\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y \end{array} \right\}}{Y} \right) \right)^{\downarrow \Downarrow}$$

$$\left(\frac{\textbf{the}_u \left(\lambda X g h \colon |G_v| = 1 \,.\, \bigcup \{ [\,]\, g^{v \mapsto Y}\, h \mid \mathsf{sq}\, Y,\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y \} \right)}{\mathsf{circ's}\, X \wedge \mathsf{in}\, Y\, X} \right)^{\downarrow \Downarrow}$$

$$\left(\frac{\lambda g h \colon |G_v| = 1 \,.\, \bigcup \left\{ [\,]\, g^{\substack{u \mapsto X \\ v \mapsto Y}}\, h^{\mathbf{1}_u \circ \mathbf{M}_u} \;\middle|\; \begin{array}{l} \mathsf{sq's}\, Y,\ \mathsf{circ's}\, X,\ \mathsf{in}\, Y\, X, \\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y \end{array} \right\}}{X} \right)^{\Downarrow}$$

$$\frac{\lambda g h \colon |G_v| = |G'_u| = 1 \,.\, \bigcup \left\{ [\,]\, g^{\substack{u \mapsto X \\ v \mapsto Y}}\, h \;\middle|\; \begin{array}{l} \mathsf{sq}\, Y,\ \mathsf{circ}\, X,\ \mathsf{in}\, Y\, X, \\ \forall Z \colon \mathsf{sq's} \,.\, Y \not\sqsubset Z, \\ \forall Z \,.\, \mathsf{circ's}\, Z \wedge \mathsf{in}\, Y\, Z \Rightarrow X \not\sqsubset Z \end{array} \right\}}{X}$$

- The inner definite is reset, freezing its supposition as above. $\mathbf{M}_v$ will throw out any outputs that do not assign v to the largest sum of circles, leaving $G = \{\langle Y, g^{v \mapsto Y}, \mathbf{1}_v \circ \mathbf{M}_v \rangle \mid \mathsf{sq's}\, Y,\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y\}$. When the input assignment g is eventually inserted, $\mathbf{1}_v$ will guarantee that this individual is constant across possible outputs (i.e., unique).
- When the host DP is reset, its maximality supposition $\mathbf{M}_u$ is applied to the set of outputs $G' = \left\{ \left\langle x, g^{\substack{u \mapsto X \\ v \mapsto Y}}, \mathbf{1}_u \circ \mathbf{M}_u \right\rangle \;\middle|\; \mathsf{sq's}\, Y,\ \mathsf{circ's}\, X,\ \mathsf{in}\, Y\, X,\ \forall Z \colon \mathsf{sq's} \,.\, Z \not\sqsupset Y \right\}$. This is the set out assignments that map v to a (the) maximal sum of squares, and v to a sum of circles that are (cumulatively) in that sum. The $\mathbf{M}_u$ filter will discard from this set any outputs that do not map u to as big a sum as possible, i.e., the maximal sum of circles that are in Y. Finally $\mathbf{1}_u$ will guarantee that this maximal sum is unique.
- For this whole update to be defined, it must be the case that every square has at least one circle in it.

Relative Plural (Haddock) Reading

⟦the circles in the squares⟧

$$\left(\frac{\mathbf{the}_u\,(\lambda X\,.\,[\,])}{X} \,\Big\|\, \left(\frac{[\,]}{\mathbf{circles}} \,\Big\|\, \frac{[\,]}{\mathbf{in}} \,\Big/\!\!\Big/\, \left(\frac{\mathbf{the}_v\,(\lambda Y\,.\,[\,])}{Y} \,\Big\|\, \frac{[\,]}{\mathbf{squares}}\right)^{\downarrow}\right)\right)^{\downarrow\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda X\,.\,[\,])}{X} \,\Big\|\, \left(\frac{[\,]}{\mathbf{circles}} \,\Big\|\, \frac{[\,]}{\mathbf{in}} \,\Big/\!\!\Big/\, \frac{\lambda gh\,.\,\bigcup\left\{[\,]\,g^{v\mapsto Y}\,h^{\mathbf{1}_v\circ\mathbf{M}_v} \,\middle|\, \text{sq's}\,Y\right\}}{y}\right)\right)^{\downarrow\Downarrow}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda Xgh\,.\,\bigcup\left\{[\,]\,g^{v\mapsto Y}\,h^{\mathbf{1}_v\circ\mathbf{M}_v} \,\middle|\, \text{sq's}\,Y\right\}\right)}{\text{circ's}\,X \wedge \text{in}\,Y\,X}\right)^{\downarrow\Downarrow}$$

$$\left(\frac{\lambda gh\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto X\\ v\mapsto Y}}\,\left(h^{\mathbf{1}_u\circ\mathbf{M}_u}\right)^{\mathbf{1}_v\circ\mathbf{M}_v} \,\middle|\, \text{sq's}\,Y,\ \text{circ's}\,X,\ \text{in}\,Y\,X\right\}}{X}\right)^{\Downarrow}$$

$$\frac{\lambda gh\colon |G_v| = |G_u| = 1\,.\,\bigcup\left\{[\,]\,g^{\substack{u\mapsto X\\ v\mapsto Y}}\,h \,\middle|\, \text{sq's}\,Y,\ \text{circ's}\,X,\ \text{in}\,Y\,X\right\}}{X}$$

- This double-barreled cumulation and concomitant cardinatlity checking is exactly as (is intended) in Brasoveanu 2012. This brings definites in line with other cardinality post-supposing expressions like ‘exactly three’.

- As before, the only difference here is that we do not reset the inner DP, which staves off its postsup until more information is accumulated in its scope. So when the outer DP is eventually reset, it ends up fixing the postsup of both definites to the same set of potential outputs.
- For any input g, the set of outputs G that $\mathbf{1}_u \circ \mathbf{M}_u \circ \mathbf{1}_v \circ \mathbf{M}_v$ will target is given below. All of the outputs in this set will map u to some plurality of circles and v to some plurality of squares (cumulatively) containing those circles.
$G = \left\{\left\langle x,\, g^{\substack{u\mapsto X\\ v\mapsto Y}},\, \mathbf{1}_u \circ \mathbf{M}_u^{\mathbf{1}_v\circ\mathbf{M}_v}\right\rangle \,\middle|\, \text{sq's}\,X,\ \text{circ's}\,Y,\ \text{in}\,Y\,X\right\}$.
- $\mathbf{M}_v$ will throw out any candidate output that is dominated by another in its choice for v, and $\mathbf{1}_v$ will guarantee (trivially, as usual) that there is exactly one such undominated candidate. That is, $\mathbf{M}_v$ will filter out any outputs that do not assign v to the maximal sum of circle-containing squares.
- Likewise, $\mathbf{M}_u$ will then throw out any candidates that are dominated in their choice for u; that is, any candidates that do not assign u to the maximal sum of circles that are contained in the sum total of circle-containing squares. So by the end, we’ll have a set of outputs all of which assign v to the totality of squares that contain (*any* circles) and u to the totality of circles that are in (*any*) squares.

4 DP-Internal Superlatives

$$\begin{aligned}
\mathbf{older} &:= \lambda xy\,.\,\text{age}\,x > \text{age}\,y\\
\mathbf{the}_u &:= \lambda\mathcal{M}ckgh\,.\,\bigcup\left\{k\,x\,g'\,h' \,\middle|\, \langle\mathbf{T}, g', h'\rangle \in c\,x\,g^{u\mapsto x}\,h^{\mathbf{1}_u\circ\mathcal{M}}\right\}\\
\mathbf{M}_u^f &:= \lambda G\,.\,\{\langle\cdot, g, \cdot\rangle \in G \mid \neg\exists\langle\cdot, g', \cdot\rangle \in G\,.\,f\,(g'\,u)\,(g\,u)\}\\
\mathbf{est}_u &:= \lambda f\,.\,\mathbf{M}_u^f\\
\mathbf{oldest}_u = \mathbf{est}_u\,\mathbf{older} &= \lambda G\,.\,\{\langle\cdot, g, \cdot\rangle \in G \mid \neg\exists\langle\cdot, g', \cdot\rangle \in G\,.\,\text{age}\,(g'\,u) > \text{age}\,(g\,u)\}
\end{aligned}$$

- **est** abstracts over the ordering function that $\mathbf{M}$ uses to compare individuals. In the case of pluralities, $\mathbf{M}_u$ filters away any output that assigns u to a *smaller sum* than it could have (ie., a smaller sum than one of the other outputs assigns to u). In the case of **oldest**, $\mathbf{M}_u$ filters outputs that assign u to a *younger* individual than they could have.
- And now here’s the swim move: **the** absorbs the max operator and then carries it along for the ride. “Absolute” definite become absolute superlatives; “Haddock” definites become relative superlatives. This part is my favorite.

Superlative Reset

⟦the oldest squirrel⟧

$$\rightsquigarrow \left(\frac{\mathbf{the}_v\,\mathbf{M}_v^{\text{ag}}\,(\lambda y\,.\,[\,])}{y} \,\Big\|\, \frac{[\,]}{\mathbf{squirrel}}\right)^{\downarrow\Downarrow} \rightsquigarrow \left(\mathbf{the}_v\,\mathbf{M}_v^{\text{ag}}\,(\lambda ygh\,.\,\{\langle\text{sq}\,y, g, h\rangle\})\right)^{\Downarrow}$$

$$\rightsquigarrow \left(\frac{\lambda gh\,.\,\bigcup\left\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v\circ\mathbf{M}_v^{\text{ag}}} \,\middle|\, \text{sq}\,y\right\}}{y}\right)^{\Downarrow}$$

$$\rightsquigarrow \frac{\lambda gh\colon |G_v| = 1\,.\,\bigcup\{[\,]\,g^{v\mapsto y}\,h \mid \text{sq}\,y,\ \forall z\colon\text{sq}\,.\,\neg\,\text{older}\,z\,y\}}{y}$$

- Resetting the superlative DP has two effects: (1) it fixes the set of individuals that the superlative operator $\mathbf{M}$ compares, in this case squirrels with respect to age; and (2) it freezes the postsup associated with the definite article.
- Here $G = \{\langle y, g^{v\mapsto y}, \mathbf{1}_v \circ \mathbf{M}_v\rangle \mid \text{sq}\,y,\ \forall z\colon\text{sq}\,.\,\neg\,\text{older}\,z\,y\}$. This set could in principle have multiple winners (multiple squirrels with the same greatest age), but the definite determiner rules that out; all the $g \in G$ need to point v to *the same* squirrel.

Absolute DP-Internal Superlatives

⟦the circus with the oldest squirrel⟧

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \mathbin{\backslash\!\backslash} \left(\frac{[\,]}{\mathbf{circus}} \mathbin{\|} \frac{[\,]}{\mathbf{with}} \mathbin{/\!/} \left(\frac{\mathbf{the}_v\,\mathbf{M}_v^{\mathrm{ag}}\,(\lambda y\,.\,[\,])}{y} \mathbin{\backslash\!\backslash} \frac{[\,]}{\mathbf{squirrel}}\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \mathbin{\backslash\!\backslash} \left(\frac{[\,]}{\mathbf{circus}} \mathbin{\|} \frac{[\,]}{\mathbf{with}} \mathbin{/\!/} \frac{\lambda gh \colon |G_v| = 1\,.\bigcup\left\{[\,]\,g^{v\mapsto y}\,h \;\middle|\; \begin{array}{l}\mathsf{sq}\,y,\\ \forall z\colon \mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda xgh\colon |G_v| = 1\,.\bigcup\{[\,]\,g^{v\mapsto y}\,h \mid \mathsf{sq}\,y,\ \forall z\colon \mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\}\right)}{\mathsf{circ}\,x \wedge \mathsf{with}\,y\,x}\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\lambda gh\colon |G_v| = 1\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h^{\mathbf{1}_u} \;\middle|\; \begin{array}{l}\mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x,\\ \forall z\colon \mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{x}\right)^{\downharpoonleft\!\upharpoonright}$$

$$\frac{\lambda gh\colon |G'_u| = |G_v| = 1\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h \;\middle|\; \begin{array}{l}\mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x,\\ \forall z\colon \mathsf{sq}\,.\,\neg\,\mathsf{older}\,z\,y\end{array}\right\}}{x}$$

- When the inner DP is reset, it has the effect just demonstrated: the set of outputs is restricted to those that map v to the (unique) oldest squirrel.
- The outer definite has no explicit maximization operator. For simplicity, for now, I assume that that the $\mathcal{M}$ argument to **the** is optional. But even max-less, the definite still imposes a uniquness condition on the values assigned to u.
- Altogether, after the host DP has been reset, we will have an essentially nondeterministic (continuized) individual with two presuppositions: (1) there is exactly one squirrel with no elders; and (2) there is exactly one circus that he is with.

Relative DP-Internal Superlatives

⟦the circus with the oldest squirrel⟧

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \mathbin{\backslash\!\backslash} \left(\frac{[\,]}{\mathbf{circus}} \mathbin{\|} \frac{[\,]}{\mathbf{with}} \mathbin{/\!/} \left(\frac{\mathbf{the}_v\,\mathbf{M}_v^{\mathrm{ag}}\,(\lambda y\,.\,[\,])}{y} \mathbin{\backslash\!\backslash} \frac{[\,]}{\mathbf{squirrel}}\right)^{\downarrow}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,(\lambda x\,.\,[\,])}{x} \mathbin{\backslash\!\backslash} \left(\frac{[\,]}{\mathbf{circus}} \mathbin{\|} \frac{[\,]}{\mathbf{with}} \mathbin{/\!/} \frac{\lambda gh\,.\bigcup\left\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v\circ\mathbf{M}_v^{\mathrm{ag}}} \;\middle|\; \mathsf{sq}\,y\right\}}{y}\right)\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\mathbf{the}_u\,\left(\lambda xgh\,.\bigcup\left\{[\,]\,g^{v\mapsto y}\,h^{\mathbf{1}_v\circ\mathbf{M}_v^{\mathrm{ag}}} \;\middle|\; \mathsf{sq}\,y\right\}\right)}{\mathsf{circ}\,x \wedge \mathsf{with}\,y\,x}\right)^{\downarrow\,\downharpoonleft\!\upharpoonright}$$

$$\left(\frac{\lambda gh\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,\left(h^{\mathbf{1}_u}\right)^{\mathbf{1}_v\circ\mathbf{M}_v} \;\middle|\; \mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x\right\}}{x}\right)^{\downharpoonleft\!\upharpoonright}$$

$$\frac{\lambda gh\colon |G_u| = |G_v| = 1\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto x\\ v\mapsto y}}\,h \;\middle|\; \begin{array}{l}\mathsf{sq}\,y,\ \mathsf{circ}\,x,\ \mathsf{with}\,y\,x\\ \forall z\colon \mathsf{sq}\,.\,(\exists a\,.\,\mathsf{with}\,z\,a) \Rightarrow \neg\,\mathsf{older}\,z\,a\end{array}\right\}}{x}$$

- The only difference here is that we do not reset the inner DP, which staves off its postsup until more of the context is accumulated in its scope. By the time the postsup is actually evaluated, all of the potential output assignments will map v to squirrles *that are in some circus.*
- Resetting the outer DP fixes the suppositions of *both* definites, bolting them down to the same set of outputs.
- For any input g, $G = \left\{\left\langle x, g^{\substack{u\mapsto x\\ v\mapsto y}}, \mathbf{1}_u^{\mathbf{1}_v\circ\mathbf{M}_v}\right\rangle \;\middle|\; \mathsf{sq}\,x,\ \mathsf{circ}\,y,\ \mathsf{with}\,y\,x\right\}$ is the set out outputs that assign v to a squirrel and v to some circus employing it.
- Before either uniqueness check is enforced, $\mathbf{M}_v$ will filter out any assignments that are dominated v-wise — that is, any assignments that send v to a younger circus squirrel than they could have.
- Then, $\mathbf{1}_v$ will guarantee that there is exactly one such v across all the surviving gs, in effect guaranteeing that there is exactly one circus squirrel that is older than all the others. Subsequently, $\mathbf{1}_u$ will enforce that squirrel's no-compete clause with his particular circus. Altogether, this will ensure that one circus squirrel is older than all the others, and that he works for exactly one circus.

5 Focus

$$\begin{aligned}\mathbf{M}_u^f &= \lambda G\,.\,\{\langle \mathsf{T}, g, h\rangle \mid \langle\langle \mathsf{T}, \cdot\rangle, g, h\rangle \in G,\ \ \neg\exists\langle\langle \cdot, \beta\rangle, g', \cdot\rangle \in G\,.\,\bigvee\beta \wedge f\,(g\,u)\,(g'\,u)\}\\ \textbf{oldest}_u &= \textbf{est}_u\ \textbf{older}\\ &= \lambda G\,.\left\{\langle \mathsf{T}, g, h\rangle \,\middle|\, \begin{array}{l}\langle\langle \mathsf{T}, \cdot\rangle, g, h\rangle \in G,\\ \neg\exists\langle\langle \cdot, \beta\rangle, g', \cdot\rangle \in G\,.\ \bigvee\beta \wedge \mathsf{age}\,(g\,u) < \mathsf{age}\,(g'\,u)\end{array}\right\}\end{aligned}$$

- Note that **the** is exactly as before. It is neutral between focus-sensitive and focus-insensitive "maximizers".

- Superlatives are apparently focus-sensitive. 'John sold MARY the most expensive painting' means he sold Mary a more expensive painting than he sold anyone else.
- To prepare for this, the entry for **est** here expects to see outputs with focus-pairs. It filters out the assignments that either fail to satisfy the "ordinary" component of their prejacent, or fail to assign u to a value whose f-degree is greater than that of every other acceptable assignment's choice of u.

Derivations

⟦John drew the oldest square⟧

$$\left(\dfrac{\dfrac{(\eta\,\mathsf{j})^{\triangleright} \star (\lambda j\,.\,[\,])}{\mathsf{j}^{\mathbf{F}} \star (\lambda x\,.\,[\,])}}{x} \;\Big\|\; \dfrac{\dfrac{[\,]}{[\,]}}{\mathsf{drew}} \;\Big\|\; \dfrac{\dfrac{\lambda gh\,.\bigcup\left\{[\,]\,g^{v\mapsto y}\ h^{\mathbf{1}_u\circ\mathbf{M}_v^{\mathsf{ag}}} \,\middle|\, \mathsf{sq}\,y\right\}}{[\,]}}{y}\right)^{\Downarrow\!\Uparrow}$$

$$\left(\dfrac{\dfrac{\lambda gh\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto \mathsf{j}\\ v\mapsto y}}\ h^{\mathbf{1}_u\circ\mathbf{M}_v^{\mathsf{ag}}} \,\middle|\, \mathsf{sq}\,y\right\}}{\mathsf{j}^{\mathbf{F}} \star (\lambda x\,.\,[\,])}}{\mathsf{drew}\,y\,x}\right)^{\Downarrow\!\Uparrow}$$

$$\lambda gh\colon |G_u| = 1\,.\bigcup\left\{[\,]\,g^{\substack{u\mapsto \mathsf{j}\\ v\mapsto y}}\ h \,\middle|\, \begin{array}{l}\mathsf{sq}\,y,\ \ \mathsf{drew}\,y\,\mathsf{j},\\ \forall z\colon \mathsf{sq}\,.\,(\exists a\,.\,\mathsf{drew}\,z\,a) \Rightarrow \neg\,\mathsf{older}\,z\,y\end{array}\right\}$$

- The superlative needs to outscope focus so that it can filter out squares drawn by John's alternatives. This is effectively guaranteed by stashing the superlative operator in the postsuppositional cubby, but we want it to out-tower focus, just so the reset operator works out (though this can probably be generalized).
- Here for the first time, we see the index on **the** coming apart from that on $\mathbf{M}$. This is because, intuitively, to me anyway, the 'the' of relative 'the oldest' targets *John*, not the squares that people have drawn. It says that John is *the winner* of the square-drawing contest, regardless of how many winning squares he has drawn.
- $\mathbf{M}_v$ keeps all of the gs that map v to a square that (i) John drew, and (ii) is at least as big as any square that anybody drew. If there aren't any such squares, the sentence is false. If there are any such squares, the sentence is guaranteed to succeed because $\mathbf{1}_u$ can't fail. It checks that all of the outputs map u to the same individual, but of course they do; they all map u to John (by design).

$$\begin{aligned}\mathcal{F}\,\alpha &\coloneqq \sigma \to \{\alpha * \sigma\} * \{\alpha * \sigma\}\\ \eta\,x &\coloneqq \lambda g\,.\,\langle\{\langle x, g\rangle\}, \{\langle x, g\rangle\}\rangle\\ m \star f &\coloneqq \lambda g\,.\,\left\langle\bigcup\{(f\,x\,g')_1 \mid \langle x, g'\rangle \in (m\,g)_1\},\ \bigcup\{(f\,x\,g')_2 \mid \langle x, g'\rangle \in (m\,g)_2\}\right\rangle\end{aligned}$$

- Rolling focus and state simultaneously to grease the wheels

6 Adjectival Exclusives

$$\begin{aligned}\mathsf{true} &\coloneqq \lambda G\,.\,\bigvee\{\alpha \mid \langle \alpha, g\rangle \in G\}\\ \textbf{not} &\coloneqq \lambda m g\,.\,\eta\,(\neg(m\,g))\,g\\ \textbf{only}_u &\coloneqq \lambda G\colon \mathsf{true}\,G\,.\,\mathbf{M}_u\,G\\ \textbf{the}_u &\coloneqq \lambda \mathcal{M} c k g\,.\,|G_u| = 1\,.\,G,\\ &\qquad \text{where } G = \mathcal{M}_u \bigcup\{k\,x\,g' \mid \langle \mathsf{T}, g'\rangle \in c\,x\,g^{u\mapsto x}\}\end{aligned}$$

- true and **not** are bog standard dynamic booleans: an update is "true" iff it generates at least one successful output
- Adjectival **only** is just $\mathbf{M}$ plus a presupposition (cf. Coppock and Beaver 2012, 2014)!
- So here's the swim move: feed **only** into **the**, and then let the scope of the exclusivity ride the scope of the determiner. This will predict relative readings for adj **only**.