

$$F :: (a \rightarrow b) \rightarrow \text{SG}a \rightarrow \text{SG}b$$

$$F = \lambda f m. \{\lambda r. f(\alpha r) \mid \alpha \in m\}$$

$$H :: (a \rightarrow b) \rightarrow \text{GS}a \rightarrow \text{GS}b$$

$$H = \lambda f m r. \{f x \mid x \in m r\}$$

$$\varepsilon :: \text{SG}a \rightarrow \text{GS}a$$

$$\varepsilon = \lambda m r. \{\alpha r \mid \alpha \in m\}$$

$$\begin{array}{ccc} \text{SG}a & \xrightarrow{Ff} & \text{SG}b \\ \downarrow \varepsilon & & \downarrow \varepsilon \\ \text{GS}a & \xrightarrow{Hf} & \text{GS}b \end{array}$$

$$\begin{aligned} \varepsilon \circ Ff &= \lambda m. \varepsilon(Ff m) \\ &= \lambda m. \varepsilon\{\lambda r. f(\alpha r) \mid \alpha \in m\} \\ &= \lambda m r. \{\beta r \mid \beta \in \{\lambda r. f(\alpha r) \mid \alpha \in m\}\} \\ &= \lambda m r. \{f(\alpha r) \mid \alpha \in m\} \end{aligned}$$

$$\begin{aligned} Hf \circ \varepsilon &= \lambda m. Hf(\varepsilon m) \\ &= \lambda m. Hf(\lambda r. \{\alpha r \mid \alpha \in m\}) \\ &= \lambda m r. \{f x \mid x \in \{\alpha r \mid \alpha \in m\}\} \\ &= \lambda m r. \{f(\alpha r) \mid \alpha \in m\} \end{aligned}$$

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$$\delta :: \text{GS}a \rightarrow \text{SG}a$$

$$\delta = \lambda m. \{\beta \mid \forall r. \beta r \in m r\}$$

$$\begin{aligned} \delta \circ Hf &= \lambda m. \delta(Hf m) \\ &= \lambda m. \delta(\lambda r. \{f x \mid x \in m r\}) \\ &= \lambda m. \{\beta \mid \forall r. \beta r \in \{f x \mid x \in m r\}\} \end{aligned}$$

$$\begin{aligned} Ff \circ \delta &= \lambda m. Ff(\delta m) \\ &= \lambda m. Ff\{\beta \mid \forall r. \beta r \in m r\} \\ &= \lambda m. \{\lambda r. f(\beta r) \mid \beta \in \{\beta \mid \forall r. \beta r \in m r\}\} \\ &= \lambda m. \{\lambda r. f(\beta r) \mid \forall r. \beta r \in m r\} \end{aligned}$$