

$$F :: (a \rightarrow b) \rightarrow \text{SG}a \rightarrow \text{SG}b$$

$$F = \lambda f m. \{\lambda r. f(\alpha r) \mid \alpha \in m\}$$

$$H :: (a \rightarrow b) \rightarrow \text{GS}a \rightarrow \text{GS}b$$

$$H = \lambda f m r. \{f x \mid x \in m r\}$$

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$$\varepsilon :: \text{SG}a \rightarrow \text{GS}a$$

$$\varepsilon = \lambda m r. \{\alpha r \mid \alpha \in m\}$$

$$\begin{array}{ccc} \text{SG}a & \xrightarrow{F f} & \text{SG}b \\ \downarrow \varepsilon & & \downarrow \varepsilon \\ \text{GS}a & \xrightarrow{H f} & \text{GS}b \end{array}$$

$$\begin{aligned} \varepsilon \circ F f &= \lambda m. \varepsilon (F f m) \\ &= \lambda m. \varepsilon \{\lambda r. f(\alpha r) \mid \alpha \in m\} \\ &= \lambda m r. \{\beta r \mid \beta \in \{\lambda r. f(\alpha r) \mid \alpha \in m\}\} \\ &= \lambda m r. \{f(\alpha r) \mid \alpha \in m\} \end{aligned}$$

$$\begin{aligned} H f \circ \varepsilon &= \lambda m. H f(\varepsilon m) \\ &= \lambda m. H f(\lambda r. \{\alpha r \mid \alpha \in m\}) \\ &= \lambda m r. \{f x \mid x \in \{\alpha r \mid \alpha \in m\}\} \\ &= \lambda m r. \{f(\alpha r) \mid \alpha \in m\} \end{aligned}$$

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$$\delta :: \text{GS}a \rightarrow \text{SG}a$$

$$\delta = \lambda m. \{\lambda r. \sigma(m r) \mid \sigma \in \text{CF}\}$$

$$\begin{array}{ccc} \text{SG}a & \xrightarrow{F f} & \text{SG}b \\ \uparrow \delta & & \uparrow \delta \\ \text{GS}a & \xrightarrow{H f} & \text{GS}b \end{array}$$

$$\begin{aligned} \delta \circ H f &= \lambda m. \delta (H f m) \\ &= \lambda m. \delta (\lambda r. \{f x \mid x \in m r\}) \\ &= \lambda m. \{\lambda r. \sigma \{f x \mid x \in m r\} \mid \sigma \in \text{CF}\} \\ &= \lambda m. \{\lambda r. f(\sigma(m r)) \mid \sigma \in \text{CF}\} \end{aligned}$$

$$\begin{aligned} F f \circ \delta &= \lambda m. F f(\delta m) \\ &= \lambda m. F f \{\lambda r. \sigma(m r) \mid \sigma \in \text{CF}\} \\ &= \lambda m. \{\lambda r. f(\beta r) \mid \beta \in \{\lambda r. \sigma(m r) \mid \sigma \in \text{CF}\}\} \\ &= \lambda m. \{\lambda r. f(\sigma(m r)) \mid \sigma \in \text{CF}\} \end{aligned}$$