$$F :: (a \to b) \to \mathrm{SG}a \to \mathrm{SG}b \qquad\qquad H :: (a \to b) \to \mathrm{GS}a \to \mathrm{GS}b$$

$$F = \lambda fm.\{\lambda r.\,f(\alpha\,r) \mid \alpha \in m\} \qquad\qquad H = \lambda fmr.\{f\,x \mid x \in m\,r\}$$

$$\varepsilon :: \mathrm{SG}a \to \mathrm{GS}a$$

$$\varepsilon = \lambda mr.\{\alpha\,r \mid \alpha \in m\}$$

$$
\begin{array}{ccc}
\mathrm{SG}a & \xrightarrow{\;F\,f\;} & \mathrm{SG}b \\
\downarrow{\scriptstyle\varepsilon} & & \downarrow{\scriptstyle\varepsilon} \\
\mathrm{GS}a & \xrightarrow{\;H\,f\;} & \mathrm{GS}b
\end{array}
$$

$$
\begin{aligned}
\varepsilon \circ F\,f &= \lambda m.\,\varepsilon\,(F\,f\,m) \\
&= \lambda m.\,\varepsilon\,\{\lambda r.\,f(\alpha\,r) \mid \alpha \in m\} \\
&= \lambda mr.\,\{\beta\,r \mid \beta \in \{\lambda r.\,f(\alpha\,r) \mid \alpha \in m\}\} \\
&= \lambda mr.\,\{f(\alpha\,r) \mid \alpha \in m\}
\end{aligned}
$$

$$
\begin{aligned}
H\,f \circ \varepsilon &= \lambda m.\,H\,f\,(\varepsilon\,m) \\
&= \lambda m.\,H\,f\,(\lambda r.\,\{\alpha\,r \mid \alpha \in m\}) \\
&= \lambda mr.\,\{f\,x \mid x \in \{\alpha\,r \mid \alpha \in m\}\} \\
&= \lambda mr.\,\{f(\alpha\,r) \mid \alpha \in m\}
\end{aligned}
$$

....

$$\delta :: \mathrm{GS}a \to \mathrm{SG}a$$

$$\delta = \lambda m.\{\beta \mid \forall r.\,\beta\,r \in m\,r\}$$

$$
\begin{aligned}
\delta \circ H\,f &= \lambda m.\,\delta\,(H\,f\,m) \\
&= \lambda m.\,\delta\,(\lambda r.\,\{f\,x \mid x \in m\,r\}) \\
&= \lambda m.\,\{\beta \mid \forall r.\,\beta\,r \in \{f\,x \mid x \in m\,r\}\}
\end{aligned}
$$

$$
\begin{aligned}
F\,f \circ \delta &= \lambda m.\,F\,f\,(\delta\,m) \\
&= \lambda m.\,F\,f\,\{\beta \mid \forall r.\,\beta\,r \in m\,r\} \\
&= \lambda m.\,\{\lambda r.\,f(\beta\,r) \mid \forall r.\,\beta\,r \in m\,r\}
\end{aligned}
$$