2.4. Efficiency of various types of algorithms

(1) Algorithms with Linear loops

```
K = 1
while $K \le 1000$
        some code
        K = K + 1
end loop
```

```
K = 1
while $I \le 1000$
        some code
        K = K + 2
end loop
```

In the first algorithm the loop is repeated 1000 times whilst in the second algorithm the loop is repeated 500 times. In both algorithms the number of iterations is directly proportional to a factor. The efficiency is proportional to number of iterations.

In general: Efficiency is linear – O(n)

(2) Algorithms with logarithmic loops

```
K = 1
while $K < 1000$
        some code
        K = K * 2
end loop
```

In this algorithm the loop is repeated 10 times.
The number of iterations is a function of the multiplier

Here multiplier is 2
Efficiency is $\log_2 1000$ approx = 10

In general: Efficiency is logarithmic – O($\log_2 n$)

(3) Algorithms with nested loops (1)

```
K = 1
while $K \le 10$
        M = 1
        while $M \le 10$
                some code
                M = M + 1
        end loop
        K = K + 1
end loop
```

The outer loop is executed 10 times
The inner loop is executed 10 times
The efficiency is 10*10 = 100

In general: Efficiency is quadratic - O(n^2)

(4) Algorithms with nested loops (2)

```
K = 1
while K ≤ 10
        M = 1
        while M ≤ K
                some code
                M = M + 1
        end loop
        K = K + 1
end loop
```

Inner loop is **dependent** on outer loop for one of its factors.

Inner loop is executed:

- once the 1st time,
- twice the 2nd time, etc.

Hence total number iterations for inner loop = 55
Average number iteration of the inner loop = 55/10 = 5.5

In general:

- The average number of iterations for the inner loop is: (n+1)/2
- Efficiency for this algorithm is n*(n+1)/2

Efficiency is quadratic - $O(n^2)$

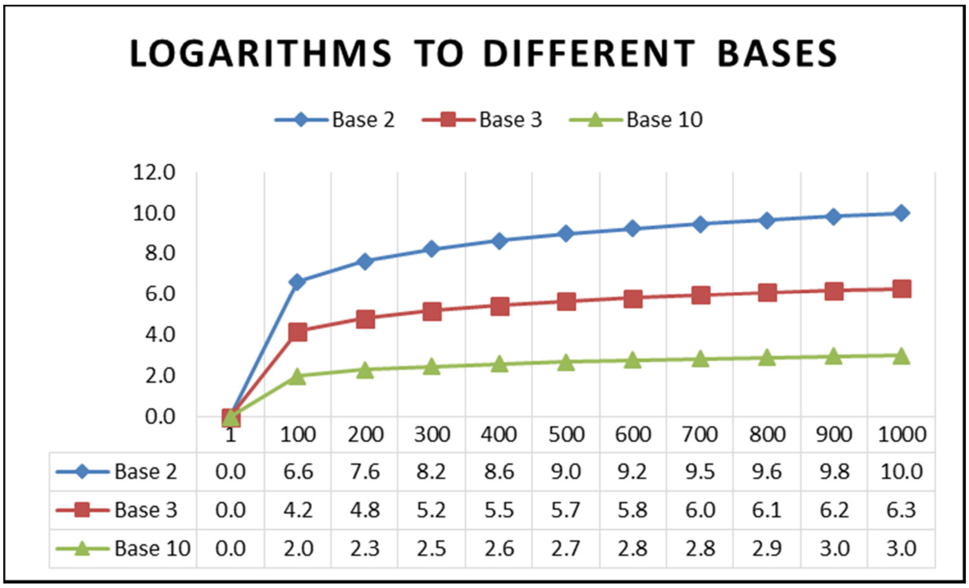
(5) Algorithms with nested loops (3)

```
K = 1
while K ≤ 10 loop
        M = 1
        while M ≤ 10 loop
                some code
                M = M * 2
        end loop
        K = K + 1
end loop
```

Inner and outer loops are **independent** of each other.

The number of iterations is: $10 \log_2 10$

In general: Efficiency - $O(n \log_2 n)$



	1	100	200	300	400	500	600	700	800	900	1000
Base 2	0.0	6.6	7.6	8.2	8.6	9.0	9.2	9.5	9.6	9.8	10.0
Base 3	0.0	4.2	4.8	5.2	5.5	5.7	5.8	6.0	6.1	6.2	6.3
Base 10	0.0	2.0	2.3	2.5	2.6	2.7	2.8	2.8	2.9	3.0	3.0