# Research into Fuzzy Systems for the Safe Control of Pressurised Water Reactor Nuclear Power Plants

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***Abstract* — This paper aims to effectively review research methods involved in the case study of the safe control of a pressurised water reactor (PWR) nuclear power plant (NPP), especially pertaining to the use of fuzzy systems. The paper then develops a new Sugeno-type SFS controller based on information gained from reviewing previous literature, and evaluates this controller against an existing controller.**

***Keywords:*** *Fuzzy Logic, Fuzzy Systems, Nuclear Power, Nuclear Power Plant Control, MATLAB.*

## I. Introduction

SINCE the advent of nuclear power, the various ways in which a nuclear power plant can be safely controlled has been of significant concern to those interested in the development of lower-carbon-emission energy production. As Uhrig et al. (1994) point out, there exists clear evidence that incidents and accidents involving NPPs are often due to "operator error". This would suggest that means of automated control of NPPs are worth if studying. Fuzzy logic, and consequently fuzzy systems, are one of the ways in which this automated control process can be achieved in that operational variables that describe the status of nuclear reactors such as pressure and control rod position can be described in fuzzy sets in order to develop the membership functions which build up a fuzzy control system.

There already exists a wealth of literature published since at least 1991 which attempts to apply an understanding of fuzzy systems to the domain of NPP control. By reviewing the most pertinent academic resources, an understanding of the role fuzzy systems play in regards to controlling NPPs can be gained; such that improved fuzzy systems can be developed by building on what is already understood within the field.

This paper employs a research method based approach that considers the different means by which a nuclear power plant can be safely controlled using fuzzy logic or a fuzzy system and then conducts the development of fuzzy controller within the context of an existing case study. In section II a background to the problem domain is given in order to contextualize the paper, in section III the paper tracks the development of a proposed Sugeno-type fuzzy controller, in section IV the paper discusses the properties of the proposed controller in a comparison and evaluation against an existing controller, and in section V the paper is concluded.

## II. Background and Contextual Importance

### *A. Fuzzy Systems*

Any Fuzzy systems can be described as a system which takes its inputs and outputs in the form of linguistic terms, and executes a process of fuzzification-inference-defuzzification (FID) in order to operate. They are built on the foundation of fuzzy logic, sets and relations, which allows for values to exist within a set to a certain membership degree from the ranges of 0 to 1 (e.g.: 0.25, 0.33, 0.653). This, in turn, allows for the consideration of logical uncertainty, as opposed to more classical 'crisp' sets which only allows for elements to either belong or not belong to a certain set (Ross, 2004).

Fuzzy systems expand on fuzzy set theory in order to develop systems which can take the membership degrees of values in fuzzy sets to plot membership functions so that crisp values can be mapped to linguistic term inputs (fuzzification), which can be inferred in various ways, and then linguistic fuzzy outputs can be mapped to crisp output values (defuzzification).

Types of fuzzy system can include: Standard Fuzzy Systems, Chained/Hierarchical Fuzzy Systems, and Networked Fuzzy Systems. Each type is characterised differently in order to cater to resolving the different attributes of systemic complexity such as non-linearity, uncertainty, dimensionality, and structure. The types of fuzzy system vary greatly and are characterized as such:

#### *1) Standard Fuzzy System (SFS):*

An SFS features one rule base with one or many inputs and one or many outputs. It is good for accurately modelling the system output, and as such is suitable for tasks that require non-linearity and uncertainty, but can less effectively represent systems of higher dimensionality and of more complex structure (Gegov, 2010).

#### *2) Chained/Hierarchical Fuzzy System (C/HFS):*

A CFS features multiple connected rule bases of an arbitrary structure, however an HFS must follow the restriction that it's rule bases can only have two inputs. With this is can be seen how that C/HFSs might be used to re-represent a SFS in order to improve on transparency and efficiency on the system at the cost of accuracy (Gegov, 2010).

#### *3) Networked Fuzzy System/Fuzzy Network (NFS/FN):*

An NFS features multiple connected rule bases in a 2-dimensional grid structure of layers progressing from left to right and levels progressing from top to bottom. As fuzzy networks can be considered to be a generalization of fuzzy systems as a whole and at a higher level of abstraction compared to other types of fuzzy system, SFSs and C/HFSs can be represented as specialized versions of fuzzy networks.

There are different ways of representing fuzzy networks such as block schemes and topological expression or through representing the nodes (each of which represent a single rule base) with a series of IF-THEN rules, an integer table, Boolean matrix, or binary relation (Gegov, 2010).

An example node using conjunctive rules with two inputs ($x_1$, $y_2$) and one output (y), with each input having two linguistic terms (BIG/SMALL and LONG/SHORT) and the output having three possible linguistic terms (HIGH/AVERAGE/LOW) would have $2^2$ (=4) permutations of possible rules and could be represented as such:

*a) By IF-THEN Rules*

IF $x_1$ is BIG and $x_2$ is LONG THEN y is LOW
IF $x_1$ is BIG and $x_2$ is SHORT THEN y is AVERAGE
IF $x_1$ is SMALL and $x_2$ is LONG then y is AVERAGE
IF $x_1$ is SMALL and $x_2$ is SHORT then y is HIGH

*b) By an Integer Table*

Where BIG = 1, SMALL = 2; LONG = 1, SHORT = 2; HIGH = 1, AVERAGE = 2, LOW = 3.

TABLE I
INTEGER TABLE FOR EXAMPLE RULE BASE

| $x_1$ | $x_2$ | y |
|---|---|---|
| 1 | 1 | 3 |
| 1 | 2 | 2 |
| 2 | 1 | 2 |
| 2 | 2 | 1 |

*c) By a Boolean Matrix*

Where 1, 2, and 3 in the first row represents the output: y.

TABLE II
BOOLEAN MATRIX FOR EXAMPLE RULE BASE

| $x_1$, $x_2$ / y | 1 | 2 | 3 |
|---|---|---|---|
| 11 | 0 | 0 | 1 |
| 12 | 0 | 1 | 0 |
| 21 | 0 | 1 | 0 |
| 22 | 1 | 0 | 0 |

*d) By a Binary Relation, R*

$$R=\{(11,3),(12,2),(21,2),(22,1)\}$$

An example fuzzy network using 3 rule bases with one input and one output could look like:

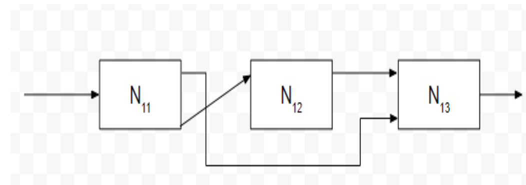



Fig. 1. Example fuzzy network with three layers and one level

However, this can be simplified to a single equivalence node by output permuting $N_{11}$, creating an identify node in layer 2 level 2, vertically merging $N_{12}$ and the identity node $I_{22,}$ then horizontally merging remaining nodes to a single equivalent node $N_E$.

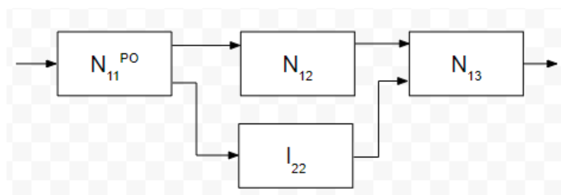



Fig. 2. Example fuzzy network after output permutation and identity node creation.

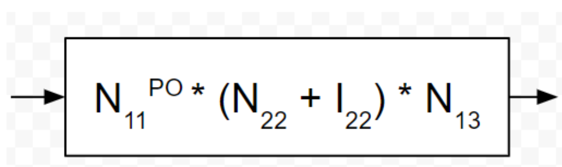



Fig. 3. Example fuzzy network after vertically merging $N_{12}$ and $I_{22}$ and horizontally merging $N_{11}^{PO}$ with ($N_{22}$ + $I_{22}$), and then horizontally merging $N_{11}^{PO}$ * ($N_{22}$ + $I_{22}$) with $N_{13}$ to form the single equivalent node represented by the equation $N_E = N_{11}^{PO} * (N_{22} + I_{22}) * N_{13}$.

Finally, fuzzy systems can also be described by the way in which they conduct they conduct operation of the FID sequence. Mamdani-type fuzzy systems are one of the most common types of fuzzy systems. They are characterized as being completely fuzzy in that fuzziness is maintained from the inputs to the outputs. The consequences of this mean that Mamdani-type systems are widely accepted and intuitive. A Sugeno-type fuzzy system can be described as a fuzzy/crisp-hybrid system in that its inputs are fuzzy and its outputs are represented as polynomial functions. This brings computational efficiency and a better ability to work with linear techniques. Tsukamoto-type fuzzy systems are described as Mamdani-Sugeno hybrids in that they feature crisp inputs and outputs (like Mamdani systems), but the outputs are gained by a function mapping (as in Sugeno systems). Tsukamoto-type fuzzy systems have only niche applications (Gegov, 2007).

### *B. Nuclear Power*

Fuzzy systems have developed an established foothold in the domain of energy systems and power control. It is well understood that nuclear power has the potential to produce large amounts of energy with lower carbon emissions than conventional fossil fuel plants and as of 2017 nuclear power accounts for 10.2% of the world's power and 29% of the world's low-carbon power ("Nuclear Power in the World Today", 2020). However, due to the risks involved in operating a Nuclear Power Plant (NPP), the means by which an NPP can be safely controlled warrant being understood.

The most common type of nuclear reactor in operation are pressurised water reactors, which produce energy by the chained fission reaction of uranium. The rate of reaction and, consequently, the safety of the nuclear reactor is determined by control rods. As control rods are inserted into the reactor pressure vessel they absorb neutrons preventing them from causing further fission reactions thus slowing the rate of reaction, conversely as they are retracted, the neutrons are able to continue the chain reaction with the fuel rods which sustains the nuclear reaction and thus energy production (Graetzer, 2019). From this, it can be seen that a system that can be tolerant towards uncertainty, like systems based on fuzzy logic, offer great potential in the control of a PWR whose variables such as reactor pressure can be controlled by the movement of control rods. This sentiment is furthered by Ruan (1996) when it is stated how "the application of fuzzy logic has, despite the ominous sound of the word "fuzzy" to nuclear engineers, a number of very desirable advantages over classical methods, e.g., its robustness and the capability to include human experience into the controller."

### *C. A Review of Literature on Extant Research Methods*

Historically, power plants (both nuclear and non-nuclear) would be controlled by non-fuzzy control systems such as PID systems which, as Wang et al. (2017) discuss, "have numerous assets" but show "poor control performance when faults occurred", which poses a significant potential risk to safe operation. As this paper is expressly interested in the role of fuzzy systems, non-fuzzy systems are not reviewed further.

As early as 1991 it can be seen that systems based on fuzzy logic have been employed in the control of NPPs. Akin & Altin (1991) present a fuzzy rule-based controller comprised of 22 conjunctive IF-THEN rules with two inputs being 'Power Error' (with eight linguistic terms) and 'Change in Power Error' (with seven linguistic terms) and a single output of 'Change in Steam Flow Rate' (with seven linguistic terms). They use the controller on a mathematical model of a reactor core that considers neutronics, core heat transfer, the steam generator, and piping. They then produce results that suggest their controller would follow and optimum controller well.

Mydlo and Schreiber (2011) develop a fuzzy PI type controller for an NPP model based on a pressure equation such that the pressure value is at 0% when the reactor is shut down, at 100% when the reactor is running optimally and at 200% when the reactor is critical; with the inputs for the controller as "the pressure in the nuclear reactor" and "the change of this reactor pressure"; and the output is "the velocity of the control rods in the nuclear reactor" (i.e. should the control rods be inserted or retracted and at what speed?). The results they attain show how that their controller successfully maintains an optimum pressure for the duration of their simulation with variations away from the optimum due to pressure drain being corrected by the controller.

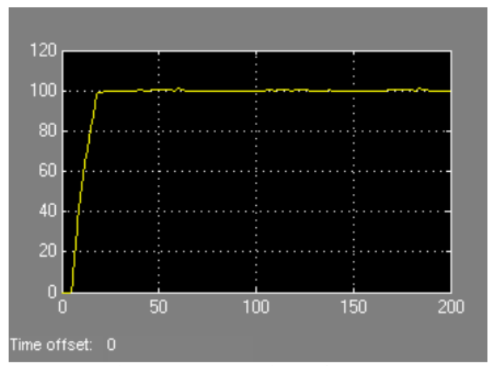



Fig. 4. Results obtained by Mydlo and Schreiber (2011) maintaining an optimum at 100%.

Wang et al. (2017) develop on the work presented by Mydlo and Schreiber (2011) by using a very similar rule base for their controller and pressure equation for their reactor model; but they also consider the faults that could occur within the system such as 'sensor faults', 'actuator faults', and 'plant faults'. They produce results similarly successful results to Mydlo and Schreiber (2011) in that an optimum is well maintained but also is able to correct changes in reactor pressure when each of the faults are applied.

The effectiveness of both Mydlo and Schreiber, and Wang et al.'s results suggest how that their case study is particularly worthy of study over the case study presented by Akin and Altin, particularly due to Wang et al.'s use of faults.

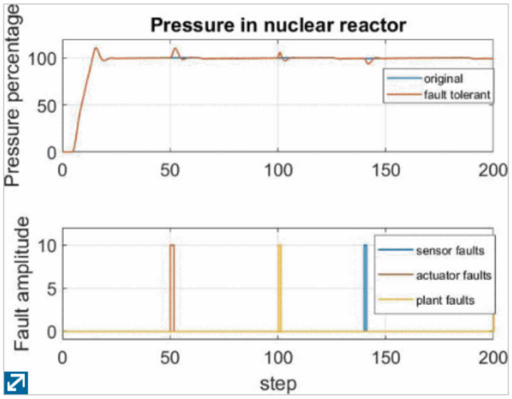


Fig. 5. Results obtained by Wang et al. (2017) maintaining an optimum at 100% while making corrective changes after faults.

### D. The Case Study

The case study for the paper, in a general sense, considers the optimum safe operation of a PWR NPP. The case study should, however be specified in a formal manner such that the model for the NPP is simple, ubiquitous and justifiable so that further work on prospective controllers can be developed in a fashion that is conducive to evaluation and comparison against other systems.

The specific case study description considered by this paper is taken from a paper by Wang et al. (2017) which can be seen to be directly inspired by Mydlo & Schreiber (2011) (see section II.C.) where the model for the NPP is derived by a pressure formula for nuclear reactors.

$$p(t)=p(t+1)+\int v(t)-p_1-p_2-p_3$$

Where: p(t) is the reactor pressure; v(t) is the control rod speed; $p_1$ is the pressure affected by the drain of the reactor; $p_2$ is the pressure applied to the control rod fully inserted into the reactor; and $p_3$ is the pressure loss of the reactor (Wang et al. 2017). As this formula has been used to an effective degree by previous researchers it stands that following work, including work presented in this paper, should adopt this case study implementation in order to further the knowledge of how a fuzzy system can be applied to the nuclear power plant control domain. The case study here also considers the use of faults within the system whereas the one presented Mydlo does not, therefore it can be seen that a system concerned with the safe operation of a PWR plant should follow the example set by Wang et al. as it is unreasonable and unrealistic to suggest that the plant can function correctly without experiencing any fault. Without a control system being able to consider faults, it cannot be considered to be able to fully and safely control a NPP.

## III. Methodology

### A. Extant Controller

The controller developed by Wang et al. (2017) was demonstrated to be quite effective in the optimum control of the NPP model, and as such is taken as an example effective extant controller for the proposed new controller to be compared against. Through interrogating the extant controller, this paper attempts to propose means by which the controller can be improved by reporting the development of a proposed controller.

The extant controller can be described as an SFS as it consists of one rule base consisting of 25 rules, due to having 5 linguistic terms for 2 input variables ($5^2$ = 25). These rules are summarized by Wang et al. in a table that abbreviates the linguistic terms for inputs and output: Very Negative (VN), Negative (N), Balance (B), Positive (P), Very Positive (VP). It is important to note that positive velocity of a control rod means

| | ***VN*** | ***N*** | ***B*** | ***P*** | ***VP*** |
|---|---|---|---|---|---|
| ***VN*** | VN | VN | VN | N | B |
| ***N*** | VN | VN | N | B | P |
| ***B*** | VN | N | B | P | VP |
| ***P*** | N | B | P | VP | VP |
| ***VP*** | B | P | VP | VP | VP |

Fig. 6. Control Table summarization of rules from Wang et al.'s (2017) controller, where the first column represents values for the first input: pressure, and the first row represents the values for the second input: change in pressure.

This table can be represented in by an integer table, Boolean matrix and binary relation where: VN = 1, N = 2, B = 3, P = 4, and VP = 5.

TABLE III
INTEGER TABLE FOR WANG ET AL. RULE BASE

| Pressure (p) | Change in Pressure (dp) | Control Rod Velocity (v) |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 3 | 1 |
| 1 | 4 | 2 |
| 1 | 5 | 3 |
| 2 | 1 | 1 |
| 2 | 2 | 1 |
| 2 | 3 | 2 |
| 2 | 4 | 3 |
| 2 | 5 | 4 |
| 3 | 1 | 1 |
| 3 | 2 | 2 |
| 3 | 3 | 3 |
| 3 | 4 | 4 |
| 3 | 5 | 5 |
| 4 | 1 | 2 |
| 4 | 2 | 3 |
| 4 | 3 | 4 |
| 4 | 4 | 5 |
| 4 | 5 | 5 |
| 5 | 1 | 3 |
| 5 | 2 | 4 |
| 5 | 3 | 5 |
| 5 | 4 | 5 |
| 5 | 5 | 5 |

TABLE IV
BOOLEAN MATRIX FOR WANG ET AL. RULE BASE

| p, dp / v | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 11 | 1 | 0 | 0 | 0 | 0 |
| 12 | 1 | 0 | 0 | 0 | 0 |
| 13 | 1 | 0 | 0 | 0 | 0 |
| 14 | 0 | 1 | 0 | 0 | 0 |
| 15 | 0 | 0 | 1 | 0 | 0 |
| 21 | 1 | 0 | 0 | 0 | 0 |
| 22 | 1 | 0 | 0 | 0 | 0 |
| 23 | 0 | 1 | 0 | 0 | 0 |
| 24 | 0 | 0 | 1 | 0 | 0 |
| 25 | 0 | 0 | 0 | 1 | 0 |
| 31 | 1 | 0 | 0 | 0 | 0 |
| 32 | 0 | 1 | 0 | 0 | 0 |
| 33 | 0 | 0 | 1 | 0 | 0 |
| 34 | 0 | 0 | 0 | 1 | 0 |
| 35 | 0 | 0 | 0 | 0 | 1 |
| 41 | 0 | 1 | 0 | 0 | 0 |
| 42 | 0 | 0 | 1 | 0 | 0 |
| 43 | 0 | 0 | 0 | 1 | 0 |
| 44 | 0 | 0 | 0 | 0 | 1 |
| 45 | 0 | 0 | 0 | 0 | 1 |
| 51 | 0 | 0 | 1 | 0 | 0 |
| 52 | 0 | 0 | 0 | 1 | 0 |
| 53 | 0 | 0 | 0 | 0 | 1 |
| 54 | 0 | 0 | 0 | 0 | 1 |
| 55 | 0 | 0 | 0 | 0 | 1 |

$R_E$ = {(11,1), (12,1), (13,1), (14,2), (15,3),
(21,1), (22,1), (23,2), (24,3), (25,4),
(31,1), (32,2), (33,3), (34,4), (35,5),
(41,2), (42,3), (43,4), (44,5), (45,5),
(51,3), (52,4), (53,5), (54,5), (55,5)}

Fig. 7. Binary relation, $R_E$, for the controller developed by Wang et al. (2017)

The ranges for the input and output variables for this controller are described by Wang et al. in membership function plots for the variables (see appendix C).

As this controller features only one rule base it can be represented as a single node with two inputs and a single output, recognized in a fuzzy network context by the topological expression: $[N_{11}]_{(p,dp\,|\,v)}$, or the block scheme:

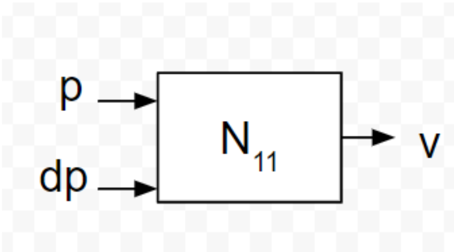



Fig. 8. A block scheme for the controller by Wang et al. (2017)

As Wang et al. do not present a control surface for their controller, the author replicates their system using the MATLAB Fuzzy Logic Designer by following the ranges from the membership function plots and control table included in their paper (see III.A). See Appendix A for rules for the Extant Controller as implemented with the Fuzzy Logic Designer Rule Editor.

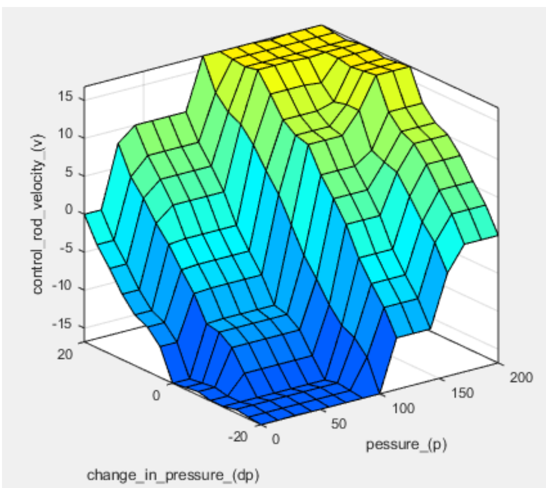



Fig. 9. The control surface for the fuzzy controller proposed by Wang et al. (2017) gained by implementing their rules using the MATLAB Fuzzy Logic Designer.

*B. New Proposed Controller*

In order to develop a new controller within the chosen case study (see section II.D.), the controller must use the same inputs and outputs so that it is compatible with the operation of the NPP model.

One way in which the extant controller could be changed in order to reduce its complexity would be to attempt to split the rule base across multiple rule bases in a CFS or a FN such that each node has fewer rules which could make the controller more transparent. However, as the controller has only two inputs and one output, it can be seen how that decomposing the controller into a hierarchical or chained fuzzy system would provide only minimal improvements to complexity overall.

Another way to reduce complexity within the current controller would be to reduce the number of linguistic terms as this would reduce the number of permutations of rules required by the controller. As the number of rules in any rule base is given by the equation: $x^n$ where x is the number of linguistic terms and n is the number of inputs, it can be seen how that the total number of rules can become rapidly unmanageable as the number of inputs and linguistic terms increases (Gegov, 2007). As the number of inputs for a new controller must be kept the same in order to conform to the case study, it becomes clear how an effective means for reducing the complexity of the controller is to reduce the number of linguistic terms that the inputs can take, and as such the proposed SFS controller is developed along the principles of reducing the number of linguistic terms.

The reduction of linguistic terms is facilitated by deleting the membership function for the linguistic terms Very Negative and Very Positive and adjusting the parameters for the membership functions for the linguistic terms Negative and Positive so that their ranges more appropriately cover the ranges previously covered by the deleted membership functions. This leaves 3 linguistic terms for each input value which only requires $3^2$ = 9 rules. In doing this for the 'change in pressure' variable, the triangular membership functions are made into trapezoidal membership functions.

The rules for the proposed controller are described by acquired by following a similar patterning of rules presented in Fig. 6 where the extreme linguistic terms VN and VP are placed in the top left and bottom right cells respectively, and then the diagonal pattern of rules is continued throughout:

- the control table:

| **p \ dp** | **N** | **B** | **P** |
|---|---|---|---|
| **N** | VN | N | B |
| **B** | N | B | P |
| **P** | B | P | VP |

Fig. 10. Control table for proposed controller

Just as in the extant controller, the proposed controller can be described by an integer table, Boolean matrix and binary relation, where the linguistic variable inputs N = 1, B = 2, P = 3; and outputs VN = 1, N = 2, B = 3, P = 4, VP = 5, as such:

TABLE V

INTEGER TABLE FOR PROPOSED CONTROLLER RULE BASE

| Pressure (p) | Change in Pressure (dp) | Control Rod Velocity (v) |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 2 |
| 1 | 3 | 3 |
| 2 | 1 | 2 |
| 2 | 2 | 3 |
| 2 | 3 | 4 |
| 3 | 1 | 3 |
| 3 | 2 | 4 |
| 3 | 3 | 5 |

TABLE VI

BOOLEAN MATRIX FOR WANG ET AL. RULE BASE

| p, dp / v | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 11 | 1 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1 | 0 | 0 | 0 |
| 13 | 0 | 0 | 1 | 0 | 0 |
| 21 | 0 | 1 | 0 | 0 | 0 |
| 22 | 0 | 0 | 1 | 0 | 0 |
| 23 | 0 | 0 | 0 | 1 | 0 |
| 31 | 0 | 0 | 1 | 0 | 0 |
| 32 | 0 | 1 | 0 | 1 | 0 |
| 33 | 0 | 0 | 0 | 0 | 1 |

$$R_P = \{(11,1), (12,2), (13,3), (21,2), (22,3), (23,4), (31,3), (32,2), (33,5)\}$$

Fig. 11. Binary relation $R_P$ for the proposed controller.

Fuzzy Logic Designer Rule Editor is used to implement the rules for the controller in MATLAB:

```
1. If (pessure_(p) is N) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is VN) (1)
2. If (pessure_(p) is N) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is N) (1)
3. If (pessure_(p) is N) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is B) (1)
4. If (pessure_(p) is B) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is N) (1)
5. If (pessure_(p) is B) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is B) (1)
6. If (pessure_(p) is B) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is P) (1)
7. If (pessure_(p) is P) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is B) (1)
8. If (pessure_(p) is P) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is P) (1)
9. If (pessure_(p) is P) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is VP) (1)
```

Fig. 12. Proposed controller rules implemented using the MATLAB Fuzzy Logic Designer Rule Editor

The proposed system is a Sugeno fuzzy inference system which was chosen as it is "computationally efficient" and works "well with linear techniques, such has PID control". This is done by first creating the rules and membership functions in the MATLAB Fuzzy Logic Designer as if the proposed controller were a Mamdani system, then the Mamdani proposed controller is loaded into the Workspace from a file by the Command Window and then converted to a Sugeno system using the 'convertToSugeno' function. The converted Sugeno controller will then have constant output membership functions that correspond to the centroids of the Mamdani controllers output membership functions ("Mamdani and Sugeno Fuzzy Inference Systems", 2019). See appendix B for examples of how this output membership function conversion works.

```
>> mamController = readfis

mamController =

  mamfis with properties:

                       Name: "proposed_npp_fis"
                  AndMethod: "min"
                   OrMethod: "max"
          ImplicationMethod: "min"
          AggregationMethod: "max"
      DefuzzificationMethod: "centroid"
                     Inputs: [1×2 fisvar]
                    Outputs: [1×1 fisvar]
                      Rules: [1×9 fisrule]
    DisableStructuralChecks: 0

>> sugController = convertToSugeno(mamController)

sugController =

  sugfis with properties:

                       Name: "proposed_npp_fis"
                  AndMethod: "min"
                   OrMethod: "max"
          ImplicationMethod: "prod"
          AggregationMethod: "sum"
      DefuzzificationMethod: "wtaver"
                     Inputs: [1×2 fisvar]
                    Outputs: [1×1 fisvar]
                      Rules: [1×9 fisrule]
    DisableStructuralChecks: 0
```

Fig. 13. MATLAB Command Window output when converting Mamdani system to Sugeno.

Membership functions for the proposed controller are manipulated using the MATLAB Fuzzy Logic Designer. The base for the triangular 'Balance' membership function is narrowed such that the range of values for the inputs that are serving towards the optimum pressure value of the reactor is reduced. This is done in order to make sure that the controller is less accepting of input values that would cause the reactor to operate under less than optimal pressure.

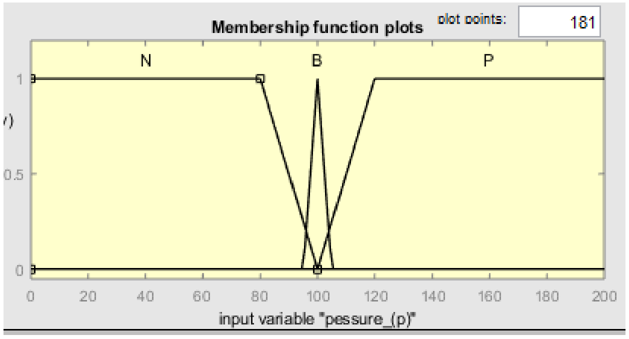



Fig. 14. Proposed system membership function plots for ‘pressure’ input variable.

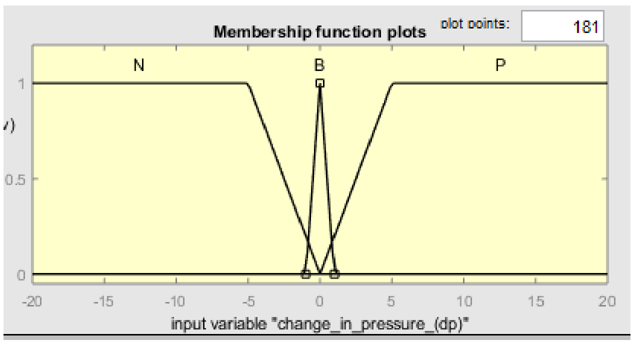



Fig. 15. Proposed system membership function plots for ‘change in pressure’ input variable.

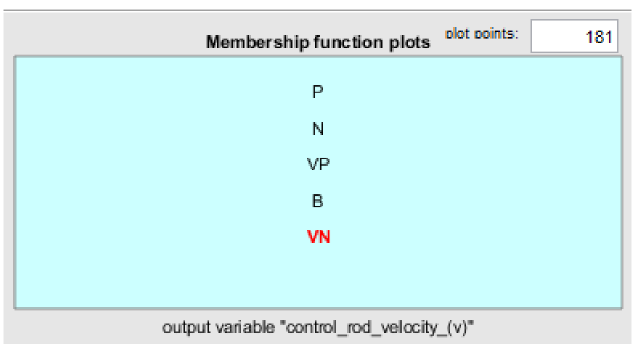



Fig. 16. MATLAB representation of constant output membership functions for the output variable ‘control rod velocity where VN has the parameter -16.8, N has the parameter -10, B has the parameter 0, P has the parameter 10, and VP has the parameter 16.8.

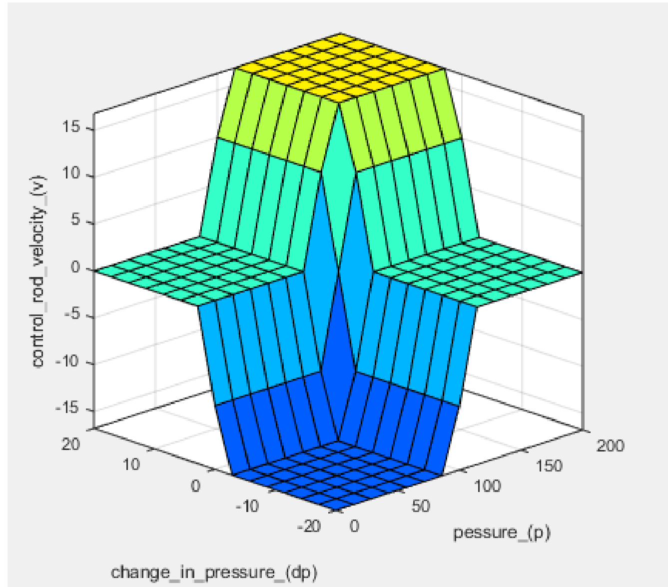



Fig. 17. Control surface for proposed controller.

## IV. Results and Evaluation

As both the extant and proposed controllers are SFSs with two inputs and a single output it can be seen how that the structural complexity when comparing the two controllers is unchanged, and as such, both will have equal Feasibility, Accuracy, and Transparency Indexes.

The proposed controller reduces the number of rules by a factor of 2.78 (25/9) which could be regarded as a benefit for any person that may need to interact with, maintain or adjust the controller in the future as there exist fewer rules that need to be considered. As the proposed controller is a Sugeno type inference system, it will operate with greater efficiency, but how this effects the controller’s ability to consistently maintain optimum reactor pressure remains to be tested.

When comparing the control surfaces output by the extant controller (Fig. 9.) and the proposed controller (Fig. 17.), it can be seen how that the proposed controller is less accurate but also less complex as it features fewer discrete points.

The minor changes found when evaluating the proposed controller against the extant controller proposed by Wang et al. (2017) suggests that for more improvements to be discovered in the process of controlling a PWR NPP, more radical structural changes may need to be made to the controller. This could have included attempting to implement the controller as a HFS which would have possibly seen improvements in feasibility, efficiency, and transparency, with perhaps a loss to accuracy (Gegov, 2010)

## V. Conclusion and Further Work

In the development of a simplified Sugeno-type SFS for the purpose controlling a PWR NPP within the context of existing fuzzy research methods, this paper has reviewed current knowledge on the application of fuzzy systems to the case study of NPP control and then compares this new

Sugeno-type SFS controller against an existing effective controller presented by previous researchers.

The next stage for future work in the case study would be to test the proposed controller within the Simulink environments used by Wang et al. (2017) and by Mydlo and Schreiber (2011) in order to see how it compares in maintaining optimum pressure within the nuclear reactor model, and whether the reduced number of rules has major effects on the efficacy of controlling pressure.

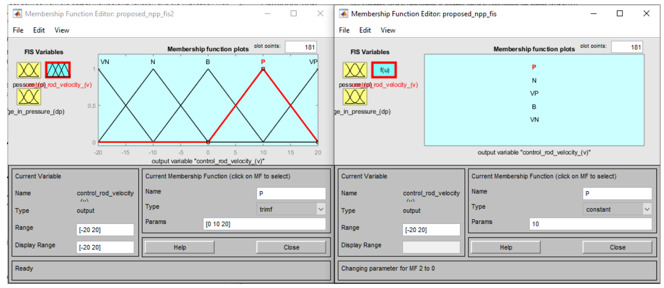
As well as this, further research and literature review could be conducted within the case study of controlling and maintaining nuclear fusion reactors, which also offers low-carbon power production with less nuclear waste production than traditional PWR NPPs.

## APPENDIX

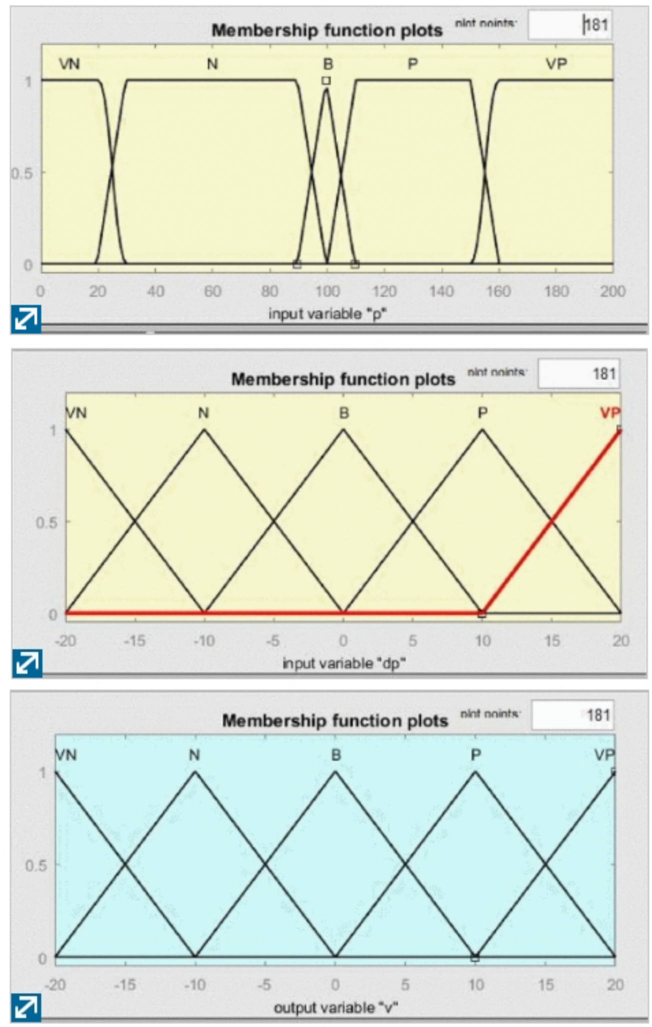
*A. Extant controller rules in Fuzzy Logic Designer Rule Editor*

1. If (pessure_(p) is VN) and (change_in_pressure_(dp) is VN) then (control_rod_velocity_(v) is VN) (1)
2. If (pessure_(p) is VN) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is VN) (1)
3. If (pessure_(p) is VN) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is VN) (1)
4. If (pessure_(p) is VN) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is N) (1)
5. If (pessure_(p) is VN) and (change_in_pressure_(dp) is VP) then (control_rod_velocity_(v) is B) (1)
6. If (pessure_(p) is N) and (change_in_pressure_(dp) is VN) then (control_rod_velocity_(v) is VN) (1)
7. If (pessure_(p) is N) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is VN) (1)
8. If (pessure_(p) is N) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is N) (1)
9. If (pessure_(p) is N) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is B) (1)
10. If (pessure_(p) is N) and (change_in_pressure_(dp) is VP) then (control_rod_velocity_(v) is P) (1)
11. If (pessure_(p) is B) and (change_in_pressure_(dp) is VN) then (control_rod_velocity_(v) is VN) (1)
12. If (pessure_(p) is B) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is N) (1)
13. If (pessure_(p) is B) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is B) (1)
14. If (pessure_(p) is B) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is P) (1)
15. If (pessure_(p) is B) and (change_in_pressure_(dp) is VP) then (control_rod_velocity_(v) is VP) (1)
16. If (pessure_(p) is P) and (change_in_pressure_(dp) is VN) then (control_rod_velocity_(v) is N) (1)
17. If (pessure_(p) is P) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is B) (1)
18. If (pessure_(p) is P) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is P) (1)
19. If (pessure_(p) is P) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is VP) (1)
20. If (pessure_(p) is P) and (change_in_pressure_(dp) is VP) then (control_rod_velocity_(v) is VP) (1)
21. If (pessure_(p) is VP) and (change_in_pressure_(dp) is VN) then (control_rod_velocity_(v) is B) (1)
22. If (pessure_(p) is VP) and (change_in_pressure_(dp) is N) then (control_rod_velocity_(v) is P) (1)
23. If (pessure_(p) is VP) and (change_in_pressure_(dp) is B) then (control_rod_velocity_(v) is VP) (1)
24. If (pessure_(p) is VP) and (change_in_pressure_(dp) is P) then (control_rod_velocity_(v) is VP) (1)
25. If (pessure_(p) is VP) and (change_in_pressure_(dp) is VP) then (control_rod_velocity_(v) is VP) (1)

*B. Output membership function centroid conversion from Mamdani to Sugeno fuzzy inference system done with the MATLAB 'convertToSugeno' function.*



*C. Wang et al.'s (2017) controller variables membership plots.*

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