

A. The idea of your program

Serial version:

```

num_city: number of cities to connect
2opt_swap(start, end);

for (i = 1; i < num_city - 1; ++i)
    for (j = i + 1; j < num_city; ++j)
        2opt_swap(i, j);

```

Can be changed to:

```

start from (1) to (num_city - 1)
    2opt_swap with *depth* (2) to (num_city - i)

for (i = 1; i < num_city - 1; ++i)
    for (depth = 1; depth < num_city - i; ++depth)
        2opt_swap(i, i + depth);

```

depth is the length between the 2opt_swap indices

Idea: Split *depth* across threads

```

maximum_depth = num_city - 1
depth_for_each_thread = maximum_depth / num_thread

for (depth = thread_depth_start[n]; depth < thread_depth_end[n]; ++depth)
    for (i = 1; i < num_city - depth; ++i)
        2opt_swap(i, i + depth);

2opt_swap(start, end):
    _read_lock
        create new_route
    _unlock
    assert(distance(new_route) < distance(current_route))
    _write_lock
        current_route = new_route
    _unlock
end

```

Note:

- A race condition may occur when a thread pass through assertion, but not yet change the `current_route`; a better `current_route` may be overwritten, so the assertion have to be done again after applying `_write_lock`
- The aforementioned race condition also prevents partial update of the `current_route`, so a new array(`new_route`) is created every operation.
- `distance(current_route)` can be cached.
- **Distance calculation can be optimized by calculate only the update segment of the route.** (Tests marked as Dist. Calc. optimized)
- There are also two ways to split the *depth*

Example 9 depth, 3 threads

Chunk:

Thread 1: 1/2/3 depth

Thread 2: 4/5/6 depth

Thread 3: 7/8/9 depth

Balanced:

Thread 1: 1/4/7 depth

Thread 2: 2/5/8 depth

Thread 3: 3/6/9 depth

B. Performance (run-time) analysis with 1, 2, 4, 8, 16 core(s)

| # of 2opt_swap called in 10 mins | | | | |
|----------------------------------|--|----------|-----------------------|--------------|
| | Before distance calculation is optimized | | Dist. Calc. optimized | |
| Th | Chunk | Balanced | Chunk.opt | Balanced.opt |
| 1 | 24856 | 25119 | 66276 | 63110 |
| 2 | 46971 | 49701 | 80953 | 129261 |
| 4 | 76105 | 85136 | 96509 | 261605 |
| 8 | 167108 | 201120 | 85412 | 541035 |
| 16 | 239296 | 405542 | 71206 | 857152 |

C. Record your distance every 30 seconds with **test11**

| | Before distance calculation is optimized | | Dist. Calc. optimized | |
|---------|--|---------------|-----------------------|---------------|
| Time | Chunk | Balanced | Chunk.opt | Balanced.opt |
| 0m 30s | 5236217548888 | 5236767376441 | 5239234504229 | 5233894202422 |
| 1m 00s | 5231959394787 | 5233243836456 | 5238367721581 | 5228312584841 |
| 1m 30s | 5227752503221 | 5229821611984 | 5237506418954 | 5223445466794 |
| 2m 00s | 5223639454509 | 5226335277762 | 5236697033023 | 5218777773759 |
| 2m 30s | 5219355854175 | 5222886988838 | 5235881056704 | 5214000570217 |
| 3m 00s | 5214973671001 | 5219493109615 | 5235048453474 | 5209375645966 |
| 3m 30s | 5210569949525 | 5215917028459 | 5234220435439 | 5204544189800 |
| 4m 00s | 5206257153593 | 5212576696538 | 5233415665273 | 5199606540301 |
| 4m 30s | 5201869454315 | 5209312681293 | 5232583414345 | 5194835256033 |
| 5m 00s | 5197506692526 | 5206066426828 | 5231731245305 | 5190038327006 |
| 5m 30s | 5193214637330 | 5202621568215 | 5230898861426 | 5185116719534 |
| 6m 00s | 5188657856652 | 5199072196711 | 5230066163885 | 5180219477646 |
| 6m 30s | 5184220766738 | 5195754086165 | 5229208916041 | 5175506414432 |
| 7m 00s | 5179929194095 | 5192451415552 | 5228305331581 | 5170612659967 |
| 7m 30s | 5175611197172 | 5189208714037 | 5227440837385 | 5165726046679 |
| 8m 00s | 5171157164455 | 5185829449116 | 5226608781609 | 5160854768811 |
| 8m 30s | 5166698519634 | 5182298344208 | 5225715564559 | 5156041475717 |
| 9m 00s | 5162187441111 | 5178791170286 | 5224870726305 | 5151151922781 |
| 9m 30s | 5157769845727 | 5175618917479 | 5224035406388 | 5146242747622 |
| 10m 00s | 5153487487696 | 5172455395479 | 5223038216906 | 5141353455597 |

D. Discussion

The number of race condition happening between rd_lock and wr_lock scales with thread count:

| % of 2opt_call with race condition | | | | |
|------------------------------------|--|----------|-----------------------|--------------|
| | before distance calculation is optimized | | Dist. Calc. optimized | |
| Th | Chunk | Balanced | Chunk.opt | Balanced.opt |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 2.38 | 4.79 | 7.79 | 12.00 |
| 4 | 9.17 | 7.76 | 17.55 | 19.55 |
| 8 | 22.55 | 10.21 | 28.09 | 23.59 |
| 16 | 31.63 | 12.82 | 33.78 | 25.04 |

The race condition caused the whole work done in that 2opt_swap() call to be wasted.

The reason for Balanced algo. to scale well and performed better with distance calculation optimization is the shorter average length of updated route comparing to Chunk algo. :

| Avg. length of updated route | | | | |
|------------------------------|--|----------|-----------------------|--------------|
| | before distance calculation is optimized | | Dist. Calc. optimized | |
| Th | Chunk | Balanced | Chunk.opt | Balanced.opt |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 367913.65 | 2.58 | 51417.19 | 2.53 |
| 4 | 423225.60 | 3.35 | 167824.67 | 3.60 |
| 8 | 460001.93 | 5.16 | 326708.33 | 5.68 |
| 16 | 470879.20 | 9.28 | 419019.19 | 9.76 |

Different job distribution lead to different allocation of resource for CPU. Balanced algo., on average, spend more time working on shorter route, so the distance calculation optimization is actually helpful.