Synchrotron radiation in DYNAC (V6.0) for electrons in bending magnets

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1. Introduction

A facility introduced in the code DYNAC allows for computations of synchrotron radiation emitted by electrons crossing a bending magnet. In this paper, one firstly evokes the basic definitions of radiation excitation. Secondly, one explains the method used in DYNAC for simulating synchrotron radiation in bending magnets.

2. Basic definitions for radiation excitation (ref. SLAC-R-121, M. Sand).

2.1 Instantaneous radiation power P_γ

The quantity P_γ stands for the rate of loss of energy by radiation:

$$P_\gamma = \frac{2}{3}\frac{r_e c}{\left(mc^2\right)^3} E^2 F_\perp^2 \tag{1}$$

Where:

c : Speed of light in vacuum ($c = 2.998\,10^8\ m - s^{-1}$)
m : Rest mass of electron $(m = 511\,KeV)$
r_e : Classical electron radius ($r_e = 10^{-15}\ m$)
E : Total energy of the electron
$F_\perp = ecB$: Magnetic force acting on the electron (B is the local magnetic field strength and e is representing the electron charge in coulomb).

For convenience, one introduces the following constant:

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{\left(mc^2\right)^3} = 8.85\times10^{-5}\ (m - GeV^{-3}) \tag{2}$$

One can rewrite eq.1 in the form:

$$P_\gamma = \frac{e^2c^3}{2\pi} C_\gamma E^2 B^2 \quad \left(GeV - \sec^{-1}\right) \tag{3}$$

Usually, one expresses the local magnetic force B in terms of the local bending radius ρ (meter) of the central trajectory. Thus, with $B\rho = E/ec$, eq.3 takes on the form:

$$P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2} \quad \left(GeV - \sec^{-1}\right) \tag{4}$$

2.2 The radiated power spectrum $\Im(\omega)$

Consider the synchrotron radiation emitted by an electron in the finite time-interval Δt. From the classical point of view, the synchrotron radiation is emitted within a continuous spectrum of frequencies ω for each direction in space. Despite the fact that the frequency spectrum will be different in each direction, we can average the frequency spectrum over all directions so as to define a radiated power spectrum $\Im(\omega)$ (ev). The quantity $\Im(\omega)d\omega\,\Delta t$ represents the total energy radiated in time interval Δt with angular frequencies between: [ω, $\omega + d\omega$]. This definition only makes sense if Δt is taken sufficiently large such that most of the energies are found in frequencies greater than $1/\Delta t$. To obtain a suitable value of Δt, one uses the fact that the radiation is emitted inside the angle $1/\gamma$ of the electron velocity vector and that this angle is swept out in time-interval $\Delta\tau = \frac{\rho}{\gamma c}$ (ρ represents the local bending radius of the trajectory and c is the speed of light in vacuum). Hence, the time-interval $\Delta\tau$ contains most of the impulse radiations and can represent a suitable value for Δt. We also permit that $\Im(\omega)$ is slowly varying with respect to time (i.e., that $\Im(\omega)$ does not change appreciably in the time-interval Δt). Taking into account these conditions, we can consider that $\Im(\omega)$ is an 'instantaneous' power spectrum whose integral over ω is the instantaneous radiated power P_γ in eq.4:

$$P_\gamma = \int_0^\infty \Im(\omega) d\omega \qquad (5)$$

The power spectrum $\Im(\omega)$ has been computed as a parametric function of the instantaneous radiated power P_γ by Schwinger and Jackson:

$$\Im(\omega) = \frac{P_\gamma}{\omega_c} S\left(\frac{\omega}{\omega_c}\right) \qquad (6)$$

The constant ω_c (the so-called *critical frequency)* is given by:

$$\omega_c = \frac{3}{2}\frac{c\gamma^2}{\rho} \quad (\sec^{-1}) \qquad (7)$$

The function $S\left(\frac{\omega}{\omega_c}\right)$ is a pure algebraic function; it is expressed in the form:

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi \int_\xi^\infty K_{5/3}(\bar{\xi})\, d\bar{\xi} \qquad (8)$$

where $K_{5/3}(\xi)$ is the modified Bessel function of order $\nu = 5/3$ (see the graph in fig.1, in annex 4.1)
It follows from eq.5 and eq.6 that $S(\xi)$ is normalized such that:

$$\int_0^\infty S(\xi)\, d\xi = 1 \qquad (9)$$

The function $S(\xi)$ is referenced to as the *spectral function* (see the graph in fig.2 , annex 4.2).

2.3 The quanta distribution function $n(u)$.

The electromagnetic radiation at the angular frequency ω is emitted in quanta of energy u such that:

$$u = \hbar\omega \qquad (10)$$

Where $\hbar$ is the Plank's constant reduced by 2π, i.e. $\hbar = 6.85\,10^{-16}\ eV - \sec$

We define $n(u)\,du$ as the number of quanta emitted per unit time between u and $u + du$. In this case, the power emitted in these quanta is $un(u)du$. This power must be the same as the one emitted in the frequency interval $d\omega = du/\hbar$ at the frequency $\omega = u/\hbar$, i.e.:

$$un(u)du = \Im(u/\hbar)\, du/\hbar \qquad (11)$$

From $\Im(\omega)$ in eq.6, the quanta distribution function can be written as:

$$n(u) = \frac{P_\gamma}{u_c^2} F\left(\frac{u}{u_c}\right) \qquad (12)$$

with:

$$u_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c\gamma^3}{\rho} \qquad (13)$$

and:

$$F(\xi) = \frac{1}{\xi} S(\xi) \quad , \quad \xi = u/u_c \qquad (14)$$

The quantity u_c is referred to as the *'critical quanta energy'*, $n(u)$ is expressed in units $(eV)^{-1} \times \sec^{-1}$.
The distribution $n(u)$ in eq.12 is a parametric function of the variable $\xi = u/u_c$ via the function $F(\xi)$ (see the graph of $F(\xi)$ in fig.3, annex 4.3).

2.4 The rate of emission of quanta and the mean quantum energy.

Denote $\mathrm{N}(u_0, u_1)$ as the rate of emission of quanta between the quanta energies u_0 and u_1. By definition $\mathrm{N}(u_0, u_1)$ is the number of quanta emitted between u_0 and u_1 per unit time:

$$N(u_0, u_1) = \int_{u_0}^{u_1} n(u)du \quad (\sec^{-1}) \tag{15}$$

From eq.12, one obtains:

$$N(u_0, u_1) = \frac{P_\gamma}{u_c} \int_{u_0}^{u_1} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right) \tag{16}$$

Looking at the total rate of emission of quanta (of all energies), one can write:

$$\mathrm{N}(0, \infty) = \int_0^\infty n(u)du$$

From eq.12 and eq.14, it can be rewritten as:

$$\mathrm{N}(0, \infty) = \frac{P_\gamma}{u_c} \int_0^\infty F(u/u_c) d\left(\frac{u}{u_c}\right) = \frac{P_\gamma}{u_c} \int_0^\infty \bar{\xi}^{-1} S(\bar{\xi}) d(\bar{\xi}) \tag{17}$$

From the asymptotic behavior of the spectral function $S(\xi)$, i.e.:

$$\xi >> 1 \,, \quad S(\xi) \approx \frac{9\sqrt{3}}{8\sqrt{2\pi}} \xi^{1/2} e^{-\xi}$$

$$\xi << 1 \,, \quad S(\xi) \approx 1.34 \xi^{1/3}$$

Jointly with eq.8, one obtains:

$$\int_0^\infty \bar{\xi}^{-1} S(\bar{\xi}) d(\bar{\xi}) \approx 15\sqrt{3}/8$$

from which eq.17 takes on the form:

$$\mathrm{N}(0, \infty) \approx \frac{15\sqrt{3}}{8} \frac{P_\gamma}{u_c} \tag{18}$$

The mean quantum energy $< u / u_0 \le u \le u_1 >$ between quanta impulses u_0 and u_1 is defined as:

$$< u / u_0 \le u \le u_1 >= \frac{1}{\mathrm{N}(u_0, u_1)} \int_{u_0}^{u_1} un(u)du \tag{19}$$

From eq.12 and eq.16, it can be rewritten in the form:

$$< u / u_0 \le u \le u_1 >= u_c \frac{\int_{u_0}^{u_1} \frac{u}{u_c} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right)}{\int_{u_0}^{u_1} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right)} \tag{20}$$

The graphs of integrals $\int_{\xi_i}^{\xi_{i+1}} \xi F(\xi) d\xi$ and $\int_{\xi_i}^{\xi_{i+1}} F(\xi) d\xi$ (with $\xi = u/u_c$) are shown in the annexes, fig.4 and fig.5.

Looking at all quanta energies (i.e., $0 \le u \le \infty$) the integral $\int_0^\infty un(u)du$ is just P_γ, so from eq.19 the mean quantum energy for all quanta energies is given by:

$$< u / 0 \le u \le \infty >= \frac{1}{\mathrm{N}(0, \infty)} \int_0^\infty un(u)du = \frac{8}{15\sqrt{3}} u_c \tag{21}$$

3. Radiation excitation in an elementary magnetic sector.

Consider an elementary magnetic sector of bend angle $\Delta\varphi$, ρ_0 being the local radius of the central trajectory in the magnetic mid-plane. The path length of the central trajectory is $\Delta l_0 = \rho_0 \Delta\varphi$ and the momentum $B\rho_0 = E_0/(ec) = p_0/e$.

Let x_0 and θ_0 be the displacement and the angle of an arbitrary trajectory at the entrance of the magnetic sector with respect to the central trajectory, its momentum is $p_0 + \Delta p$. In the par-axial ray approximation and with TRANSPORT notations (see K.L. Brown, SLAC-R-75), the path length Δl is given by:

$$\Delta l = R_{51}x_0 + R_{52}\theta_0 + R_{56}\delta + \Delta l_0 + \textit{higher order terms} \tag{22}$$

with:

$$\begin{aligned} \delta &= \frac{\Delta p}{p_0} \\ R_{51} &= \cos\Delta\varphi \\ R_{52} &= \rho_0(1-\cos\Delta\varphi) \\ R_{56} &= \rho_0(\Delta\varphi - \sin\Delta\varphi) \\ \Delta l_0 &= \rho_0\Delta\varphi \end{aligned} \tag{23}$$

The path length Δl of the electron is covered in time $\Delta\tau = \Delta l / c$, $\mathrm{N}(u_0, u_n)$ (see eq.16) being the number of quanta emitted between quanta impulses u_i and u_{i+1} per unit time, the total number of these quanta emitted by the electron in the magnetic sector Δl is:

$$\Gamma(u_i, u_{i+1}) = \Delta\tau \times \mathrm{N}(u_i, u_{i+1}) = \frac{\Delta l P_\gamma}{c\, u_c} \int_{u_i}^{u_{i+1}} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right) \tag{24}$$

We start from the fact that most of the significant quanta are radiated in the interval $0.1 \le u/u_c \le 3$. One looks at a suitable partition of quanta energies between $u_0 = 0.1\, u_c$ and $u_n = 3\, u_c$ (i.e. $u_0, u_1, u_2, ..., u_n$) in such a way that one can assume that all quanta emitted between u_i and u_{i+1} possess in average the same mean quantum energy (see eq.20).

Let $\delta E(u_i, u_{i+1})$ be the energy lost by the electron from quanta impulses between u_i and u_{i+1}. From eq.20 and eq.24 one obtains:

$$\delta E(u_i, u_{i+1}) = \Gamma(u_i, u_{i+1}) \times < u / u_i \le u \le u_{i+1} > = \frac{\Delta l P\gamma}{c} \int_{u_i}^{u_{i+1}} \frac{u}{u_c} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right) \tag{25}$$

By adding up all energies $\delta E(u_i, u_{i+1})$, one obtains the total energy $\delta E(u_0, u_n)$ lost from impulse radiations:

$$\delta E(u_0, u_n) = \frac{\Delta l P\gamma}{c} \sum_{i=0}^{n-1} \int_{u_i}^{u_{i+1}} \frac{u}{u_c} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right) \tag{26}$$

One takes into account that (see annex 4.4):

$$\sum_{i=0}^{n-1} \int_{u_i/u_c}^{u_{i+1}/u_c} \frac{u}{u_c} F\left(\frac{u}{u_c}\right) d\left(\frac{u}{u_c}\right) = 0.86967 \tag{27}$$

Therefore, one obtains:

$$\delta E(u_0, u_n) = 0.86967 \frac{\Delta l P \gamma}{c} \tag{28}$$

Let E_i and E_f be the energy of the electron at the entrance and at the exit of the elementary magnetic sector respectively, one has:

$$E_f = E_i - \delta E(u_0, u_n)$$

The change $\Delta\delta$ of the momentum is:

$$\Delta\delta \approx -\frac{\delta E(u_0, u_n)}{E_i}$$

The changes δx of x and $\delta\theta$ of θ due to radiations are given by:

$$\delta x \approx -R_{16} \frac{\delta E(u_0, u_n)}{E_i} \quad , \quad \delta\theta \approx -R_{26} \frac{\delta E(u_0, u_n)}{E_i}$$

with:

$$R_{16} = \rho_0 (1 - \cos\Delta\varphi) \quad , \quad R_{26} = \sin\Delta\varphi$$

At the output of the elementary magnetic sector Δl, one will have:

$$x_s^* = x_s + \delta x \quad , \quad \theta_s^* = \theta_s + \delta\theta$$

where x_s and θ_s are the horizontal displacement and angle at the output of the sector Δl, without radiations.

The technique consists of dividing the bending magnet of bend angle φ and of radius ρ_0 (i.e. of length $l = \rho_0\varphi$) in a succession of small elementary magnetic sectors of length $\Delta l = \rho_0 \Delta\varphi$. Next, for each trajectory and for each elementary sector, one computes the changes $\delta E(u_0, u_n)$, $\Delta\delta$, δx, $\delta\theta$ and, thus, step-by-step one obtains the final energy E_f, the horizontal displacement x_f and angle θ_f of trajectories at the exit of the bending magnet.

4. ANNEXES

4.1 The modified Bessel function $K_{5/3}(\xi)$.

The general expression of the function $K_{5/3}(z)$ is given by:

$$K_\nu(z)=\frac{\Gamma\left(\nu+\frac{1}{2}\right)(2z)^\nu}{\sqrt{\pi}}\int_0^\infty \frac{\cos t}{\left(t^2+z^2\right)^{\nu+1/2}}dt \text{ , with } \nu=5/3$$

For large values of z ($z \geq 2.5$), one may use the asymptotic relation:

$$K_\nu(z)\rightarrow\sqrt{\frac{\pi}{2z}}\,e^{-z}$$

To compute $K_{5/3}(z)$ a convenient way consists of utilizing the Airy function (see Abramovitch, 10.4.2, page 446):

$$Ai(z)=c_1 f(z)-c_2 g(z)$$

with:

$$c_1=.35502 \quad , \qquad c_2=.25881$$

and:

$$f(z)=1+\frac{1}{3!}z^3+\frac{1.4}{6!}z^6+\frac{1.4.7}{9!}z^9+\ldots\ldots \quad , \quad g(z)=z+\frac{2}{4!}z^4+\frac{2.5}{7!}z^7+\frac{2.5.8}{10!}z^{10}+\ldots\ldots$$

In terms of the Bessel functions, one can write:

$$K_{\pm 1/3}(\zeta)=\pi\left(\sqrt{3/z}\right)Ai(z) \text{ , } K_{\pm 2/3}(\zeta)=\pi\left(\sqrt{3}/z\right)Ai'(z) \text{, with } \zeta=\frac{3}{2}z^{3/2}$$

In addition, one makes use of the recurrence relation:

$$K_{\nu+1}(\zeta)=K_{\nu-1}(\zeta)+\frac{2\nu}{\zeta}K_\nu(\zeta) \text{ , with } \nu=2/3$$

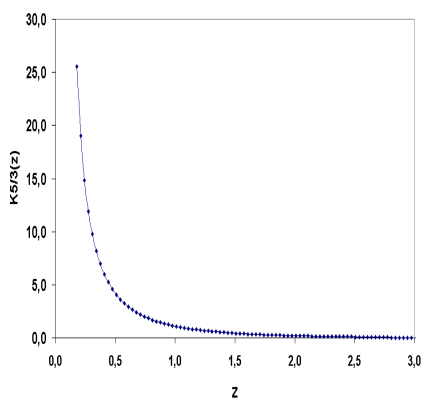

Fig.1. The modified Bessel function $K_{5/3}(z)$ is depicted as a parametric function of z.

4.2 The spectral function $S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_{\xi}^{\infty} K_{5/3}(\bar{\xi})\, d\bar{\xi}$ (eq.8).

The spectral function $S(\xi)$ is shown in following fig.2 with respect to $\xi = \omega/\omega_c = u/u_c$; ω_c is the critical frequency (see eq.7) and u_c the critical quanta energy (see eq.13).

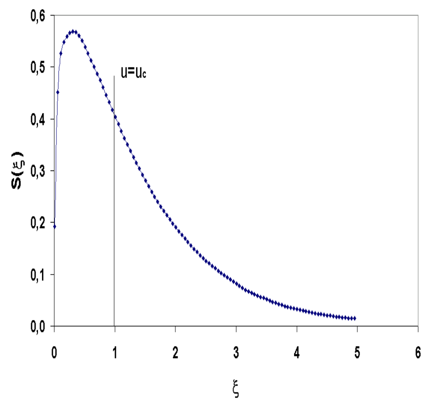

Fig.2. The portrait of the spectral function $S(\xi)$

One can write the spectral function $S(\xi)$ under the form of a Chebitcheff polynomial series as:

$$S(\xi) = \sum_{k=0}^{k=21} A(k)\,\xi^k$$

The coefficients $A(k)$ are shown in the following table 1:

$k+1$	A(k)
1	.94300913930510433E-01
2	.10872132036214680E+02
3	-.11716935553615622E+03
4	.73847592244266707E+03
5	-.29795391698982567E+04
6	.81901663761547534E+04
7	-.16092370350163837E+05
8	.23414130655837827E+05
9	-.25888511759608577E+05
10	.22166493006014440E+05
11	-.14896165588936965E+05
12	.79272736270849655E+04
13	-.33574848629852449E+04
14	.11331009578850217E+04
15	-.30386434084435871E+03
16	.64273368595982987E+02
17	-.10581084803508167E+02
18	.13265529012623767E+01
19	-.12227831733369790E+00
20	.78085800385687392E-02
21	-.30846637114439259E-03

Table 1

4.3 The function $F(\xi)=\frac{1}{\xi}S(\xi)$

The quanta distribution $n(u)=\frac{P_\gamma}{u_c^2}F\left(\frac{u}{u_c}\right)$ is depending on $\xi=u/u_c$ via the function $F(\xi)=\frac{1}{\xi}S(\xi)$.

By Using the Chebitcheff polynomial series, one obtains:

$$F(\xi)=\frac{A(0)}{\xi}+\sum_{k=1}^{k=21}A(k)\,\xi^{k-1}$$

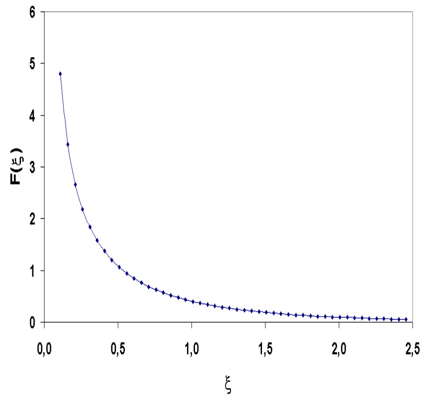

Fig.3 The function $F(\xi)=\frac{1}{\xi}S(\xi)$ is depicted with respect to $\xi=u/u_c$.

4.4 Computation of the integrals $\int_{\xi_i}^{\xi_{i+1}}\xi\,F(\xi)\,d\xi$ **and** $\int_{\xi_i}^{\xi_{i+1}}F(\xi)\,d\xi$**.**

We start from the fact that most of significant quanta are radiated in the interval $0.1\le\xi=\frac{u}{u_c}\le 3$. One divides this interval in suitable successions of identical elementary intervals $[\xi_{i+1}-\xi_i]$ ($\xi_0=0.1,\xi_1,\xi_2,....,\xi_n=3$). For each of these elementary intervals, one can write from the Chebitcheff polynomial series of $S(\xi)$:

$$\int_{\xi_i}^{\xi_{i+1}}\xi\,F(\xi)\,d\xi=\int_{\xi_i}^{\xi_{i+1}}S(\xi)\,d\xi=\sum_{k=0}^{k=21}A(k)\frac{1}{k+1}\left(\xi_{i+1}^{k+1}-\xi_i^{k+1}\right)$$

In following fig.4, the integral $\int_{\xi_i}^{\xi_{i+1}}\xi\,F(\xi)\,d\xi$ is depicted with respect to $\xi_i=i\,h$ ($h=0.12$).

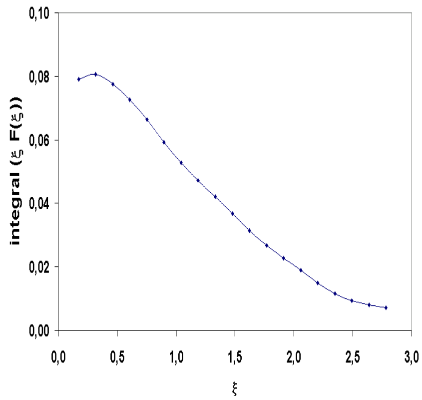

Fig.4 the integral $\int_{\xi_i}^{\xi_{i+1}} \xi\, F(\xi)\, d\xi$ is shown with respect to $\xi_i = i\, h$

By dividing the size $0.1 \le \xi \le 3$ in 30 identical elementary intervals and in adding up the different partial integrals $\int_{\xi_i}^{\xi_{i+1}} \xi\, F(\xi)\, d\xi$, one has:

ξ_i	ξ_{i+1}	$\int_{\xi_i}^{\xi_{i+1}} \xi\, F(\xi)\, d\xi$	$\sum \int_{\xi_i}^{\xi_{i+1}} \xi\, F(\xi)\, d\xi$
.10000E+00	.19667E+00	.52454E-01	.52454E-01
.19667E+00	.29333E+00	.54469E-01	.10692E+00
.29333E+00	.39000E+00	.54901E-01	.16182E+00
.39000E+00	.48667E+00	.53655E-01	.21548E+00
.48667E+00	.58333E+00	.51447E-01	.26693E+00
.58333E+00	.68000E+00	.49039E-01	.31597E+00
.68000E+00	.77667E+00	.46611E-01	.36258E+00
.77667E+00	.87333E+00	.44066E-01	.40664E+00
.87333E+00	.97000E+00	.41393E-01	.44803E+00
.97000E+00	.10667E+01	.38712E-01	.48675E+00
.10667E+01	.11633E+01	.36159E-01	.52291E+00
.11633E+01	.12600E+01	.33781E-01	.55669E+00
.12600E+01	.13567E+01	.31541E-01	.58823E+00
.13567E+01	.14533E+01	.29383E-01	.61761E+00
.14533E+01	.15500E+01	.27286E-01	.64490E+00
.15500E+01	.16467E+01	.25278E-01	.67017E+00
.16467E+01	.17433E+01	.23406E-01	.69358E+00
.17433E+01	.18400E+01	.21698E-01	.71528E+00
.18400E+01	.19367E+01	.20144E-01	.73542E+00
.19367E+01	.20333E+01	.18707E-01	.75413E+00
.20333E+01	.21300E+01	.17344E-01	.77147E+00
.21300E+01	.22267E+01	.16033E-01	.78751E+00
.22267E+01	.23233E+01	.14782E-01	.80229E+00
.23233E+01	.24200E+01	.13614E-01	.81590E+00
.24200E+01	.25167E+01	.12550E-01	.82845E+00
.25167E+01	.26133E+01	.11588E-01	.84004E+00
.26133E+01	.27100E+01	.10705E-01	.85074E+00
.27100E+01	.28067E+01	.98684E-02	.86061E+00
.28067E+01	.29033E+01	.90553E-02	.86967E+00

Table 2

Therefore, looking at the last value in the last colon of table 2, one has (see eq.27):

$$\int_{\xi_0=0.1}^{\xi_n\approx 3}\frac{u}{u_c}F(\xi)d(\xi)=0.86967$$

Looking now at the integrals $\int_{\xi_i}^{\xi_{i+1}}F(\xi)\,d\xi$, it is a simple matter to get from the Chebitcheff polynomial series of $S(\xi)$:

$$\int_{\xi_i}^{\xi_{i+1}}F(\xi)d\xi=A(0)[\log(\xi_{i+1})-\log(\xi_i)]+\sum_{k=1}^{k=21}\frac{A(k)}{k}\left(\xi_{i+1}^k-\xi_i^k\right)$$

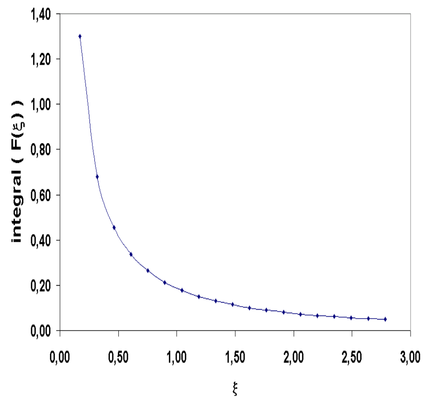

Fig.5 The integral $\int_{\xi_i}^{\xi_{i+1}}F(\xi)\,d\xi$ is shown with respect to $\xi_i=i\,h$ (with $h=0.12$).

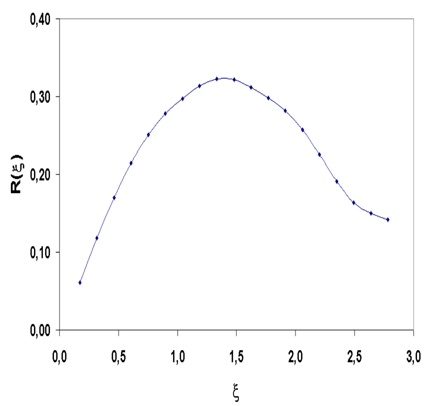

Fig.6 The rate $R(\xi_i^*)=\dfrac{\int_{\xi_i}^{\xi_{i+1}}\xi\,F(\xi)d\xi}{\int_{\xi_i}^{\xi_{i+1}}F(\xi)d\xi}$ is represented with respect to $\xi_i=i\,h$ ($h=0.12$).

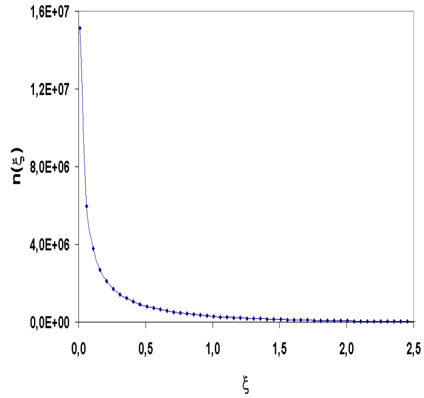

Fig.7 The quanta distribution function $n(\xi)=\frac{P_\gamma}{u_c^2}F(\xi)$ ($eV^{-1}\sec^{-1}$) is shown as a parametric function of $\xi = u/u_c$

5. Example.
To fix the ideas with an example, consider an electron of energy 1 GeV crossing a bending magnet of bend angle $\Delta\phi = 57.29$ deg and radius $\rho = 5$ m (central trajectory).

One has:

$\gamma = 1.957\,10^3$

Path length: $\Delta l = 5$ m

Time of flight: $\Delta\tau = \Delta l / c = 1.666\,10^{-8}$ sec

Instantaneous radiation power: $P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2} = 1.69\,10^2\ GeV\sec^{-1}$

Critical quanta energy: $u_c = \frac{3}{2}\frac{\hbar c\gamma^3}{\rho} = 462\,eV$

Total rate of emission of quanta: $\mathrm{N}(0,\infty) \approx \frac{15\sqrt{3}}{8}\frac{P_\gamma}{u_c} = 1.17\,10^9\ \sec^{-1}$

The following table 3 gives the energy loss $\delta E(u_i,u_{i+1}) = \frac{\Delta l P\gamma}{c}\int_{u_i}^{u_{i+1}}\frac{u}{u_c}F\left(\frac{u}{u_c}\right)d\left(\frac{u}{u_c}\right)$ (ev) between the quanta impulses u_i and u_{i+1}.

$u_i\ ev$	$u_{i+1}\ ev$	$\delta E(u_i, u_{i+1})$
4,62E+01	1,13E+02	2,22E+02
1,13E+02	1,80E+02	2,27E+02
1,80E+02	2,47E+02	2,18E+02
2,47E+02	3,14E+02	2,04E+02
3,14E+02	3,81E+02	1,86E+02
3,81E+02	4,48E+02	1,66E+02
4,48E+02	5,15E+02	1,48E+02
5,15E+02	5,82E+02	1,33E+02
5,82E+02	6,49E+02	1,18E+02
6,49E+02	7,16E+02	1,03E+02
7,16E+02	7,83E+02	8,82E+01
7,83E+02	8,50E+02	7,52E+01
8,50E+02	9,17E+02	6,39E+01
9,17E+02	9,84E+02	5,30E+01
9,84E+02	1,05E+03	4,22E+01
1,05E+03	1,12E+03	3,27E+01
1,12E+03	1,19E+03	2,61E+01
1,19E+03	1,25E+03	2,23E+01
1,25E+03	1,32E+03	1,99E+01

Table 3

It follows that the total energy $\delta E = \sum_i \delta E(u_i, u_{i+1})$ lost from radiations by the electron in the magnetic sector is $\approx 2148\ eV$ and the variation of the momentum is $\Delta\delta \approx -\delta E / E = -2.148\,10^{-6}$.

Suppose now that the energy of the electron is 5GeV. In this case, the instantaneous radiation power is $P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2} = 1.056\,10^5\ Gev\,\sec^{-1}$ (i.e. $5^4 = 6.25\,10^2$ times greater than the previous one), the total energy δE lost from impulse radiations is of the order $\approx 1.34\,10^6\ eV$ and the variation of momentum is $\Delta\delta \approx -\delta E / E = -2.7\,10^{-4}$.