The motion of particles throughout an RFQ as applied in DYNAC V6.0R13 and later

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1. Introduction

The six types of RFQ cells in the code DYNAC are:

A) Standard accelerating cell (type 0 in PARMTEQ and DYNAC).
B) Fringe-field cell region (type F in PARMTEQ, type 4 in DYNAC)
C) Transition region delta-m cell (type T in PARMTEQ, type 1 in DYNAC)
D) Entry transition cell before the first accelerating cell (type E in PARMTEQ, type 2 in DYNAC)
E) Transition region m=1 cell (type M in PARMTEQ, type 3 in DYNAC)
F) Radial Matching Section cell (type R in PARMTEQ, type 5 or type 6 in DYNAC)

The specified cell length L is divided in equal-length segments Δs. Each of these segments is separated in two half lengths of $\Delta s/2$. The beam matrix is followed through the segment by a sequence of drift-impulse-drift transformations. The transverse and longitudinal forces are calculated and applied at the centre $\xi = \Delta s/2$ of the segment.
In standard cells the length L is divided into 18 equal-length segments. When the cell is representing the fringe-field region at the end of the RFQ, its length L is divided into 36 L / ($\beta\lambda$) segments, where β = v/c and λ is the free-space wavelength of the RF. The computations are made by using a step-by-step integration with regard to the elementary segments Δs (similar to codes like PARMTEQ and TRACE 3-D).
In the Radial Matching Section cell, the length L_{RMS} is divided into 126 equal-length segments.

2. Definitions

a : Minimum distance from the vane to the axis

ma : Maximum distance from the vane to the axis

r_0 : Mean aperture between the vanes

L : Cell length

V : Inter-vane potential difference

$r = \sqrt{x^2 + y^2}$

tan θ = y / x

ρ_0 : Transverse radius of curvature of the vane tip in the middle of the cell.

3. Summary of the fields equations used in the RFQ.

A) Standard accelerating cells (type 0 in DYNAC)

The potential function satisfying the Laplace equations in cylindrical coordinates can be written up to the second order term as:

$U(r, \theta, z) = V / 2\, [A_{10}\, I_0(kr) \cos(kz) + A_{01}\, r^2 \cos(2\theta) + A_{12}\, I_4(kr) \cos(kz) \cos(4\theta) + A_{03}\, r^6 \cos(6\theta)]$ (A-1)

with the propagation constant: $k = \pi / L$

Electric field components (cylindrical coordinates):

$$\overline{E}_r = -\frac{\partial U}{\partial r} \quad , \quad \overline{E}_\theta = -\frac{1}{r}\frac{\partial U}{\partial \theta} \quad , \quad \overline{E}_z = -\frac{\partial U}{\partial z}$$

$$\overline{E}_r = -\frac{V}{2}\left[2rA_{01}\cos(2\theta) + k\{A_{01} I_1(kr) + 0.5 A_{12} I_3(kr)\cos(4\theta)\}\cos(kz)\right] \quad \text{(A-2)}$$

$$\overline{E}_\theta = \frac{V}{2}\left[2rA_{01}\sin(2\theta) + 4A_{12} I_4(kr)\sin(4\theta)\cos(kz)/r\right] \quad \text{(A-3)}$$

$$\overline{E}_z = \frac{V}{2}\left[A_{10} I_0(kr) + A_{12} I_4(kr)\cos(4\theta)\right]k\sin(kz) \quad \text{(A-4)}$$

Where:

$r = \sqrt{x^2 + y^2}$

$\tan\theta = y / x$

Limiting the Bessel functions expansion to the sixth order, one has:

$I_0(s) \approx 1 + s^2/4 + s^4/64 + s^6/2304$

$I_1(s) \approx s/2 + s^3/16 + s^5/128$

$I_3(s) \approx s^3/48 + s^5/768$

$I_4(s) \approx s^4/384$

Coefficients:

$$A_{01} = \frac{3(5\rho_0 + r_0)}{2(7\rho_0 + r_0)\, r_0^2} \quad \text{(A-5)}$$

$$A_{10} = \frac{K_1 I_4(kma) - K_2 I_4(ka)}{I_0(ka) I_4(kma) - I_0(kma) I_4(ka)} \quad \text{(A-6)}$$

$$A_{12} = \frac{K_2 I_0(ka) - K_1 I_0(kma)}{I_0(ka) I_4(kma) - I_0(kma) I_4(ka)} \quad \text{(A-7)}$$

Where:

$$K_1 = 1 - (A_{01} a^2 + A_{03} a^6)$$

$$K_2 = A_{01}(ma)^2 + A_{03}(ma)^6 - 1$$

$$A_{03} = \frac{-(\rho_0 + r_0)}{2(7\rho_0 + r_0) r_0^6} \quad \text{(A-8)}$$

The mean aperture of the vane r_0 is given by:

$$r_0 = \frac{a}{\sqrt{\chi}}$$

With:

$$\chi = \frac{I_0(ka) + I_0(kma)}{m^2 I_0(ka) + I_0(kma)}$$

Returning to the transverse fields in Cartesian coordinates (x, y), one has:

$$\overline{E}_x = \overline{E}_r \cos\theta - \overline{E}_\theta \sin\theta$$

$$\overline{E}_y = \overline{E}_r \sin\theta + \overline{E}_\theta \cos\theta$$

NOTES:

- Apart from the accelerating coefficient A_{10} , which is the one given in the PARMTEQ output file, all the other coefficients above are computed in DYNAC. A possibility exists in the code DYNAC permitting to compute the coefficient A_{10} .
- For simulating PARMTEQ cells, odd number cells have VA_{01} positive and even cells have VA_{01} negative. Since in DYNAC the cells numbers may be different from the ones given in the PARMTEQ output file, a specific flag 'IFLAG' will identify the sign of VA_{01}. IFLAG = 0 is corresponding to an even cell number in PARMTEQ and IFLAG = 1 to an odd cell number.

If $\rho_0 = -r_0$, one will have:

$$A_{03} = A_{12} = 0$$

$$A_{10} = \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}$$

$$A_{01} = \chi / a^2$$

$$r_0 = 1/\sqrt{A_{01}}$$

$$(A_{01}V) = V / r_0^2$$

These coefficients are the ones used in the code TRACE 3D [3].

B) Fringe-field cell region (type F in PARMTEQ, type 4 or 7 in DYNAC)

Propagation constant: $k_f = \pi / (2L)$

For cell type 4, the profile along the azimuthal direction z is given from the following equation:

$$\frac{8}{(Kr_0)^2} T(r,z)\cos\theta = 1$$

Where:

$$T(r,z) = \frac{3}{4}\left[I_2(Kr)\cos(Kz) + \frac{1}{27} I_2(3Kr)\cos(3Kz) \right]$$

With $\theta = 0$, $I_2(Kr) = (Kr)^2/8$ it is a simple matter obtaining:

$$r(z) = \frac{2r_0}{\sqrt{3}\left[\cos(Kz) + \frac{\cos(3Kz)}{3}\right]^{1/2}}$$

r_0 : Average radius of the previous cell

Linearized electric field components (see [3]):

$$\overline{E_x} = \left[-\frac{VC_1(z)}{r_0^2} - \frac{k_f^2 A_{10} V}{4} C_2(z)\right] x \qquad \text{(B-1)}$$

$$\overline{E_y} = \left[\frac{VC_1(z)}{r_0^2} - \frac{k_f^2 A_{10} V}{4} C_2(z)\right] y \qquad \text{(B-2)}$$

$$\overline{E_z} = \frac{k_f A_{10} V}{2} S(z) \qquad \text{(B-3)}$$

With:

$$S(z) = 0.75\,[\sin(k_f\, z) + \sin(3k_f\, z)] \qquad \text{(B-4)}$$

$$C_1(z) = 0.75\,[\cos(k_f\, z) + \cos(3k_f\, z)/3] \qquad \text{(B-5)}$$

$$C_2(z) = 0.75\,[\cos(k_f\, z) + 3\cos(3k_f\, z)] \qquad \text{(B-6)}$$

NOTES:
The term $A_{10}V$ is representing the product of acceleration efficiency and maximum inter-vane voltage. The coefficient A_{10} is obtained from the PARMTEQ output file. In most cases this coefficient is null, this entailing $\overline{E}_z = 0$.

The beam dynamics in the fringe field may also be computed based on the shape of the fringe field profile (cell type 7); the approach is essentially the one described in section F, paragraph 2 for the RMS.

C) Transition region delta-m cell (type T in PARMTEQ, type 1 in DYNAC)

This T-cell **must follow** a standard accelerating cell (type 0 in DYNAC) and be followed by a transition region m=1 cell (type M in PARMTEQ, type 3 in DYNAC) or by a fringe field region (type F in PARMTEQ, type 4 in DYNAC).

Propagation constant: $k = \pi / (2L)$ (the cell-period is 4L)

C-1) the previous standard cell is an odd number cell in PARMTEQ or with IFLAG = 1 in DYNAC.

At the entrance of the cell one will have:

$$x - vane = ma \qquad y - vane = a$$

Potential function:

$$U(r,\theta, z) = (V / 2)\,[(r/r_0)^2 \cos(2\theta) - A_{10}\, I_0(kr) \cos(kz) - A_{30}\, I_0(3kr) \cos(3kz)] \qquad \text{(C-1)}$$

Coefficients:

$$A_{10} = \frac{m^2-1}{T_{10}(k) + \alpha(k) T_{30}(k)/3} \qquad \text{(C-2)}$$

$$A_{30} = \frac{\alpha(k)}{3} A_{10} \tag{C-3}$$

With:

$$T_{10}(k) = m^2 I_0(ka) + I_0(mka)$$

$$T_{30}(k) = m^2 I_0(3ka) + I_0(3mka)$$

$$\alpha(k) = \frac{I_0(kr_0)}{I_0(3kr_0)}$$

Electric fields components (in polar coordinates):

$$\overline{E}_r = -\partial U / \partial r = -V\left[(r / r_0^2)\cos 2\theta - \frac{1}{2} kA_{10} I_1(kr)\cos(Kz) - \frac{3}{2} kI_1(3kr)\cos(3kz)\right] \tag{C-4}$$

$$\overline{E}_\theta = -(1/r)\,\partial U / \partial\theta = V\left(r / r_0^2\right)\sin 2\theta \tag{C-5}$$

$$\overline{E}_z = -\partial U / \partial z = -(V/2)\left[kA_{10} I_0(kr)\sin(kz) + 3kA_{30} I_0(3kr)\sin(3kz)\right] \tag{C-6}$$

C-2) the previous standard cell is an even number cell in PARMTEQ or with IFLAG = 0 in DYNAC.

At the entrance of the cell on will have

$x - vane = a \qquad y - vane = ma$

Potential function:

$U(r,\theta, z) = (V / 2)\,[(r/r_0)^2 \cos(2\theta) + A_{10}\, I_0(kr) \cos(kz) + A_{30}\, I_0(3kr) \cos(3kz)]$ (C-7)

Coefficients:

A_{10} , A_{30} : Like eq. (C-2) and (C-3) above

Electric fields components:

$$\overline{E_r} = -\frac{\partial U}{\partial r} = -V\left[\left(\frac{r}{r_0^2}\right)cos(2\theta) + \frac{1}{2}kA_{10}I_1(kr)cos(kz) + \frac{3}{2}kI_1(3kr)cos(3kz)\right] \tag{C-8}$$

$\overline{E}_\theta$, $\overline{E}_z$: Like in eq. (C-5) and eq. (C-6)

Remark: The field components in Cartesian coordinates are given by:

$$\overline{E}_x = \overline{E}_r \cos\theta - \overline{E}_\theta \sin\theta$$

$$\overline{E}_y = \overline{E}_r \sin\theta + \overline{E}_\theta \cos\theta$$

D) Entry transition cell before the first accelerating cell and after the R.M.S (type E in PARMTEQ, type 2 in DYNAC)

At the entrance of this cell one always has:

X-vane = Y-vane = r_0

Propagation constant: $k = \pi / (2L)$ (the cell-period is 4L)

D-1) the following accelerating cell is an even number cell in PARMTEQ or with IFLAG = 0 in DYNAC

At the exit of the cell (i.e. z = L) one will have:

x – vane = ma *y – vane = a*

Potential function:

$U(r,\theta, z) = (V / 2)\,[(r/r_0)^2 \cos(2\theta) - A_{10}\, I_0(kr) \sin(kz) - A_{30}\, I_0(3kr) \sin(3kz)]$ (D-1)

Coefficients:

$$A_{10} = \frac{m^2-1}{T_{10}(k)+\alpha(k)T_{30}(k)/3} \quad \text{(D-2)}$$

$$A_{30} = -\frac{\alpha(k)}{3} A_{10} \quad \text{(D-3)}$$

with:

$$T_{10}(k) = m^2 I_0(ka) + I_0(mka)$$

$$T_{30}(k) = m^2 I_0(3ka) + I_0(3mka)$$

$$\alpha(k) = \frac{I_0(kr_0)}{I_0(3kr_0)}$$

Electric fields components (polar coordinates):

$$\overline{E}_r = -\partial U / \partial r = -V\left[(r/r_0^2)\cos 2\theta - \frac{1}{2}kA_{10}I_1(kr)\sin(kz) - \frac{3}{2}kI_1(3kr)\sin(3kz)\right] \quad \text{(D-4)}$$

$$\overline{E}_\theta = -(1/r)\,\partial U / \partial\theta = V\left(r/r_0^2\right)\sin 2\theta \quad \text{(D-5)}$$

$$\overline{E}_z = -\partial U / \partial z = -(V/2)\left[kA_{10}I_0(kr)\cos(kz) + 3kA_{30}I_0(3kr)\cos(3kz)\right] \quad \text{(D-6)}$$

D-2) the following accelerating cell is an odd number cell in PARMTEQ or with IFLAG = 1 in DYNAC.

In this case, at the exit of the cell:

x – vane = a *y – vane = ma*

Potential function: as in eq. (D-1)

Coefficients:

$$A_{10} = \frac{1-m^2}{T_{10}(k)+\alpha(k)T_{30}(k)/3} \quad \text{(D-7)}$$

$$A_{30} = -\frac{\alpha(k)}{3} A_{10} \quad \text{(D-8)}$$

Electric fields components: like in eq. (D-4) to eq. (D-6)

E) Transition region m=1 cell (type M in PARMTEQ, type 3 in DYNAC)

This cell follows the transition region delta-m cell (type T in PARMTEQ, type 1 in DYNAC).

Propagation constant: $k = 2\pi / L$ (the cell-period is L)

At the entrance (i.e. $z = 0$) and at the exit (i.e. $z = L$) of the cell, one has:

$$x = y = r_0$$

Potential function:

$$U(r,\theta,z) = (V/2)(r/r_0)^2\cos 2\theta \quad \text{(E-1)}$$

Electric fields components (polar coordinates):

$$\overline{E}_r = -\partial U / \partial r = -V(r/r_0^2)\cos 2\theta \quad \text{(E-2)}$$

$$\overline{E}_\theta = -(1/r)\,\partial U / \partial\theta = V\left(r/r_0^2\right)\sin 2\theta \quad \text{(E-3)}$$

$$\overline{E}_z = -\partial U / \partial z = 0 \quad \text{(E-4)}$$

F) Radial Matching Section (Type 5 or type 6 in DYNAC)

Although PARMTEQ typically lists the Radial Matching Section (R.M.S.) of the RFQ as consisting of a grand total of 4, 6 or 8 elementary cells (usually an even number, specified in the PARMTEQ output file by Type R), each of these cells being computed like an accelerating cell having m = 1 (this entailing an energy gain null), the effective R.M.S. must be considered as one complete section with a length equal to the total length of these elementary cells. It is this last option of the R.M.S. which has been retained in DYNAC.

The beam dynamics in DYNAC for the R.M.S. can be chosen to be based on electrical field components (Type 5) or on the shape of the electrodes (Type 6).

Potential function:

$$U(r,\theta,z) = (V/2)\left[A_q q(r,z)\cos 2\theta + A_d d(r,z)\cos 6\theta\right] \qquad \text{(F-1)}$$

With:

$$q(r,z) = I_2(kr)\sin(kz) - \frac{1}{27}I_2(3kr)\sin(3kz) \qquad \text{(F-2)}$$

$$d(r,z) = I_6(kr)\sin(kz) - \frac{1}{3^7}I_6(3kr)\sin(3kz)$$

$$k = \pi/(2L_{RMS})$$

The radius r_0 being the distance between the axis and the vane at the position $z = L_{RMS}$, we define:

$$q_0 = q(r_0, L_{RMS}) \;,\; d_0 = d(r_0, L_{RMS}) \qquad \text{(F-3)}$$

From the boundary condition $U(r_0, 0, L_{RMS}) = V/2$ and eq. (F-1) we obtain the relationship between the coefficients A_q, A_d :

$$d_0 A_d + q_0 A_q = 1 \qquad \text{(F-4)}$$

Limiting the power of the radius r to the second order, one has:

$$I_2(Kr) = k^2r^2/8$$

$$I_2(3Kr) = 9k^2r^2/8 \qquad \text{(F-5)}$$

This entailing that one has assumed that: $d(r,z) \cong 0$

In this case, the potential function is reduced to the following simpler expression:

$$U(r,\theta,z) = (V/2)A_q q(r,z)\cos 2\theta \qquad \text{(F-6)}$$

From eq. (F-4) and with $d_0 \approx 0$:

$$A_q = 1/q_0 \qquad \text{(F-7)}$$

From eq. (F-2), one obtains:

$$q_0 = I_2(kr_0)\sin(kL_{RMS}) - \frac{1}{27}I_2(3kr_0)\sin(3kL_{RMS})$$

which can be rewritten in limiting the power of the radius r to the second order,

$$q_0 = \frac{1}{6} k^2 r_0^2 \tag{F-8}$$

and:

$$q(r,z) \approx \frac{k^2 r^2}{8} \sin kz - \frac{k^2 r^2}{24} \sin 3kz \tag{F-9}$$

1) The dynamics in the RMS based on the electric fields components (type 5 in DYNAC)

Components of the electric fields:

$$\overline{E}_r = -\partial U / \partial r = -\frac{V}{2} A_q \frac{\partial q(r,z)}{\partial r} \cos 2\theta = -\frac{V}{8} A_q k^2 r \left[\sin(kz) - \frac{1}{3} \sin(3kz) \right] \cos 2\theta \tag{F-10}$$

$$\overline{E}_\theta = -(1/r)\, \partial U / \partial \theta = V A_q \frac{q(r,z)}{r} \sin 2\theta \tag{F-11}$$

$$\overline{E}_z = -\partial U / \partial z = -\frac{V}{2} A_q \frac{\partial q(r,z)}{\partial z} \cos 2\theta = -\frac{V}{16} A_q k^3 r^2 \left[\cos kz - \cos 3kz \right] \cos 2\theta \tag{F-12}$$

The electric fields acting on particles are given by:

$$E_r(z) = q\, E_r \sin \varphi(z)\ ,\ \ E_\theta(z) = q\, E_\theta \sin \varphi(z)\ ,\ \ E_z(z) = q\, E_z \sin \varphi(z) \tag{F-13}$$

where $\varphi(z)$ is the RF phase with respect to the azimuthal direction (see next chapter).

The transverse field components E_x , E_y in the Cartesian coordinates are given by:

$$E_x = E_r \cos \theta - E_\theta \sin \theta \tag{F-14}$$

$$E_y = E_r \sin \theta + E_\theta \cos \theta \tag{F-15}$$

2) The dynamics in the RMS based on the shape of vanes (type 6 in DYNAC)

Profile of the RMS vanes:

The equipotential surface defining the shape of the electrodes satisfies the equation:

$$A_q q(r,z) \cos 2\theta = 1 \tag{F-16}$$

Consider the vane profile along the azimuthal direction z with $\theta = 0$, from eq. (F-9):

$$\frac{K^2 r^2}{8} \sin Kz - \frac{K^2 r^2}{24} \sin 3Kz = 1 / A_q \tag{F-17}$$

From eq. (F-17), it is a simple matter to obtain the vane profile $r(z)$ with respect to the azimuthal direction z.

The transverse beam dynamics in the R.M.S. can be described by the following equations:

$$\frac{d^2x}{d\tau^2} = -\frac{qV}{E_0\gamma a(z)^2}\sin(\phi)\, x \qquad \text{(F-18)}$$

$$\frac{d^2y}{d\tau^2} = \frac{qV}{E_0\gamma a(z)^2}\sin(\phi)\, y \qquad \text{(F-19)}$$

V : Inter-vane voltage. $\tau = ct$. E_0 : Rest mass. $a(z)$: Current radius of the channel at z position, ϕ : Current RF phase $\phi = 2\pi f\tau + \phi_0$

Since in the azimuthal direction one has $z = \beta\tau$, one can write in the x-direction:

$$\frac{d^2x}{d\tau^2} = \beta^2 \frac{d}{dz}\left(\frac{dx}{dz}\right) = \beta^2 \frac{\Delta x'}{dz}$$

Therefore, from eq. (F-18) and eq. (F-19) the changes in x' and y' that are caused by the RFQ fields acting over the distance dz are;

$$\Delta x' = -\frac{qVdz}{E_0\beta^2\gamma a(z)^2}\sin(\phi)\, x \qquad \text{(F-20)}$$

$$\Delta y' = \frac{qVdz}{E_0\beta^2\gamma a(z)^2}\sin(\phi)\, y \qquad \text{(F-21)}$$

4. Computation of the dynamics in the azimuthal direction.

The length Δs being taken adequately small, one may assume that $\beta \approx \overline{\beta}$ and $\gamma \approx \overline{\gamma}$, where $\overline{\beta}$ and $\overline{\gamma}$ are the relativistic parameters corresponding to the energy at the middle $\xi = \Delta s / 2$ of the segment Δs . In this case, in each $\Delta s / 2$ drift, the phase ϕ is incremented by the term $\frac{\omega}{\overline{\beta} c}\xi$, where $\omega = 2\pi f$, f is the RF frequency. In addition, due to the impulse at the middle of the segment, one has a jump $\Delta\phi$ in phase:

$$\Delta\phi = \left(\frac{q}{m_0c^2}\right)\frac{\omega}{c}\frac{E_z}{\overline{\beta}^3\overline{\gamma}^3}\int_0^{\Delta s} z\,dz \qquad \text{(4-1)}$$

The electric field E_z is the one at the middle position $\xi = \Delta s / 2$, it is specified by $E_z = \overline{E}_z \sin\phi$.

The phase ϕ_i being the one at the entrance of the segment Δs and ϕ_e the one at the end of this segment, one will have:

$$\phi_e = \phi_i + \frac{2\omega}{\overline{\beta} c}\xi + \Delta\phi \qquad \text{(4-2)}$$

The energy W_f at the end of the segment Δs is given by:

$$W_f = W_i + |q|\overline{E}_z \cos\phi\, \Delta s \qquad \text{(4-3)}$$

The energy $\overline{W}$ at the middle $\xi = \Delta s / 2$ may be approximated by:

$$\overline{W} = W_i + |q|\overline{E}_z \cos\phi\, \Delta s / 2 \qquad \text{(4-4)}$$

This allows computing the relativistic parameters $\overline{\beta}$ and $\overline{\gamma}$ at the middle $\xi = \Delta s / 2$.

We caution that the above approximations are valid provided that the length Δs has been taken sufficiently small.

5. Computation of the dynamics in the transverse directions.

The changes in $x^{'}$ and $y^{'}$ caused by the RFQ fields acting over the distance Δs are [1],[2],[3]:

$$\Delta\left(x^{'}\right) = \frac{q \Delta s \sin \phi}{m_0 c^2 \overline{\beta}^2 \overline{\gamma}} \overline{E}_x \tag{5-1}$$

$$\Delta\left(y^{'}\right) = \frac{q \Delta s \sin \phi}{m_0 c^2 \overline{\beta}^2 \overline{\gamma}} \overline{E}_y \tag{5-2}$$

Since the beam matrix is followed through the segment Δs by a sequence of drift-impulse-drift, after the first drift of length $\xi = \Delta s / 2$, one has:

$$x_m = x_i + \xi \, x_i^{'} \tag{5-3}$$

$$y_m = y_i + \xi \, y_i^{'} \tag{5-4}$$

Due to impulses, the changes in deviations $x^{'}$ and $y^{'}$ are as follows:

$$x_f^{'} = \frac{(\beta\gamma)_i}{(\beta\gamma)_f} x_i^{'} + \Delta\left(x^{'}\right) \tag{5-5}$$

$$y_f^{'} = \frac{(\beta\gamma)_i}{(\beta\gamma)_f} y_i^{'} + \Delta\left(y^{'}\right) \tag{5-6}$$

With: $(\beta\gamma) = \sqrt{\gamma^2 - 1}$

The relativistic parameters β_i, γ_i and β_f, γ_f are the ones at the entrance and at the exit of the segment Δs , respectively.

Pertaining to the second drift of length $\xi = \Delta s / 2$, at the exit of the segment one will have:

$$x_f = x_m + \xi \, x_f^{'} \tag{5-7}$$

$$y_f = y_m + \xi \, y_f^{'} \tag{5-8}$$

6. References

[1] C. Biscari, Computer Programs and Methods for the Design of High Intensity RFQs, CERN/PS 85-67, Sep 15, 1985

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[3] Trace 3D document, LA-UR-97-886