A Bioeconomic model of Scallop Cultures

Exercises Accompanying the Course Environmental Modelling

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Problem formulation

In an aquaculture farm in the Baja California, scallops (Pecten) are grown in 100 big trays that lie on the bottom. The manager of the firm wants to optimize the time of harvesting, so as to have maximal profit. You will make a bioeconomic model to help her reach this decision.

Assumptions

- Growth of these animals is described in terms of density (N) and length (L).
- After seeding the trays with 5×10^4 individuals, the population density declines at a first-order rate described by the mortality rate constant $z = 0.5\ yr^{-1}$.
- Individual growth for scallops is represented using the von Bertalanffy formulation, which describes the change in length using the following differential equation:

$$\frac{dL}{dt} = k \cdot (L_\infty - L),$$

  where k is the growth rate constant ($k = 0.8\ yr^{-1}$) and L_∞ is the asymptotic length ($L_\infty = 100\ mm$).
- The length of the 'seedlings' is 20 mm.
- It is straightforward to estimate individual weight, W (g), from the length using an allometric relation:

$$W = aL^b$$

  For the particular scallop, the coefficients are: a $= 8 \times 10^{-6}$, $b = 3.4$.
- The cost to maintain a scallop batch (or tray) is 150 dollar per year. This is money for labor, and you can assume that this money is set aside continuously. Scallops can only be sold if they weigh more than 30 grams; they are sold at a price of 1.4 dollar per kg.

Tasks

Implement this model and use it to answer the following questions:

- What is the recommendation to the manager: when should the scallops be sold, and how much profit will be made?
- Would it be worthwhile to move the firm to another area, where the scallops grow much better, at a rate of 1.0 yr^{-1}, but where the maintenance cost is twice as high?

Tip: The ecological and biogeochemical problems that we solved thus far were defined in terms of the increase in biomass or concentrations as a function of the sources and sinks. In economics, problems can be defined similarly, but here one is interested in the changes in costs and/or profits.

If you have time:

- It is not realistic to assume that small-sized scallops are sold at the same price as big ones. Assume that 1.4 dollar per kg is the price you get for a small individual (30 g), and an extra 0.1 dollar per kg adds on top of that for every gram that the individual is heavier. How do your conclusions change?
- The firm now owns 100 trays. What is the maximal profit the firm can make? The manager also considers to double the number of trays, but this increases the cost to maintain the scallops with 20% per tray. The investment costs (e.g., for buying the trays) are 25000 dollars. What would be your advice to her?

Answers

The equations describing the ecological part of this problem are given:

$$\frac{dL}{dt} = k \cdot (L_\infty - L)$$

$$\frac{dN}{dt} = -z \cdot N$$

where the density of the scallops continuously declines due to mortality. The initial conditions are: $L_{(t=0)} = 20\ mm$ and $N_{(t=0)} = 5 \times 10^4$.

For the economical part, we take the *cost* as a state variable that increases with the daily cost of culturing:

$$\frac{dCost}{dt} = Culturecost$$

We assume $Cost_{(t=0)} = 0$.

At each time step, we can estimate the price the manager would get if she sold the Scallops. Substracting the current costs from this revenue then gives the current profit or loss that would be obtained if they were sold.

To test whether it is worthwhile to invest in new trays, the cost is augmented, at the beginning with the investment cost, i.e., the initial condition is changed $Cost_{(t=0)} = InvestCost$. Also the initial condition for the density is doubled: $N_{(t=0)} = 10 \times 10^4$.

R implementation

Model implementation

```
require(deSolve)  # package with solution methods
```

```
## Loading required package: deSolve
```

```
# state variables
state <- c(L = 20,          # Scallop length, [mm]
           N = 5*10^4,      # Scallop number  [ind]
           Cost = 0)        # Integrated cost [dollars]

# parameters
parms <- c(
  k    = 0.8,             # growth rate constant        [/yr]
  Linf = 100,             # asymptotic length           [mm]
```

```r
  z    = 0.5,             # mortality rate constant    [/yr]
  a    = 8E-6,
  b    = 3.4,             # allometric coefficients to estimate weight
  numtrays    = 100,      # number of trays
  minWeight   = 30,       # Minimal weight at which Scallops can be sold
  culturecost = 150,      # cost for culturing           [/yr]
  price_kg    = 1.43,     # price per/KG
  price_a     = 0.1       # increased price per gram over minW
)

#The derivative function
Scallops <- function(t, state, params){
  with (as.list(c(state, params)), {

    dL    <- k*(Linf-L)
    dN    <- -z*N
    dCost <- culturecost          # daily cost for culturing adds to total

    weight <- a * L^b             # [g ind-1]
    biom   <- weight*N/1000       # biomass, [kg]
    price  <- (price_kg + price_a*(weight-minWeight))*(weight > minWeight)
    profit <- (biom*price - Cost)* numtrays # total profit = price minus integrated cost

    return (list(c(dL, dN, dCost),
                 weight = weight,
                 biomass = biom,
                 price = price,
                 profit = profit))
  })
}
```

Model runs

```r
# output times
outtimes <- seq(from = 0, to = 3, length.out = 100)

#solve the equation
out <- ode(y = state, parms = parms, func = Scallops, times = outtimes)
head(out)
```

```
##             time        L        N      Cost    weight  biomass price     profit
## [1,] 0.00000000 20.00000 50000.00  0.000000 0.2121251 10.60625     0     0.0000
## [2,] 0.03030303 21.91609 49248.13  4.545455 0.2895238 14.25851     0  -454.5455
## [3,] 0.06060606 23.78626 48507.57  9.090909 0.3824715 18.55277     0  -909.0909
## [4,] 0.09090909 25.61165 47778.15 13.636364 0.4917876 23.49670     0 -1363.6364
## [5,] 0.12121212 27.39332 47059.70 18.181818 0.6181328 29.08914     0 -1818.1818
## [6,] 0.15151515 29.13232 46352.05 22.727273 0.7620176 35.32107     0 -2272.7273
```

```r
max(out[,"profit"] )
```

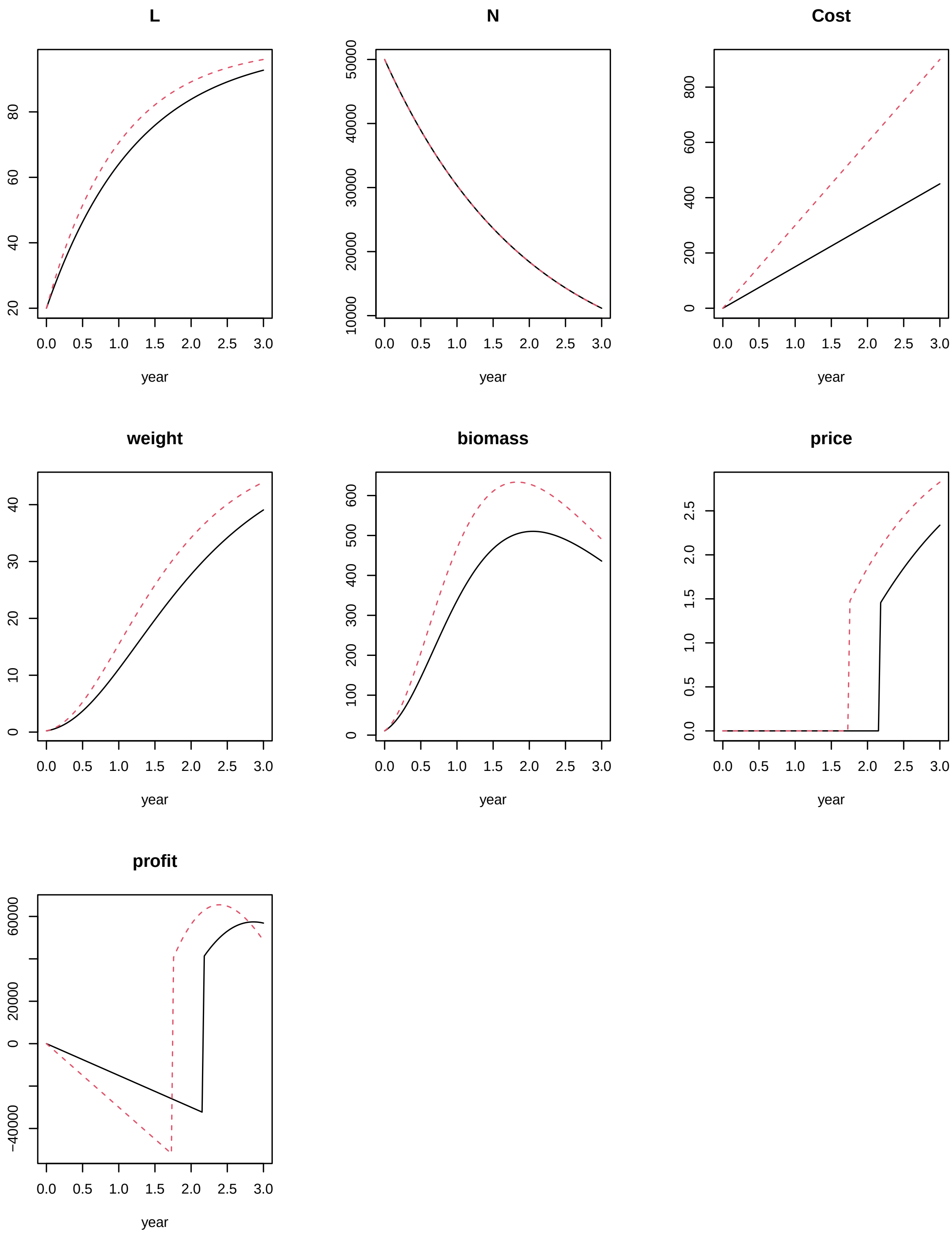
```
## [1] 57419.3
```

```r
# ----------------------------------------------------
# second run - a warmer climate, but more expensive
```

```r
# ---------------------------------------------------
pars <- parms
pars["k"] <- 1
pars["culturecost"] <- pars["culturecost"]*2
out2 <- ode(y = state, parms = pars, func = Scallops, times = outtimes)
plot(out, out2, xlab = "year")
max(out[,"profit"])
```

```
## [1] 57419.3
```

```r
# ... and without the extra profit for larger scallops
p2 <- parms
p2["price_a"] <- 0
outb <- ode(y = state, parms = p2, func = Scallops, times = outtimes)
plot(out, outb, xlab = "year")
```



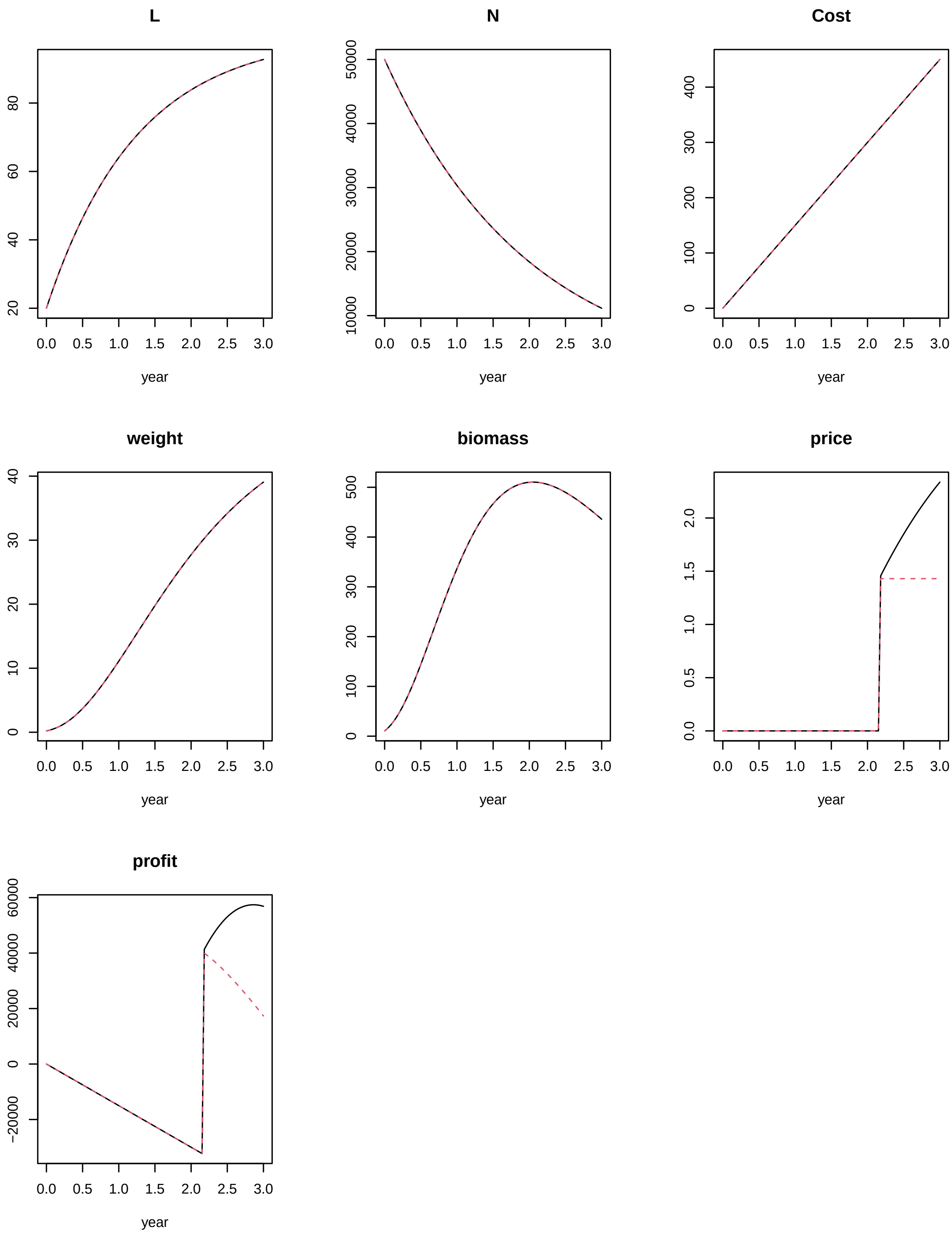

```
# ------------------------------------------------------
# third run - increase the size of the farm?
# ------------------------------------------------------
# first year
```

```r
stateb <- state
stateb["Cost"] <- 250   # initial investment cost

parsb <- parms
parsb["culturecost"] <- parms["culturecost"]*1.2   # maintenance is more expensive
parsb["numtrays"] <- 2*parms["numtrays"]   # number of trays * 2

out3 <- ode(y = stateb, parms = parsb, func = Scallops, times = outtimes)

# second year: no investment cost
statec <- stateb; statec["Cost"] <- 0
out4 <- ode(y = statec, parms = parsb, func = Scallops, times = outtimes)
```




```r
par(mfrow = c(1, 2))
plot(out, out2, which = "profit", lwd = 2, xlab = "year", ylab = "dollar", mfrow = NULL)
legend("bottomright", col = 1:2, lwd = 2, lty = 1:2, c("original", "warmer"))
```

```
plot(out, out3, out4, which = "profit", lwd = 2, xlab = "year", ylab = "dollar",
     mfrow = NULL)
legend("topleft", col = 1:3, lwd = 2, lty = 1:3,
       c("original", "doubling, 1st year", "doubling, 2nd year"))
```