



# Reactive Transport in the Hydrosphere

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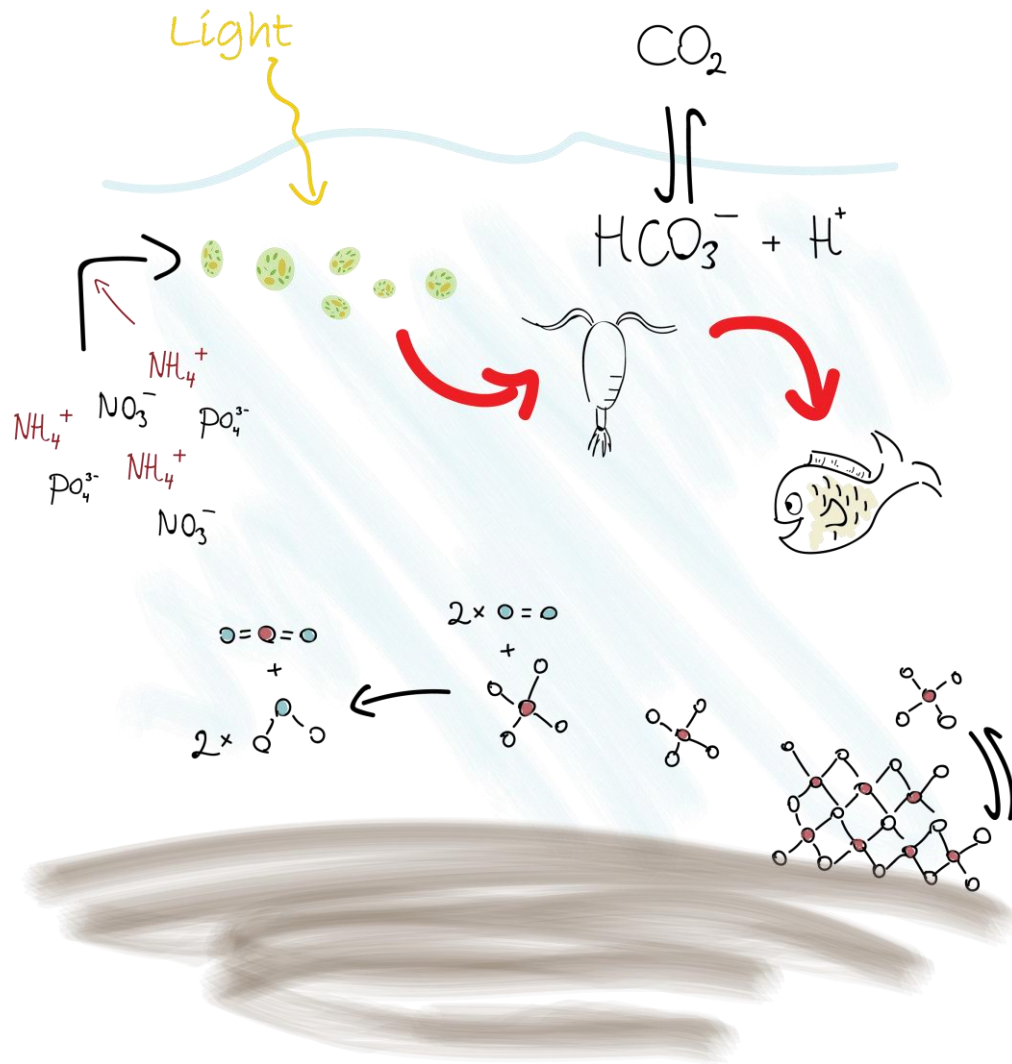
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# Rate laws for . . .



## Chemical reactions

- Irreversible
- Reversible
- Enzyme-catalyzed (metabolic)
- Substrate limitation
- Substrate inhibition
- Rate saturation

## Large-scale models

## Partitioning between phases

- Mineral dissolution / precipitation
- Gas exchange

## Ecological interactions

- Grazing, predator-prey type

## Transport



# Rate laws for ecological interactions

## General form



$$\text{Interaction} = \text{maxRate} \cdot \text{WORKER} \cdot \text{RateLimitation} \cdot \text{RateInhibition}$$

- Workers do not interact with each other
- More workers = more work

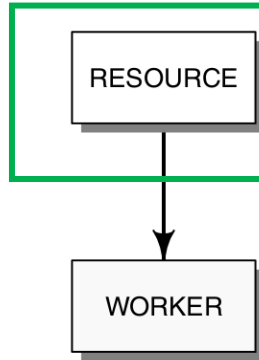
$$\text{NutrientUptakeRate} = r \cdot \text{ALGAE} \cdot \text{RateLimitingTerm}$$

$$\text{PredationRate} = r \cdot \text{PREDATOR} \cdot \text{RateLimitingTerm}$$

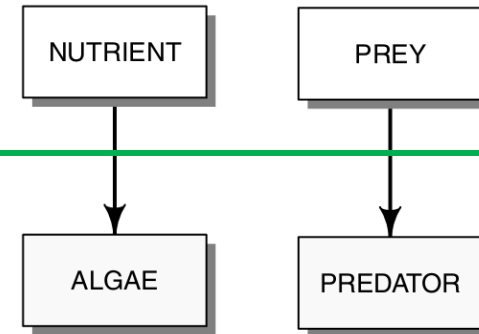


# Ecological interactions

## General form



## Examples



$$\text{Interaction} = \text{maxRate} \cdot \text{WORKER} \cdot \text{RateLimitation} \cdot \text{RateInhibition}$$

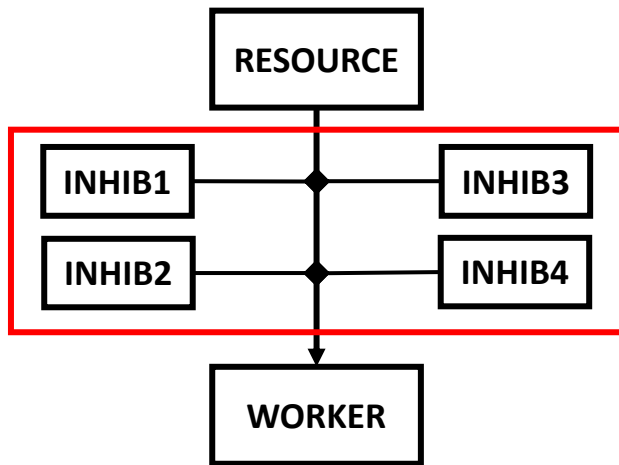
- Limitation by the resource
- Etc.

- Presence of competitors
- Inhibitor, Pollution, Etc.

$$\text{NitrateUptake} = r \cdot \frac{[NO3]}{[NO3] + K_{NO3}} \cdot \frac{K_{NH3}}{[NH3] + K_{NH3}} \cdot \text{ALGAE}$$



# Lumping of multiple growth-inhibiting factors



Impact of the effects is lumped:

***K*** = ecosystem carrying capacity (maximum size that the population can reach given abundant resources)

Logistic growth model:

$$\frac{dN}{dt} = r \cdot N \cdot \overbrace{\left(1 - \frac{N}{K}\right)}^{\text{Inhibition term}}$$

Population size of the worker



# Logistic growth model

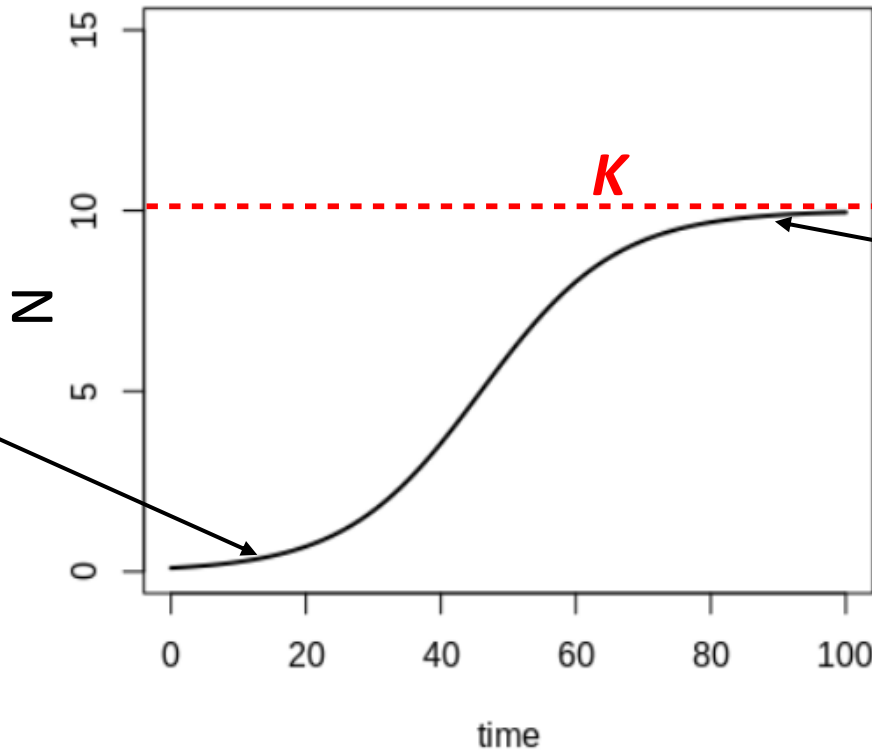
$$\frac{dN}{dt} = r \cdot N \cdot \left(1 - \frac{N}{K}\right)$$

Optimal growth

$$N \ll K$$

$$\left(1 - \frac{N}{K}\right) \approx 1$$

$$N = N_0 \cdot \exp(rt)$$



Growth stops

$$N \approx K$$

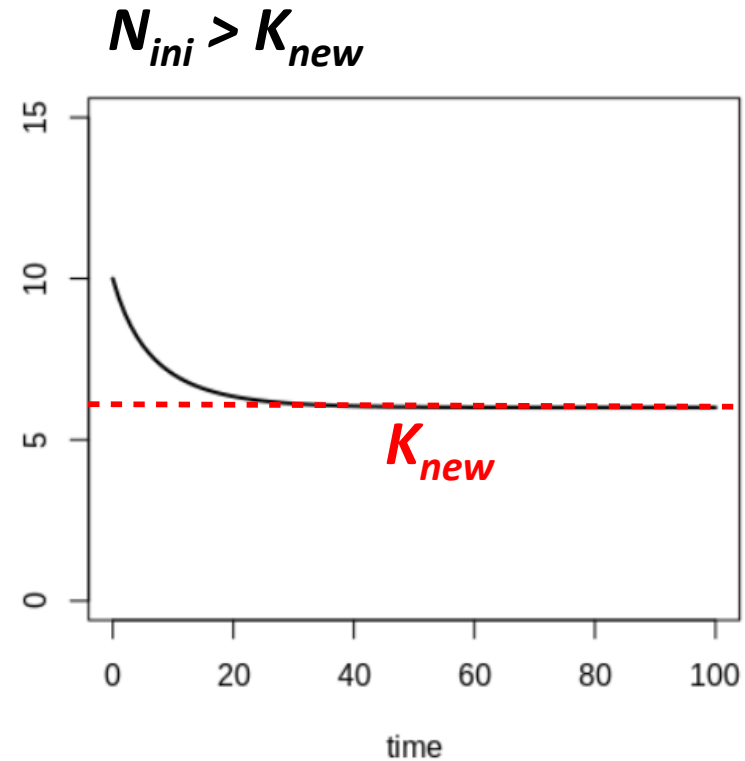
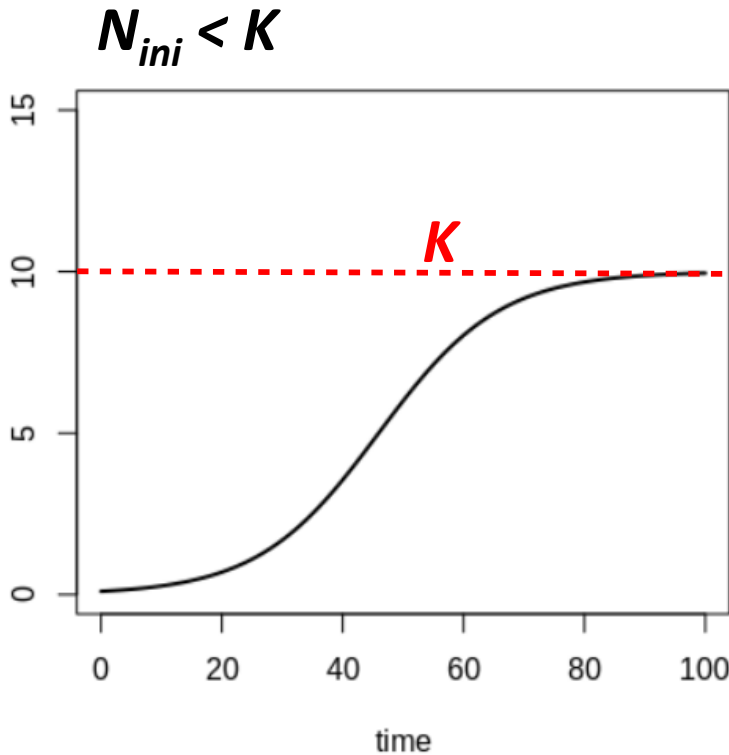
$$\left(1 - \frac{N}{K}\right) \approx 0$$

$$\frac{dN}{dt} \approx 0$$



# Logistic growth model

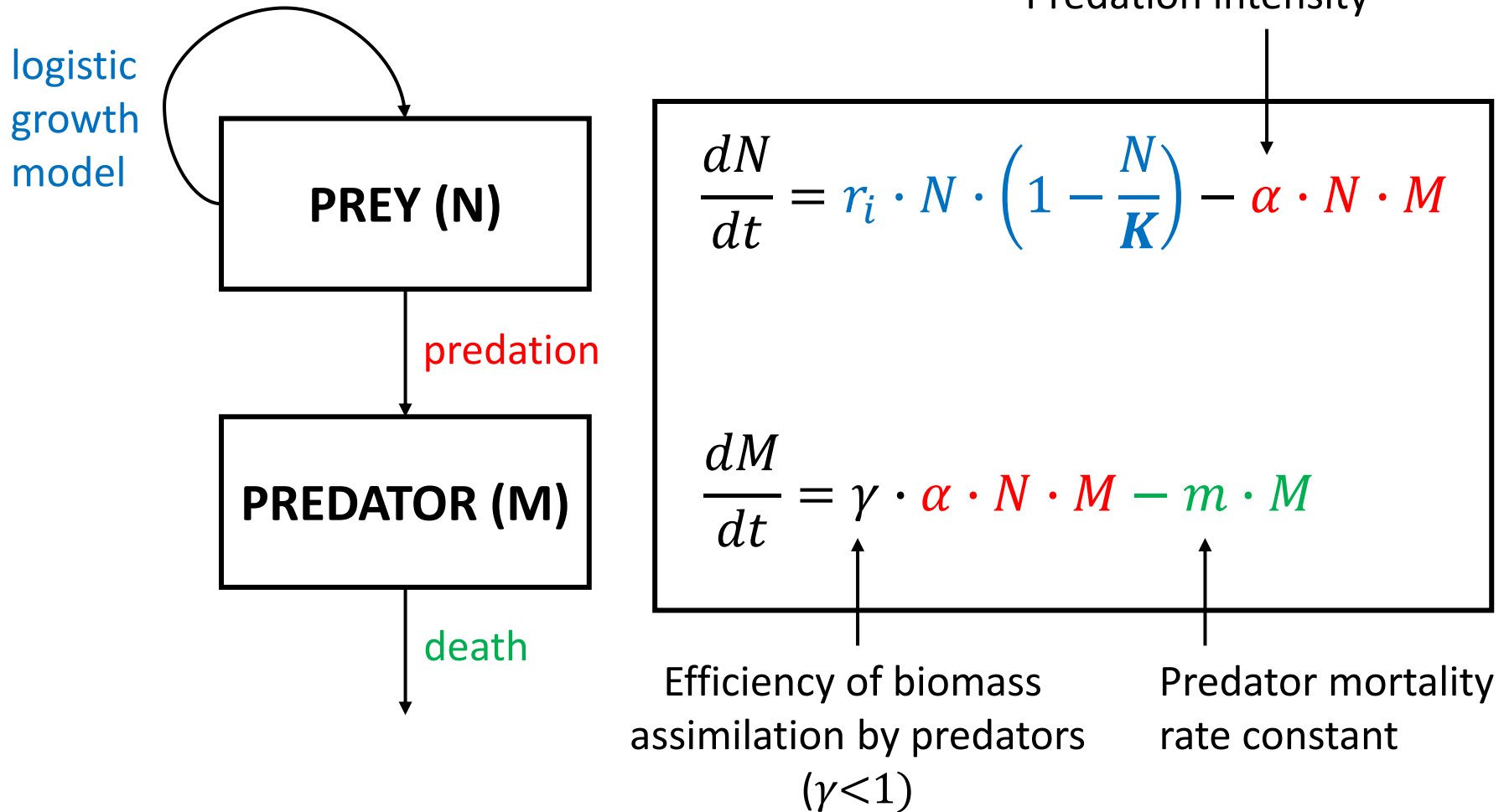
$$\frac{dN}{dt} = r \cdot N \cdot \left(1 - \frac{N}{K}\right)$$



$K \rightarrow K_{new}$  due to, e.g., habitat destruction



# Predator-prey interactions – Lotka-Volterra model



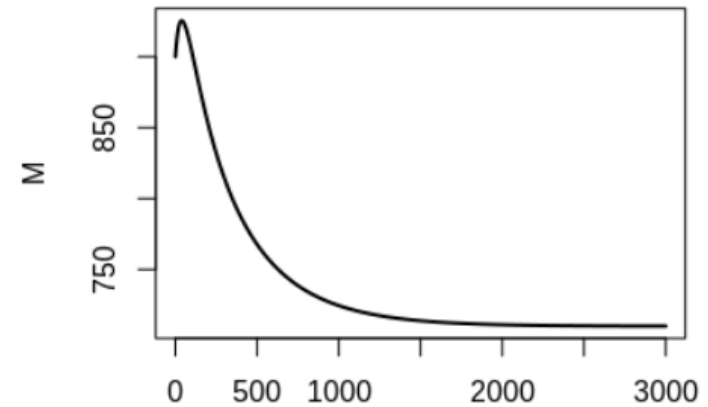
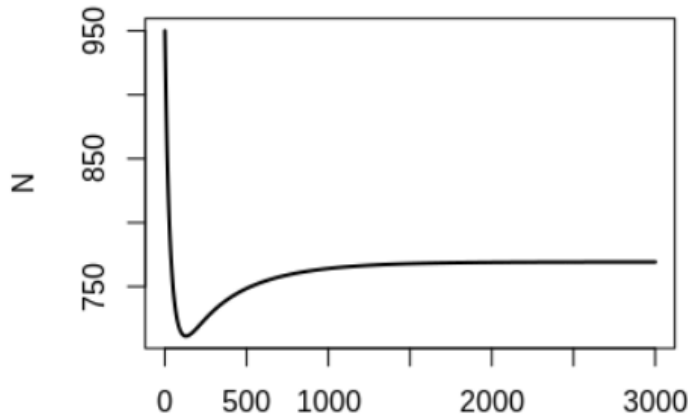


# Predator-prey interactions – Lotka-Volterra model

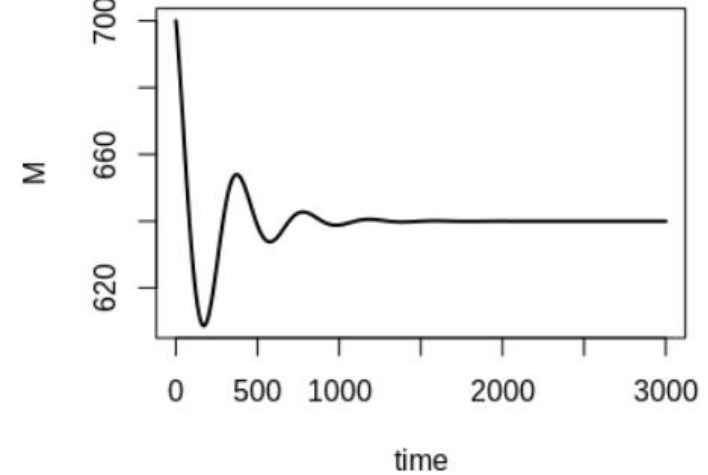
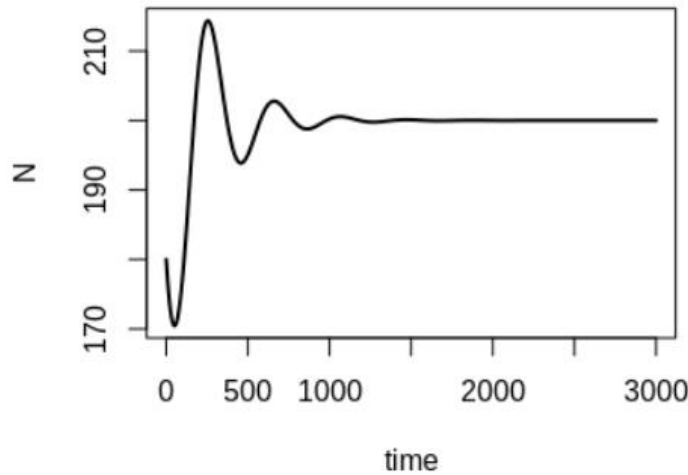
$$\frac{dN}{dt} = r_i \cdot N \cdot \left(1 - \frac{N}{K}\right) - \alpha \cdot N \cdot M$$

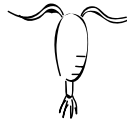
$$\frac{dM}{dt} = \gamma \cdot \alpha \cdot N \cdot M - m \cdot M$$

$K=1000$   
 $r_i=0.04$   
 $\alpha=1.3e-5$   
 $\gamma=0.8$   
 $m=0.2 \cdot r_i$



$K=1000$   
 $r_i=0.04$   
 $\alpha=5e-5$   
 $\gamma=0.8$   
 $m=0.2 \cdot r_i$





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