Earth's global C cycle: the CO_2 Problem and Mitigation Strategies

Exercises Accompanying the Course Reaction Transport Modelling in the Hydrosphere

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Introduction

In a seminal work, Sarmiento and Gruber (2002) published a global carbon budget of the Earth, where C is exchanged among 7 large compartments (*Figure* 1): atmosphere, marine biota, terrestrial biota (vegetation, soil and detritus), surface of the ocean, intermediate and deep ocean, surface sediments in the ocean, and sediments and rocks (comprising organic carbon, limestone and methane hydrate).

In this exercise, you will make a dynamic model of the Earth's global carbon cycle based on this budget. You will then use this model to assess the response of the Earth's carbon cycle to perturbations, tackling the following questions: How, and how fast, does the Earth's carbon cycle respond to perturbations? What is the new steady-state condition? What are the consequences of various carbon reduction scenarios?

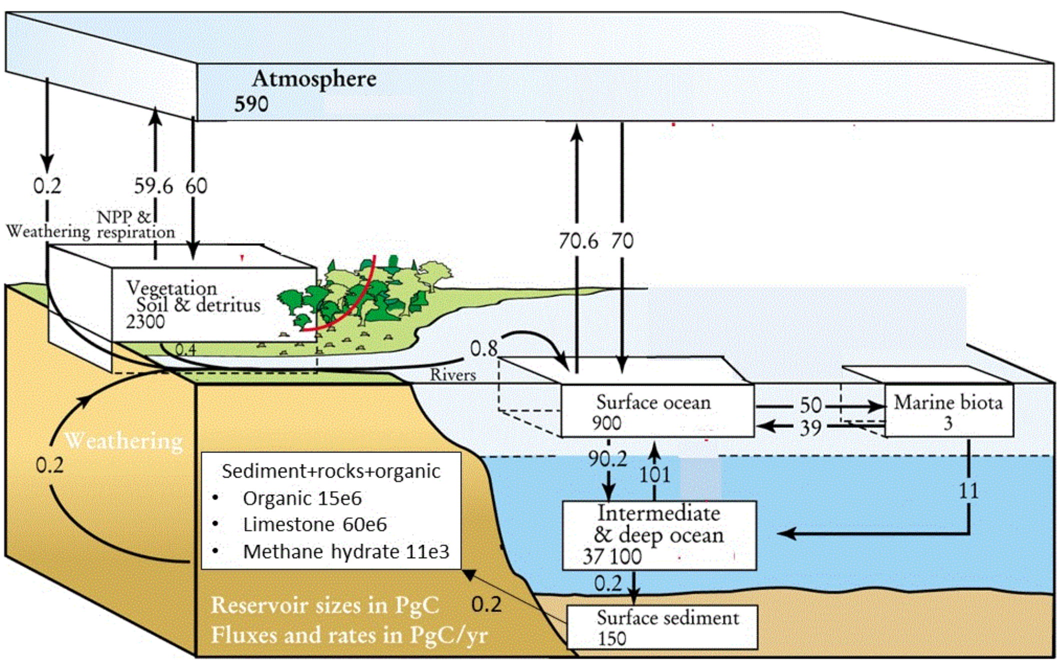


Figure 1: Earth's global carbon budget. Adapted from: Sinks for anthropogenic carbon. Jorge L. Sarmiento and Nicolas Gruber. Physics Today 55(8), 30 (2002); doi: 10.1063/1.1510279.

Problem formulation

The ultimate 7 compartment model is best explained based on a simple example where we only consider *two compartments*: the atmosphere, with the total mass of carbon M.ATM, and the biosphere, with the total mass of carbon M.BIO.[1]

At steady-state the mass of carbon is 590 and 2300 $Pg\ C$ (Peta gram) in the atmosphere and biosphere, respectively, while the fluxes between the two compartments are 60 $Pg\ C\ yr^{-1}$ ($Figure$ 1).[2]

In the first approximation, we assume that carbon is transferred between the atmosphere and the biosphere at a rate that is *first-order* with respect to the *source compartment*, i.e., linearly proportional to the size of the source. Thus, the mass balance equations in this simplified 2-box model have the form

$$\frac{dM.ATM}{dt} = -F_{AB} + F_{BA} = -r_{AB} \times M.ATM + r_{BA} \times M.BIO,$$
$$\frac{dM.BIO}{dt} = +F_{AB} - F_{BA} = r_{AB} \times M.ATM - r_{BA} \times M.BIO,$$

where F_{AB} and F_{BA} are the fluxes from A to B and vice versa (units of $Pg\ C\ yr^{-1}$), and r_{BA} and r_{BA} are the corresponding first-order rate constants (units of yr^{-1}). Note that the formulation $F_{AB} = r_{AB} \times M.ATM$ means that the rate constant r_{AB} represents the fraction of the source compartment transferred per year. We use this mathematical formulation to simulate what happens under transient conditions, such as when the system is perturbed from the steady state.

At steady-state, both the carbon mass ($M.ATM$, $M.BIO$) and the total fluxes (F_{AB}, F_{BA}) are known ($Figure$ 1), so we use that information to estimate the first order removal rate constants:

$$r_{AB} = F_{AB}/M.ATM = 60/590\, yr^{-1},$$
$$r_{BA} = F_{BA}/M.BIO = 60/2300\, yr^{-1}.$$

Obviously, if this model is run dynamically with the steady-state values as initial conditions, then the carbon masses should stay constant. Thus, the first steps in the implementation of this model in R must include the verification that this is true.

Task 1: 2-compartment model of the Earth's global carbon cycle

Before you embark on the development of the more complex model, implement and apply the simplified 2-compartment model described above. Fluxes are expressed in $Pg\ yr^{-1}$, mass in each compartment is expressed in $Pg\ C$, the parameter values are

parameter	value	unit
r_{AB}	60/590	yr^{-1}
r_{BA}	60/2300	yr^{-1}

as estimated above. You can start with the R-markdown template file *RTM_0D.Rmd* to implement this model.[3]

- Run the model for 100 years, with initial conditions that represent the steady-state (*M.ATM = 590*, *M.BIO = 2300*). The system should be at steady-state, i.e., none of the pools should change in time.

[1] Note the use of dot (".") in the name of the state variable.
[2] P stands for Peta, which denotes 10^{15}.
[3] You can obtain this file from Rstudio: File → new File → Rmarkdown → from template → RTM_0D. Save this file under a different name, e.g., Ccycle_2box.Rmd. Do not forget to change the heading of this file.

- Now assume that, at the start of the simulation there has been a transfer of 50 $Pg\ C$ from the biosphere to the atmosphere. Such a transfer might represent *deforestation* where the forest is burned to CO_2. Run the model for 100 years. What happens to the perturbed system? Does it return to the same steady-state as before? On which time scale do the changes occur?
- Now implement the burning of 10 $Pg\ C$ of *fossil fuel* at the start. As fossil fuel is *not* explicitly modeled, this means that 10 $Pg\ C$ is added to the atmospheric pool at the start of the simulation. How does the system behave now? What can you say about the steady state in this perturbed system? On which time scale do the changes occur? Support your numerical results with a logical explanation.

Task 2: Complex representation of the Earth's carbon cycle with 7 compartments

Now you are ready to develop and implement a more realistic carbon cycle model based on the schematic diagram shown in $Figure$ 1.

- Based on the diagram, create the mass balance equations for each carbon reservoir. Number the reservoirs and give them sensible names, e.g., M1.ATMOSPHERE, M2.SURFACEOCEAN, etc. Give sensible names to the fluxes that represent the source and sink, e.g., $F1.2$ is the flux from M1.ATMOSPHERE to M2.SURFACEOCEAN, and $F2.1$ is the reverse flux, etc.
- Assume first-order kinetics for each flux, i.e., $F1.2 = r1.2 \times M1.ATMOSPHERE$, etc. Estimate the corresponding rate constants using the steady-state situation depicted in $Figure$ 1.
- Implement the model in R. Save it in a different file, e.g., Ccycle_7box.Rmd.
- Solve the model for 1000 years, using steady-state masses as initial conditions. This model should be at steady-state, else you have made an error that you need to fix!
- Run the model to study the consequences of the following scenarios.

Scenario 1: Instantaneous deforestation

Assess the response of the system to an instantaneous deforestation (similar as for the 2-compartment model).

- Assume that, at the start of the simulation, there has been a transfer of 100 $Pg\ C$ from the biosphere to the atmosphere.
- Run the model for 1000 years. Plot the result of this model, together with the steady-state results.
- What happens to the perturbed system?
- Where does the carbon end up after 1000 years?

Scenario 2: Burning of fossil fuel at a constant rate

- Implement in your model fossil fuel burning as an extra—constant—flux to the atmosphere. Think from which reservoir this carbon originates.
- The current fossil fuel emission is 10 $Pg\ C\ yr^{-1}$. Use your model to estimate the response of the different carbon reservoirs to emissions equal to 5, 10 and 20 $Pg\ C\ yr^{-1}$ over a time interval of 1000 years. Assume that the emissions would stay constant over this time scale.
- What fraction of the emitted carbon will be present in the atmosphere after 10, 100 and 1000 years?
- Where is most of the emitted carbon stored after 10, 100 and 1000 years?

Scenario 3: Time-dependent burning of fossil fuel for 200 years

The burning of fossil fuel has not been constant over time. A more realistic scenario assumes that the emission rates increase linearly with time.

Assume that the rate of fossil fuel emission to the atmosphere increased linearly over a period of 200 years, and that afterwards the mankind has managed to completely eliminate these emmissions. To implement in R a rate that first increases linearly and then abruptly drops to 0, you can write:

```
F.fuel <- a.fuel * t * (t < t_em)
```

where a.fuel describes the linear increase in the emission rate and t_em defines the time at which emissions drop to 0. Note that the term "(t < t_em)", which is a logical expression, will be 1 as long as the condition is true (logical TRUE), whereas it will be 0 if the condition is not fulfilled (logical FALSE). Assume a.fuel = 0.05 $Pg\ C\ yr^{-2}$ and t_em = 200 yr.

- Run this model for 1000 years. Where will most of the carbon end up after 1000 years?
- Run this model also for 1e6 years. Discuss the response of the system on time-scales ranging from hundreds to million years.

Scenario 4: CO_2-mitigation by increased oceanic production

Several "engineering" fixes have been proposed to mitigate the consequences of anthropogenic CO_2 emissions. One approach is to stimulate primary production in the surface ocean (so-called "ocean fertilization' ', by adding a limiting nutrient such as iron).

- How do you implement this strategy in the model?
- Use the status of the system after 200 years of linearly increasing burning of fossil fuels (scenario 3) as the initial condition for this scenario. Then run the model for 1000 years with fossil fuel emissions set to zero. Experiment by changing the value of the relevant model parameter(s) to study the impact of different degrees of ocean fertilization on the removal of the atmospheric CO_2.
- Compare the model runs with the steady-state condition and with the scenario where no mitigation has been implemented (scenario 3).

Scenario 5: CO_2-mitigation by sequestration of atmospheric CO_2 into deep ocean

Another proposed mitigation approach is to sequester CO_2 from the atmosphere and inject it directly to the deep ocean. This can be done, for example, by stimulating primary production in the surface ocean, harvesting this extra production, and injecting it into the deep ocean. In this exercise, we can simplify this approach by by-passing this "D-tour" and assuming that atmospheric CO_2 is directly transferred into the deep ocean.

- How do you implement this strategy in the model?
- Use the status of the system after 200 years of linearly increasing burning of fossil fuels (scenario 3) as the initial condition for this scenario. Then run the model for 1000 years with fossil fuel emissions set to zero. Experiment by changing the value of the relevant model parameter(s) to study the impact of different rates of CO_2 sequestration on the removal of the atmospheric CO_2.
- Compare the model runs with the steady-state condition and with the scenarios 3 and 4.

Answers

Global carbon model with *two* compartments

Model implementation

Units used:

- Time : year
- Mass : Pg C

Assumptions:

- C reservoirs on Earth are represented as *homogeneous* boxes.
- Total C in each reservoir is modelled.
- All natural fluxes are *first-order* with respect to the *source* reservoir.
- Perturbations due to anthropogenic activity are modeled by *changing the initial condition.*

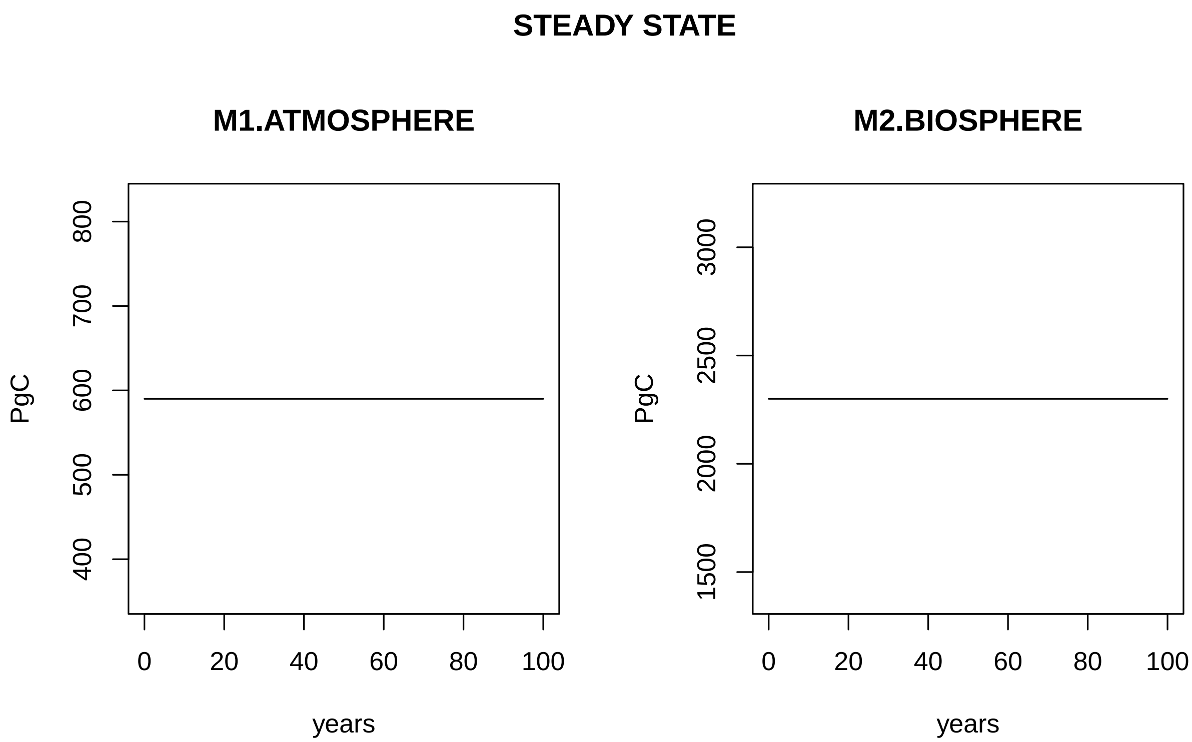
```
library(deSolve)

# Initial conditions, corresponding to the equilibrium (steady state)
state.eq <- c(
 M1.ATMOSPHERE     = 590,          # carbon mass in atmosphere, PgC
 M2.BIOSPHERE      = 2300          # carbon mass in biosphere,  PgC
)

# Model parameters
parms <- with (as.list(state.eq), c(
  # rate constants, yr-1 (first-order kinetics for all fluxes)
  k1.2 = 60 / M1.ATMOSPHERE,    #  atmosphere -> biosphere, [1/yr]
  k2.1 = 60 / M2.BIOSPHERE  )   #  biosphere  -> atmosphere [1/yr]
)

# model equations
Cmodel_2comp  <- function(t, C, p) {       # 2-compartment carbon model
  with(as.list (c(C, p)),{

    # rate laws (first-order kinetics)
    F1.2 <- k1.2*M1.ATMOSPHERE    # atm  -> bio
    F2.1 <- k2.1*M2.BIOSPHERE     # bio  -> atm

    # mass balance equations
    dM1.ATMOSPHERE    <- F2.1 - F1.2
    dM2.BIOSPHERE     <- F1.2 - F2.1

    return(list(c(dM1.ATMOSPHERE, dM2.BIOSPHERE)))
  })
}  # end of model equations
```

Model application

Steady-state

Run the model for 100 years and plot the results.

```
time_seq <- 0:100   # run the model for 100 years
out1 <- ode(y = state.eq, times = time_seq, func = Cmodel_2comp, parms = parms)

par(oma = c(0,0,2,0)) # for wider margins
plot (out1, xlab = "years", ylab = "PgC")
title(main = "STEADY STATE", outer = TRUE)
```




```
out1[nrow(out1),]  # look at last values
```

```
##          time M1.ATMOSPHERE  M2.BIOSPHERE
##           100           590          2300
```

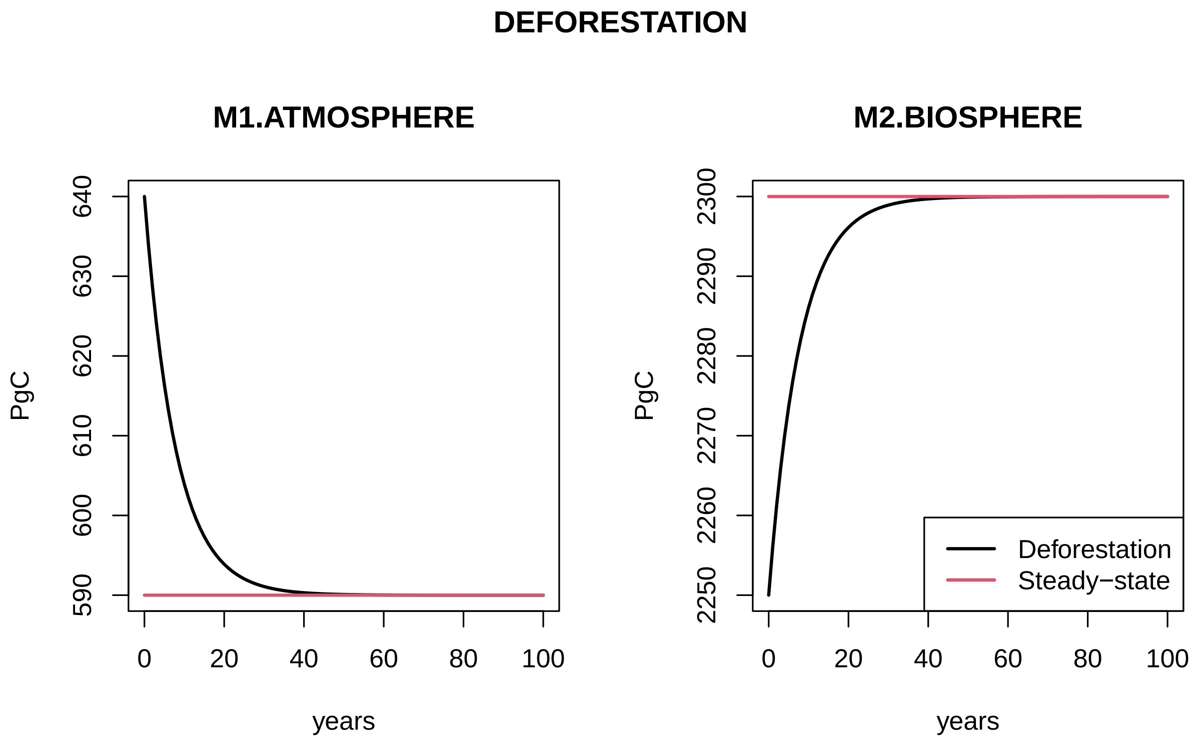
The masses stay constant, hence the model is correctly implemented.

Scenario 1: instantaneous DEFORESTATION

50 Pg C is transferred from M2.BIOSPHERE to M1.ATMOSPHERE. This is implemented by *changing the initial condition* while keeping the *same model parameters*.

```
Cdeforest <- 50

# define new initial state
state2 <- state.eq
state2["M1.ATMOSPHERE"] <- state2["M1.ATMOSPHERE"] + Cdeforest
```

```r
state2["M2.BIOSPHERE"]  <- state2["M2.BIOSPHERE"]  - Cdeforest

# Calculate, plot and print the solution, including also the original solution
out2 <- ode(y = state2, times = time_seq, func = Cmodel_2comp, parms = parms)

par(oma = c(0,0,2,0))   # for wider margins
plot(out2, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
legend("bottomright", legend = c("Deforestation", "Steady-state"),
       col = 1:2, lwd = 2, lty = 1)
title(main= "DEFORESTATION ", outer = TRUE)
```



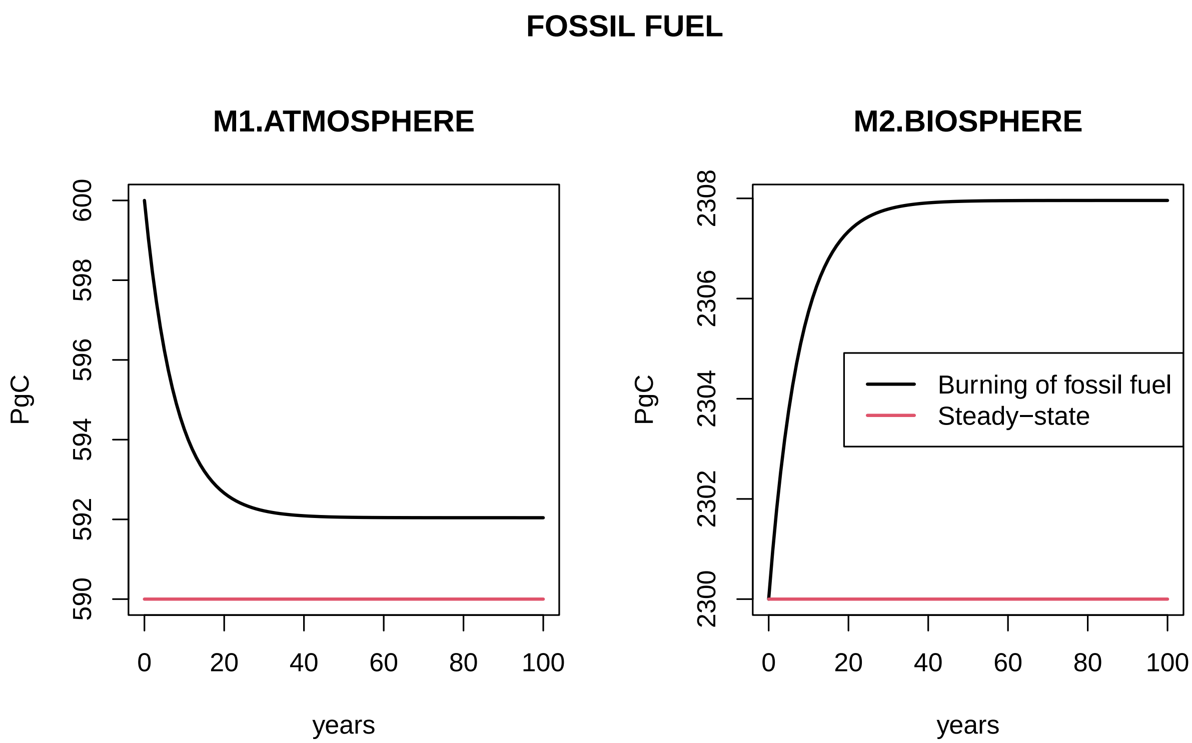
The system returns to the *original* steady state after about 40 years.

Scenario 2: instantaneous burning of FOSSIL FUEL

10 Pg C is transferred to M1.ATMOSPHERE from a compartment that is *not* part of the model. This is implemented by *changing the initial condition* (M1.ATMOSPHERE only) while keeping the *same model parameters*.

```r
FossilFuel <- 10 # Pg C
state3     <- state.eq    # start from the steady state
state3["M1.ATMOSPHERE"] <- state3["M1.ATMOSPHERE"] + FossilFuel

out3 <- ode(y = state3, times = time_seq, func = Cmodel_2comp, parms = parms)

par(oma = c(0,0,2,0))
plot(out3, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
legend("right", legend = c("Burning of fossil fuel", "Steady-state"),
```

```
       col = 1:2, lwd = 2, lty = 1)
title(main = "FOSSIL FUEL", outer = TRUE)
```



The system reaches an equilibrium after about 40 years. However, in this scenario, the equilibrium values are *different* compared to those in scenario 1. This is because, in scenario 2, carbon was *added* to the system. In the equilibrium, the fluxes F_{12} and F_{21} are *equal*, so that the time-derivative of each state variable is zero. Based on the rate laws used, this yields the relationship

$$k_{12} \cdot M1.ATM^* = k_{21} \cdot M2.BIO^* \quad \rightarrow \quad \frac{M1.ATM^*}{M2.BIO^*} = \frac{k_{21}}{k_{12}} = \frac{60}{2300} \cdot \frac{590}{60} = \frac{590}{2300},$$

where * denotes the equilibrium value. We note that the *total* carbon in the system remains *constant*, as follows from the summation of the mass balance equations for both state variables $[d(M1.ATM + dM2.BIO)/dt = 0]$. Therefore, the amount of carbon in the system at an equilibrium is *the same* as at the beginning, i.e.,

$$M1.ATM^* + M2.BIO^* = 2300 + 10 + 590.$$

Using the two last equations and some simple algebra, we can calculate the equilibrium amounts of C in both compartments. We obtain values $M1.ATM^* = 592.0415$ and $M2.BIO^* = 2307.958$, which are identical to those obtained by the model, as shown by the command below.

```
out3[nrow(out3),]  # look at last values
```

```
##          time M1.ATMOSPHERE  M2.BIOSPHERE
##      100.0000      592.0415     2307.9585
```

We conclude that the 10 Pg C initially added to the system was *redistributed* such that about 20% ends up in the atmosphere and 80% ends up in the biosphere. More C ends up in the biosphere because the rate constant describing the flux from the biosphere to the atmosphere is lower.

Global carbon cycle model with *seven* compartments

Model implementation

Units used:

- Time : year
- Mass : PgC

Assumptions:

- C reservoirs on Earth are represented as homogeneous boxes.
- Total C in each reservoir is modelled.
- All fluxes are first-order with respect to the source reservoir, except for the anthropogenic fluxes, which are modeled as described in text.

Note before you start reading the code:

- Implementation of the model is very *neat*. This makes the model much easier to *read an understand* by someone who has not done the modelling. Additionally, it makes the model much easier to *modify or expand*, by you or by anyone else. Do not underestimate this aspect of R-coding.

- We develop *one* model that can handle *all* scenarios. The different scenarios are implemented by *changing initial conditions or model parameters*, and *not* by implementing a separate model function for each scenario.

```
# define names of state variables and their initial values
state.eq <- c(
 M1.ATMOSPHERE     = 590,             # atmosphere
 M2.SURFACEOCEAN   = 900,             # surface ocean
 M3.MARINEBIOTA    = 3,               # marine biota
 M4.DEEPOCEAN      = 37100,           # deep ocean
 M5.SEDIMENTOCEAN  = 150,             # ocean sediment
 M6.LANDBIOTA      = 2300,            # land biosphere
 M7.ROCKS_FOSSIL   = 60e6+15e6+11e3, # sediments & rocks & fossil
 M8.EMITTED        = 0                # total emitted fossil C (auxillary)
)

#==============================================================================
# Model parameters:
#==============================================================================

parms <- with (as.list(state.eq), c(

  # rate constants, yr-1 (first-order kinetics for all natural fluxes)
  k1.2 = 70.2 /M1.ATMOSPHERE,    # atm     -> surfoc (Uptake+weathering)
  k2.1 = 70.6 /M2.SURFACEOCEAN,  # surfoc  -> atm
  k2.3 = 50   /M2.SURFACEOCEAN,  # surfoc  -> marbio
  k3.2 = 39   /M3.MARINEBIOTA,   # marbio  -> surfoc
  k3.4 = 11   /M3.MARINEBIOTA,   # marbio  -> deepoc
  k4.2 = 101  /M4.DEEPOCEAN,     # deepoc  -> surfoc
  k2.4 = 90.2 /M2.SURFACEOCEAN,  # surfoc  -> deepoc
  k4.5 = 0.2  /M4.DEEPOCEAN,     # deepoc  -> sed
  k5.7 = 0.2  /M5.SEDIMENTOCEAN, # sed     -> rocks
  k6.2 = 0.4  /M6.LANDBIOTA,     # landbio -> surfoc
  k1.6 = 60   /M1.ATMOSPHERE,    # atm     -> landbio
  k6.1 = 59.6 /M6.LANDBIOTA,     # landbio -> atm
```

```
  k7.2 = 0.2  /M7.ROCKS_FOSSIL,  # rocks_fossil   -> surface ocean
  k7.4 = 0.0  /M7.ROCKS_FOSSIL,  # rocks_fossil   -> deep ocean

  # constants for additional input of C by anthropogenic burning of
  # fossil fuels, which is described as an extra flux
  # F14 <- (aFuel*t + bFuel)*(t<tmax_input)
  aFuel = 0,          # rate of increase of fuel burning, PgC yr-2
  bFuel = 0,          # baseline fuel burning, PgC yr-1
  tmax_input = 1000,  # after this period, fossil fuel burning stops, yr

  # constant mitigation flux of C from atmosphere directly into the deep ocean
  Fatm2deep = 0       # PgC yr-1
  )
) # end of with(as.list(...

#==============================================================================
# model function
#==============================================================================

Cmodel  <- function(t, C, p) {

  with(as.list (c(C, p)),{

    # define expressions for all fluxes
    F1.2 <- k1.2*M1.ATMOSPHERE    # atm     -> surfoc   (Uptake+weathering)
    F1.6 <- k1.6*M1.ATMOSPHERE    # atm     -> landbio
    F2.1 <- k2.1*M2.SURFACEOCEAN  # surfoc  -> atm
    F2.3 <- k2.3*M2.SURFACEOCEAN  # surfoc  -> marbio
    F2.4 <- k2.4*M2.SURFACEOCEAN  # surfoc  -> deepoc
    F3.2 <- k3.2*M3.MARINEBIOTA   # marbio  -> surfoc
    F3.4 <- k3.4*M3.MARINEBIOTA   # marbio  -> deepoc
    F4.2 <- k4.2*M4.DEEPOCEAN     # deepoc  -> surfoc
    F4.5 <- k4.5*M4.DEEPOCEAN     # deepoc  -> sed
    F5.7 <- k5.7*M5.SEDIMENTOCEAN # sed     -> rocks
    F6.1 <- k6.1*M6.LANDBIOTA     # landbio -> atm
    F6.2 <- k6.2*M6.LANDBIOTA     # landbio -> surfoc
    F7.2 <- k7.2*M7.ROCKS_FOSSIL  # rocks   -> surface ocean

    # fossil fuel burning
    F.fuel <- (bFuel + aFuel*t) * (t <= tmax_input)

    # mass balance equations for each state variable
    dM1.ATMOSPHERE    <- F2.1 + F6.1 - F1.2 - F1.6 + F.fuel - Fatm2deep
    dM2.SURFACEOCEAN  <- F1.2 + F3.2 + F4.2 + F6.2 + F7.2 - F2.1 - F2.3 - F2.4
    dM3.MARINEBIOTA   <- F2.3 - F3.2 - F3.4
    dM4.DEEPOCEAN     <- F2.4 + F3.4 - F4.2 - F4.5 + Fatm2deep
    dM5.SEDIMENTOCEAN <- F4.5 - F5.7
    dM6.LANDBIOTA     <- F1.6 - F6.1 - F6.2
    dM7.ROCKS_FOSSIL  <- F5.7 - F7.2 - F.fuel
    # track the amount of emitted and sequestered C
    dM8.EMITTED       <- F.fuel - Fatm2deep

    return(list(c(
```

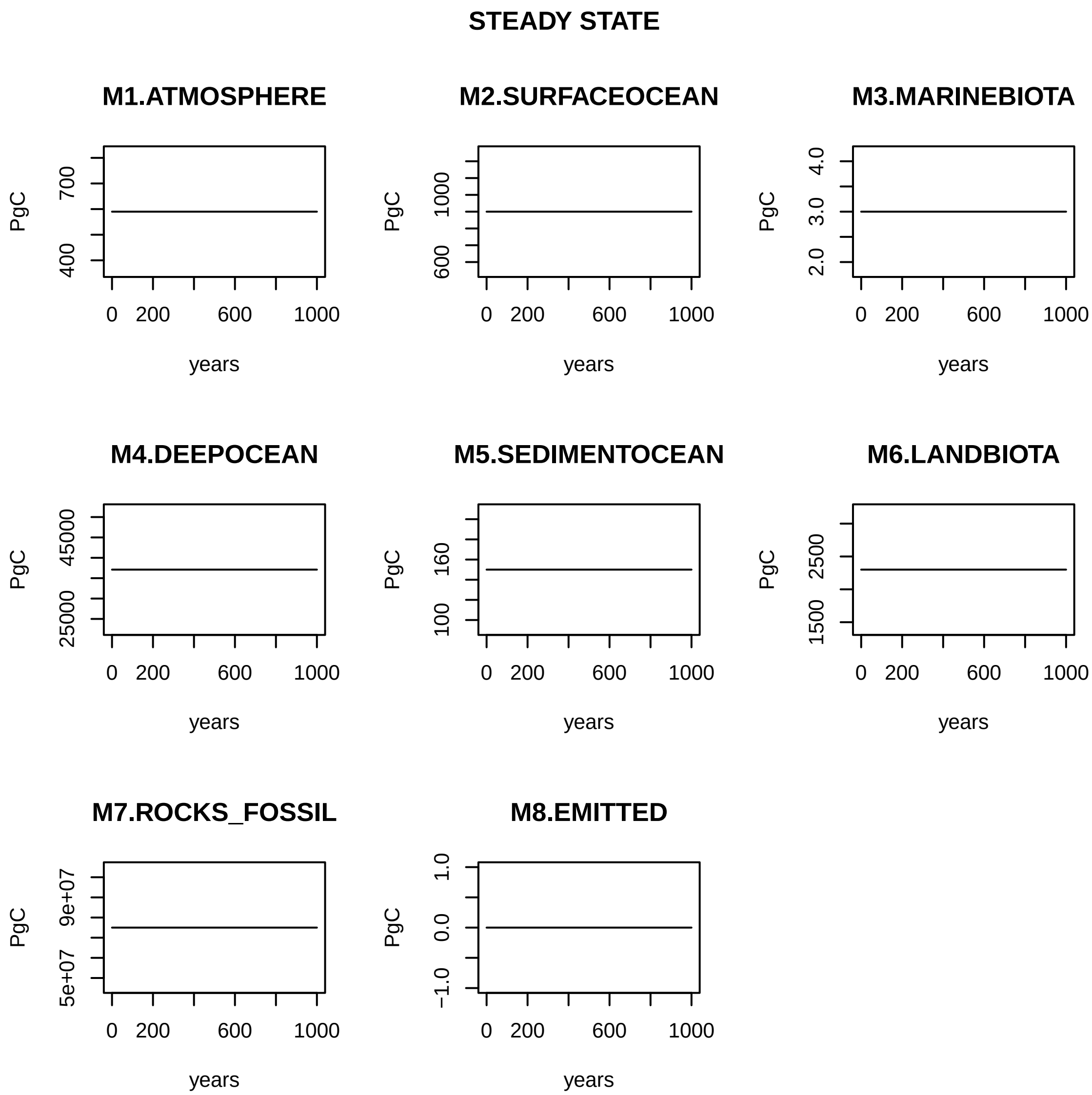
```
      dM1.ATMOSPHERE, dM2.SURFACEOCEAN, dM3.MARINEBIOTA, dM4.DEEPOCEAN,
      dM5.SEDIMENTOCEAN, dM6.LANDBIOTA, dM7.ROCKS_FOSSIL, dM8.EMITTED )))
  })
}  # end of model equations
```

Scenarios

Steady-state

```
time_seq <- 0:1000   # run the model for 1000 years
par(oma = c(0,0,2,0))
out1 <- ode(y = state.eq, times = time_seq, func = Cmodel, parms = parms)
plot(out1, xlab = "years", ylab = "PgC")
title(main = "STEADY STATE", outer = TRUE)
out1[nrow(out1),]
```

```
##             time    M1.ATMOSPHERE  M2.SURFACEOCEAN   M3.MARINEBIOTA
##             1000              590              900                3
##      M4.DEEPOCEAN M5.SEDIMENTOCEAN     M6.LANDBIOTA  M7.ROCKS_FOSSIL
##            37100              150             2300         75011000
##       M8.EMITTED
##                0
```



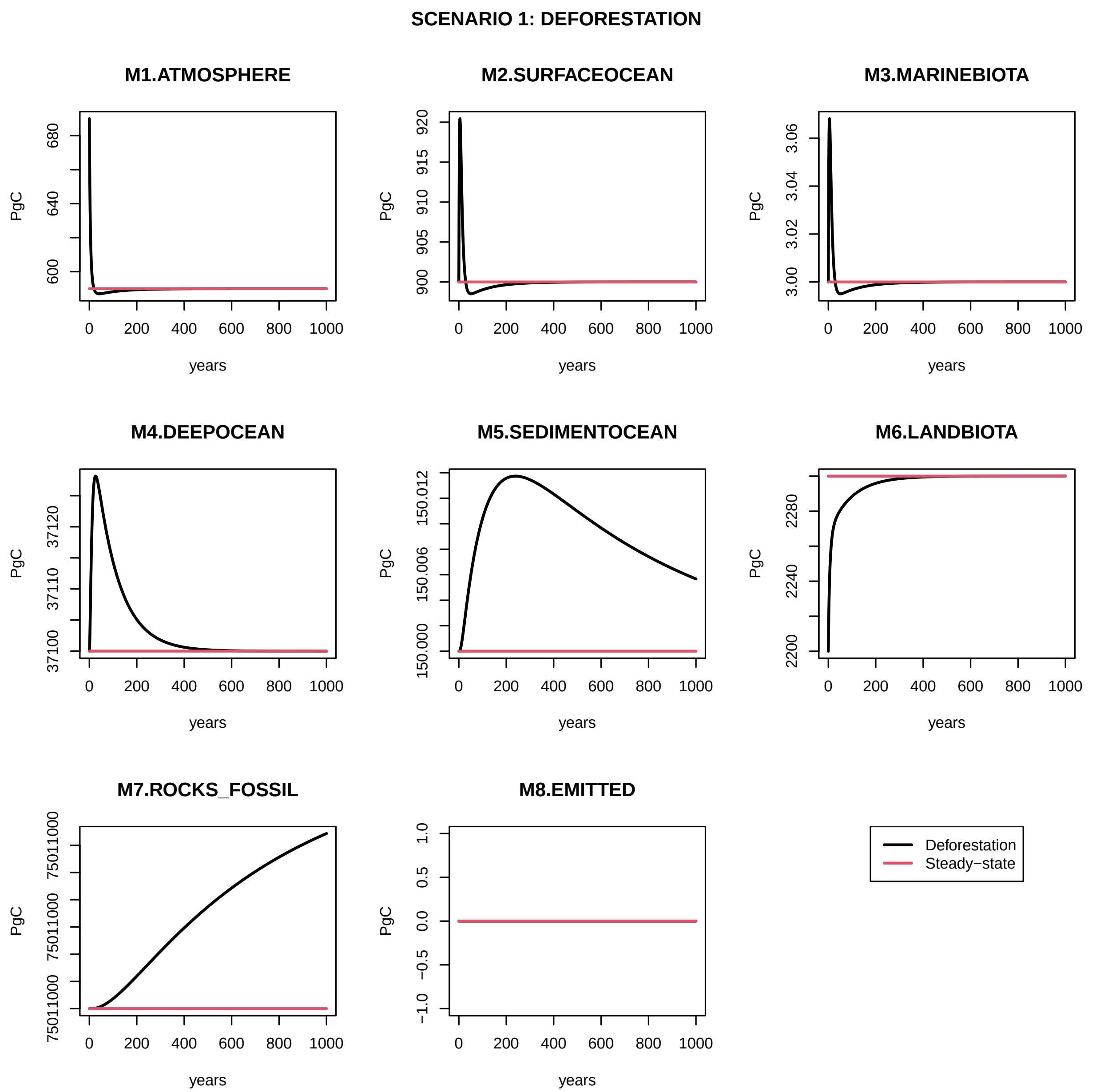
As expected, there is no change in the C content in any of the modeled compartments when the initial conditions corresponding to the steady-state are used.

Scenario 1: Instantaneous deforestation

100 Pg C is transferred from M6.LANDBIOTA to M1.ATMOSPHERE. This is implemented by *changing the initial condition.*

```
Cdeforest <- 100       # Pg C transferred from M6.LANDBIOTA to M1.ATMOSPHERE
time_seq <- 0:1000     # run the model for 1000 years

# new initial state
state2 <- state.eq
state2["M1.ATMOSPHERE"] <- state2["M1.ATMOSPHERE"] + Cdeforest
state2["M6.LANDBIOTA"]  <- state2["M6.LANDBIOTA"]  - Cdeforest

# Calculate, plot and print the solution, adding the original solution
```

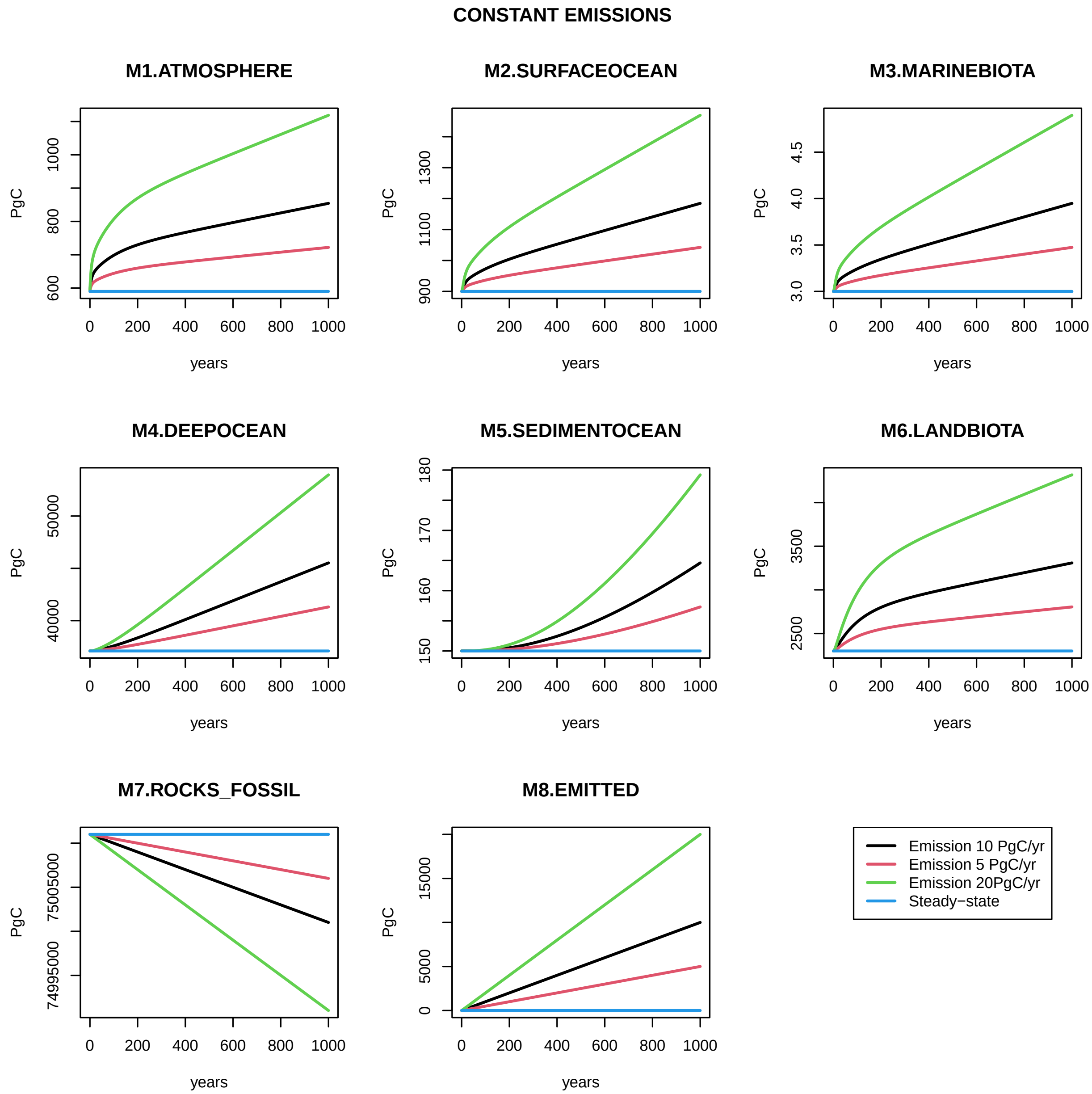
```
out2 <- ode(y = state2, times = time_seq, func = Cmodel, parms = parms)
par(oma = c(0,0,2,0))
plot(out2, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top", legend = c("Deforestation", "Steady-state"),
       col = 1:2, lwd = 2, lty = 1)
title(main= "SCENARIO 1: DEFORESTATION ", outer = TRUE)
```



Scenario 2: Constant emissions

Constant emissions are implemented by changing one model parameter ("bFuel") and leaving the rest of the model unchanged. Different outputs are produced for different emission rates.

```r
emission_rate <- 10 # Pg C yr-1
time_seq <- 0:1000  # run the model for 1000 years

state3 <- state.eq  # initial state corresponds to the stead state

# define new set of model parameters and run the model
parms3 <- parms
parms3["bFuel"] <- emission_rate
out3 <- ode(y = state3, times = time_seq, func = Cmodel, parms = parms3)

# half the emission rate
parms3b <- parms
parms3b["bFuel"] <- emission_rate/2
out3b <- ode(y = state3, times = time_seq, func = Cmodel, parms = parms3b)

# double the emission rate
parms3c <- parms
parms3c["bFuel"] <- emission_rate*2
out3c <- ode(y = state3, times = time_seq, func = Cmodel, parms = parms3c)

# display results
par(oma = c(0,0,2,0))
plot(out3, out3b, out3c, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top",
       legend = c("Emission 10 PgC/yr", "Emission 5 PgC/yr",
                  "Emission 20PgC/yr",  "Steady-state"),
       col = 1:4, lwd = 2, lty = 1)
title(main = "CONSTANT EMISSIONS", outer = TRUE)
```



As expected, the total amount of C emitted into the atmosphere, which is tracked by the state variable M8.EMITTED, increases linearly with time. For example, the emission rate of 10 PgC/yr yields 10,000 PgC of emitted carbon over 1000 years, consistent with what the black line in the lower-right graph shows.

Calculation of the change in the amount of C in each compartment after 10, 100 and 1000 years is implemented with a new function *CalcChange*. The function is then called for the output of the scenario with the emission rate of 10 PgC/yr (out3).

```
# function to calculate reservoir changes
CalcChange <- function(out, time){
  time_seq <- out[,1]    # times of the solution
  Cini <- out[1, 2:9 ]   # initial condition
  Cend <- out[which(time_seq == time), 2:9] # C at time
  return(Tot <- Cend - Cini) # Change relative to initial condition
}
```

```
cat (" after  10 years, of the", CalcChange(out3, 10)["M8.EMITTED"],
     "PgC emmitted, ", CalcChange(out3, 10)["M1.ATMOSPHERE"],
     " PgC is present in the atmosphere\n")
```

```
##  after  10 years, of the 100 PgC emmitted,  45.62966  PgC is present in the atmosphere
```

```
cat (" after 100 years, of the", CalcChange(out3, 100)["M8.EMITTED"],
     "PgC emmitted, ", CalcChange(out3, 100)["M1.ATMOSPHERE"],
     " PgC is present in the atmosphere\n")
```

```
##  after 100 years, of the 1000 PgC emmitted,  108.3922  PgC is present in the atmosphere
```

```
change <- CalcChange(out3, 1000)
cat(" after 1000 years, the % of emitted carbon in each reservoir is:\n")
```

```
##  after 1000 years, the % of emitted carbon in each reservoir is:
```

```
round(change[1:7]/change[8]*100,1)
```

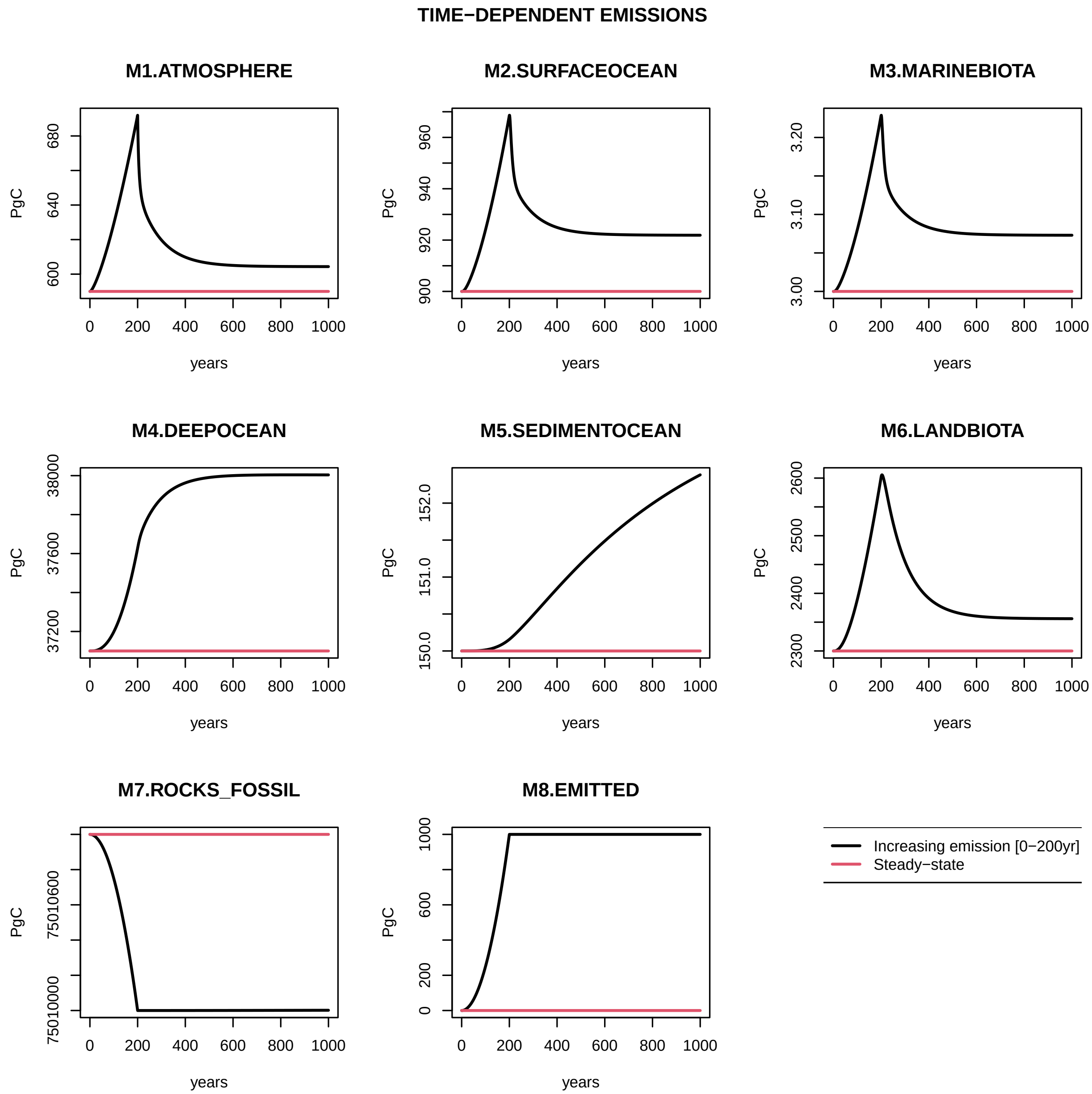
```
##    M1.ATMOSPHERE  M2.SURFACEOCEAN   M3.MARINEBIOTA     M4.DEEPOCEAN
##              2.6              2.8              0.0             84.2
## M5.SEDIMENTOCEAN      M6.LANDBIOTA  M7.ROCKS_FOSSIL
##              0.1             10.1            -99.9
```

We conclude that after 1000 years of constant emissions, about 84% of the emitted C ends up in the deep ocean, 10% in the land biota, and minor fractions in the other compartments.

Scenario 3: Linearly increasing emissions

Emissions are now linearly increasing with time, starting from 0 and they stop after 200 years.

```
emission_rate         <- 0    # Pg C yr-1
emission_acceleration <- 0.05 # Pg C yr-2

parms5 <- parms       # new set of model parameters
parms5["bFuel"] <- emission_rate
parms5["aFuel"] <- emission_acceleration
parms5["tmax_input"] <- 200   # emissions stop after 200 years
state5 <- state.eq

out5 <- ode(y = state5, times = time_seq, func = Cmodel, parms = parms5)

par(oma = c(0,0,2,0))
plot(out5, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top", legend = c("Increasing emission [0-200yr]", "Steady-state"),
      col = 1:2, lwd = 2, lty = 1)
title(main = "TIME-DEPENDENT EMISSIONS", outer = TRUE)
```



Now we use the function *CalcChange* to calculate changes in the C contents (absolute and relative) after 200 and 1000 years.

```
change200 <- CalcChange(out5, 200)
change1000 <- CalcChange(out5, 1000)
change200
```

```
##    M1.ATMOSPHERE  M2.SURFACEOCEAN   M3.MARINEBIOTA      M4.DEEPOCEAN
##      101.9607876       68.6139959        0.2286092       527.3388468
## M5.SEDIMENTOCEAN      M6.LANDBIOTA   M7.ROCKS_FOSSIL       M8.EMITTED
##        0.1566373      301.6913690     -999.9902456       999.9998961
```

```
round(change200[1:7]/change200[8]*100,1)
```

```
##    M1.ATMOSPHERE  M2.SURFACEOCEAN   M3.MARINEBIOTA      M4.DEEPOCEAN
##             10.2              6.9              0.0              52.7
```

```
## M5.SEDIMENTOCEAN      M6.LANDBIOTA  M7.ROCKS_FOSSIL
##               0.0              30.2           -100.0
```

```
round(change1000[1:7]/change1000[8]*100,1)
```

```
##    M1.ATMOSPHERE  M2.SURFACEOCEAN   M3.MARINEBIOTA     M4.DEEPOCEAN
##              1.4              2.2              0.0             90.4
## M5.SEDIMENTOCEAN      M6.LANDBIOTA  M7.ROCKS_FOSSIL
##              0.2              5.6            -99.8
```

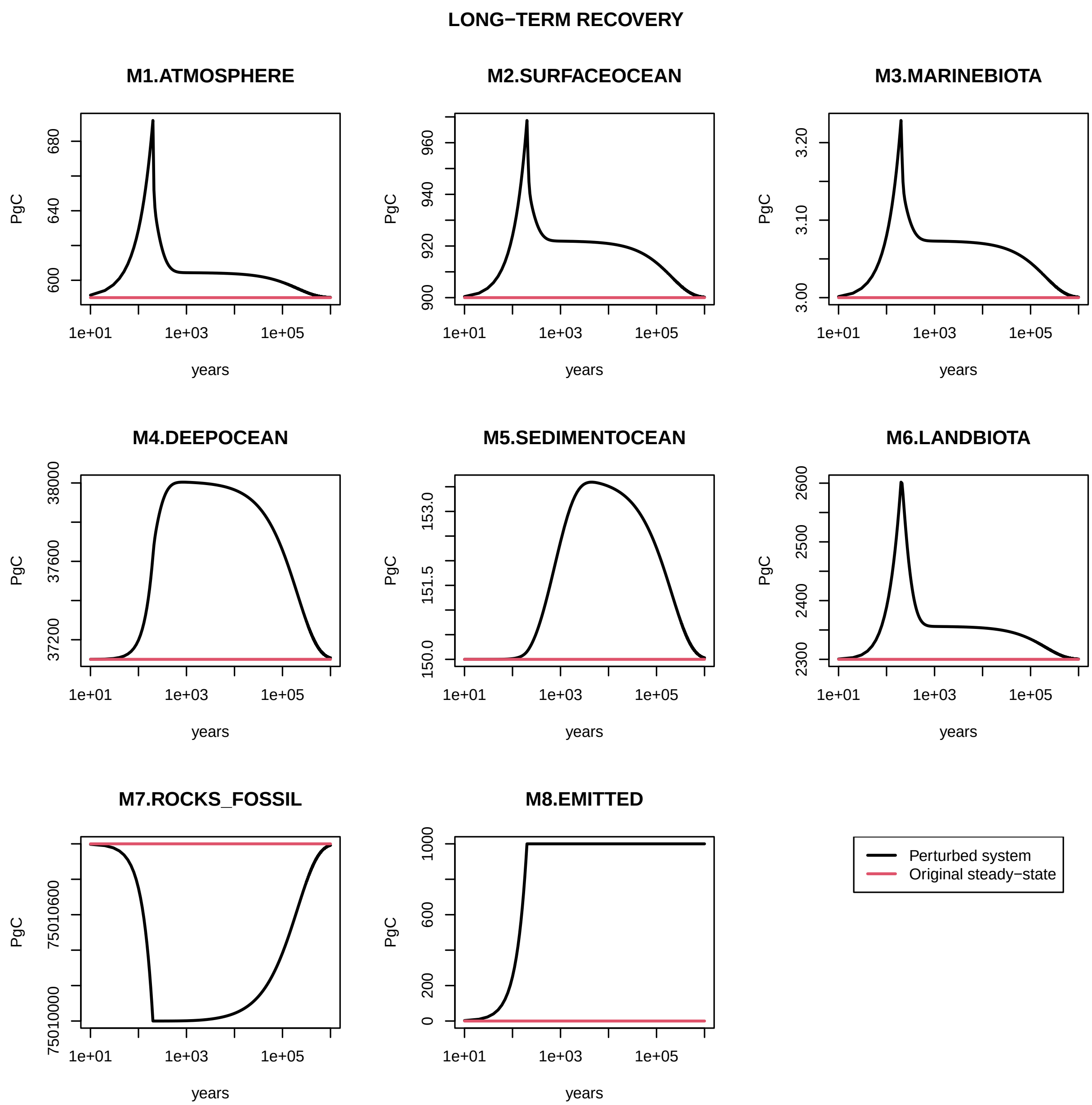
We conclude that if the emissions linearly increase for 200 years (from 0, with an acceleration of 0.05 PgC yr^{-2}) and then abruptly stop, the total emitted C amounts to 1000 PgC (consistent with the expected value of the integrated emission rate: $0.05 \times t^2/2 = 0.05/2 \times 200^2 = 1000$). At the end of the 200 years period, about 10% of the emitted C ends up in the atmosphere, 7% in the surface ocean, 53% in the deep ocean, and 30% in the land biota. If the emissions continue to be zero after the initial 200 year period, after 1000 years most of the emitted carbon will be present in the deep ocean (90%), a small part in the land biota (5.6%), and minor parts in the other compartments.

Scenario 3': Long-term recovery after 200 years of linearly increasing fossil fuel burning

Here we explore the long-term dynamics of the global C cycle following the perturbation caused by the linearly increasing rate of fossil fuel emission over 200 years. Thus, we run the same model as in the previous section but over 1 million years.

Note how the time sequence is defined to allow capture of both the faster dynamics during the initial 1000 years and the slower dynamics afterwards. If we kept the step size of 10 years for the entire duration of 1 million years, the output would be unnecessarily large (100,000 values!), which would lead to unnecessarily large output figures (and long R-markdown "knitting"). The time-axis is displayed in a log-scale to visualize both the fast and slow dynamics.

```
time_seq1M <- c(seq(from=0, to=1000, by=10),             # denser first 1000 years
                seq(from=1000, to=1e6, length.out = 5000)) # less dense afterwards

# linearly increasing emission rate until 200 years, then stop
out5_1M  <- ode(y = state.eq, times = time_seq1M, func = Cmodel, parms = parms5)
# also run the model in steady state
out_1M <- ode(y = state.eq, times = time_seq1M, func = Cmodel, parms = parms)

par(oma = c(0,0,2,0))
plot(out5_1M, out_1M, log="x", lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top", legend = c("Perturbed system", "Original steady-state"),
  col = 1:2, lwd = 2, lty = 1)
title(main = "LONG-TERM RECOVERY", outer = TRUE)
```

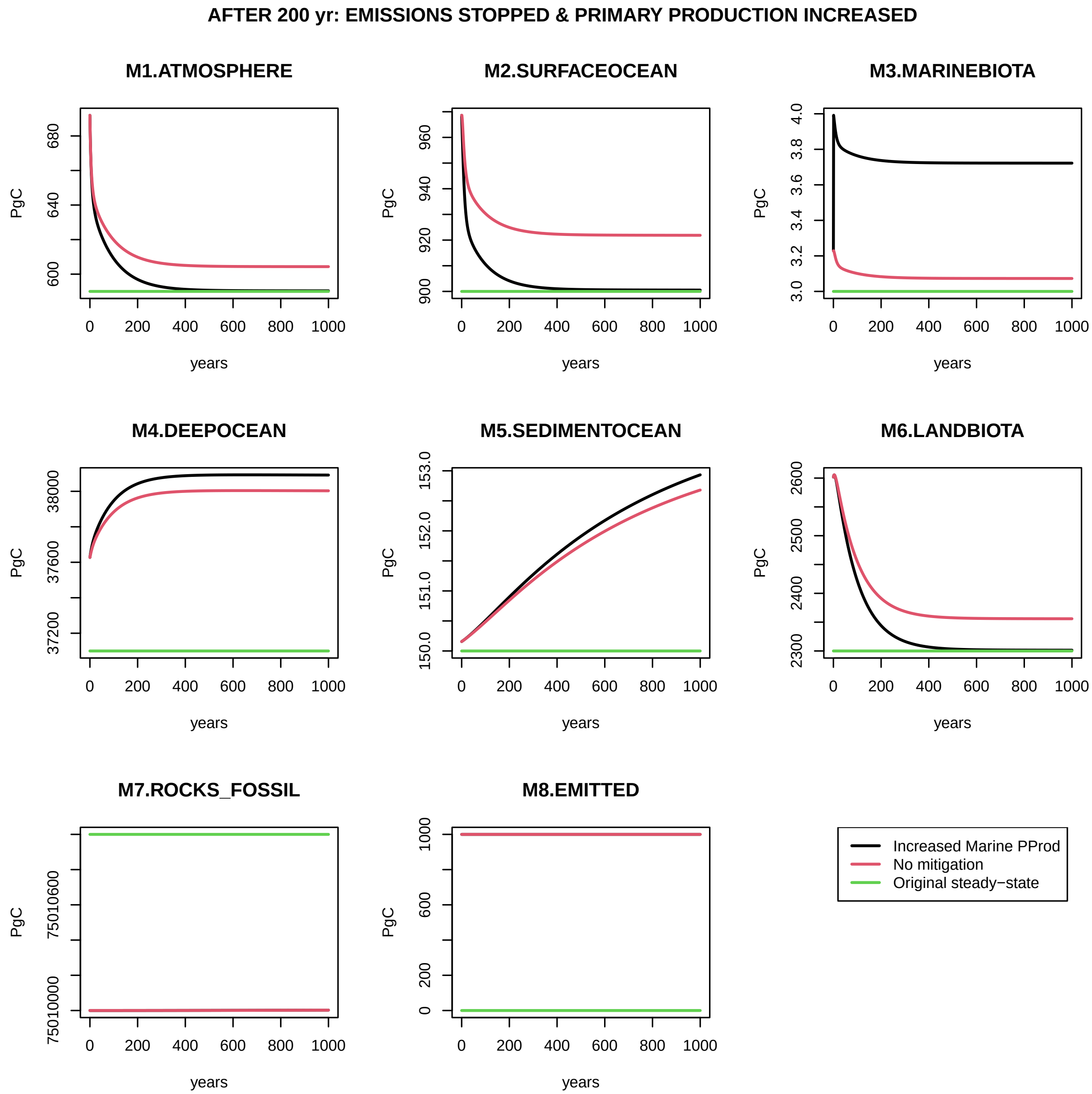


We see that the *response* of the global C cycle to the perturbation by fossil fuel burning over 200 years occurs largely on *two time scales*. As discussed above, during the initial approximately 1000 years, most (90%) of the carbon emitted to the atmosphere is transferred to the deep ocean. Then, it takes around 1 million years until the system reaches the original steady state (assuming, of course, that no rate constants change over time). This is due to the combination of two effects: (i) there is an intense recycling of carbon between the deep ocean, surface ocean and marine biota; (ii) the carbon burial flux is very small (0.2 PgC yr^{-1} in steady state, slightly larger if the size of the deep sea compartment increases) compared to most of the other fluxes.

Scenario 4: Mitigation by increasing oceanic primary production

First, we consider that emissions linearly increased for 200 years and then abruptly stopped. We use the status of the system after the 200 years of such emissions as the *initial condition* for modelling the effects of the mitigation strategy involving stimulation of the oceanic primary production. This mitigation strategy is implemented by *increasing the parameter* $k2.3$, which defines the magnitude of the flux from M2.SURFACEOCEAN to M3.MARINEBIOTA. We use trial end error to vary the stimulation parameter (*ppFraction*) so that the atmospheric C content decreases towards the original level after 1000 years. We compare the results with the scenario where this mitigation strategy is not employed (using original model parameters but a perturbed initial conditions; scenario 3).

```
# start from the status after 200 years of linearly increasing emissions
state7 <- state.eq + CalcChange(out5, 200)

# use the original model parameters (no emissions), but stimulate oceanic
# primary productivity (surface->marinebiota) by a factor ppFraction
parms7 <- parms
ppFraction <- 0.24
parms7["k2.3"]  <- parms["k2.3"]*(1 + ppFraction)

# mitigation implemented (parms7)
out7  <- ode(y = state7, times = time_seq, func = Cmodel, parms = parms7)
# no mitigation implemented (parms)
out7a <- ode(y = state7, times = time_seq, func = Cmodel, parms = parms)

par(oma = c(0,0,2,0))
plot(out7, out7a, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top", legend = c("Increased Marine PProd", "No mitigation",
                         "Original steady-state"), col = 1:3, lwd = 2, lty = 1)
title(main = "AFTER 200 yr: EMISSIONS STOPPED & PRIMARY PRODUCTION INCREASED",
      outer = TRUE)
```



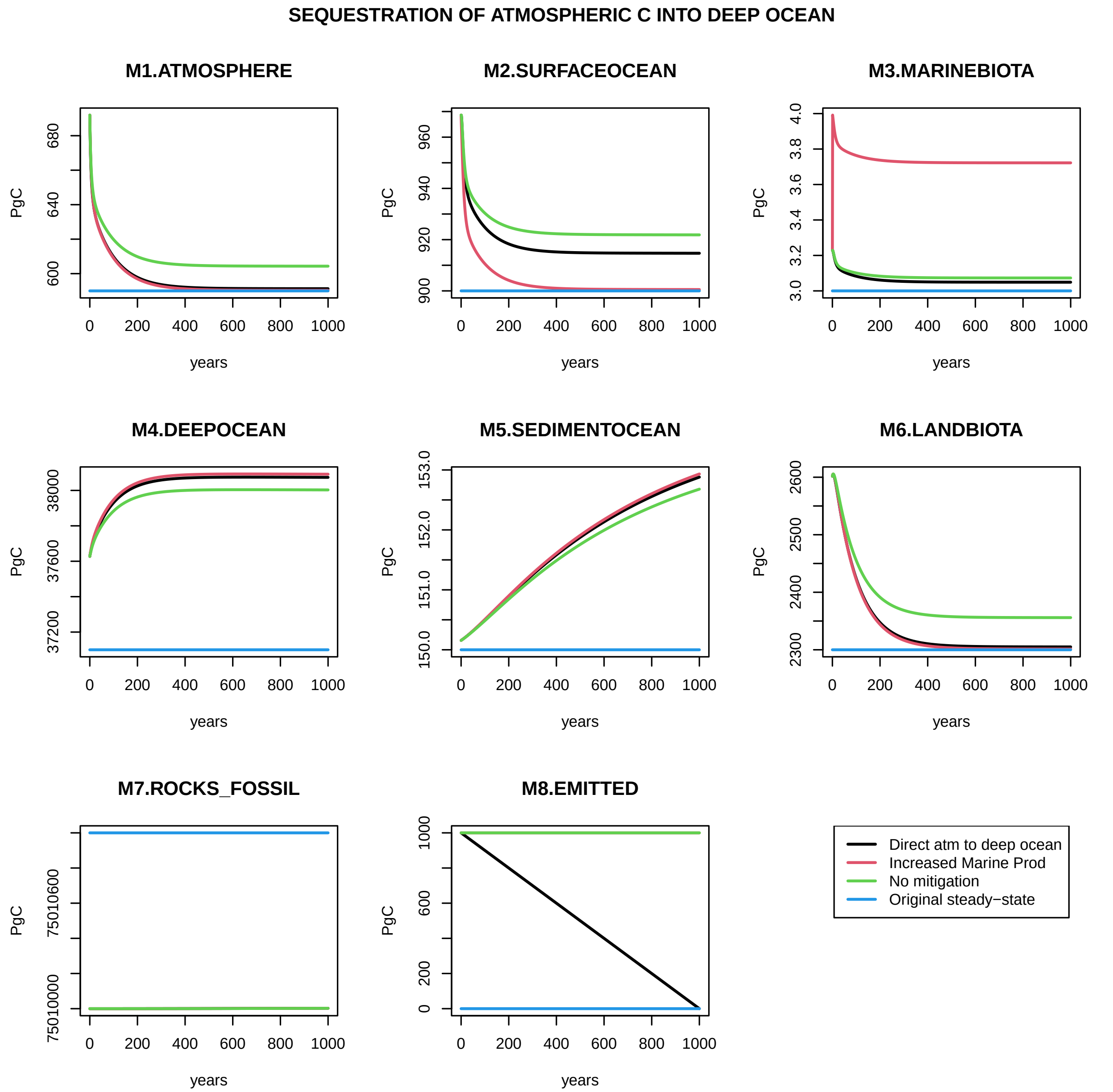
The above graphs show that the stimulation of the oceanic productivity leads to an increased C content of the marine biota, the deep ocean and marine sediments, and a decrease in the C content of the atmosphere, surface ocean and terrestrial biota (black lines). Within about 400 years, the atmospheric C content will reach the original steady state level (i.e., prior to anthropogenic emissions); however, this will require a *continuous* stimulation of the oceanic primary productivity by *24%*! In contrast, if no such mitigation strategy were employed, the decrease in the atmospheric C would proceed in a similar way, albeit to a level that is by about 15 PgC higher than the original steady state level (i.e., 2.5% higher). Notably, the initial rapid decrease in the atmospheric C (by about 50 PgC) following the abrupt stop in fossil fuel emissions is similar with or without the mitigation strategy employed.

Together, this suggests that an immediate stopping of fossil fuel emissions is the most important factor to reduce atmospheric C. The system will "heal itself" to similar levels within about 400 years, regardless whether oceanic primary productivity is stimulated or not (at least based on this model, which is still quite simple). At this point, most of the emitted fossil fuel C will end up in the deep ocean. As discussed above, it will take about 1 million years until a "full recovery" of the system to the original state.

Scenario 5: Sequestration of atmospheric C directly into the deep ocean

Again, we first consider that emissions linearly increased for 200 years and then abruptly stopped. We use the status of the system after the 200 years of such emissions as the *initial condition* for modelling the effects of this second mitigation strategy. This mitigation strategy is implemented by setting the sequestration flux (model parameter *Fatm2deep*) to 1 PgC yr^{-1}. Over 1000 years, this sequestration rate would remove the same amount of C from the atmosphere as the total amount added by fossil fuel burning over the period of 200 years with the linearly increasing rate (scenario 3). We compare the results with the scenario where this mitigation strategy is not employed (scenario 3) as well as with the one employing stimulation of oceanic productivity (scenario 4).

```
# start from the status after 200 years of linearly increasing emissions
state8 <- state.eq + CalcChange(out5, 200)

parms8 <- parms          # original model parameters
parms8["Fatm2deep"] <- 1 # sequestration flux 1 PgC yr-1

# mitigation implemented (parms8)
out8  <- ode(y = state8, times = time_seq, func = Cmodel, parms = parms8)
# no mitigation implemented (parms)
out8a <- ode(y = state8, times = time_seq, func = Cmodel, parms = parms)

par(oma = c(0,0,2,0))
plot(out8, out7, out8a, out1, lty = 1, lwd = 2, xlab = "years", ylab = "PgC")
plot.new()
legend("top", legend = c("Direct atm to deep ocean", "Increased Marine Prod",
                         "No mitigation", "Original steady-state"),
       col = 1:4, lwd = 2, lty = 1)
title(main = "SEQUESTRATION OF ATMOSPHERIC C INTO DEEP OCEAN", outer = TRUE)
```



We see that the effects of this second mitigation approach are very much the same as of the first mitigation approach (compare black and red lines in the above graphs), except for the compartments of the surface ocean and marine biota, which follow a similar path as if no mitigation strategy were employed. This is, clearly, due to the fact that in this modelling approach we have by-passed the "D-tour" via the surface ocean and marine biota on the way from the atmosphere to the deep ocean.

References

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Karline Soetaert, Thomas Petzoldt, R. Woodrow Setzer (2010). Solving Differential Equations in R: Package deSolve. Journal of Statistical Software, 33(9), 1–25. URL http://www.jstatsoft.org/v33/i09/ DOI

10.18637/jss.v033.i09