Farmer's dilemma—optimisation of crop cultures in the presence of weeds and in response to eutrophication

Exercises Accompanying the Course Reaction Transport Modelling in the Hydrosphere

Karline Soetaert and Lubos Polerecky, Utrecht University

March 2021

Introduction

In the chemical, biogeochemical and ecological models that you have solved thus far, state variables were defined in terms of *concentrations* (volumetric or areal) or *biomass*, and the models were formulated through differential equations that describe the rate of change in these state variables as a function of the sinks and sources. In economics, problems can be formulated in a similar way—as differential equations—but here one is interested in the changes in costs and profits. In this exercise, you will implement the model of crops and weeds that you have seen earlier, and expand it to additionally include an *economic* perspective.

Note that this exercise should be considered as a demonstration of how to give models an economic "touch". It is *not* meant to be very realistic, or turn you into a millionaire.[1] Our choices of the parameters for the P dynamics are very roughly estimated, and the costs assumed in the model may well be totally wrong.

Problem formulation

A farmer wants to optimise his harvest of lettuce. He needs to decide whether to fertilize his land with phosphate once, before planting, or at regular occasions after the lettuce seedlings have been planted, and how much of the fertilizer to add.

The problem he has is that the field of lettuce is invaded by a weed that (i) grows slightly slower than the lettuce, and (ii) has a much deeper root system and a higher affinity for phosphorus than the lettuce. While the roots of lettuce penetrate 7 cm deep into the soil, the weed's roots penetrate down to a depth of 14 cm.

Once the fertilizer is added to the soil, it percolates, due to the rain, from the upper 7 cm to the deeper soil layer, at a rate of 0.5% per day.

The farmer also wants to optimize the time of harvesting, so as to have a maximal profit. You will make a bio-economic model to help reach this decision.

The biological part

In one of the previous exercises, you have already devised a conceptual model that could be used to mimic this agriculture (Figure 1).

To account for the different root lengths of the crop and weed, the soil is subdivided into an upper layer (0–7 cm), where the roots of both plants take up nutrients, and a deeper layer (7–14 cm), where only the weed has

[1] Although if it does, we hope that you will remember how you got there! ;-)

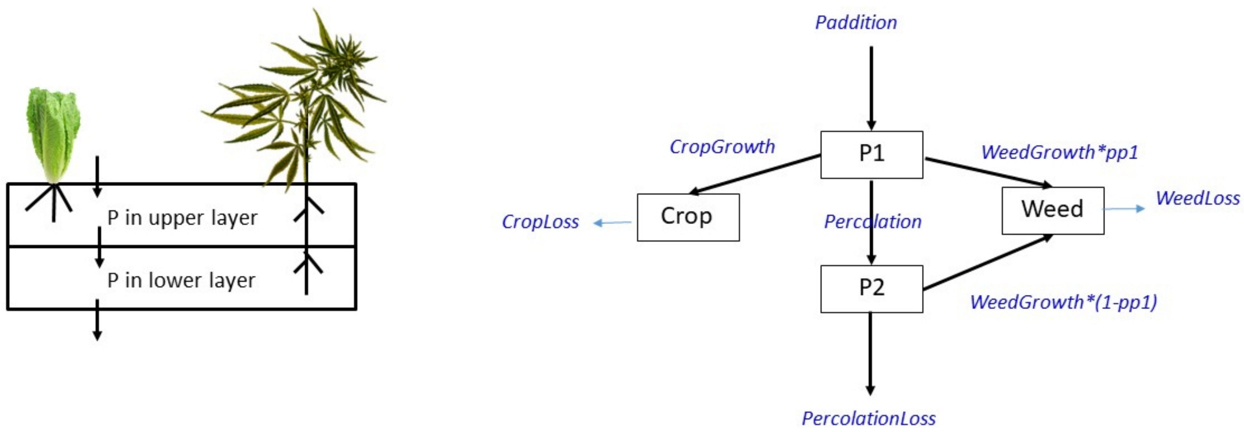


Figure 1: Diagram of the crop-and-weed model.

access to nutrients. The rain induces a flow of nutrients from the upper layer to the deeper layer, and from the deeper layer to the soil below 14 cm.

Given that phosphorus is the limiting nutrient, the state variables are expressed in areal P concentrations for each layer ($mol\ P\ m^{-2}$).

The continuous P addition is implemented as a parameter (*Paddition*) while the fertilization at the start is imposed as the initial condition, i.e., by having a larger concentration of P in the upper soil layer.

Tasks

First, create suitable rate expressions for the flows of P in the system.

- Assume that, in addition to a competition for phosphorus, there is also a competition for space, i.e., the weed plants may limit the growth of the crop (and vice versa) because they block the sunlight. You may model this using the concept of *ecosystem carrying capacity*, assuming that the maximum *total* plant concentration (summed biomass of the weeds and crops, expressed in $mol\ P\ m^{-2}$) that can be sustained (ktot) is 0.3 $mol\ P\ m^{-2}$.
- The following table provides the parameters that you need to use. (tip: based on the units of the parameters, you should be able to deduce the corresponding rate expressions).

Name	Value	Description	Unit
Paddition	0.9/90	Rate of P supply to P1	$mol\ P\ m^{-2}d^{-1}$
percolation	0.05	Dilution by rain	d^{-1}
ksCrop	2e-3	Monod ct for P uptake by crop	$mol\ P\ m^{-2}$
ksWeed	0.5e-3	Monod ct for P uptake by weed	$mol\ P\ m^{-2}$
ktot	0.3	Carrying capacity	$mol\ P\ m^{-2}$
rGcrop	0.125	Max. growth rate, crop	d^{-1}
rGweed	0.1	Max. growth rate, weed	d^{-1}
rMcrop	0.0	Loss rate (mortality)	d^{-1}
rMweed	0.0	Loss rate (mortality)	d^{-1}
N	25	Density of crop plants	$ind\ m^{-2}$
P2WW	62000	Convert P to wet weight	$g\ ww\ (mol\ P)^{-1}$

Implement the model with continuous fertilization in R, using the above parameter values. You can start with the R-markdown template model file *RTM_0D.Rmd* to implement this model.[2]

- Assume the following initial concentrations of P for the crop, weed and the two soil layers (all in units of $mol\ P\ m^{-2}$): $WEED = 0.001$, $CROP = 0.005$, $P1 = 0.1$, and $P2 = 0.1$.
- There are 25 crop plants per m^2 (parameter N). This number does not change over time.
- Calculate, at each time point, the mean *wet weight* of the lettuce plants and export it from the model as an ordinary output variable. To do this, use the parameter $P2WW$, which is estimated based on the following assumptions: (i) the content of P in plant dry weight is 0.2% of dry weight; (ii) dry weight of lettuce plants is 25% of wet weight.
- Also output the summed biomass of crops and weeds, the total P in the system, the amount of P added to the system through fertilization, and the amount of P lost from the system due to percolation (all in $mol\ P\ m^{-2}$).
- Run the model for 90 days.

An alternative culturing method is to fertilise the upper layer of the soil *before* planting the crops, so that P does not need to be administered continuously. Here, assume that the amount of P *initially added* to the top soil is the *same* as the accumulated amount of P added to the system during the *continuous fertilization* over 90 days in the previous scenario.

- Implement the model with one-time fertilization in R, using the above parameter values (except for the fertilization rate, which you should set to 0).
- Run the model for 90 days.

Compare results from both scenarios and discuss the following aspects:

[2]You can obtain this file from Rstudio: File → new File → Rmarkdown → from template → RTM_0D. Save this file under a different name. Do not forget to change the heading of this file.

- Which strategy gives a higher crop yield?
- Is the growth of the crop limited? If so, what is the limiting factor?
- How much of the added fertilizer is utilized for growth of the crop, and how much is wasted? Where is the wasted fertilizer going to?

The economic part

Now we take into account the cost of this culture and the profit that the farmer can make (if any).

Assumtions

Use the following assumptions for costs:

- The cost to maintain 1 m^2 of the field is 1 euro per week. When the field also needs to be regularly fertilised, this increases to 1.025 euro per week per m^2. This only includes the cost of labor, and you can assume that this money is set aside continuously.
- The cost of the fertiliser is 3 euro per kg of P.
- The cost for the one-time fertilisation of the soil (before planting) is 5 euro per m^2.
- The cost for planting the lettuce is 0.5 euro per m^2.

The eventual profits are determined as follows:

- The lettuce plants can only be sold if they weigh more than 300 grams (wet weight).
- The plants are sold at a price of 1 euro per kg.

Tasks

- Implement the cost as an extra state variable. Think how this increases in time. Tip: you will need to put some costs in the differential equations, other costs should be added to the initial conditions.
- Estimate the income the farmer could have from selling the lettuce (this will be 0 as long as the plants weigh less than 300 g). Subtract the cost from this potential income to estimate the profit the farmer would make if he would sell the crops at that time.

Assuming that your model parameters are realistic, discuss the following aspects:

- Is it more beneficial to fertilise once or multiple times?
- When should the crops best be sold, and how much profit will be made?
- Regarding the waste of fertilizer, what would be the environmenal impact of the agriculture approach employed by the farmer? What would you recommend if the farmer *wanted* to make his agriculture more environmentally friendly? (Base this recommendation on the fertilizer waste.) How would this affect the profits the farmer can make? (This partially explains the title of this exercise.)
- How would you *stimulate* the farmer — by modifying economic parameters (so cost) — to become more environmentally friendly?

Crops and weeds model — Answers

R-implementation — biological part

In addition to the amounts of P in the soil compartments, crops and weeds, we also introduce a state variable *Plost* to be able to trace the amount of P lost to the bottom soil due to percolation. Furthermore, we export the factors *(1-TotBiom/ktot)*, *NutCrop/(NutCrop+ksCrop)* and *NutWeed/(NutWeed+ksWeed)* to be able to trace what limits the plant growth. These two pieces of information are relevant when assessing the environmental impact of the agriculture and interpreting the P dynamics predicted by the model.

```
require(deSolve)  # package with solution methods

# state variables, units = molP/m2
state <- c(WEED = 0.001, CROP = 0.005, P1 = 0.1, P2 = 0.1, Plost = 0)

# parameters
parms <- c(
  Paddition   = 0.9/90,  # [molP/m2/d]  Rate of P supply
  percolation = 0.05,    # [/d]         Rain rate dilution parameter
  ksCrop      = 2e-3,    # [molP/m2]    Monod coefficient for P uptake
  ksWeed      = 0.5e-3,  # [molP/m2]    Monod coefficient
  ktot        = 0.300,   # [molP/m2]    Carrying capacity - space limitation
  rGcrop      = 0.125,   # [/d]         Growth rate,
  rGweed      = 0.1,
  rMcrop      = 0.0,     # [/d]         Loss rate (e.g. mortality),
  rMweed      = 0.0,
  N           = 25,      # [ind/m2]     Density of crop plants
  P2WW        = 62000    # [gWW/molP]   31/0.25/0.002
)

# Model function
Crops <- function(t, state, params) {
  with (as.list(c(state, params)), {

    # variables needed for rate expressions
    TotBiom    <- WEED + CROP       # total plant biomass
    NutWeed    <- P1 + P2           # nutrients accessible to weed
    NutCrop    <- P1                # nutrients accessible to crops
    partP1     <- P1/NutWeed        # part of P for weed from first layer

    # Rate expressions - all rates in units of [molP/m2/day]
    WeedGrowth <- rGweed * WEED*(1-TotBiom/ktot) *NutWeed/(NutWeed+ksWeed)
    CropGrowth <- rGcrop * CROP*(1-TotBiom/ktot) *NutCrop/(NutCrop+ksCrop)

    WeedLoss   <- rMweed * WEED     # death and other loss terms
    CropLoss   <- rMcrop * CROP

    Percolate  <- P1 * percolation  # transfer of P from layer 1 to layer 2
    Ploss      <- P2 * percolation  # transfer of P from layer 2 to deep soil

    # Mass balances [molP/m2/day]
    dWEED      <- WeedGrowth - WeedLoss
    dCROP      <- CropGrowth - CropLoss
    dP1        <- Paddition - Percolate - CropGrowth - WeedGrowth * partP1
```

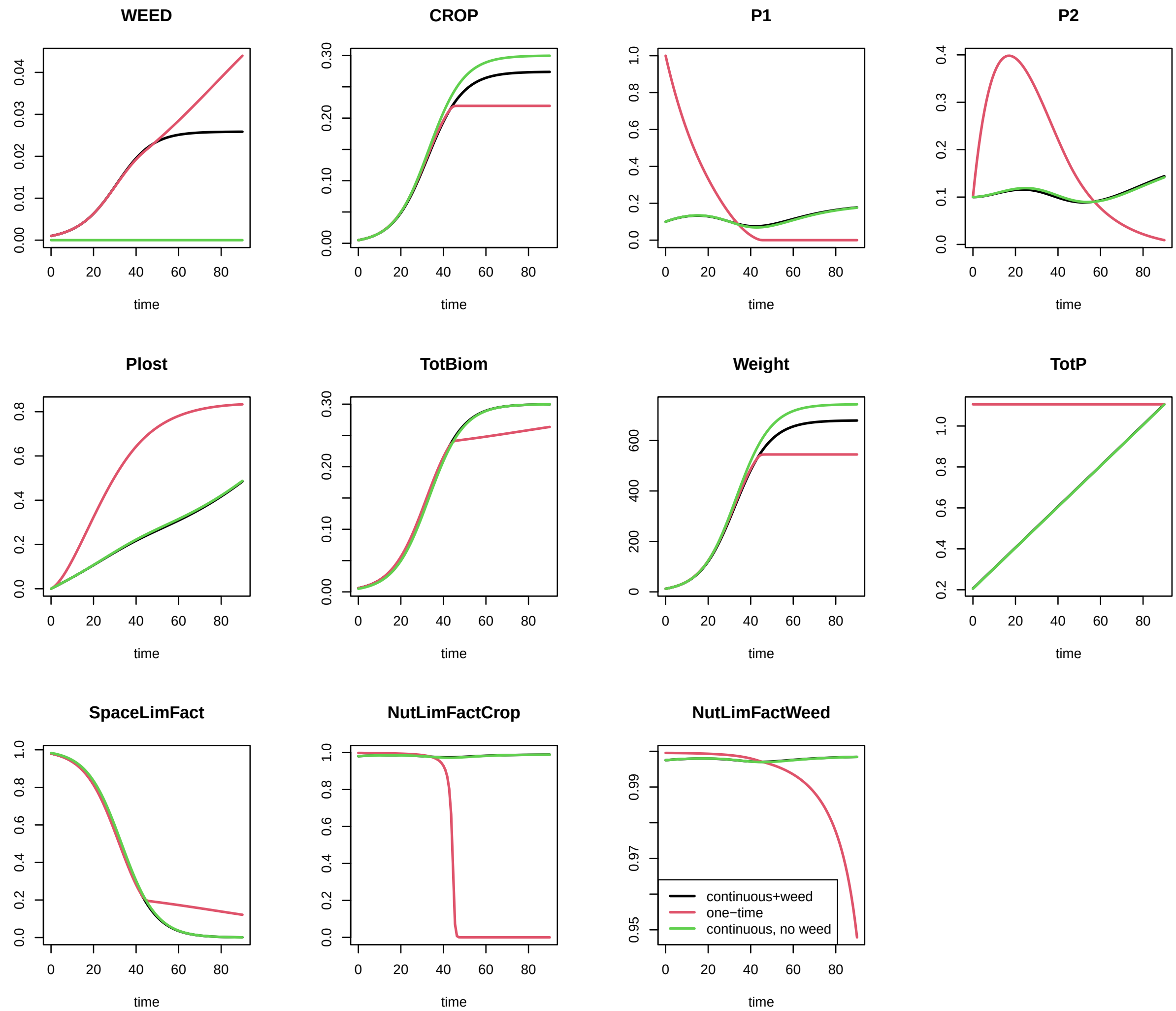
```
    dP2        <- Percolate - Ploss - WeedGrowth * (1 - partP1)
    dPlost     <- Ploss + WeedLoss + CropLoss

    # Individual weight of a crop plant
    Weight     <- CROP/N*P2WW         # wet weight, gram per plant

    list(c(dWEED, dCROP, dP1, dP2, dPlost),
      TotBiom = TotBiom, Weight = Weight,
      TotP = WEED + CROP + P1 + P2 + Plost,       # output the total P, mass balance check
      SpaceLimFact = (1-TotBiom/ktot),            # output the "space-limitation" factor
      NutLimFactCrop = NutCrop/(NutCrop+ksCrop),  # output the nutrient limitation factor
      NutLimFactWeed = NutWeed/(NutWeed+ksWeed))  # for both crop and weed
  })

}
```

The model is solved thrice over 90 days.

- Run 1 implements continuous P addition (with weeds).
- Run 2 implements one-time P fertilization (with weeds).
- Run 3 implements continuous P addition (without weeds).

To compare the models in a fair way, we assume that the one-time and continuos addition of P are done in such a way that the *total* amount of P *added* to the soil is the *same* after 90 days.

```
# output time
outtimes <- seq(from = 0, to = 90, length.out = 100)  # run for 3 months

# Continuous P-addition, with weeds
out <- ode(y = state, parms = parms, func = Crops, times = outtimes)

# One-time P ferilisation, with weeds
parms2 <- parms
parms2["Paddition"] <- 0.0       # No P-addition
state2 <- state
state2["P1"] <- 1                # Higher value for P1 at start
out2 <- ode(y = state2, parms = parms2, func = Crops, times = outtimes)

# Continuous P-addition, without weeds
state3 <- state
state3["WEED"] <- 0
out3 <- ode(y = state3, parms = parms, func = Crops, times = outtimes)

# visualise it
plot(out, out2, out3, lty = 1, lwd = 2, mfrow=c(3,4))

legend("bottomleft", c("continuous+weed",  "one-time", "continuous, no weed"),
       col = 1:3, lty = 1, lwd = 2)
```



We see that during the initial 42 days, crops and weeds grow practically at the same rate. This is because P in the soil is *not* limiting their growth. Rather, their growth is limited by space, as the space limitation factor $(1 - TotBiom/ktot)$ gradually decreases from 1 to about 0.2.

The situation changes rather abruptly after about 42 days. In the 2nd scenario, the crop becomes limited by P due to removal by percolation, whereas the continuous fertilization ensures that this does not happen in the 1st scenario. As a result, the crop continues to grow, reaching about 25% higher stock per m^2 after 90 days in the first compared to the second scenario. This is, however, *not* the case for the *weed*, which continues to grow rapidly even after 42 days in the 2nd scenario. This is, clearly, due to the fact that its roots reach deeper into the soil and thus have more access to P. Although the continuous growth of weeds has a negative impact on the growth of crops due to space limitation, it is only marginal. Without weeds, the crop weight after 90 days would be 743 $g/plant$ without weeds in comparison to 680 $g/plant$ with weeds (i.e., about 10% higher yield), as indicated by the results of the scenario 3 (green line).

Another notable result of this simulation is that *most* of the fertilizer is *lost* in this agriculture. Only about 0.27 out of 1.1 $mol\ P\ m^{-2}$ (about 25% yield on fertilizer) is converted into profitable biomass (crop) in the continuous fertilization scenario. This fraction decreases to about 0.22 out of 1.1 $mol\ P\ m^{-2}$ (20% yield) in the one-time fertilization scenario. These results illustrate the potential evironmental impact of this agriculture.

Crop and weed model including economics — Answers

To estimate the net financial gain for the farmer, we need to take into account all costs and profits associated with the cultivation and sale of the plants.

We do this by adding an extra state variable: the integrated cost. Its dynamics are quite simple:

The *Cost* increases with the daily expenditures of culturing (labor), which is either 1 euro per m^2 and per week (1 $EUR\ m^{-2}\ wk^{-1}$) if there is no continuous fertilization, or 1.025 $EUR\ m^{-2}\ wk^{-1}$ if continuous fertilization *is* performed. Additionally, the costs are augmented with the cost of the fertiliser.

We write for the balance equations:

$$\frac{dCost}{dt} = laborCost + costP \times Padded$$

Here Cost is in $EUR\ m^{-2}$, and the rates are in $EUR\ m^{-2}\ d^{-1}$.

The initial cost of planting is $Cost_{ini} = 0.5\ EUR\ m^{-2}$.

In case the fertilisation is done before planting, we need to also include the labor cost and depreciation of material used (5 $EUR\ m^{-2}$) in addition to the cost of the fertiliser.

R-implementation — economic part

```r
require(deSolve)  # package with solution methods

# state variables, units = [molP/m2] - except for COST: [Euro/m2]
state <- c(WEED = 0.001, CROP = 0.005, P1 = 0.1, P2 = 0.1, COST = 0.5, Plost = 0)

# parameters
parms <- c(
  Paddition   = 0.9/90,     # [molP/m2/d]  Rate of P supply
  percolation = 0.05,       # [/d]         Rain rate
  ksCrop      = 2e-3,       # [molP/m2]    Monod coefficient for P uptake
  ksWeed      = 0.5e-3,     # [molP/m2]    Monod coefficient
  ktot        = 0.300,      # [molP/m2]    Carrying capacity - space limitation
  rGcrop      = 1*0.125,    # [/d]         Growth rate,
  rGweed      = 1*0.1,
  rMcrop      = 0.0,        # [/d]         Loss rate (e.g. mortality),
  rMweed      = 0.0,
  N           = 25,         # [ind/m2]     Density of crop plants
  P2WW        = 62000,      # [gWW/molP]   31/0.25/0.002
  laborCost   = 1.025/7,    # [euro/m2/d]  daily cost of maintenance
  PCost       = 3/1000*31,  # [euro/molP]  cost of the fertiliser, 3 eur/kg
  priceCROP   = 1/1000      # [euro per g] price for plants > 300g, 1 eur/kg of wet weight
)

# Model function
Crops <- function(t, state, params) {
  with (as.list(c(state, params)), {

    # variables needed for rate expressions
    TotBiom    <- WEED + CROP       # total plant biomass
    NutWeed    <- P1 + P2           # nutrients accessible to weed
```

```
    NutCrop    <- P1                 # nutrients accessible to crops
    partP1     <- P1/NutWeed         # part of P for weed from first layer

    # Rate expressions - all rates in units of [molP/m2/day]
    WeedGrowth <- rGweed * WEED*(1-TotBiom/ktot) *NutWeed/(NutWeed+ksWeed)
    CropGrowth <- rGcrop * CROP*(1-TotBiom/ktot) *NutCrop/(NutCrop+ksCrop)

    WeedLoss   <- rMweed * WEED      # death and other loss terms
    CropLoss   <- rMcrop * CROP

    Percolate  <- P1 * percolation  # transfer of P from layer 1 to layer 2
    Ploss      <- P2 * percolation  # transfer of P from layer 2 to deep soil

    # Mass balances [molP/m2/day]
    dWEED      <- WeedGrowth - WeedLoss
    dCROP      <- CropGrowth - CropLoss
    dP1        <- Paddition - Percolate - CropGrowth - WeedGrowth * partP1
    dP2        <- Percolate - Ploss - WeedGrowth * (1 - partP1)
      dPlost     <- Ploss + WeedLoss + CropLoss

    # Individual weight of a crop plant
    Weight     <- CROP/N*P2WW          # gram wet weight per plant

    # Economic part
    dailyCost  <- laborCost + PCost*Paddition


    Price      <- CROP*P2WW * (Weight > 300)*priceCROP  # total price per m2 of plants
    Profit     <- Price - COST

    dCOST      <- dailyCost

    list(c(dWEED, dCROP, dP1, dP2, dCOST, dPlost),
      TotBiom = TotBiom, Weight = Weight,
      TotP = WEED + CROP + P1 + P2 + Plost,
      SpaceLimFact = (1-TotBiom/ktot),
      NutLimFactCrop = NutCrop/(NutCrop+ksCrop),
      NutLimFactWeed = NutWeed/(NutWeed+ksWeed),
        Price = Price, Profit = Profit)
  })

}
```

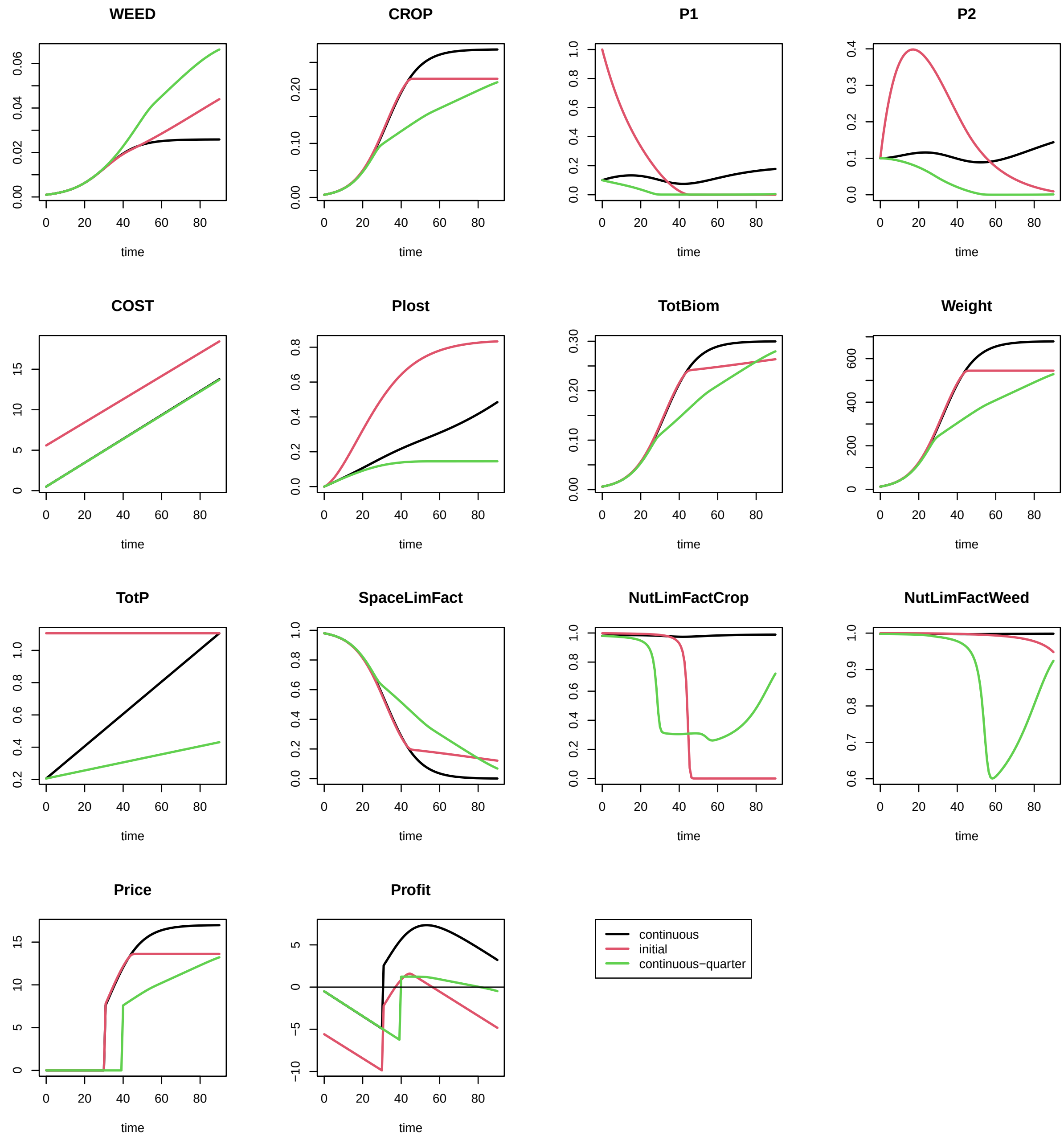
The model is solved thrice over 90 days.

- Run 1 implements continuous P addition.
- Run 2 implements one-time P fertilization.
- Run 3 implements continuous P addition at a *quarter* of the fertilization rate compared to run 1.

The 3rd run is used to assess the environmental and economic impact of reduced fertilization rate.

```
# output time
outtimes <- seq(from = 0, to = 90, length.out = 100)  # run for 3 months

# Continuous P-addition
```

```r
state["COST"] <- 0.5             # price of planting & fertilizer cost
out <- ode(y = state, parms = parms, func = Crops, times = outtimes)

# Initial P ferilisation
# Change the parameter values and initial conditions
parms2 <- parms
parms2["Paddition"] <- 0.0       # no continuous fertilization
parms2["laborCost"] <- 1./7      # slightly cheaper if no continuous fertilization

state2 <- state
state2["P1"] <- 1                # higher value for P1
state2["COST"] <- 0.5 + 5 + 0.9 * parms["PCost"] # price of planting, initial
                                                 # fertilisation (labor) & fertilizer cost
                                                 # (0.9 mol P m-2 is added)
out2 <- ode(y = state2, parms = parms2, func = Crops, times = outtimes)

# Continuous P-addition at a quarter of the original rate
state3 <- state
parms3 <- parms
parms3["Paddition"] <- parms3["Paddition"]*0.25
out3 <- ode(y = state, parms = parms3, func = Crops, times = outtimes)

# visualise results
plot(out, out2, out3, lty = 1, lwd = 2, mfrow=c(4,4))
abline(h = 0)   # above the line: true profit; below the line: loss.
plot.new()
legend("topleft", c("continuous",  "initial", "continuous-quarter"),
       col = 1:3, lty = 1, lwd = 2)
```



Based on the discussion of the previous results, one can discuss independently the economic and environmental implications of these model results — the graphs speak for themselves.

References

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Karline Soetaert, Thomas Petzoldt, R. Woodrow Setzer (2010). Solving Differential Equations in R: Package deSolve. Journal of Statistical Software, 33(9), 1–25. URL http://www.jstatsoft.org/v33/i09/ DOI 10.18637/jss.v033.i09