



Reactive Transport in the Hydrosphere

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Additional contributions: Dries Bonte, University Ghent

Audio effects: mixkit.co



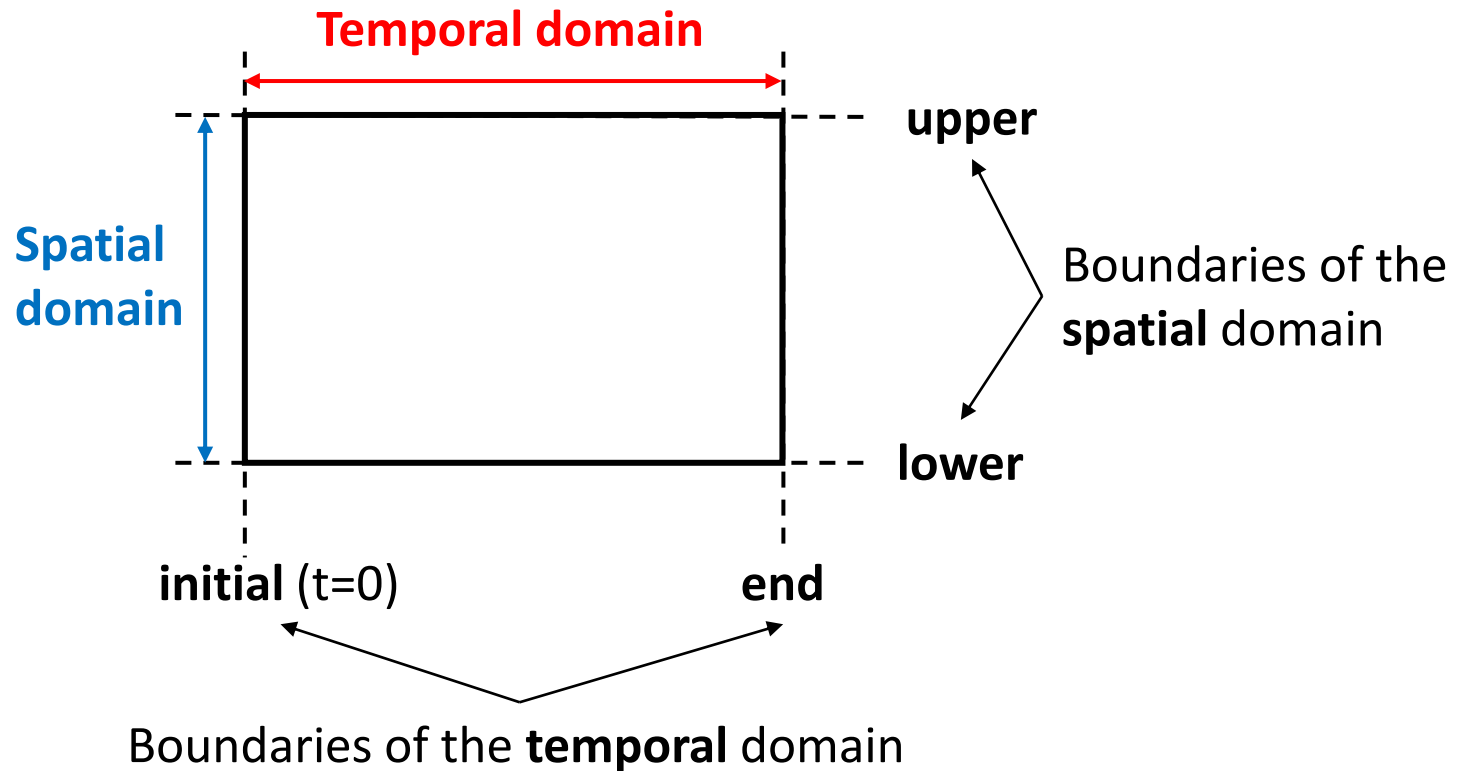
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Solving reaction-transport equation

$$\frac{\partial C_f}{\partial t} = \frac{1}{f} \cdot \frac{\partial}{\partial x} \left(f \cdot D_f \cdot \frac{\partial C_f}{\partial x} \right) - \frac{1}{f} \cdot \frac{\partial}{\partial x} (f \cdot v_f \cdot C_f) + R_f$$

f = volume fraction



We need to specify **conditions** at **some** of these **boundaries**!



Boundary conditions

Steady state:

$$0 = \frac{\partial C_f}{\partial t} = \frac{1}{f} \cdot \frac{\partial}{\partial x} \left(f \cdot D_f \cdot \frac{\partial C_f}{\partial x} \right) - \frac{1}{f} \cdot \frac{\partial}{\partial x} (f \cdot v_f \cdot C_f) + R_f$$

Only conditions at **spatial boundaries** are required

For every
component!

If diffusion is negligible:

Transport in **1** direction!

upper



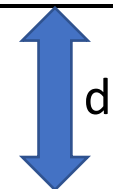
v_f = advective transport

→ **1 boundary condition required**

If diffusion is important:

Transport in **2** directions!

upper



diffusive transport

lower

→ **2 boundary conditions required**



Boundary conditions

Typically:

Upper boundary (e.g., sediment-water interface, start of a river stretch)

- Imposed **flux** for **solid** substances (e.g., particulate organic carbon, mineral particles)
- Imposed **concentration** in the liquid phase for **solutes** (e.g., O_2 , nutrients)

Lower boundary (if needed, i.e., if diffusive transport cannot be neglected)

- **Zero gradient** for both solids/solutes

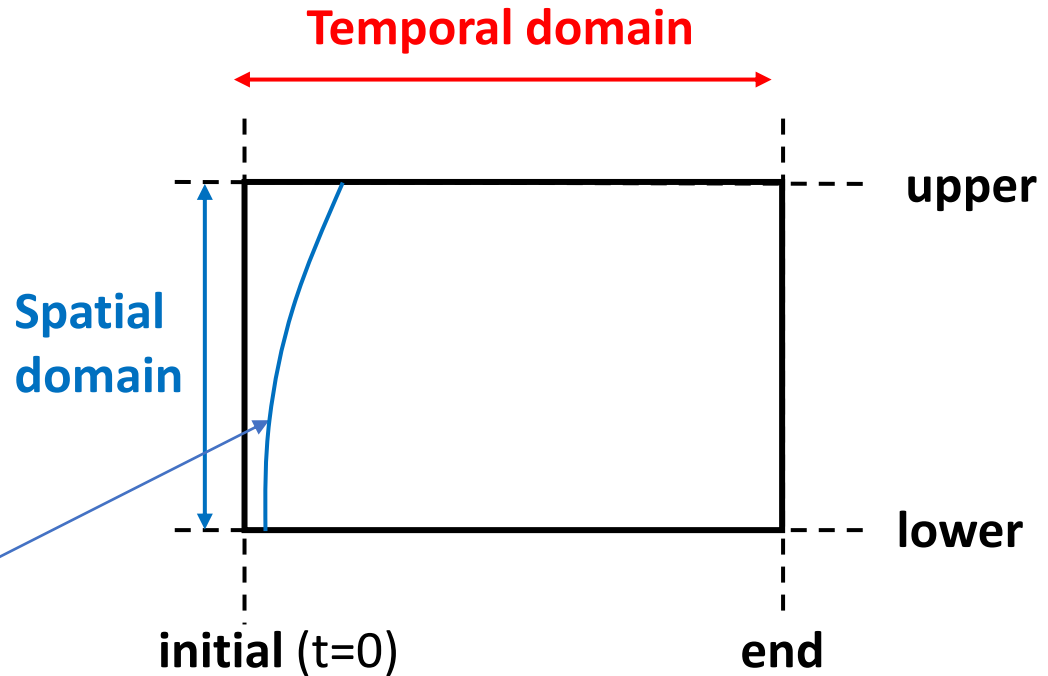
(Based on the idea that resources that drive reactions are depleted at the lower boundary -> *“nothing happens any more”* -> no spatial variation)

In general, boundary conditions must reflect **realistic** assumptions about the modeled system!



Boundary conditions

Transient state:



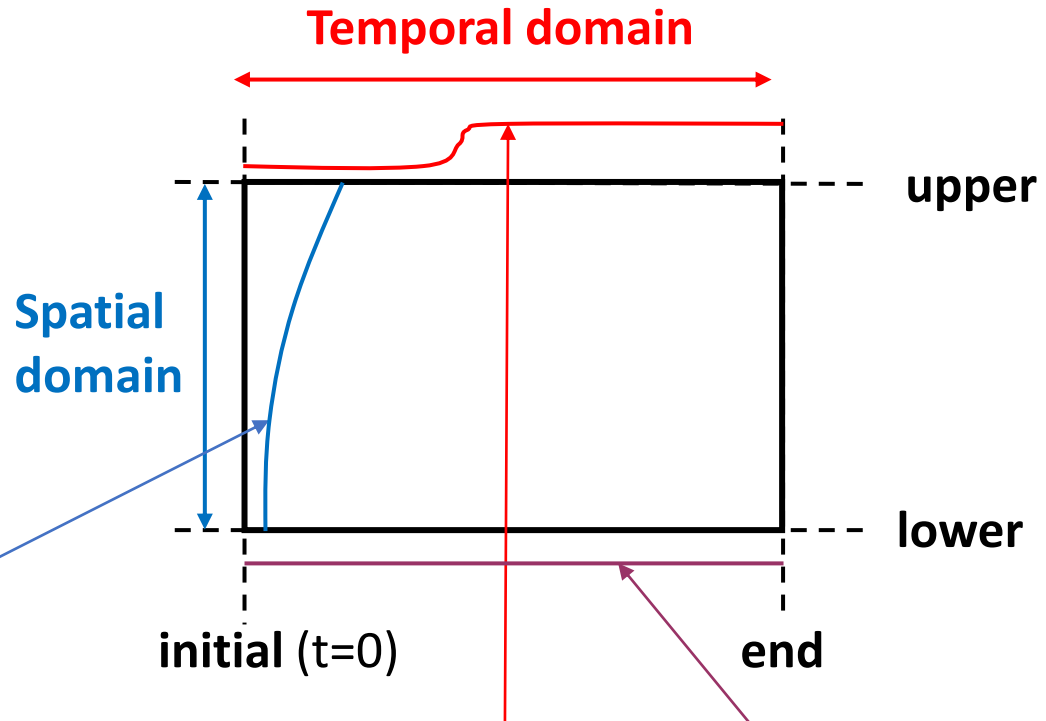
1. $C(0, x) = C_{\text{ini}}(x)$ Initial distribution for the **entire spatial** domain.



Boundary conditions

Transient state:

For every
component!



1. $C(0, x) = C_{ini}(x)$ Initial distribution for the **entire spatial** domain.

2. Imposed over the **entire temporal** domain.

If diffusion is negligible:

$C_{upper}(t)$ or $J_{upper}(t)$

If diffusion **cannot** be neglected:

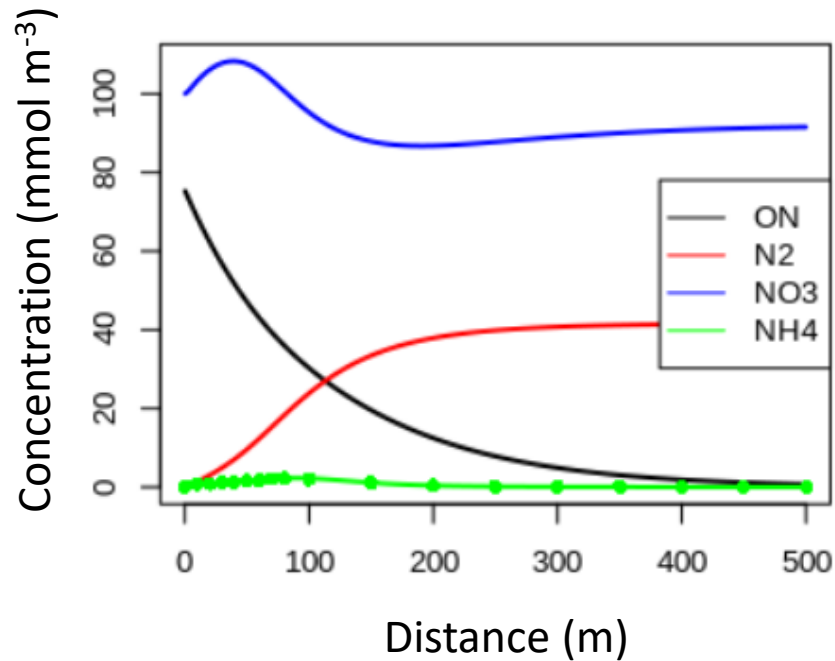
$C_{upper}(t)$ or $J_{upper}(t)$ & $C_{lower}(t)$ or $J_{lower}(t)$



Examples of solutions

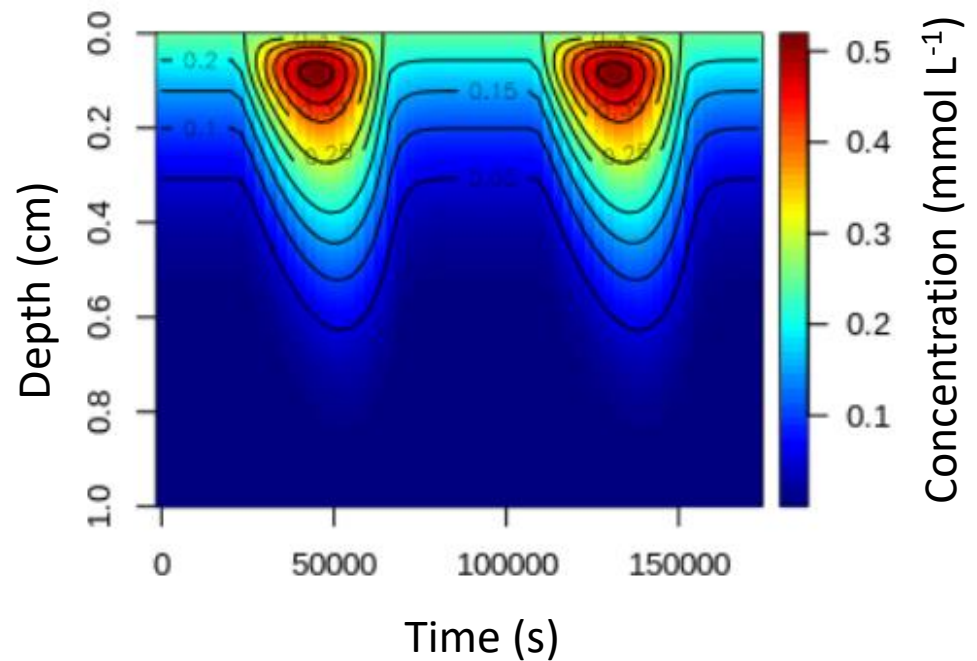
Steady state

Dissolved N species along an aquifer



Transient

O₂ in a microphytobenthic biofilm





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