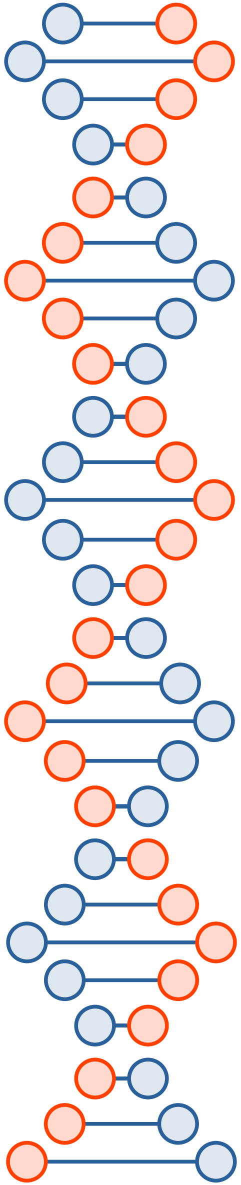

Short refreshment on the topic of UNITS

How to prevent making (mostly silly) errors

when solving quantitative problems

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- Units play an important role when solving quantitative problems.
- In this course, units will be practically ‘everywhere'. For example,
  - we will often need to **convert between different units** when setting up models or defining model parameters
  - the first **quality check** of our models will very often include a **check of units**; in most cases, it will also be the most effective one.
  - Our basic rationale is: *‘if the units are wrong, the model cannot be right’.*

Our experience:

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Our aim here:

To give you tips on **how to avoid** making such mistakes.

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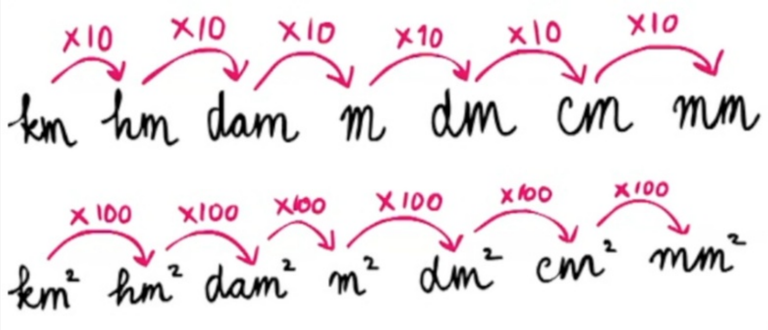

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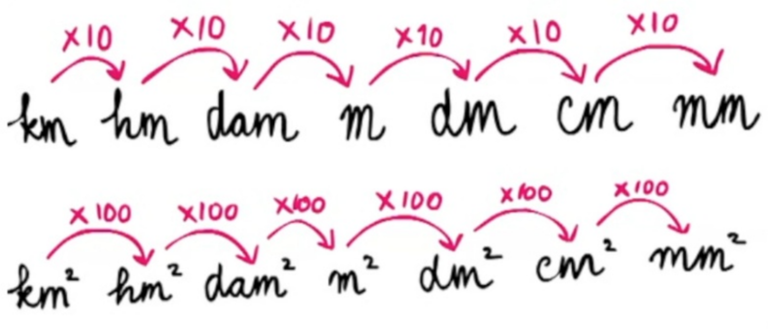

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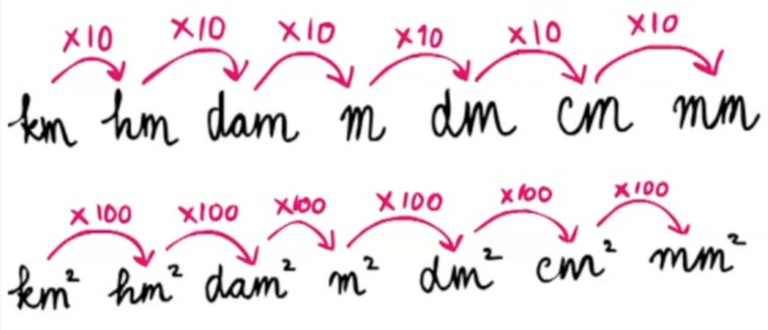

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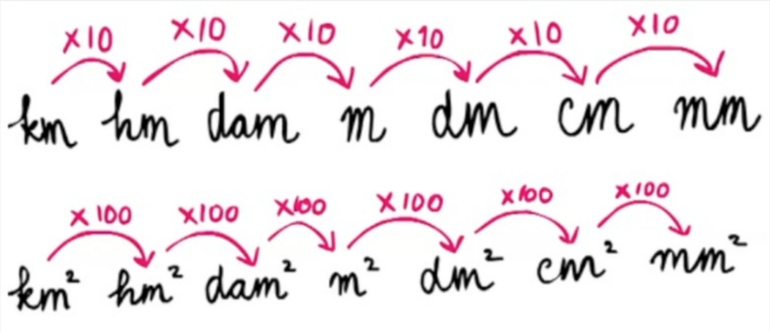

- they are only helpful in simple problems.
- In more complex problems, and more often than not, **they lead to confusion** and, ultimately, those **errors** that you want to avoid.
- Your calculation **will be much less prone to errors** if you first convert all prefixes to the form **10^n**.

Step 1

Convert all prefixes to the form $\mathbf{10^n}$, where n is an **integer** number.

The most common prefixes[1] and their meaning are summarized in this table:

Prefix name	Prefix symbol	10^n	Example
nano	n	10^{-9}	nm = 10^{-9} m
micro	μ	10^{-6}	μs = 10^{-6} s
milli	m	10^{-3}	ms = 10^{-3} s
centi	c	10^{-2}	cm = 10^{-2} m
deci	d	10^{-1}	dm = 10^{-1} m
hecto	h	10^{2}	hPa = 10^{2} Pa
kilo	k	10^{3}	km = 10^{3} m
mega	M	10^{6}	MPa = 10^{6} Pa

[1]More prefixes can be found here: https://en.wikipedia.org/wiki/Metric_prefix

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Step 2

Convert all units to **base SI units**.

Remember that these are the base SI units:

Unit	Name	Quantity
m	meter	length
s	second	time
kg	kilogram	mass
mol	mole	amount of substance
K	Kelvin	temperature
cd	candela	light
A	ampere	electric current

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Somewhat oddly, **kg** is the base SI unit of mass, **not gram** (g)!!!

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  examples: N = Newton = $kg\ m\ s^{-2}$

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- or introduced for **convenience** or “historical reasons”

examples:	L	= liter	= 1 dm^3
	min	= minute	= 60 s
	Atm	= atmosphere	= 101325 Pa
	M	= molar	= 1 mol L^{-1}

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We will, however, make **one exception**:

We will often be interested in results on **longer time scales** than seconds, for example over days or years.

Thus, we will often use **days** (**d**) or **years** (**yr**) as an acceptable unit of time.

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Simplify the expression with units using basic mathematical rules:

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Simplify the expression with units using basic mathematical rules:

- First evaluate the **powers**
- Then continue with **multiplication** or **division** of units in the expression
- Only **add** or **subtract** quantities if they have the **same** unit.

Example 1

Convert the concentration of 200 μM to mol m^{-3}.

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$$200\,\mu M = 200 \times 10^{-6}\,mol\,L^{-1} = 200 \times 10^{-6}\,\frac{mol}{L}$$

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4. Perform the final conversion using the relationships 1 and 3.

$$2 \times 10^{-5} cm^2 s^{-1} = 2 \times 10^{-5} \times \frac{cm^2}{s} = 2 \times 10^{-5} \times \frac{10^{-4} m^2}{\frac{1}{31.536 \times 10^6} yr}$$

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Example 3

Consider **N = 100** spherical amorphous silica particles suspended in a well-mixed water volume of **V = 1L**. Initially, the concentration of the dissolved silica (silicic acid) in the water is **Si_{ini} = 1 mM** and the radius of the particles is **r_{ini} = 0.1 mm** (the same for all particles). Assume the rate constant for silica dissolution of **k_d = 1.6 mol m^{-2} yr^{-1}** and for silica precipitation of **k_p = 0.8 m yr^{-1}**.

The net rate of dissolution of silica particles is calculated according to $\frac{dSi}{dt} = -k_p \times \frac{A}{V} \times (Si - Si_{eq})$

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What is the **initial rate** of increase in the concentration of silicic acid due to the dissolution of the silica particles?

1. First, we evaluate the equilibrium and initial concentrations in SI units:

$$Si_{eq} = \frac{1.6\, mol\, m^{-2}\, yr^{-1}}{0.8\, m\, yr^{-1}} = 2 \frac{mol\, m^{-2}}{m} = 2\, mol\, m^{-3} \qquad Si_{ini} = 1\, mM = 1 \times 10^{-3} \frac{mol}{L} = 1 \times 10^{-3} \frac{mol}{10^{-3}\, m^3} = 1\, mol\, m^{-3}$$

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2. Then, we substitute the values to the formula and follow the steps 1−3 described above:

$$\left(\frac{dSi}{dt}\right)_{ini} = -0.8\,\frac{m}{yr} \times \frac{100 \times 4\pi \times (0.1\,mm)^2}{1\,L} \times \left(1\,mol\,m^{-3} - 2\,mol\,m^{-3}\right)$$

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$$\approx 1000\,\frac{m}{yr} \times \frac{10^{-5}}{m}\,mol\,m^{-3}$$

$$= 0.01\,mol\,m^{-3}\,yr^{-1}$$