Solving the Heat Equation using the HHL algorithm.

Abstract

Solving the Linear System $Ax = b$ is a problem that has ubiquitous applications. While iterative Krylov Space based classical methods is an active area of research in Computational Mathematics; Harrow, Hassidim and Lloyd showed that their quantum algorithm has an exponential speed up over the classical counterparts. In this research project we intend to apply the HHL algorithm to solve the classically discretized version of the Heat Equation $u_t = u_{xx}$. For the sake of simplicity, non-adaptive finite difference approximation of the PDE is chosen. In addition, determining the Hamiltonian corresponding to the Quantum Simulation of the Linear System, and efficiently encoding the initial state of b, are also the focal points of our research. Finally, we aim for an implementation of the entire algorithm on a quantum simulator. Extending the work from this project to other discretization schemes of a PDE would be the future scope of this research.