Automatic Geometric Symmetry Detection

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1 Problem Statement

Consider a system of of N arbitrary particles with $r_i \in \mathbb{R}^p$ and $y_i \in \mathbb{N}$ where $i = 1 : N$ as the positions and labels respectively. We design a learning algorithm that identifies the geometric symmetries of the particle configuration. Further, the effect of noise on our algorithm is examined and improvements to its robustness are suggested.

2 Literature review

2.1 Symmetries

Consider a vector space X endowed with a metric m , i.e. a Metric Space $M(X, m)$.
The problem of determining the geometric symmetries of the vector space X corresponds to finding invariants of X under an arbitrary transformation T.

One immediately has to answer the following questions.

1. What is the vector space X?
2. What is the metric m that one is interested in?
3. What are the invertible transformations T defined on X?

2.1.1 Case 1

$X \subset \mathbb{R}$ defines the most trivial space. Principally, a point on X can stay where it was or be shifted by a finite or a non-finite value. The former corresponds to an Identity Transformation I while the latter corresponds to a Translation T. A useful metric that X could be equipped with is the Manhattan distance d_m. In addition, d_m in invariant under T.

2.2 Geometric Deep Learning