


A Min–Max Approach to Event- and Self-Triggered Sampling and Regulation of Linear Systems

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Abstract—This paper presents both an event- and a self-triggered sampling and regulation scheme for continuous time linear dynamic systems by using zero-sum game formulation. A novel performance index is defined wherein the control policy is treated as the first player and the threshold for control input error due to aperiodic dynamic feedback is treated as the second player. The optimal control policy and sampling intervals are generated using the saddle point or Nash equilibrium solution, which is obtained from the corresponding game algebraic Riccati equation. To determine the optimal event-based sampling scheme, an event-triggering condition is derived by utilizing the worst case control input error as the threshold. To avoid the additional hardware for the event-triggering mechanism, a near optimal self-triggering condition is derived to determine the future sampling instants given the current state vector. To guarantee Zeno-free behavior in both the event- and self-triggered closed-loop systems, the minimum intersample times are shown to be lower bounded by a nonzero positive number. Asymptotic stability of the closed-loop system is ensured using Lyapunov stability analysis. Finally, simulation examples are provided to substantiate the analytical claims.

Index Terms—Event-triggered control, linear system, optimal control, self-triggered control, zero-sum game.

I. INTRODUCTION

EVENT-TRIGGERED control (ETC) and self-triggered control (STC) schemes provide a computationally efficient approach for determining the sampling instants to implement

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control schemes on a digital platform. Traditionally, continuous-time systems use periodic, sampled-data control [1] approach, where the sensor sampling instances are separated by a fixed time interval and determined *a priori*, during the initial design phase. The sampled-data approach with fixed sampling period is widely used in the control literature because of the readily available theoretical tools for controller synthesis and stability analysis. However, periodic, time-based sampling leads to higher computational cost [2], which is a major challenge for dynamical systems with communication network in the feedback loop; referred to as the networked control systems (NCS) [3], [4].

The ETC [2], [5]–[11] and the STC [12]–[14] use state-dependent sampling and controller execution based on their respective stability and the performance requirements. The main difference between ETC [2], [5]–[11] and STC [12]–[14] is the additional hardware used, in the former case, as triggering mechanism to orchestrate the aperiodic sampling and controller execution rule. The additional hardware is replaced with a software-based sampling and controller execution rule [12]–[14], in the latter case, where the next sampling instant is predicted in advance using the current state information. Both ETC and STC schemes [2], [5]–[14] reduce computations and also the communication cost for NCS when compared to the periodic time sampling based control execution [1].

The ETC [2], [5]–[9] and the STC [12]–[14] schemes primarily deal with two aspects of control design. First, an emulation based controller [5] is designed and, second, an event- or self-triggering condition is derived such that the system stability [2], [12], [13] and performance of the controlled system are guaranteed [8], [9], [14]. The design of the triggering condition is crucial because it must ensure that the events or sampling instants are not accumulated, i.e., a Zeno-free behavior [2] of the triggering mechanism. Recently, a variety of event-triggering conditions, such as state-based static [2], [5], dynamic [6], integral [7], and performance-based [8], [9], [14], were proposed in the literature.

For instance, the ETC scheme in [2] uses an event-triggering condition that is a static function of the system state vector. On the other hand, in the dynamic event-triggered designs [6], the event-triggering mechanism is designed as a function of a dynamic variable that is adapted based on the system performance. Although the dynamic triggering conditions [6] elongate the sampling intervals when compared to its static counterpart [2], the sampling interval selection is not optimal. From the performance-based triggering point of view, an event-triggered scheme with guaranteed performance is presented in [8], [9],

and [14]. The authors analyzed the effect of the event-triggered feedback on the system performance and derived the triggering condition to guarantee \mathcal{L}_2 performance. However, the control policy and the selection of triggering instants are not generated by optimizing a predefined performance index.

From the optimal control [15] point of view, both optimal ETC [16]–[21] and optimal STC [22] designs are presented in the literature. An event-based control scheme using a quadratic value function to optimally schedule events at the sensor or actuator was proposed in [16] and [17]. Molin and Hirche [18] used certainty equivalence principle to design the controller and the sampling scheme independently given complete knowledge of the system dynamics. In contrast, the event-based adaptive/neural network based controller designs for uncertain systems using approximate dynamic programming are presented in [21] and [20], where adaptive triggering conditions are utilized to handle the uncertain dynamics. However, in all the above-mentioned optimal ETC schemes [16]–[21], an optimal control policy is obtained using a performance index whereas the sampling instants are selected independent of it.

On the other hand, a self-triggered codesign approach is presented in [22] to optimize the sampling intervals and controller performance simultaneously. This codesign method [22], with the traditional quadratic performance index, obtains the optimal control policy through Riccati equation. The sampling intervals are then maximized by evaluating and enforcing the cost to be less than the quadratic optimal value of the performance index computed assuming the continuous availability of the feedback information. Although, the codesign scheme elongates the sampling intervals when compared to the traditional self-triggering approach [13] and keeps the cost less than the optimal value, it does not guarantee an equilibrium solution [23], from the perspective of both sampling and cost, i.e., the optimal cost is not affected by the sampling scheme.

Motivated by the above-mentioned limitations, in this paper, both the control policy and the sampling intervals are generated simultaneously through a performance index. The optimal event-based sampling and control problem is formulated as a two-player zero-sum game, where the control policy is treated as the first player and the threshold for the error between continuous and ETC policy, due to the aperiodic sampling, is viewed as the second player. A novel performance index is defined using the system state and the players. The player I's objective is to minimize the cost function while the player II attempts to maximize it. The problem leads to a min–max problem [23], where the objective is to reach a saddle point solution or Nash equilibrium [23]. The Nash equilibrium solution is obtained using the game-theoretic algebraic Riccati equation (GARE) [23]. The event-based optimal control policy is designed using the solution of GARE, and optimal sampling condition is derived using the worst case control input error as threshold to maximize the sampling periods.

To eliminate the requirement of the additional piece of hardware for triggering mechanism in the event-based implementation, an STC scheme is also presented. A weaker triggering condition, derived from the event-based implementation, is utilized to determine the future sampling instants for the self-triggered design. Thus, the self-triggered sampling condition leads to a suboptimal solution. Finally, asymptotic stability of the closed-loop system for both the designs is guaranteed using the Lyapunov stability theorem, and Zeno-free behavior of the triggering mechanism is also guaranteed analytically.

The main contributions of the paper include the following: 1) formulation of the event- and self-triggered optimal sampling and control problem using zero-sum game; 2) design of the ETC and the STC schemes; 3) derivation of event-based optimal sampling and self-triggering conditions; 4) proof of asymptotic stability for both the designs.

Notations: The notations used in this paper are standard. For notational brevity, the functional arguments are dropped throughout unless needed for clarity of exposition, i.e., $x(t)$ is written as x .

The remainder of this paper is organized as follows. Section II briefly describes the background on event-based control and formulates the problem. The optimal event-based codesign is presented in Section III followed by STC design in Section IV. Simulation results are presented in Section V followed by conclusions in Section VI.

II. BACKGROUND AND PROBLEM STATEMENT

Consider a linear continuous-time system given by

$$\dot{x}(t) = Ax(t) + Bu(t), x(0) = x_0 \quad (1)$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ denote the system state and control input vector, respectively. The matrices $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the internal dynamics and input coefficient matrices, respectively, with the following standard assumption.

Assumption 1: The pair (A, B) is controllable and the state vector $x(t)$ is measurable.

Consider a traditional infinite horizon performance index for the system in (1) given by

$$J(x(0)) = \int_0^\infty (x^T H x + u^T R u) d\tau \quad (2)$$

where $H \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are symmetric positive definite matrices, and $u(t) = \mu(x(t))$, where $\mu: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is an admissible control policy. The objective is to minimize the performance index J by designing an optimal control policy u^* given by

$$u^* = \mu^*(x(t)) \triangleq \arg \min_u \int_0^\infty (x^T H x + u^T R u) d\tau. \quad (3)$$

The solution of the optimal control problem can be obtained by solving the algebraic Riccati equation (ARE) [15]. The inherent assumption of the execution of the optimal control policy is continuous availability of the system state vector.

In contrast, in the ETC and the STC formalism, the system state vector is sampled and the controller is executed aperiodically by using a sampling condition as shown in Fig. 1. Define the aperiodic sampling instants as $\{t_k\}_{k=0}^\infty$ with $t_0 = 0$. The sampled state sent to the controller at the aperiodic sampling instants can be expressed as

$$x_s(t) = x(t_k), t_k \leq t < t_{k+1} \quad (4)$$

where $x_s(t) \in \mathbb{R}^n$ is the sampled state at the controller for controller execution. The resulting state measurement error can be expressed as

$$e_{s,x}(t) = x_s(t) - x(t) = x(t_k) - x(t), t_k \leq t < t_{k+1} \quad (5)$$

where $e_{s,x}(t) \in \mathbb{R}^n$ is the state measurement error. Now, the control input with the sampled state $x_s(t)$ is expressed as $u_s(t) = \mu(x_s(t))$, which is held at the actuator by a zero-order-hold (ZOH) and applied to the system till the next update is

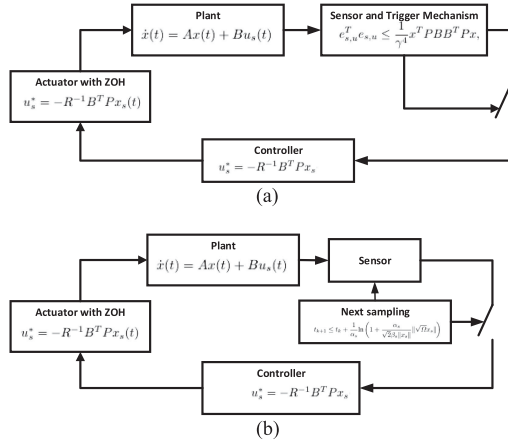


Fig. 1. (a) ETC system. (b) STC system.

reached. The system in (1) with sampled state-based input $u_s(t)$ can be expressed as

$$\dot{x}(t) = Ax(t) + Bu_s(t). \quad (6)$$

Note that the sampled control input $u_s(t) = \mu(x_s(t)) \in \mathbb{R}^m$ is a piecewise constant input signal due to ZOH.

Define the error between the continuous control input, $u(t) = \mu(x(t))$, and the sampled control input, $u_s(t)$, as

$$e_{s,u}(t) = u_s(t) - u(t) \quad (7)$$

where $e_{s,u}(t) \in \mathbb{R}^m$, the control input error due to the aperiodic feedback, is a piecewise continuous function. The sampled system (6) with (7) can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + Be_{s,u}(t). \quad (8)$$

The aperiodic sampling and controller execution introduced an error term, $Be_{s,u}(t)$, in the system dynamics (1), which can be considered as an exogenous input to the system. It is clear that larger the magnitude of $e_{s,u}(t)$, longer the aperiodic intervals.

To obtain an optimal threshold for $e_{s,u}(t)$, such that the sampling intervals can be maximized, we introduce a dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) + B\hat{e}_{s,u}(t) \quad (9)$$

where $\hat{e}_{s,u}$ is an exogenous independent signal, whose optimal value will be used as a threshold for $e_{s,u}$ in (8). Therefore, the objective is to, first, design the optimal control policy u^* for the system in (9) in the presence of the worst case exogenous signal $e_{s,u}^*$ (worst case value for $\hat{e}_{s,u}$) such that the system in (9) attains the optimal target system, $\dot{x}(t) = Ax(t) + Bu^*(t) + Be_{s,u}^*(t)$. Second, design the sampling instants and the control policy for the system in (8) such that the resulting closed-loop system emulates the target system.

Therefore, the traditional performance index in (2) for the system in (9) must be redefined to include the additional term $\hat{e}_{s,u}$, which can be maximized. This clearly leads to a zero-sum min-max game formulation. In the next section, a solution to the dynamic game is presented.

III. CO-OPTIMIZATION OF SAMPLING PERIODS AND CONTROL POLICY FOR EVENT-BASED CONTROL

In this section, a novel performance index for the event- and the self-triggered system is defined and a solution is presented to optimize the performance index.

Redefine the infinite horizon performance index (2) for an event-based control system (8) as

$$J(x(0), u, \hat{e}_{s,u}) = \frac{1}{2} \int_0^\infty (x^T(\tau) H x(\tau) + u^T(\tau) R u(\tau) - \gamma^2 \hat{e}_{s,u}^T(\tau) \hat{e}_{s,u}(\tau)) d\tau \quad (10)$$

where $H \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are constant symmetric positive definite user defined matrices and $\gamma > \gamma^*$, with γ^* is the minimum value of γ , is a constant such that the performance index in (10) is finite.

Remark 1: In the traditional H_∞ optimal control [23], [24] approach, solution of the optimal control problem leads to a saddle point optimal control input u^* and worst case exogenous signal $e_{s,u}^*$ [23]. In the event-triggered formalism, the control input error $e_{s,u}$ is a function of u , which is dependent, unlike independent disturbance in H_∞ optimal control [23], [24]. Therefore, the proposed performance index (10) is defined using an independent continuous exogenous signal, $\hat{e}_{s,u}$, based on (9). Minimization of (10) will lead to the optimal exogenous signal, $e_{s,u}^*$ which is utilized as the threshold for the triggering mechanism [defined in (21)].

To design the optimal control policy, u^* and the sampling instants $\{t_k\}_{k=0}^\infty$, which will maximize the sampling intervals, $T_k = t_{k+1} - t_k$, a saddle point solution to the min-max optimization problem [23], defined by the cost function (10) with dynamic state constraint (9), needs to be obtained. This means to find the optimal value function given by

$$\begin{aligned} V^*(x(t)) &= \min_u \max_{\hat{e}_{s,u}} J(x, u, \hat{e}_{s,u}) \\ &= \min_u \max_{\hat{e}_{s,u}} \int_t^\infty \frac{1}{2} (x^T(\tau) H x(\tau) + u^T(\tau) R u(\tau) \\ &\quad - \gamma^2 \hat{e}_{s,u}^T(\tau) \hat{e}_{s,u}(\tau)) d\tau \end{aligned} \quad (11)$$

such that given the pair $(u^*, e_{s,u}^*)$, the saddle point solution is reached, i.e., $\min_u \max_{\hat{e}_{s,u}} J(x, u, \hat{e}_{s,u}) = \max_{\hat{e}_{s,u}} \min_u J(x, u, \hat{e}_{s,u})$.

To solve the optimal control problem, begin by defining the Hamiltonian for the optimal value function (11) along the system dynamics (9). The Hamiltonian can be expressed as

$$\begin{aligned} \mathcal{H} \left(x, u, \hat{e}_{s,u}, \frac{\partial V^*}{\partial x} \right) &= \frac{1}{2} x^T H x + \frac{1}{2} u^T R u - \frac{1}{2} \gamma^2 \hat{e}_{s,u}^T \hat{e}_{s,u} \\ &\quad + \frac{\partial V^{*T}}{\partial x} (Ax + Bu + B\hat{e}_{s,u}) \end{aligned} \quad (12)$$

where $\frac{\partial V^{*T}}{\partial x} = \frac{\partial V^{*T}(x(t))}{\partial x(t)}$.

The optimal control input can be computed by using the stationarity condition ($\frac{\partial \mathcal{H}(x, u, \hat{e}_{s,u}, \frac{\partial V^*}{\partial x})}{\partial u} = 0$) and given by

$$u^* = \mu^*(x) = \arg \min_u \mathcal{H}(\cdot) = -R^{-1} B^T \frac{\partial V^*}{\partial x} \quad (13)$$

and the worst case control input error, with $\frac{\partial \mathcal{H}(x, u, \hat{e}_{s,u}, \frac{\partial V^*}{\partial x})}{\partial \hat{e}_{s,u}} = 0$, is given by

$$e_{s,u}^* = \arg \max_{\hat{e}_{s,u}} \mathcal{H}(\cdot) = \frac{1}{\gamma^2} B^T \frac{\partial V^*}{\partial x}. \quad (14)$$

The optimal value for linear systems with quadratic performance index (10) can be represented as $V^* = \frac{1}{2} x^T P x$, where

$P \in \mathbb{R}^{n \times n}$ is a symmetric positive definite kernel matrix of the GARE (to be derived in (18)). Then, the optimal control input (13) can be expressed as

$$u^* = -R^{-1}B^T Px \quad (15)$$

whereas the worst case control input error (14) is given by

$$e_{s,u}^* = \frac{1}{\gamma^2} B^T Px. \quad (16)$$

By substituting the optimal control input (15) and the worst-case control-input input error (16), the Hamilton–Jacobi–Isaacs (HJI) equation can be expressed as $\mathcal{H}(x, u^*, e_{s,u}^*, \frac{\partial V^*}{\partial x}) = -\frac{1}{2}x^T Hx - \frac{1}{2}u^{*T} Ru^* + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u}^* + \frac{\partial V^{*T}}{\partial x} (Ax + Bu^* + Be_{s,u}^*)$. Rearranging the equation reveals

$$\begin{aligned} \mathcal{H}(x, u^*, e_{s,u}^*, \frac{\partial V^*}{\partial x}) &= \frac{1}{2}x^T (A^T P + PA + H \\ &\quad - PBR^{-1}B^T P + \frac{1}{\gamma^2} PBB^T P)x \end{aligned} \quad (17)$$

where the GARE is given by

$$A^T P + PA + H - PBR^{-1}B^T P + \frac{1}{\gamma^2} PBB^T P = 0. \quad (18)$$

The next theorem states the existence of an unique solution of the GARE in the context of aperiodic sampling.

Theorem 1 (Existence of the solution of GARE): Consider the linear system (1). Let the Assumption 1 holds. Then, there exist a unique symmetric positive definite kernel matrix P satisfying the GARE (18) provided the condition $R^{-1} > \frac{1}{\gamma^2} I$ is satisfied. Furthermore, the Hamiltonian $\mathcal{H}(x, u^*, e_{s,u}^*, \frac{\partial V^*}{\partial x}) = 0$.

Proof: The GARE (18) can be rewritten as

$$A^T P + PA + H - PB \left(R^{-1} - \frac{1}{\gamma^2} I \right) B^T P = 0. \quad (19)$$

By selecting γ properly one can find a positive definite matrix \bar{R} such that $R^{-1} - (1/\gamma^2)I = \bar{R}^{-1}$. Substituting \bar{R}^{-1} , the GARE (19) becomes an ARE [15]. Since the pair (A, \sqrt{H}) is detectable, the matrix P is unique symmetric and positive definite solution of (18) [15]. Since P satisfies the GARE in (18), the Hamiltonian $\mathcal{H}(x, u^*, e_{s,u}^*, \frac{\partial V^*}{\partial x}) = 0$. ■

The sampled optimal control input, using the solution of the GARE, with sampled state (4) can be expressed as

$$u_s^* = -R^{-1}B^T Px_s. \quad (20)$$

Next, define the event-triggering condition for the system in (8) to determine sampling instants using worst case threshold in (16) as

$$e_{s,u}^T e_{s,u} \leq \frac{1}{\gamma^4} x^T PBB^T Px, \quad \forall t. \quad (21)$$

The triggering-mechanism evaluates the condition (21) and samples the system state vector when the inequality is violated. The following theorem guarantees the asymptotic stability of the sampled system.

Theorem 2 (Asymptotic stability of optimal event-triggered system): Consider the linear system (1) represented as event-sampled system in (6). Let Assumption 1 holds, γ satisfies $\gamma^2 > \lambda_{\max}(R)$ and the symmetric positive definite matrix P

satisfies (18). Then, the sampled system (6) with control policy (20) is asymptotically stable for any initial state $x_0 \in \mathbb{R}^n$ if the inequality in (21) holds.

Proof: Let $L: \mathbb{R}^n \rightarrow \mathbb{R}^+$ be a continuously differentiable positive definite Lyapunov candidate function given by $L(x) = \frac{1}{2}x^T Px = V^*(x)$, where $V^*(x)$ is the optimal value of the value function and P is a symmetric positive definite matrix that satisfies GARE (18).

The first derivative of $L(x)$ along the system dynamics (6) is given by $\dot{L} = \frac{\partial V^{*T}}{\partial x} \dot{x} = \frac{\partial V^{*T}}{\partial x} (Ax + Bu_s) = \frac{\partial V^{*T}}{\partial x} (Ax + Bu + Be_{s,u})$.

Replacing with the optimal value of control input u^* from (15), the first difference leads to

$$\dot{L} = \frac{\partial V^{*T}}{\partial x} [(Ax + Bu^* + Be_{s,u}^*) - B(e_{s,u}^* - e_{s,u})]. \quad (22)$$

Given u^* and $e_{s,u}^*$, the HJI equation in (17) $\mathcal{H}(x, u^*, e_{s,u}^*, \frac{\partial V^*}{\partial x}) = 0$. Then, the HJI equation can be rearranged as $\frac{\partial V^{*T}}{\partial x} (Ax + Bu^* + Be_{s,u}^*) = -\frac{1}{2}x^T Hx - \frac{1}{2}u^{*T} Ru^* + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u}^*$. Furthermore, from (14) we have $\gamma^2 e_{s,u}^* = B^T \frac{\partial V^*}{\partial x}$. Utilizing these facts and inserting in (22), the first difference

$$\begin{aligned} \dot{L} &= -\frac{1}{2}x^T Hx - \frac{1}{2}u^{*T} Ru^* + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u}^* \\ &\quad - \gamma^2 e_{s,u}^{*T} e_{s,u}^* + \gamma^2 e_{s,u}^{*T} e_{s,u}. \end{aligned} \quad (23)$$

Applying Young's inequality, the first difference

$$\begin{aligned} \dot{L} &\leq -\frac{1}{2}x^T Hx - \frac{1}{2}u^{*T} Ru^* + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u}^* \\ &\quad - \gamma^2 e_{s,u}^{*T} e_{s,u} + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u} + \frac{1}{2}\gamma^2 e_{s,u}^T e_{s,u}. \end{aligned} \quad (24)$$

Recalling the triggering condition (21), the condition can be rewritten as $e_{s,u}^T e_{s,u} \leq e_{s,u}^{*T} e_{s,u}^*$. Substituting this triggering condition in (24), the first derivative is upper bound as

$$\dot{L} \leq -\frac{1}{2}x^T Hx - \frac{1}{2}u^{*T} Ru^* + \frac{1}{2}\gamma^2 e_{s,u}^{*T} e_{s,u}^*. \quad (25)$$

Inserting u^* from (15) and $e_{s,u}^*$ in (16), the first derivative of the Lyapunov function can be given by

$$\begin{aligned} \dot{L} &\leq -\frac{1}{2}x^T Hx - \frac{1}{2}x^T PBR^{-1}B^T Px \\ &\quad + \frac{1}{2}\gamma^2 \left(\frac{1}{\gamma^4} x^T PBB^T Px \right) = -\frac{1}{2}x^T Mx \end{aligned} \quad (26)$$

where $M = H + PBR^{-1}B^T P - \frac{1}{\gamma^2} PBB^T P$ is a positive definite matrix since H is positive definite and by selecting γ such that $\gamma^2 > \lambda_{\max}(R)$, the matrix $R^{-1} - \frac{1}{\gamma^2} I$ will also be positive definite.

From (26), the Lyapunov first derivative $\dot{L}(x)$ is negative definite. By Lyapunov stability theorem [25], the system state $x \rightarrow 0$ as $t \rightarrow \infty$. Alternatively, the closed-loop sampled system is asymptotically stable. ■

Remark 2: Note that the set of sampling instants is determined using the equality condition in (21), i.e., $\{t_k\}_{k=0}^\infty = \{t \in \mathbb{R}_{\geq 0} | e_{s,u}^T e_{s,u} = \frac{1}{\gamma^4} x^T PBB^T Px\}$. Since the threshold for the sampled policy error is selected as it's the worst case value, the sampling intervals $T_k = t_{k+1} - t_k$ are maximized with respect to the performance index (10).

The optimal event-triggering condition (21) is a function of the control input error and the state vector of the original system. The following corollary presents a traditional triggering condition [2] using the state error (5) and the sampled state vector (4).

Corollary 1: Let the hypothesis of Theorem 2 hold. Then, the sampled system (6) with control policy (20) is asymptotically stable for any initial state $x_0 \in \mathbb{R}^n$ if the inequality

$$e_{s,x}^T \Pi e_{s,x} \leq \frac{1}{2} x_s^T \Omega x_s \forall t \in [t_k, t_{k+1}), k \in \mathbb{N} \quad (27)$$

holds, where $\Pi = \psi_\pi I + \Omega + K^T K \in \mathbb{R}^{n \times n}$, with $\Omega = \frac{1}{\gamma^4} P B B^T P \in \mathbb{R}^{n \times n}$, $K = R^{-1} B^T P \in \mathbb{R}^{m \times n}$, $\psi_\pi > 0$ is a constant, and I is the identity matrix with appropriate dimension.

Proof: To complete the proof, it suffices to show that given the inequality (27) the sampling condition (21) holds. By the definition of Ω in (27), the inequality (21) can be rewritten as $e_{s,u}^T e_{s,u} \leq x_s^T \Omega x_s$. Furthermore, the control input error $e_{s,u} = u_s - u = K x_s - K x = K(x_s - x) = K e_{s,x}$.

Expansion of the expression in (27) leads to

$$\begin{aligned} e_{s,x}^T \Pi e_{s,x} &= \psi_\pi e_{s,x}^T e_{s,x} + e_{s,x}^T \Omega e_{s,x} + e_{s,x}^T K^T K e_{s,x} \\ &= \psi_\pi e_{s,x}^T e_{s,x} + e_{s,x}^T \Omega e_{s,x} + e_{s,u}^T e_{s,u} \leq \frac{1}{2} x_s^T \Omega x_s. \end{aligned} \quad (28)$$

Rearrangement of the (28) reveals that $e_{s,u}^T e_{s,u} \leq \frac{1}{2} x_s^T \Omega x_s - e_{s,x}^T \Omega e_{s,x} - \psi_\pi e_{s,x}^T e_{s,x}$. Adding and subtracting similar terms and rearranging the equation again, we get $e_{s,u}^T e_{s,u} \leq x_s^T \Omega x_s + e_{s,x}^T \Omega e_{s,x} - 2x_s^T \Omega e_{s,x} - (\frac{1}{2} x_s^T \Omega x_s + 2e_{s,x}^T \Omega e_{s,x} - 2x_s^T \Omega e_{s,x}) - \psi_\pi e_{s,x}^T e_{s,x}$.

By completion of square, we have

$$\begin{aligned} e_{s,u}^T e_{s,u} &\leq (x_s - e_{s,x})^T \Omega (x_s - e_{s,x}) \\ &\quad - \left(\frac{1}{\sqrt{2}} x_s - \sqrt{2} e_{s,x} \right)^T \Omega \left(\frac{1}{\sqrt{2}} x_s - \sqrt{2} e_{s,x} \right) \\ &\quad - \psi_\pi e_{s,x}^T e_{s,x} \leq x_s^T \Omega x_s. \end{aligned} \quad (29)$$

From (29), it is clear that given (27) the inequality (21) holds. Consequently, by Theorem 2, the event-sampled system (6) is asymptotically stable. ■

Remark 3: The condition in (27) uses the sampled state vector x_s and can also be used as an event-triggering condition. The sampling instants can be determined by the violation of the inequality. However, the condition in (27) is a weaker condition and results in a suboptimal triggering. The main advantage of the inequality in (27) is that it only uses sampled signals and can be used to predict the next sampling instants at the controller using the current sampled state without any triggering mechanism as presented in the next section.

IV. SELF-TRIGGERED IMPLEMENTATION

In this section, the event-based design is extended to the self-triggered case by deriving a sampling condition with current states to determine the sampling instants.

The inequality in (27) can be equivalently written as

$$\|\sqrt{\Pi} e_{s,x}\| \leq \frac{1}{\sqrt{2}} \|\sqrt{\Omega} x_s\|, \forall t \in [t_k, t_{k+1}), k \in \mathbb{N} \quad (30)$$

where $\sqrt{\Pi}$ and $\sqrt{\Omega}$ are the matrix square roots of the Π and Ω , respectively. By definition, the inverse of Π and, hence, $\sqrt{\Pi}$ exists by properly selecting the value of ψ_π .

The most important condition for developing the self-triggered scheme is that the sampling periods must be bounded from below by a nonzero positive constant [14]. The following lemma states the lower boundedness of the sampling period.

Lemma 1: Consider the linear system (1) represented as sampled data system in (6) along with the sampled optimal control input (20). Let γ satisfies $\gamma^2 > \lambda_{\max}(R)$, Π is invertible, and there exists a symmetric positive definite matrix P satisfy (18). Then, the sampled system is asymptotically stable if the sampling is carried out when the condition in (30) is violated. Furthermore, the intersample times T_k are lower bounded by a nonzero positive constant.

Proof: From Corollary 1, the inequality in (27) or equivalently in (30) is weaker than (21). Consequently, by Theorem 2, the sampled system is asymptotically stable. It only remains to show the lower boundedness intersample times. Since the inequality in (30) is a weaker condition of (21), it suffices to show that the intersample times T_k determined by (30) are lower bounded.

Define an error $\bar{e}_{s,x} = \sqrt{\Pi} e_{s,x}$. The inter-sampling periods $T_k = t_{k+1} - t_k$ is implicitly defined by the equality in (30). The error $\|\bar{e}_{s,x}(t)\|$ evolves from zero at time $t_k, k \in \mathbb{N}$ to the threshold value of $\frac{1}{\sqrt{2}} \|\sqrt{\Omega} x_s\|$ at time $t_{k+1}, k \in \mathbb{N}$. The time derivative of the measurement error $\|\bar{e}_{s,x}\|$ becomes

$$\begin{aligned} \frac{d}{dt} \|\bar{e}_{s,x}\| &\leq \|\dot{\bar{e}}_{s,x}\| = \|\sqrt{\Pi}(\dot{x}_s - \dot{x})\| \\ &= \|\sqrt{\Pi} \dot{x}\| = \|\sqrt{\Pi}(A x + B u_s)\| \\ &= \|\sqrt{\Pi}(A(x_s - e_{s,x}) + B K x_s)\| \\ &\leq \|\sqrt{\Pi}(A + B K)x_s\| + \|\sqrt{\Pi} A e_{s,x}\| \\ &\leq \|\sqrt{\Pi} A \sqrt{\Pi}^{-1} \sqrt{\Pi} e_{s,x}\| + \|\sqrt{\Pi}(A + B K)x_s\|. \end{aligned}$$

Defining $\alpha_s = \|\sqrt{\Pi} A \sqrt{\Pi}^{-1}\|$ and $\beta_s = \|\sqrt{\Pi}(A + B K)\|$, we have

$$\frac{d}{dt} \|\bar{e}_{s,x}\| \leq \alpha_s \|\bar{e}_{s,x}\| + \beta_s \|x_s\|. \quad (31)$$

By comparison lemma [25], the solution of the differential inequality in (31) is upper bounded by

$$\begin{aligned} \|\bar{e}_{s,x}(t)\| &\leq \|\bar{e}_{s,x}(t_k)\| e^{\alpha_s(t-t_k)} + \int_{t_k}^t \beta_s \|x_s(\tau)\| e^{\alpha_s(t-\tau)} d\tau \\ &= \frac{\beta_s \|x_s(t)\|}{\alpha_s} \left(e^{\alpha_s(t-t_k)} - 1 \right). \end{aligned} \quad (32)$$

With $\|\bar{e}_{s,x}(t_k)\| = 0, \forall k \in \mathbb{N}$, at the next sampling instant $t_{k+1}, \forall k \in \mathbb{N}$, the error $\|\bar{e}_{s,x}(t_{k+1})\| = \frac{1}{\sqrt{2}} \|\sqrt{\Omega} x_s\|$. From (32), it holds that $\frac{1}{\sqrt{2}} \|\sqrt{\Omega} x_s\| = \|\bar{e}_{s,x}(t_{k+1})\| \leq \frac{\beta_s \|x_s\|}{\alpha_s} (e^{\alpha_s(t_{k+1}-t_k)} - 1)$, which leads to $\frac{\beta_s \|x_s\|}{\alpha_s} (e^{\alpha_s(t_{k+1}-t_k)} - 1) > \frac{1}{\sqrt{2}} \|\sqrt{\Omega} x_s\|$. Solving for the $T_k = t_{k+1} - t_k$ reveals

$$T_k > \frac{1}{\alpha_s} \ln \left(1 + \frac{\alpha_s}{\sqrt{2} \beta_s \|x_s\|} \|\sqrt{\Omega} x_s\| \right). \quad (33)$$

From (33), the inter-sample durations $T_k = t_{k+1} - t_k > 0, \forall k \in \mathbb{N}$ and, hence, the sampling periods are bounded from below by a nonzero positive constant. ■

With the guarantee in lower boundedness of the sampling periods, the following theorem states the suboptimal STC scheme.

Theorem 3: Consider the linear continuous time system (1) represented as a sampled data system in (6). Let the Assumption 1 holds, γ satisfies $\gamma^2 > \lambda_{\max}(R)$, and the symmetric positive definite matrix P satisfies (18). Then, the self-triggered system (6) with optimal control policy (20) is asymptotically stable for any initial state $x_0 \in \mathbb{R}^n$ if systems states are sampled and controller is executed at the time instants determined by the following inequality:

$$t_{k+1} \leq t_k + \frac{1}{\alpha_s} \ln \left(1 + \frac{\alpha_s}{\sqrt{2}\beta_s \|x_s\|} \|\sqrt{\Omega}x_s\| \right). \quad (34)$$

Furthermore, the self-triggered system is Zeno free.

Proof: The next sampling instant t_{k+1} for the self-triggered condition (34) uses the expression of inter-sample times T_k in (33). Alternatively, enforcing the self-triggered sampling condition (34) ensures that the event-triggering condition (30) is satisfied. Therefore, by Lemma 1, the self-triggered system is asymptotically stable.

Furthermore, from (33) of Lemma 1, $T_k > \frac{1}{\alpha_s} \ln(1 + \frac{\alpha_s}{\sqrt{2}\beta_s \|x_s\|} \|\sqrt{\Omega}x_s\|) > 0$. This implies $t_{k+1} - t_k > 0$, i.e., the sampling periods are bounded from below by a nonzero positive number and the self-triggered system does not exhibit Zeno behavior. ■

Remark 4: The sampling intervals determined by the self-triggering scheme uses a weaker condition (30) when compared to the event-triggering condition (21). Therefore, the STC results has a known degree of suboptimality from the sampling period point of view.

V. SIMULATION RESULTS

In this section, numerical results are presented using two examples to validate the analytical design presented in Sections III and IV. In addition, the results are compared with the existing method in [14].

Example 1: The unstable batch reactor example [3], [4] is considered for simulation whose dynamics are given by

$$\dot{x} = \begin{bmatrix} 1.38 & -0.207 & 6.715 & -5.676 \\ -0.581 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u.$$

The simulation parameters were selected as follows: initial state vector $x_0 = [1.8, -2.4, -4.6, 4]^T$, the matrices for the cost function $H = I_{4 \times 4}$ and $R = 0.05I_{2 \times 2}$. The value of $\gamma = 0.25$ was selected such that $\gamma^2 > \lambda_{\max}(R)$, as in Theorem 1. The simulation is run for 5 s, and the results are shown in Fig. 2 through Fig. 6.

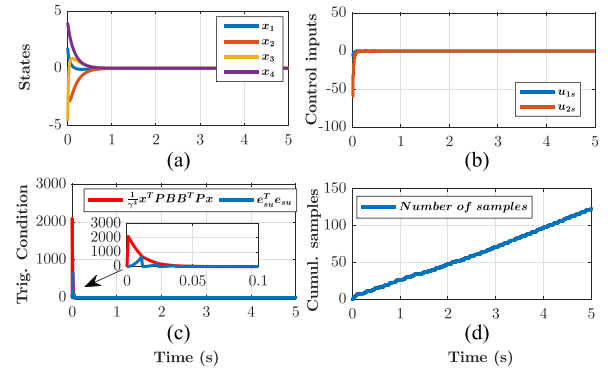


Fig. 2. Evolution of (a) the state trajectory, (b) optimal event-based control policy, (c) control input error and sampling threshold, and (d) cumulative number of sampling instants.

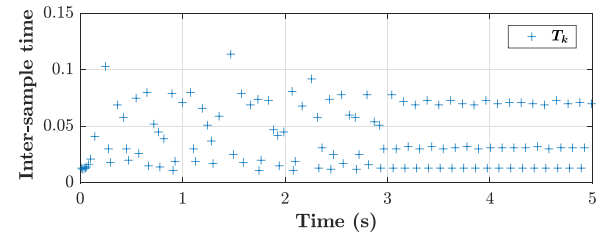


Fig. 3. Aperiodic intersample times with a minimum intersample time of 0.011 s for $\gamma = 0.25$.

Event-Triggered Control: The evolution of the system state vector and the control inputs are shown in Fig. 2(a) and (b). The optimal event-sampled control input in Fig. 2(b) is a piece wise constant function due to the event-based aperiodic execution of the control input. The control input error $e_{s,u}$ used as the event-triggering error is plotted in Fig. 2(c) along with the threshold. The plot is zoomed out for clarity of the evolution of the control input error, which resets at every sampling instants. A total of 123 sampling instants occurred during the simulation time of 5 s for $\gamma = 0.25$, as shown in Fig. 2(d). A minimum sampling time of 0.011 s is observed and the aperiodic sampling instants during the simulation are plotted in Fig. 3.

Next, the comparison results with the existing work in [14] is presented. Note that a direct comparison is not possible due to the difference between the approaches and performance requirements. Therefore, to have a fair comparison in terms of the cost, same value of $\gamma = 0.25$ is used for both the approaches and $\beta = 0.76$ was selected for evaluating the triggering condition (11) in [14] such that equal numbers of triggering occurs for both the methods. This results in 122 events, which is close to the proposed method, i.e., 123. The optimal cost trajectory for both the approaches are computed as $V^*(x) = x^T P x$ with same P matrix, which is the solution of the GARE in (18). The cumulative optimal costs are computed by integrating the current values.

The optimal cost surface and the optimal cost trajectory for the proposed and existing method [14] are plotted in Fig. 4. Since the batch reactor is a fourth-order system, for plotting the costs, only the first two states are considered at a time while the other two are made zero. It is clear that the optimal cost in the proposed method uses the shortest path resulting in lower cumulative cost, which when compared to the existing method [14], as shown in Fig. 5.

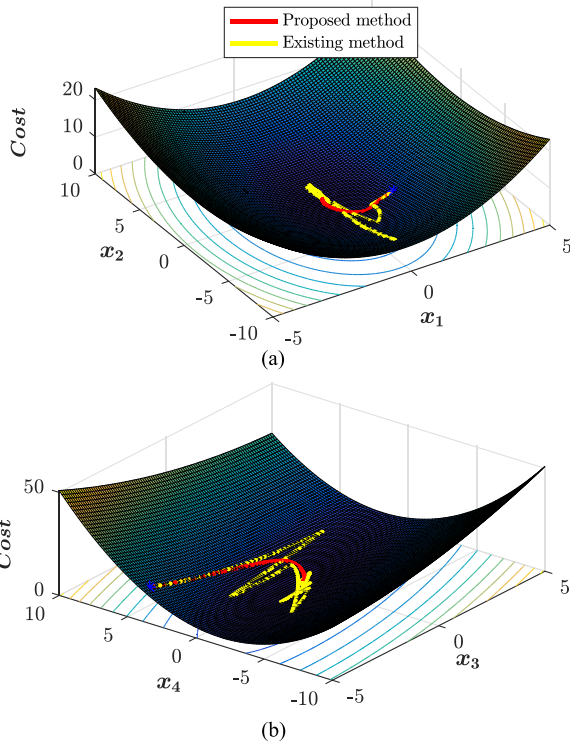


Fig. 4. Optimal cost surface and comparison of optimal cost trajectory between proposed and existing method [14]. (a) For state x_1 and x_2 , with $x_3 = x_4 = 0$. (b) For state x_3 and x_4 , with $x_1 = x_2 = 0$.

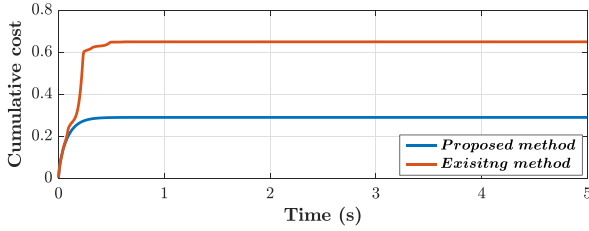


Fig. 5. Comparison of cumulative cost between proposed and existing method [14].

Self-Triggered Control: The sampling instants are determined by the controller using the self-triggering condition in (34) with $\psi_\pi = 0.001$ and all other parameters are the same as in the case of ETC.

The convergence of the system states and control input are shown in Fig. 6(a) and (b). The intersample times are shown in Fig. 6(c). Recall that the self-triggering condition (34) is derived using a weaker condition than the event-triggering condition given by (21). Therefore, intersample times are small when compared to the event-based intersample times shown in Fig. 3. A minimum intersample time of 0.001 s is observed with an averaged sampling interval of 0.0011 s.

Example 2: In this example, the inverted pendulum on a cart with dynamics as in [14] is considered. The parameters used for the simulation were $m = 1$, $g = 10$, $M = 10$, and $l = 3$, with initial state $x_0 = [0.98, 0, 0.2, 0]^T$, $H = I$, and $R = 0.05$. All other parameters are selected as in Example 1 to show the efficacy of the design for both sampling intervals and cost.

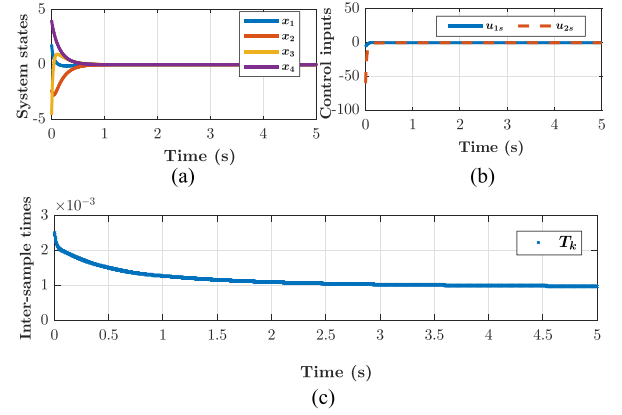


Fig. 6. Self-triggered implementation. (a) Evolution of the state trajectory. (b) Optimal event-based control policy. (c) Intersample time with $\gamma = 0.25$.

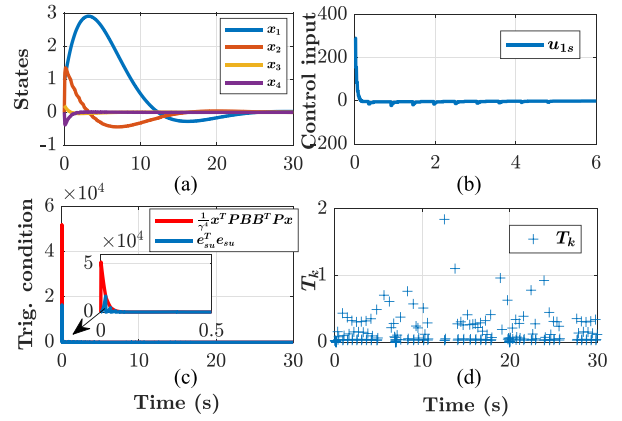


Fig. 7. (a) Evolution of the state trajectory. (b) Optimal event-based control policy. (c) Triggering condition. (d) Intersample times (T_k).

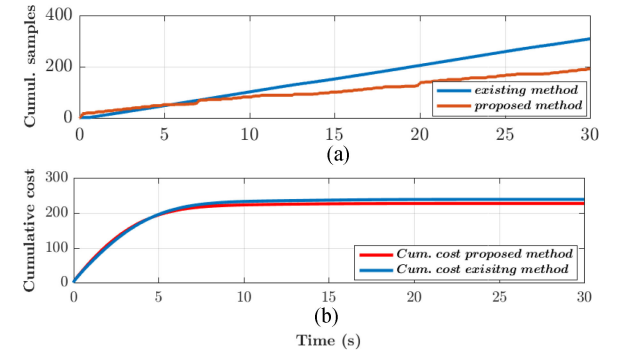


Fig. 8. Comparison of (a) cumulative sampling instants and (b) cumulative cost between the proposed and existing method [14].

The convergence of the system states and the optimal control input are shown in Fig. 7(a) and (b), respectively. The evolution of the triggering condition is shown in Fig. 7(c) and intersample times with an average intersample time of 0.156 s is shown in Fig. 7(d). The comparison results for number of samples and cumulative cost are shown in Fig. 8(a) and (b), respectively. It is clear that the number of sampling and cumulative cost of the proposed design is lower than the existing method in [14] with the selection of same $\gamma = 0.25$ and $\beta = 0.76$.

VI. CONCLUSION

In this paper, a unified design approach for both the ETC and the STC of linear continuous time system were presented. The control policy and the event-triggered intervals were optimized by using the proposed min-max optimization approach with the proposed novel performance index. The event-triggering condition directly used the worst case threshold signal from the unified formulation to elongate the sampling intervals. The self-triggering condition derived using a weaker version of the event-triggering condition led to a suboptimal solution. From the simulation results, it is observed that the STC resulted in a higher number of sampling instants, as expected, thus considered as an suboptimal solution. The comparison results showed that the proposed scheme results in lower cost. The proposed ETC and STC do not consider network constraints during the design and will be included in our future research.

REFERENCES

- [1] K. J. Åström and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*. Chelmsford, MA, USA: Courier Corporation, 2013.
- [2] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Sep. 2007.
- [3] W. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nesic, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.
- [4] G. C. Walsh, H. Ye, and L. G. Bushnell, "Stability analysis of networked control systems," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 3, pp. 438–446, May 2002.
- [5] P. Tallapragada and N. Chopra, "Decentralized event-triggering for control of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 59, no. 12, pp. 3312–3324, Dec. 2014.
- [6] A. Girard, "Dynamic triggering mechanisms for event-triggered control," *IEEE Trans. Autom. Control*, vol. 60, no. 7, pp. 1992–1997, Jul. 2015.
- [7] S. H. Mousavi, M. Ghodrati, and H. J. Marquez, "Integral-based event-triggered control scheme for a general class of non-linear systems," *IET Control Theory Appl.*, vol. 9, no. 13, pp. 1982–1988, Aug. 2015.
- [8] W. P. M. H. Heemels, G. E. Dullerud, and A. R. Teel, " \mathcal{L}_2 -gain analysis for a class of hybrid systems with applications to reset and event-triggered control: A lifting approach," *IEEE Trans. Autom. Control*, vol. 61, no. 10, pp. 2766–2781, Oct. 2016.
- [9] B. A. Khashooei, D. J. Antunes, and W. Heemels, "Output-based event-triggered control with performance guarantees," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3646–3652, Jul. 2017.
- [10] G. Guo, Z. Lu, and P. Shi, "Event-driven actuators: To zero or to hold?" *Int. J. Robust Nonlinear Control*, vol. 24, no. 17, pp. 2761–2773, 2014.
- [11] G. Guo, L. Ding, and Q.-L. Han, "A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems," *Automatica*, vol. 50, no. 5, pp. 1489–1496, 2014.
- [12] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems," *IEEE Trans. Autom. Control*, vol. 55, no. 9, Sep. 2010.
- [13] M. Mazo Jr, A. Anta, and P. Tabuada, "An ISS self-triggered implementation of linear controllers," *Automatica*, vol. 46, no. 8, pp. 1310–1314, 2010.
- [14] X. Wang and M. D. Lemmon, "Self-triggered feedback control systems with finite-gain \mathcal{L}_2 stability," *IEEE Trans. Autom. Control*, vol. 54, no. 3, pp. 452–467, Mar. 2009.
- [15] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*. New York, NY, USA: Wiley, 2012.
- [16] D. P. Borgers, V. S. Dolk, and W. Heemels, "Riccati-based design of event-triggered controllers for linear systems with delays," *IEEE Trans. Autom. Control*, vol. 63, no. 1, pp. 174–188, Jan. 2018.
- [17] R. Cogill, "Event-based control using quadratic approximate value functions" in *Proc. 48th IEEE Conf. Decis. Control, 28th Chin. Control Conf.*, 2009, pp. 5883–5888.
- [18] A. Molin and S. Hirche, "On the optimality of certainty equivalence for event-triggered control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 470–474, Feb. 2013.
- [19] C. Peng and T. C. Yang, "Event-triggered communication and h_∞ control co-design for networked control systems," *Automatica*, vol. 49, no. 5, pp. 1326–1332, 2013.
- [20] Q. Zhang, D. Zhao, and Y. Zhu, "Event-triggered h_∞ control for continuous-time nonlinear system via concurrent learning," *IEEE Trans. Syst. Man Cybern.: Syst.*, vol. 47, no. 7, pp. 1071–1081, Jul. 2017.
- [21] A. Sahoo, H. Xu, and S. Jagannathan, "Near optimal event-triggered control of nonlinear discrete-time systems using neurodynamic programming," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 9, pp. 1801–1815, Sep. 2016.
- [22] T. Gommans, D. Antunes, T. Donkers, P. Tabuada, and M. Heemels, "Self-triggered linear quadratic control," *Automatica*, vol. 50, no. 4, pp. 1279–1287, 2014.
- [23] T. Başar and P. Bernhard, *H_∞ -Optimal Control and Related Minimax Design Problems*. Cambridge, MA, USA: Birkhäuser, Boston, 1995.
- [24] X. Wei and L. Guo, "Composite disturbance-observer-based control and h_∞ control for complex continuous models," *Int. J. Robust Nonlinear Control, IFAC-Affiliated J.*, vol. 20, no. 1, pp. 106–118, 2010.
- [25] H. K. Khalil, *Nonlinear Systems* vol. 2. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996.



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