# 16-715: Advanced Robot Dynamics and Simulation

#### Fall 2021

### Course Description

This course explores the fundamental mathematics behind modeling the physics of robots, as well as state-of-the-art algorithms for robot simulation. We will review classical topics like Lagrangian mechanics and Hamilton's Principle of Least Action, as well as modern computational methods like discrete mechanics and fast linear-time algorithms for dynamics simulation. A particular focus of the course will be rigorous treatments of 3D rotations and non-smooth contact interactions (impacts and friction) that are so prevalent in robotics applications. We will use numerous case studies to explore these topics, including quadrotors, fixed-wing aircraft, wheeled vehicles, quadrupeds, humanoids, and manipulators. Homework assignments will focus on practical implementation of algorithms and a course project will encourage students to apply simulation methods to their own research.

**Prerequisites:** Strong linear algebra skills, experience with a high-level programming language like Python, MATLAB, or Julia, and basic familiarity with ordinary differential equations.

#### Instructors

Prof. Zac Manchester Email: zacm@cmu.edu
TA: Kevin Tracy Email: ktracy@cmu.edu

## Logistics

- Lectures will be held Tuesdays and Thursdays 10:10–11:30 AM Eastern time in NSH 1305.
- Office hours will be Wednesdays 11:30-12:30 PM Eastern time in NSH 1505, and Fridays 9-10 AM Eastern time on zoom. Zoom link and schedule: https://tinyurl.com/98k5hs8w.
- Homework assignments will be due at 11:59 PM Eastern time two weeks after they are assigned.
- GitHub will be used to distribute and collect assignments.
- Slack will be used for general discussion and Q&A outside of class and office hours.
- There will be no exams. Instead, each student will complete a project on a topic of their choice.

### Learning Objectives

By the end of this course, students should be able to:

- 1. Derive differential equations for simple mechanical systems using classical Newton-Euler and Lagrangian techniques
- 2. Model more complex industrial, wheeled, legged, aerial, underwater, and space robotic systems using modern computational techniques
- 3. Simulate environmental contact interactions like impacts and Coulomb friction
- 4. Understand the trade-offs and limitations of different dynamics formulations and simulation techniques
- 5. Build high-performance simulation tools that can be deployed in machine learning and control design applications

### Learning Resources

There is no textbook required for this course. Video recordings of lectures and lecture notes will be posted online. Additional references for further reading will be provided with each lecture.

#### Homework

Homework will be posted every 2 weeks and students will be given at least one full week to complete assignments. All homework will be distributed and collected using GitHub. Solutions and grades will be returned within one week of homework due dates.

## Grading

Grading will be based on:

- 50% Project
- 40% Homeworks
- 10% Participation

Attendance during lectures is not required to earn a full participation grade. Students can also participate through any combination of office hours, Slack discussions, project presentations, and by offering constructive feedback about the course to the instructors.

#### Course Policies

Late Homework: Students are allowed a budget of 6 late days for turning in homework with no penalty throughout the semester. They may be used together on one assignment, or separately on two assignments. Beyond these six days, no other late homework will be accepted.

Accommodations for Students with Disabilities: If you have a disability and are registered with the Office of Disability Resources, I encourage you to use their online system to notify me of

your accommodations and discuss your needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.

Statement of Support for Students' Health & Well-Being: Take care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, avoiding drugs and alcohol, getting enough sleep, and taking some time to relax. This will help you achieve your goals and cope with stress.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit <a href="http://www.cmu.edu/counseling">http://www.cmu.edu/counseling</a>. Consider reaching out to a friend, faculty, or family member you trust for help getting connected to the support that can help.

If you or someone you know is feeling suicidal or in danger of self-harm, call someone immediately, day or night:

CaPS: 412-268-2922

Re:solve Crisis Network: 888-796-8226

If the situation is life threatening, call the police:

On campus: CMU Police: 412-268-2323

Off campus: 911

If you have questions about this or your coursework, please let me know. Thank you, and have a great semester.

#### **Tentative Schedule**

Aug 31 Sep 2 Particles, Pendulums, and Orbits  2 Sep 7 Energy and Stability Sep 9 Numerical Solution of ODEs and Runge-Kutta Methods  3 Sep 14 No Class Sep 16 Rigid Bodies, Euler's Equation, and Lie Groups  4 Sep 21 Quaternions and Numerical Simulation Sep 23 Quaternions and Numerical Simulation Sep 24 Sep 28 Constrained Optimization Pt. 1 HW 1 Due Sep 30 The Least-Action Principle HW 2 Out  6 Oct 5 Coordinates, Constraints, and Manifolds Oct 7 D'Alembert, Virtual Work, and Generalized Forces  7 Oct 12 Lagrangian Mechanics and Manipulators HW3 Out  8 Oct 19 Momentum, Duality, and the Hamiltonian Oct 21 Lie Algebras and Euler's Equation from a Lagrangian  9 Oct 26 Discrete Mechanics and Variational Integrators HW3 Due HW4 Out	<u> </u>
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Nov 2 Impacts as Inequality Constraints	_
Nov 4   Coulomb Friction and The Maximum Dissipation Principle	
Nov 9 Legged Robots and Hybrid Systems HW4 Due	
Nov 11   Sparsity and Linear-Time Algorithms	
12 Nov 16 Case Studies	
Nov 18 Case Studies	
13 Nov 23 Case Studies	
Nov 25 No Class	
Nov 30 Project Presentations	_
Dec 2 Project Presentations	

# **Project Guidelines**

Students should work in groups of 1–4 to complete a substantial final project. The goal is for students to apply the coarse content to their own research. Project proposals will be solicited in late September and topics will be selected in consultation with the instructors.

Project grades will be based on a short presentation given during the last week of class and a final report submitted via Google drive by May 18 Anywhere on Earth. Reports should be written in the form of a 6 page (plus references) ICRA or IROS conference paper using the standard two-column IEEE format. Sections should include an abstract, introduction and/or background to motivate your problem, 2–3 main technical sections on your contributions, conclusions, and references. Grading will be based on the following criteria:

10%	Class presentation
10%	Adherence to IEEE formatting and length requirements
10%	Innovation & Creativity: Is what you did new/cool/interesting? Convince me.
30%	Clarity of presentation: Can I understand what you did from your writing + plots?
40%	Technical correctness: Are your results reasonable? Is your code correct?