

Revision work sheet Key on Polynomial

- 1) $X^{82}, x^{82} + x : x^{99}, x^{99} + 7$
- 2) $9x^2 = 72, x^2 = 72/9 = 8, x = \sqrt{8}$
- 3) i) $p(2) = 5(2)^3 - 4(2)^2 + 3 = 40 - 16 + 3 = 27$ ii) $p(-1) = 5(-1)^3 - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$
- 4) $3x + 7 = 0, 3x = -7, x = -7/3$
- 5) $2(\frac{1}{2})^3 - 4m(\frac{1}{2})^2 + 2(\frac{1}{2}) + 1 = 0$ (since $\frac{1}{2}$ is a zero of the polynomial)
 $2(1/8) - 4m(1/4) + 1 + 1 = 0, \frac{1}{4} - m + 2 = 0, m = 9/4$
- 6) Zero of $x-1$ is $x=1$, so by remainder theorem, $p(1) = 3(1)^3 - 4(1)^2 - 3(1) - 1 = 3 - 4 - 3 - 1 = -5$ is the remainder.
- 7) Zero of $3s - 2$ is $s = 2/3 : p(2/3) = 3(2/3)^3 + (2/3)^2 - 20(2/3) + 12 = 0/9 = 0$.

Therefore by factor theorem $3s - 2$ is a multiple of the polynomial

- 8) $P(x) = 2x^3 - 5x^2 + x + 2$ zeros of $x-1, x-2$ and $2x+1$ are $1, 2$ and $-1/2$
 $P(1) = 2 - 5 + 1 + 2 = 0$, therefore $x-1$ is a factor of $p(x)$, by factor theorem
 $P(2) = 2(8) - 5(4) + 2 + 2 = 16 - 20 + 4 = 0$, therefore $x-2$ is a factor of $P(X)$
 $P(-1/2) = 2(-1/8) - 5(1/4) + (-1/2) + 2 = -3/4$. Therefore $2x+1$ is not a factor

- 9) $81 - 25y^2 = (9 - 5y)(9 + 5y)$ by the identity $(a^2 - b^2) = (a + b)(a - b)$
 $3xy - 243xy^5 = 3xy(1 - 81y^4) = 1^2 - (9y^2)^2 = (1 + 9y^2)(1 - 9y^2) = (1 + 9y^2)(1 + 3y)(1 - 3y)$
- 10) i) $16\sqrt{5}x^2 - 40x - 10\sqrt{5} = 8\sqrt{5}(2x - \sqrt{5}) - 5(2x - \sqrt{5}) = (2x - \sqrt{5})(8\sqrt{5} - 5)$
 ii) $x^2 + 4x - 3x - 12 = x(x+4) - 3(x+4) = (x+4)(x-3)$
- 11) $(2x - y/3)^3 = 8x^3 - 4x^2y + 2/3xy^2 - y^3/27$
 $(x - y/2 + z/3)^2 = x^3 + y^3/4 + z^3/9 + 2x(-y/2) + 2(-y/2)z/3 + 2z/3x = x^3 + y^3/4 + z^3/9 - xy - zy/3 + 2/3zx$

- 12) Area of rectangle = $25x^2 - 35x + 12$

We know, area of rectangle = length \times breadth

So, by factoring $25x^2 - 35x + 12$, the length and breadth can be obtained.

$$25x^2 - 35x + 12 = 25x^2 - 15x - 20x + 12$$

$$\Rightarrow 25x^2 - 35x + 12 = 5x(5x - 3) - 4(5x - 3)$$

$$\Rightarrow 25x^2 - 35x + 12 = (5x - 3)(5x - 4)$$

So, the length and breadth are $(5x - 3)(5x - 4)$.

Now, perimeter = $2(\text{length} + \text{breadth})$

$$\begin{aligned}\text{So, perimeter of the rectangle} &= 2((5x - 3) + (5x - 4)) \\ &= 2(5x - 3 + 5x - 4) = 2(10x - 7) = 20x - 14\end{aligned}$$

$$\text{So, the perimeter} = 20x - 14$$

$$13. (9a^2 + 12ab) = 3a(3a + 4b)$$

$$\begin{aligned}14) \text{ Given } a = 2 - p, \text{ then } \text{LHS} &= p^3 + 6pa + a^3 - 8 = p^3 + 6p(2-p) + (2-p)^3 - 8 \\ &= p^3 + 12p - 6p^2 + 8 - 12p + 6p^2 - p^3 - 8 = 0 = \text{RHS}\end{aligned}$$

$$\begin{aligned}15) 1 + a + b - c + a - b - c - a - a - b - c &= 1 - c + a - c + a + b - b - c + a - a - b - c \\ &= 1(1-c) + a(1-c) + b(1-c) + a - b - c \\ &= (1-c)(1+a+b+ab) = (1-c)[1 + (1+a)+b(1+a)] \\ &= (1-c)[(1+a)(1+b)]\end{aligned}$$

$$16) (4p + 5q)^3 = 64p^3 + 240p^2q + 300pq^2 + 25q^3$$

$$17) 100 - 9a^2 = (10 + 3a)(10 - 3a)$$

$$18) \text{ Consider the equation } 3x + 2y = 12$$

Now, square both sides:

$$(3x + 2y)^2 = 12^2$$

$$\Rightarrow 9x^2 + 12xy + 4y^2 = 144$$

$$\Rightarrow 9x^2 + 4y^2 = 144 - 12xy$$

From the questions, $xy = 6$

So,

$$9x^2 + 4y^2 = 144 - 72 = 72$$