Parallel Techniques for Ant-Based Optimization Algorithms

T. Bui (Penn State University)
T. Nguyen (University of New Mexico)
J. Rizzo (Concurrent Technologies)

Outline

- Introduction
- Sequential and Parallel Framework of Ant-Based Optimization (ABO)
- An Example – The Maximum Clique Problem
- Experimental Results
- Conclusion

Introduction

- Motivations
  - Ant algorithms provide good heuristics for many NP-Hard problems
  - They make good candidates for parallelization
  - Multi-core machines are prevalent and affordable
- Ant-Based Optimization (ABO) and Shared Memory Parallelism
  - a variant of Ant Algorithms
  - is amenable to multi-core (shared-memory) parallel model
- Test problem
  - Max-Clique
  - Results show run-time improvements of almost linear with a dual-core and up to a factor of 6 on an eight-core processor

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ACO vs. ABO

- **Ant-Colony Optimization (<u>ACO</u>)**
  - A set of ants is used
  - Each ant, one at a time, solves the entire problem
  - Each ant passes information about its solution to the other ants by using pheromone
  - Each ant works alone, using information from previous ants to help it solve the problem

- **Ant-Based Optimization (<u>ABO</u>)**
  - A set of ants is used
  - No ants solve or see the entire problem
  - Ants pass information among themselves through pheromone and their locations
  - Ants work together to explore the search space, identify promising regions and local optimization is used extract solution

Shared and Distributed Memory

- **Shared-Memory (SM)**
  - Is a collection of identical processors sharing main memory
  - Communication: threads-based
  - Advantage: simple conversion to parallelism
  - Disadvantage: deadlocks, race conditions, shared access
  - Examples: multi-processor, multi-core

- **Distributed-Memory (DM)**
  - Composes of stand-alone machines, each with its own processor and memory set
  - Communication: message passing
  - Advantage: scalability
  - Disadvantage: need to redesign algorithms
  - Examples: Beowulf clusters

Generic ABO Algorithm (Sequential)

Find starting solution **S** using a quick heuristic method

Distribute ants based on **S**

repeat

Exploration

Each ant performs some tasks, moves to another location, and lays down some pheromone

Exploitation

Construct a new configuration **C** based on ants' locations

A local optimization technique is used to extract a solution **S'** from **C**

Keep the best solution found so far.

Jolting

Periodically perturb the current configuration

until some criteria are met

return the best solution found

Generic ABO Algorithm (Parallel)

- **Find starting solution S using *several* heuristic methods**

Distribute ants based on S

repeat

Exploration

- **Ants do work in parallel where applicable**

Exploitation

- **Construct *several* configurations C based on ants' locations**
- **Use *several* local optimization methods to extract a solution S' from C**

Keep the best solution found so far.

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Periodically perturb the current configuration

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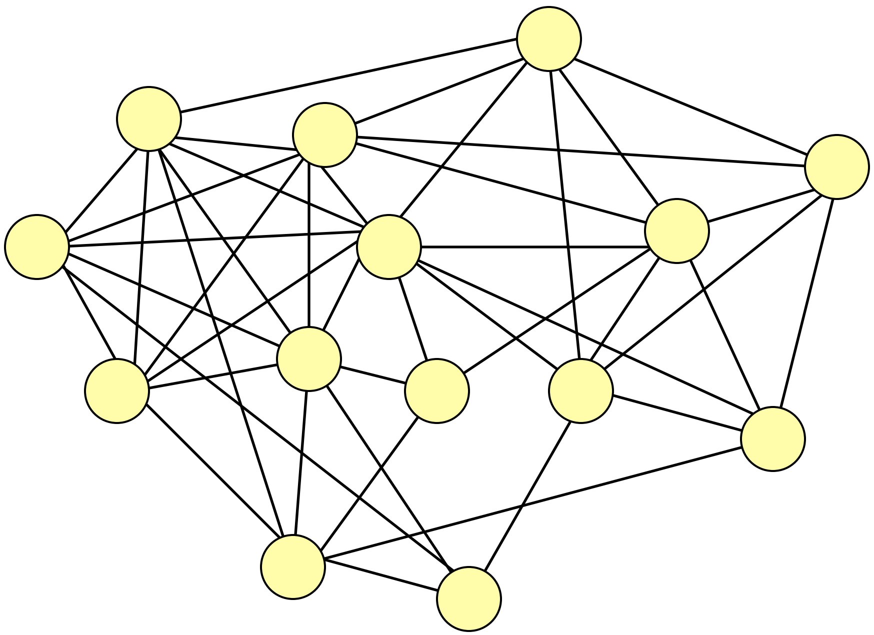
Max-Clique Problem

Max-Clique Problem

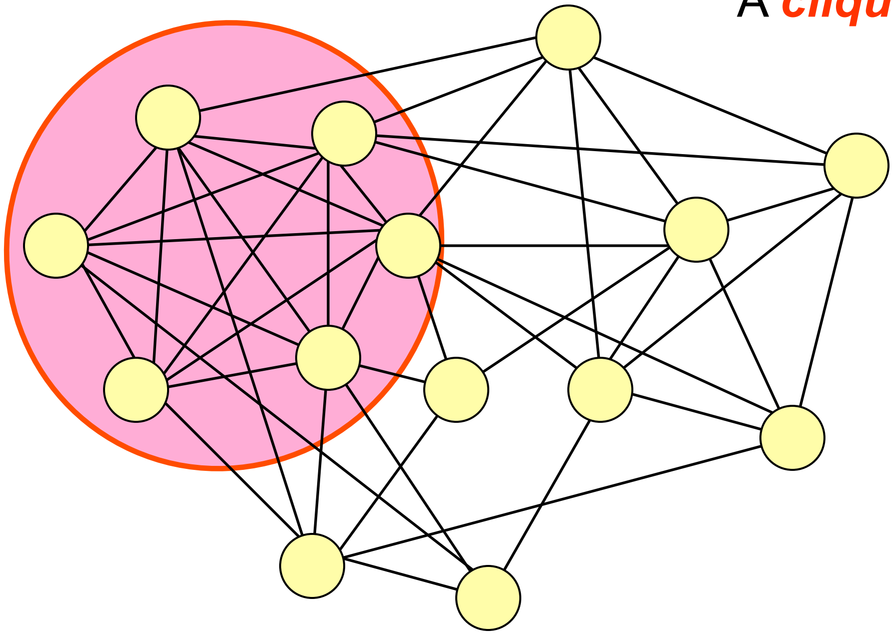
- **Input**: an undirected graph G



Max-Clique Problem

- **Input**: an undirected graph G
- **Output**: the ***maximum*** clique of G

A ***clique*** is a complete sub-graph



ABO for Max Clique

Use a greedy method to get a clique **K** from input graph **G**
Distribute ants on **G** with heavier concentration in **K**

repeat

Exploration

Ants make decision where to move

Ants move and deposit pheromone along the way

Small amount of pheromone close to ants' previous locations are evaporated

Exploitation

Find a promising region **R** in the graph based on ants' locations and pheromones

Extract a clique **K** from **R** and grow it

Jolting

Periodically shuffle ants

until some criteria are met

return the largest clique found

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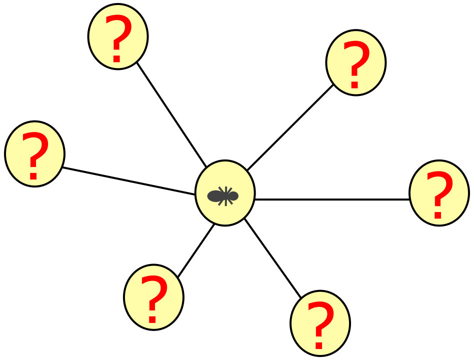

Ant Movement

- Each ant can choose to stay put, move to a random neighboring vertex or

- It can choose to move to a neighboring vertex **v** based on *three* components:
  1. **pheromone** on edge leading to **v**
  2. **population** of ants on **v**
  3. **connectivity** (degree) of **v**

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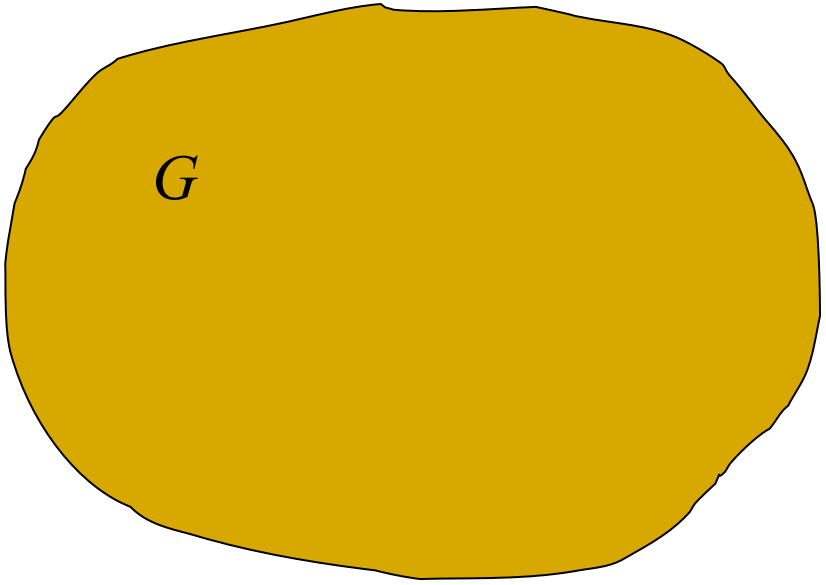
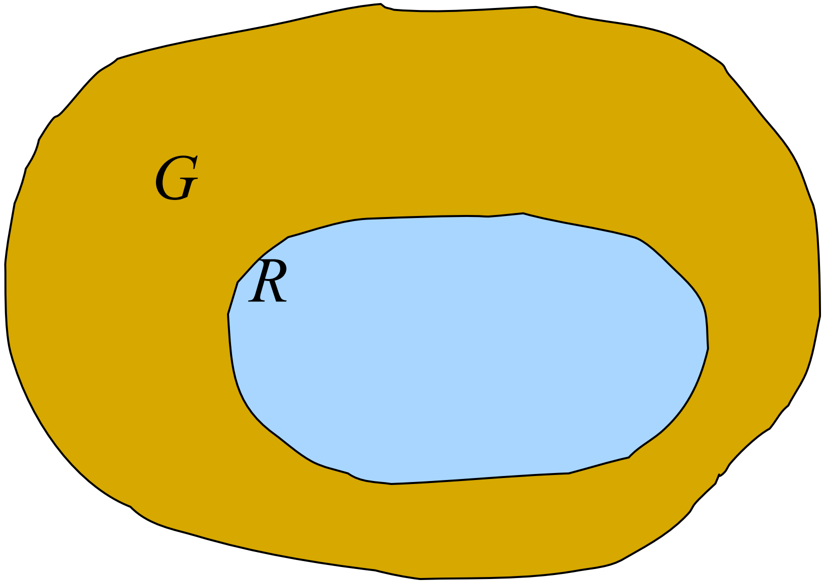
Find and Grow Clique



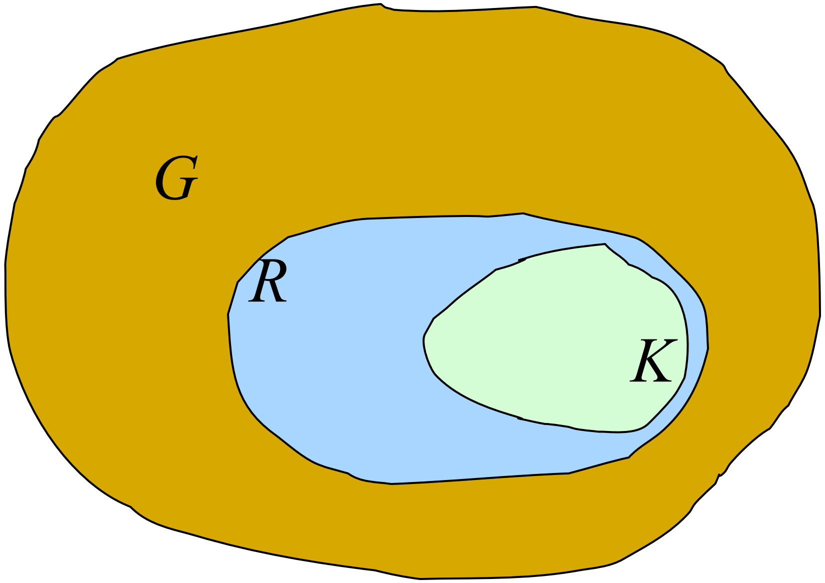
Find and Grow Clique

- Identify interesting region **R** based on ant configuration



Find and Grow Clique

- Identify interesting region **R** based on ant configuration
- Extract clique **K** from **R** using greedy technique



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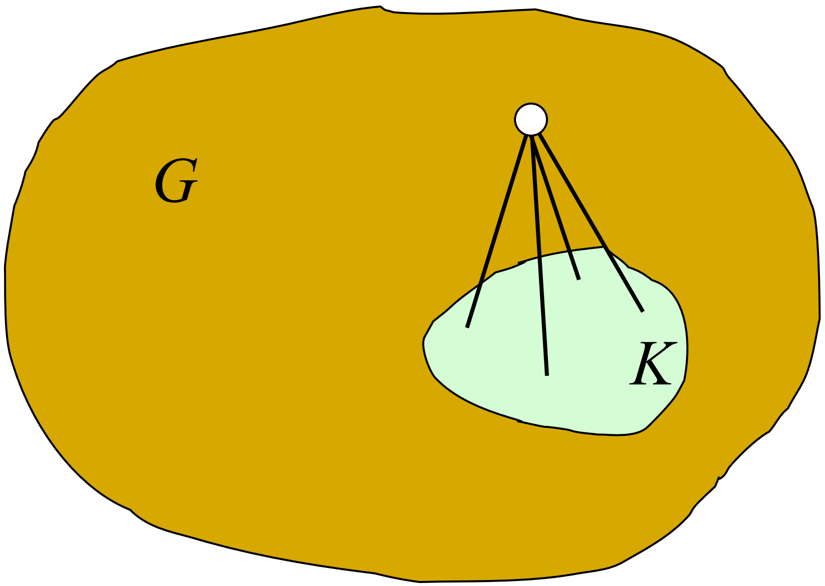

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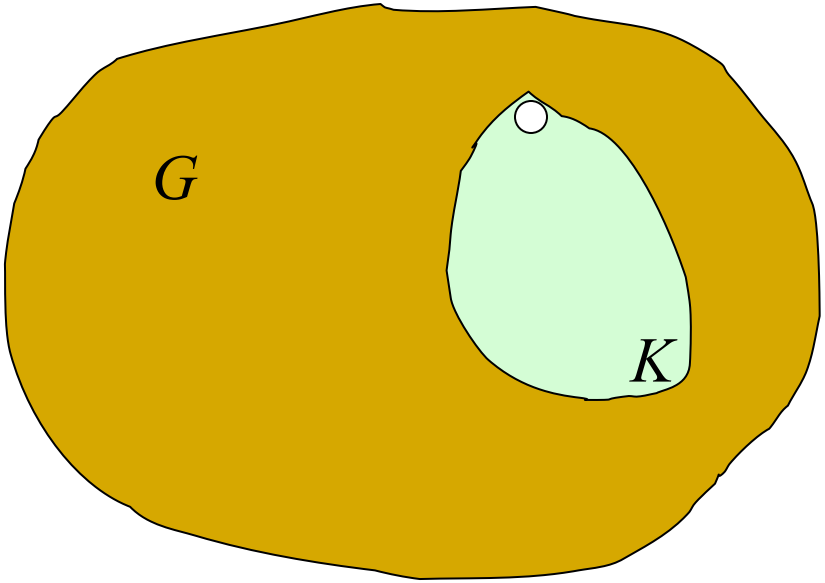


ABO for Max Clique (Parallel)

- Obtain *several* cliques by running multiple instances of the same greedy algorithm with different seeds and keep the largest clique K

Distribute ants on **G** with heavier concentration in **K**

repeat

Exploration

- Ants Operations are done in parallel
- Pheromone Evaporations are done in parallel

Exploitation

- Find *several* promising regions R's in the graph based on ants' locations and pheromones
- Extract cliques K's from regions R's, grow them, and keep the largest one

Jolting

Periodically shuffle ants

until some criteria are met

return the largest clique found

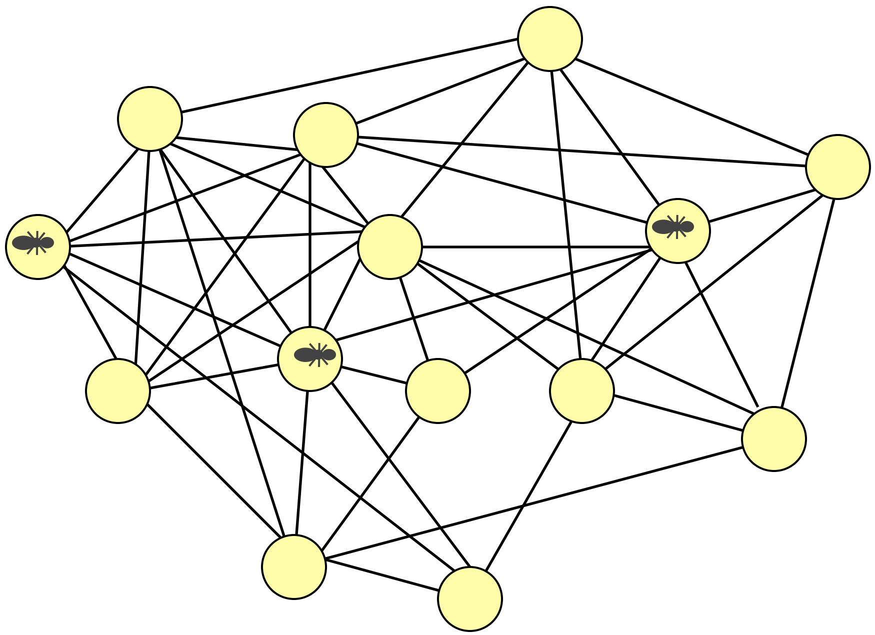
How to Parallelize Ants' Operations

Ants Operations: make decision on where to move and move there next

- Sequential: at each time cycle, ants operate iteratively, and action of an ant influences other ants
- Parallel: split the actions into two loops and parallelize both
  - At each time cycle, all ants make decision on where to move in parallel
  - At each time cycle, all ants move to their destinations in parallel

Ants Movements (Sequential)

```
at time t
for each ants a do
   ant a makes a decision where
   to move to and move there
done
```



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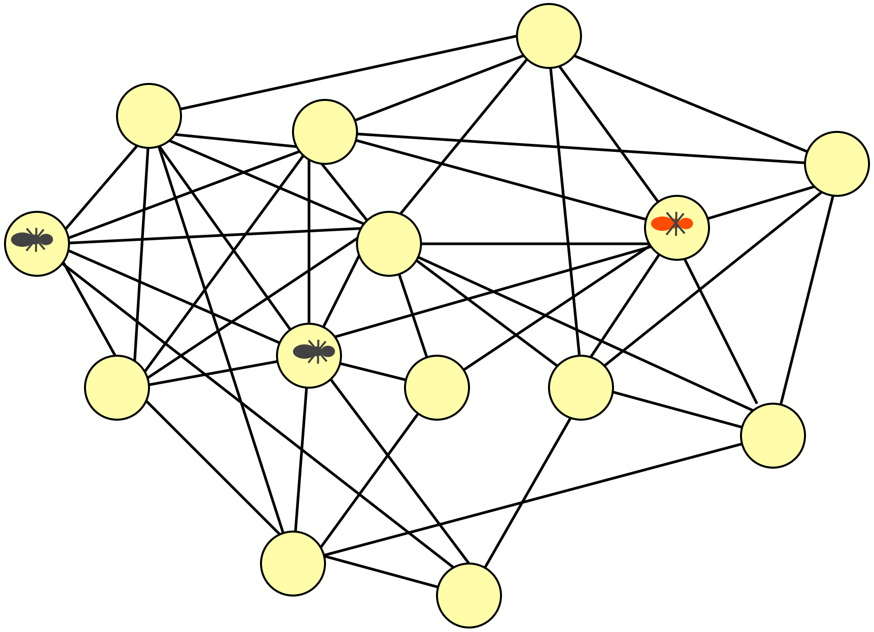

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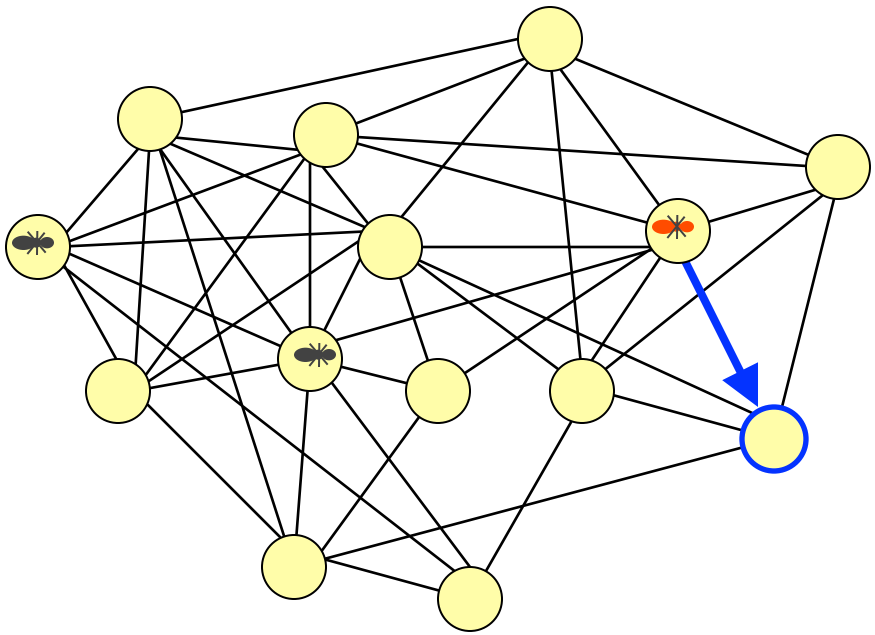

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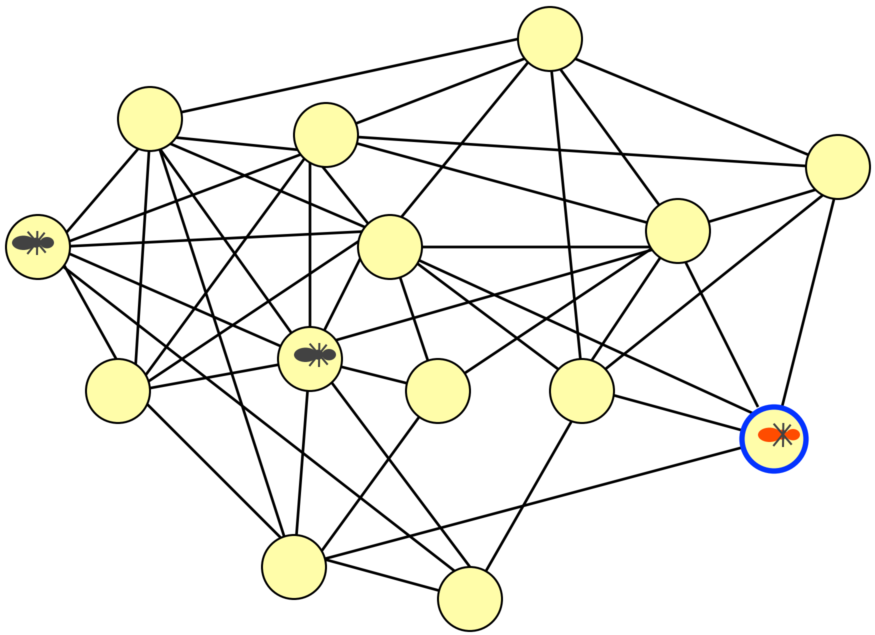

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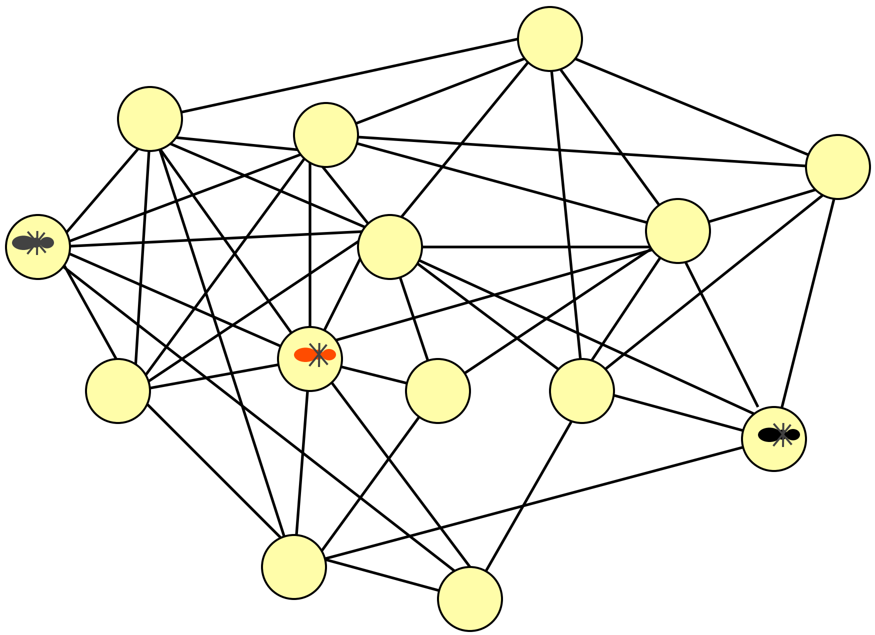

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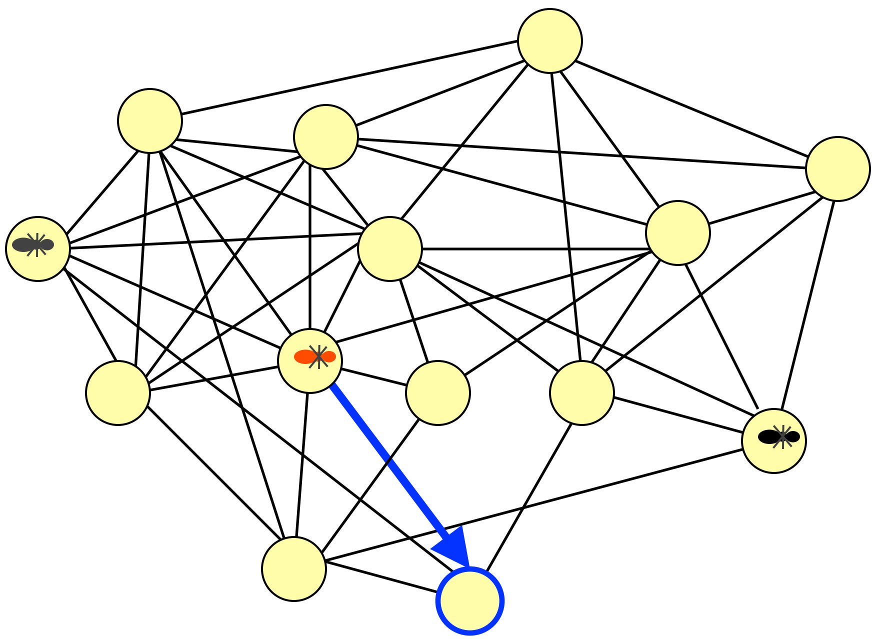

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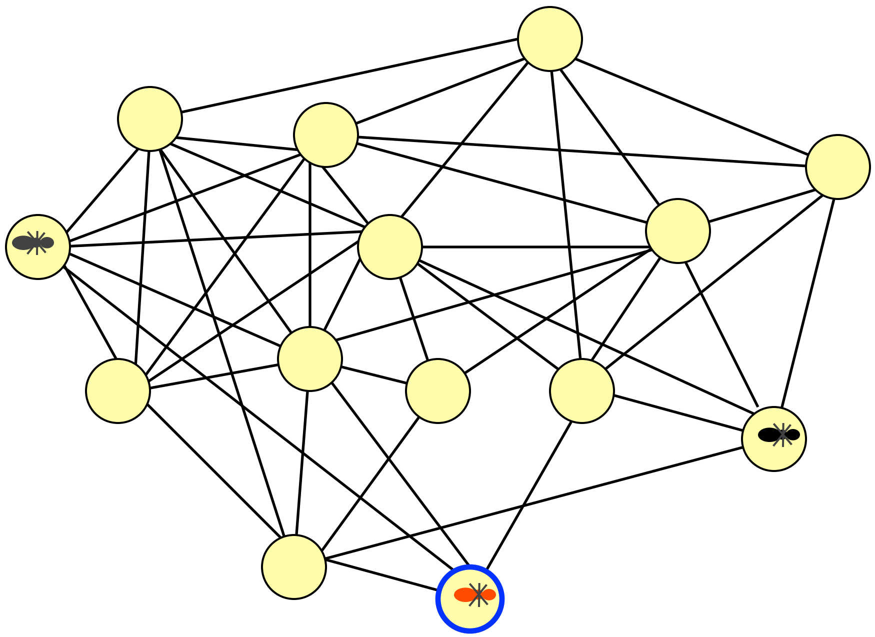

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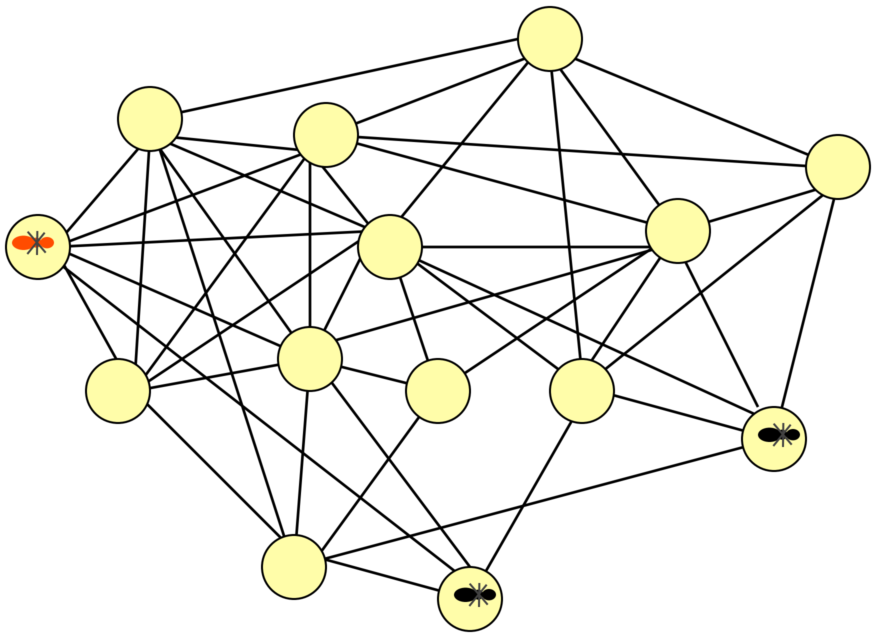

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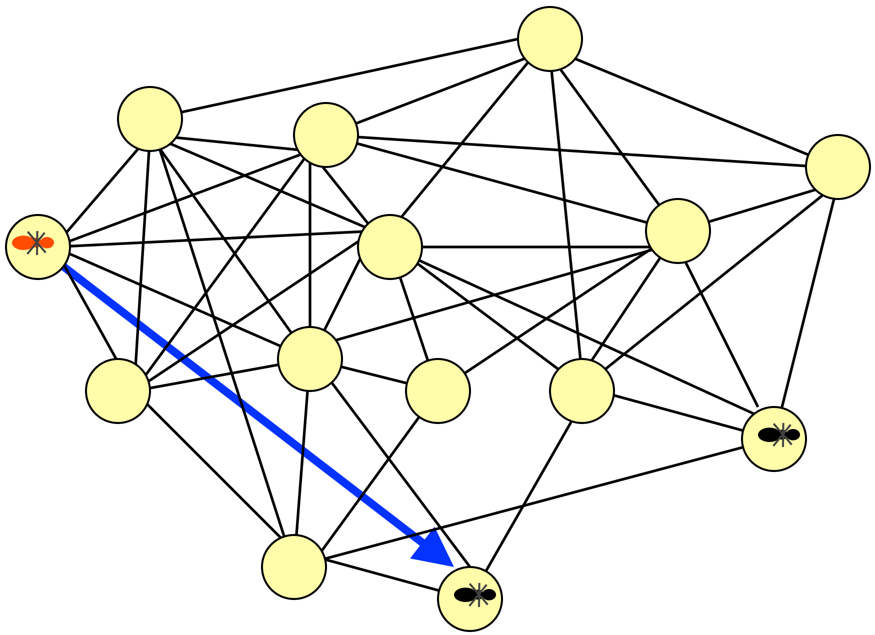

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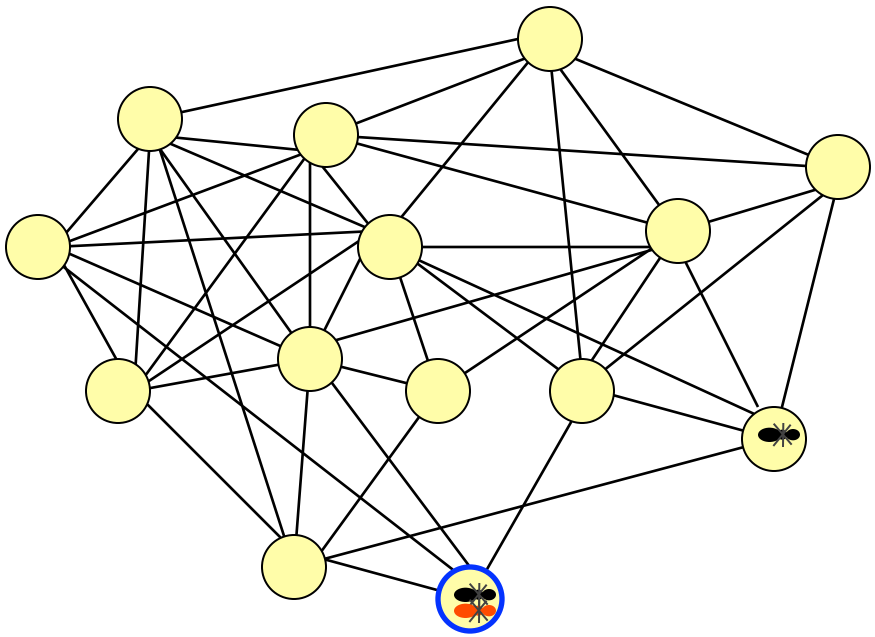
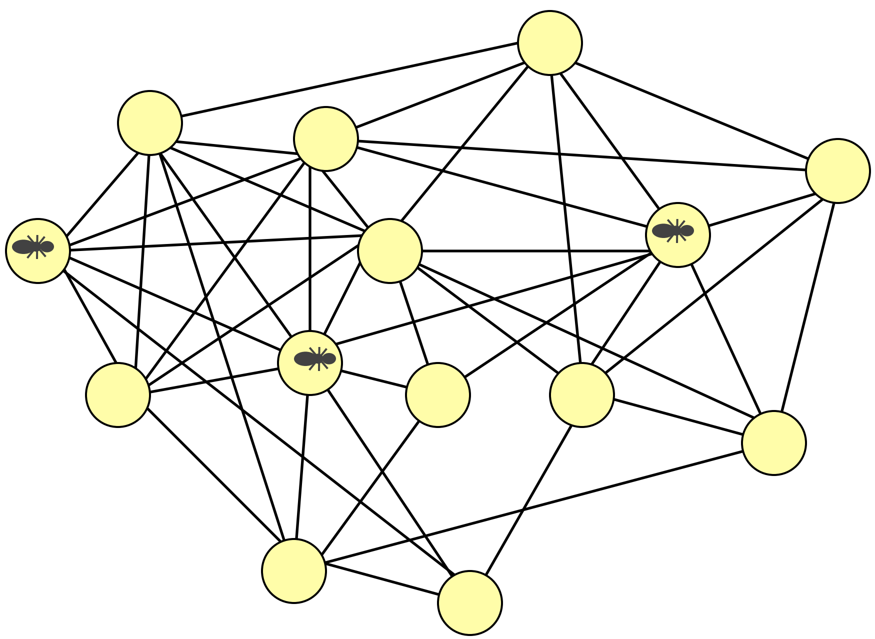


Ants Movements (Parallel)

```
at time t
for each ants a do
   ant a makes a decision where to move in parallel
done

for each ants a do
   ant a moves to its destination in parallel
done
```



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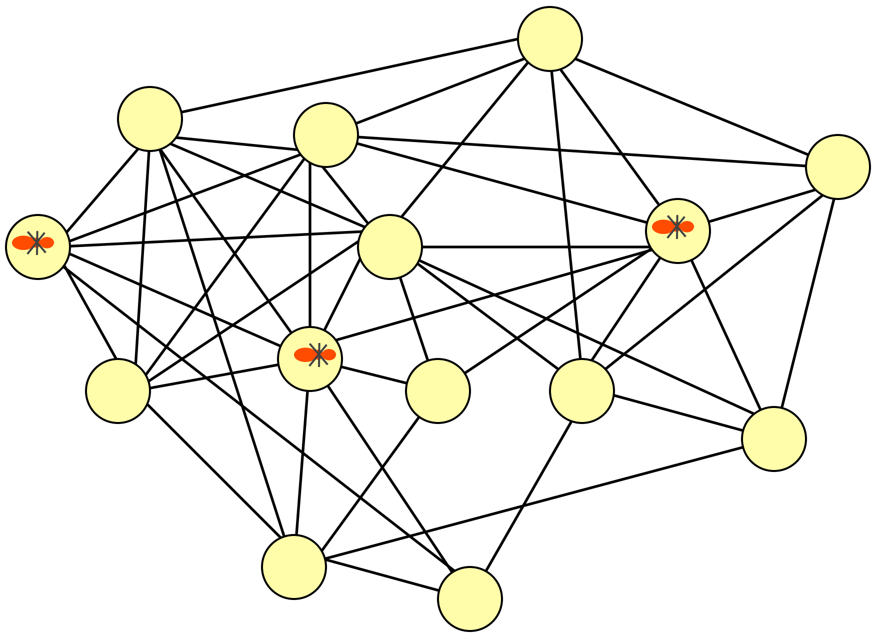

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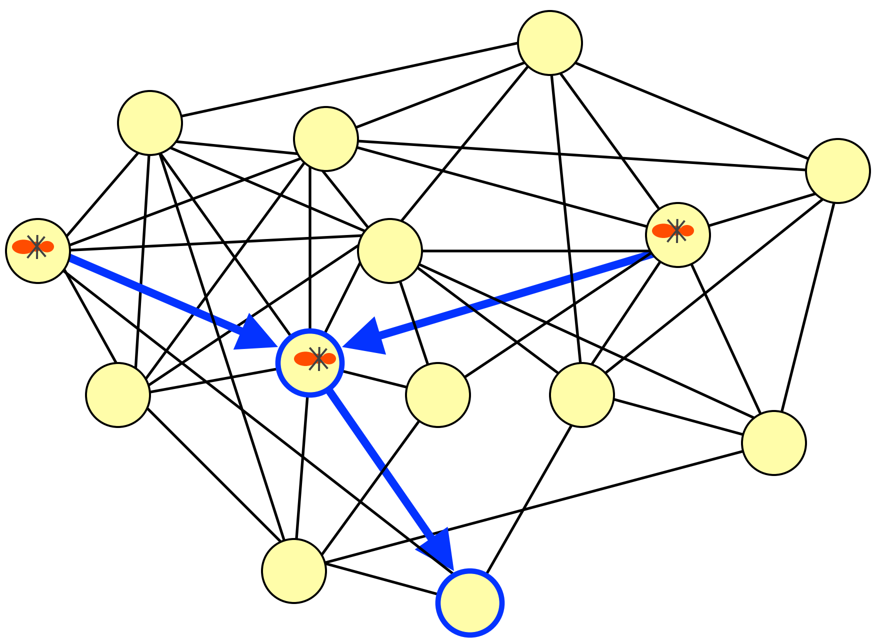

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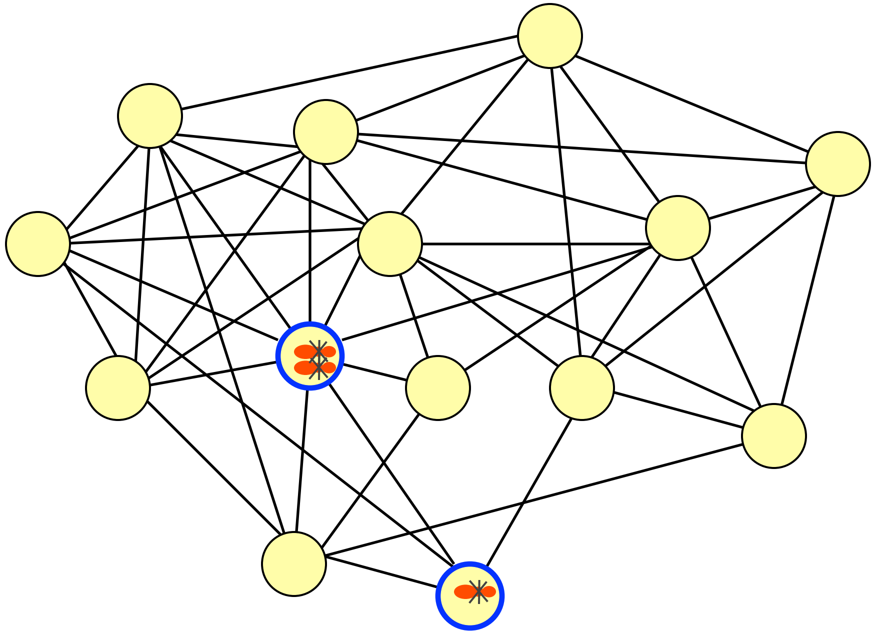

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Testing details

- Implementation details
  - C++
  - 8-core 2.2 GHz Intel, 16 GB RAM, Linux

- Benchmark Details
  - 120 Benchmark instances
    - 20+ vertices to 4000+ vertices
    - 200+ edges to 7+ million edges
  - 100 runs per instance

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	19.59 / 0.55	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

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brock200_1	21	21	**19.53 / 0.52**	19.57 / 0.57	19.59 / 0.55	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	**19.57 / 0.57**	19.59 / 0.55	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	**19.59 / 0.55**	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	19.59 / 0.55	**19.68 / 0.53**	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	19.59 / 0.55	19.68 / 0.53	**19.59 / 0.51**
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

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Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	**19.53 / 0.52**	**19.57 / 0.57**	**19.59 / 0.55**	**19.68 / 0.53**	**19.59 / 0.51**
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
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c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
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mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
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Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	19.59 / 0.55	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
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Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

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			C=1	C=2	C=4	C=6	C=8
Small instances							
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brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
frb56-25-1	56	48	44.84 / 0.91	44.91 / 0.75	45.07 / 0.89	45.05 / 0.79	45.14 / 0.93
frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
keller6	59	48	43.27 / 1.45	43.19 / 1.47	43.74 / 1.30	43.71 / 1.34	43.57 / 1.31
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

Experimental Results: Solution Quality

Graph	Opt	ABO-MC Best	Solution Avg / StdDev when run on C cores				
			C=1	C=2	C=4	C=6	C=8
Small instances							
brock200_1	21	21	19.53 / 0.52	19.57 / 0.57	19.59 / 0.55	19.68 / 0.53	19.59 / 0.51
brock400_2	29	25	22.69 / 0.72	22.84 / 0.61	22.83 / 0.62	22.85 / 0.65	22.79 / 0.67
johnson8-2-4	14	14	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00	14.00 / 0.00
johnson16-2-4	8	8	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00	8.00 / 0.00
Medium instances							
c2000.5	16	16	13.98 / 0.55	14.09 / 0.38	14.16 / 0.46	14.18 / 0.46	14.17 / 0.43
frb53-24-1	53	45	42.41 / 0.84	42.31 / 0.88	42.51 / 0.78	42.55 / 0.73	42.56 / 0.86
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frb59-26-1	59	51	47.66 / 0.84	47.90 / 0.78	47.84 / 0.86	47.80 / 0.82	47.99 / 0.89
Large instances							
c4000.5	18	16	14.95 / 0.50	15.15 / 0.48	15.00 / 0.32	15.10 / 0.30	15.10 / 0.44
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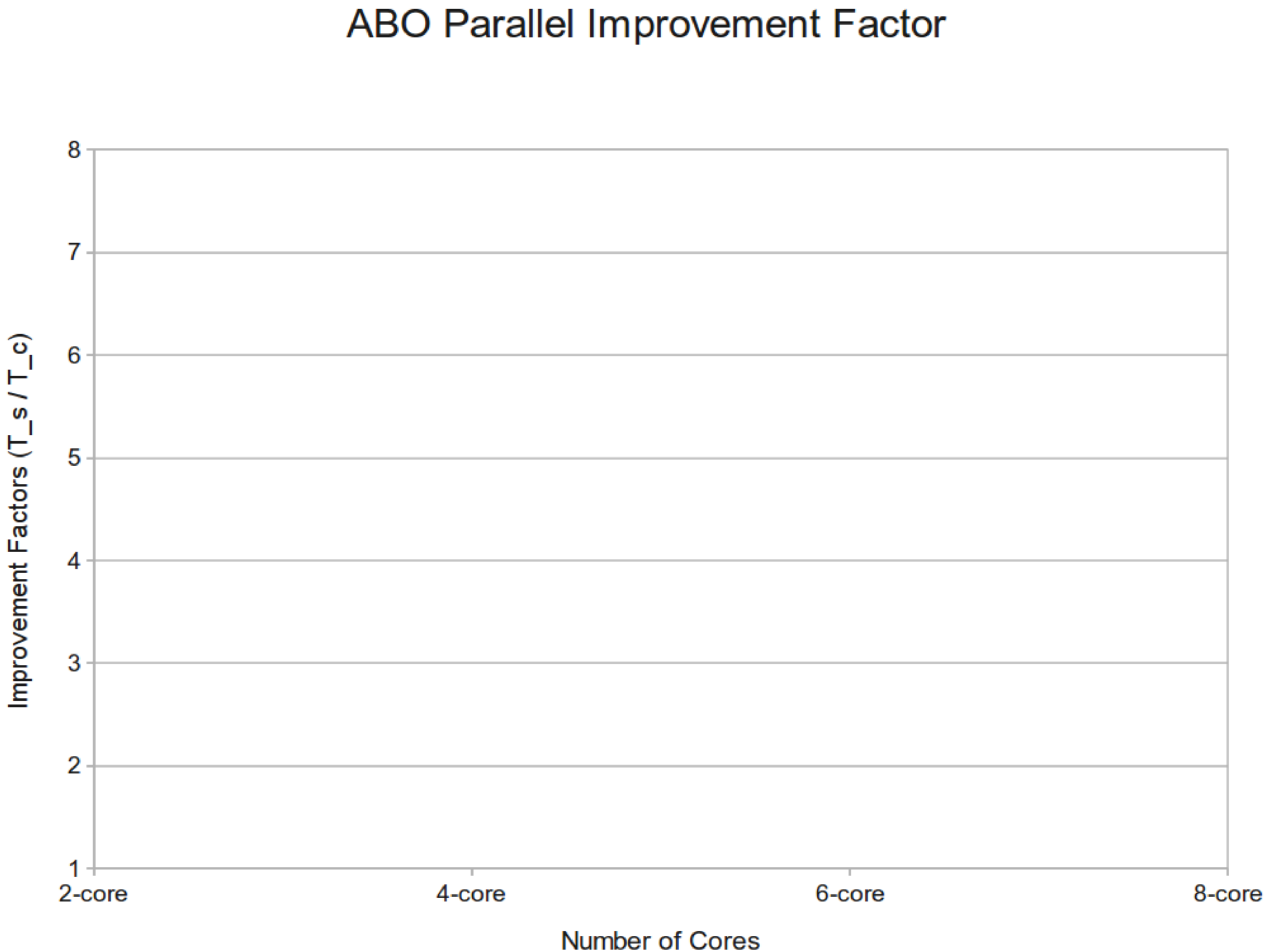
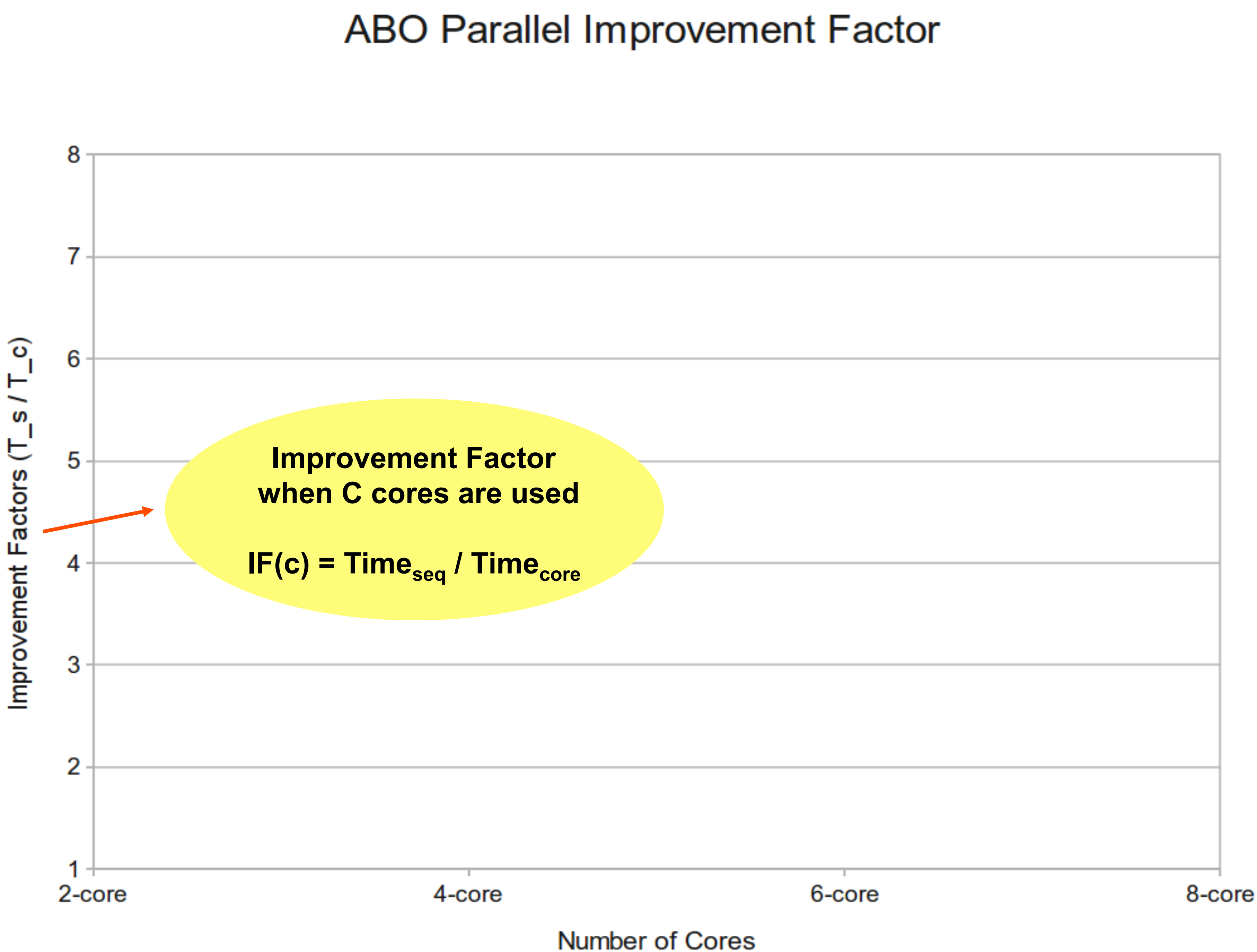
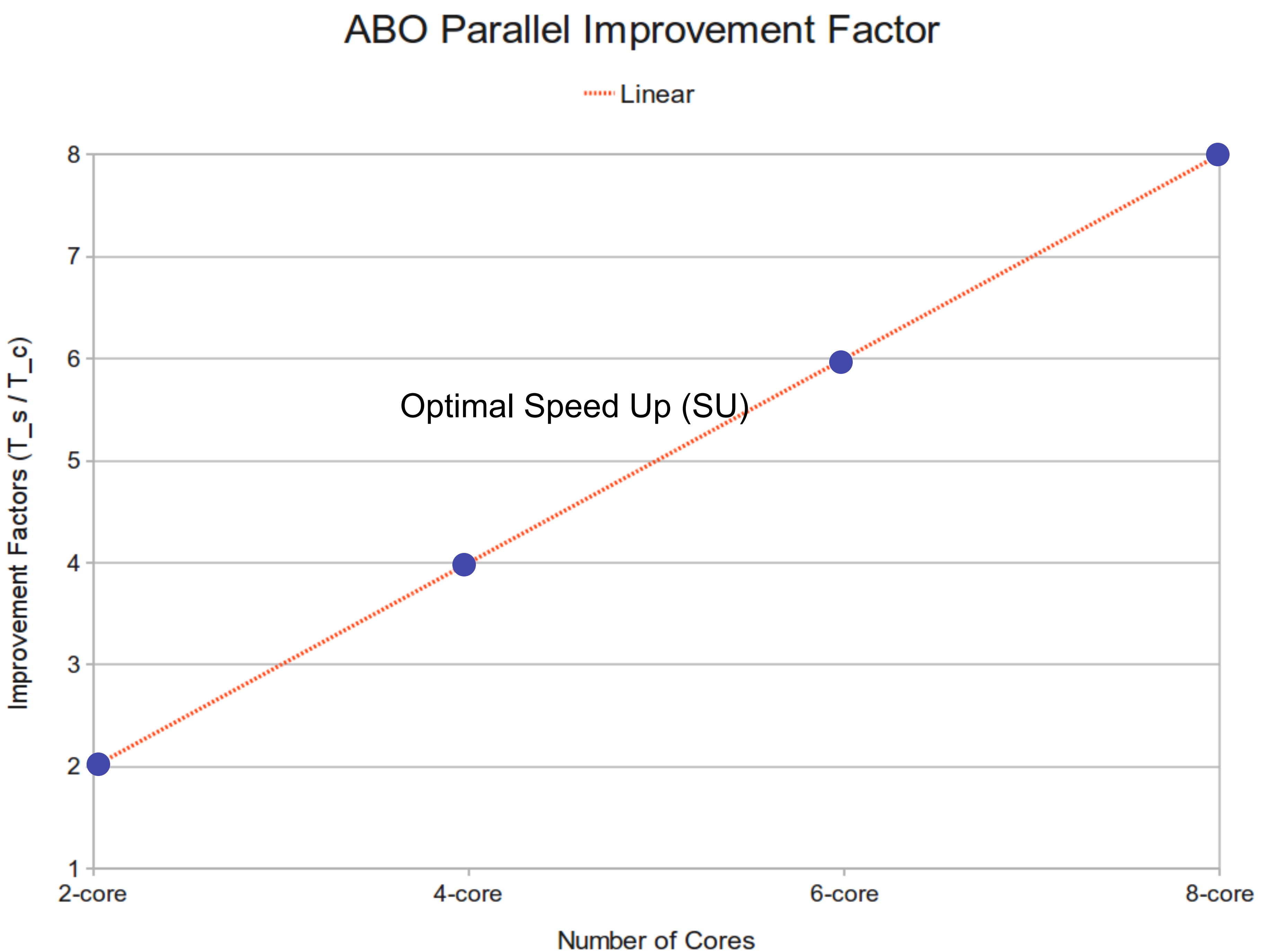
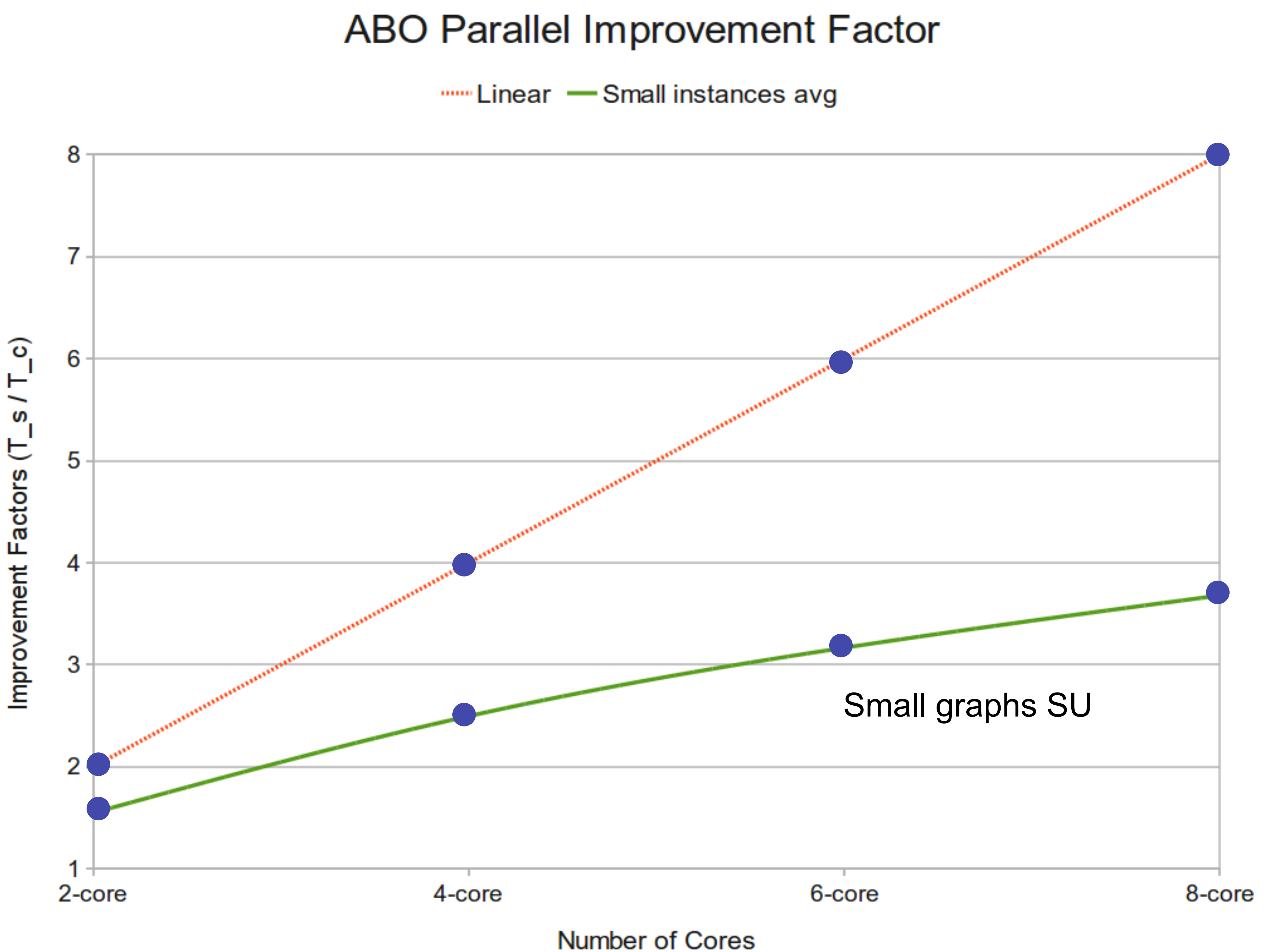
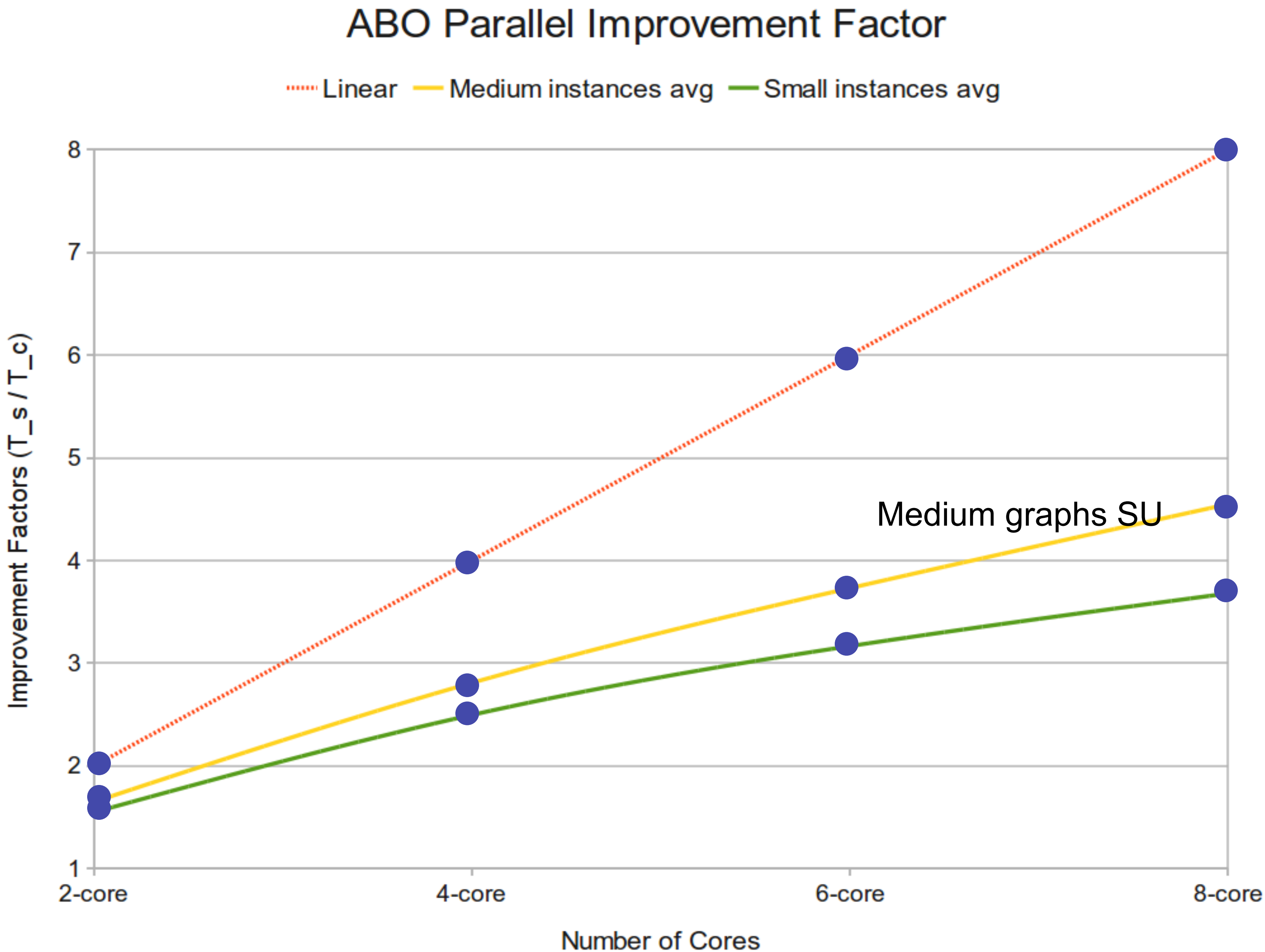
mann_a81	1100	1096	1096.0 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0	1096.00 / 0.0
frb100-40	100	82	77.06 / 1.05	77.37 / 1.04	77.55 / 0.95	77.66 / 1.13	77.63 / 1.08

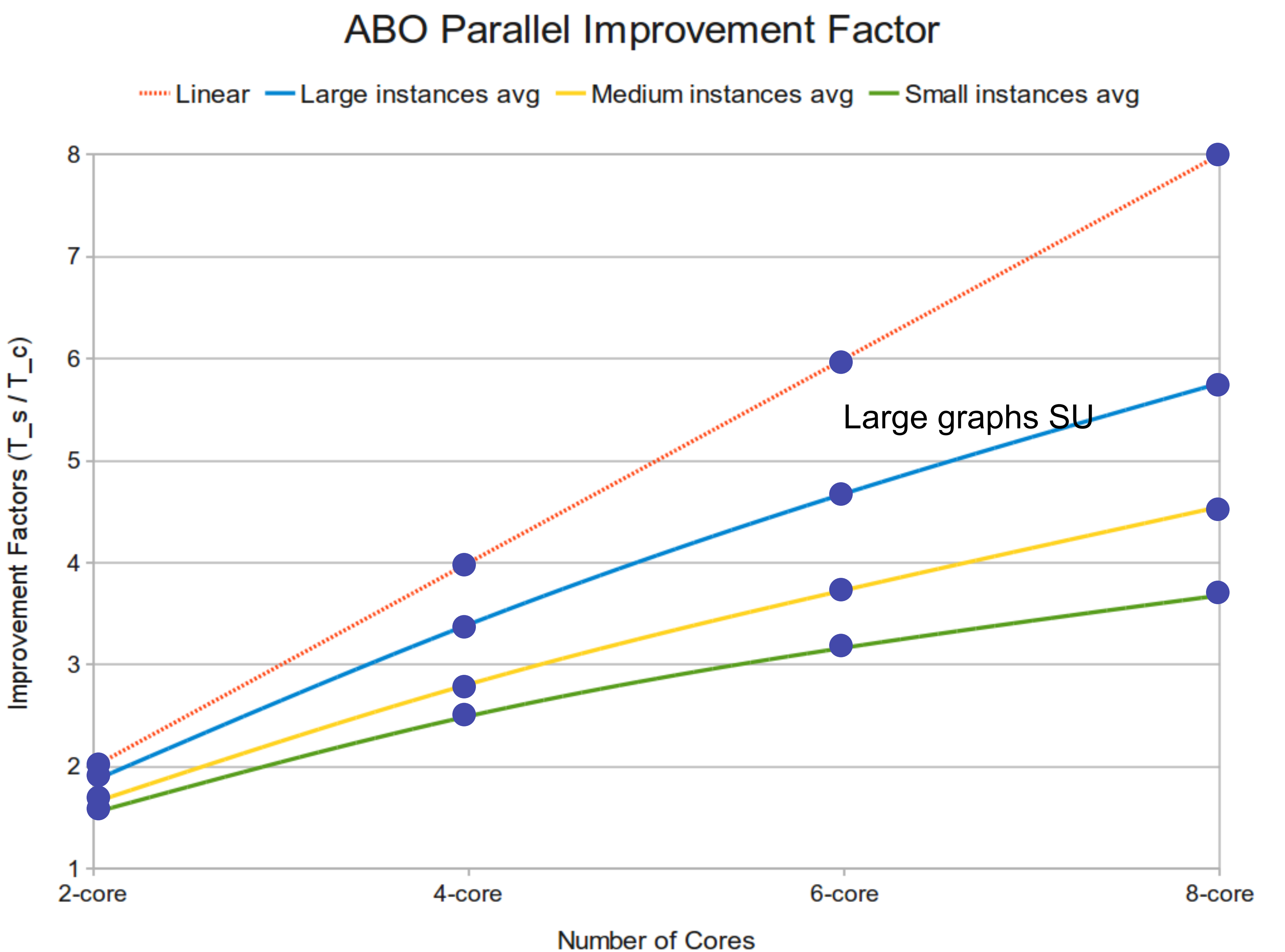

ABO Parallel Improvement Factor
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
2-core
4-core
6-core
8-core
Number of Cores


ABO Parallel Improvement Factor
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
2-core
4-core
6-core
8-core
Number of Cores
Improvement Factor
when C cores are used
IF(c) = $Time_{seq}$ / $Time_{core}$


ABO Parallel Improvement Factor
Linear
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
Optimal Speed Up (SU)
2-core
4-core
6-core
8-core
Number of Cores


ABO Parallel Improvement Factor
Linear
Small instances avg
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
2-core
4-core
6-core
8-core
Number of Cores
Small graphs SU


ABO Parallel Improvement Factor
Linear
Medium instances avg
Small instances avg
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
Medium graphs SU
2-core
4-core
6-core
8-core
Number of Cores



ABO Parallel Improvement Factor
Linear
Large instances avg
Medium instances avg
Small instances avg
Improvement Factors (T_s / T_c)
8
7
6
5
4
3
2
1
Large graphs SU
2-core
4-core
6-core
8-core
Number of Cores

Improvement Factor in the Running Time, IF(c)

Graph	Vertices / Edges	95 % Confidence Interval for IF(c)			
		C=2	C=4	C=6	C=8
Small instances					
hamming6-4	64 / 704	(1.071, 1.231)	(1.000, 1.364)	(0.938, 1.250)	(0.938, 1.231)
johnson8-2-4	28 / 210	(0.833, 1.200)	(0.714, 0.857)	(0.625, 0.833)	(0.625, 0.750)
Medium instances					
c-fat500-5	500 / 23191	(1.504, 1.560)	(2.083, 2.457)	(2.735, 3.116)	(2.804, 3.398)
gen400_p0.9_55	400 / 71820	(1.523, 1.569)	(2.468, 2.646)	(3.118, 3.542)	(3.752, 4.157)
Large instances					
keller6	3361 / 4619898	(1.908, 1.983)	(3.535, 3.672)	(4.904, 5.096)	(6.111, 6.320)
hamming10-2	1024 / 518656	(1.805, 1.890)	(3.268, 3.382)	(4.473, 4.632)	(5.494, 5.703)

Improvement Factor in the Running Time, IF(c)

Graph	Vertices / Edges	95 % Confidence Interval for IF(c)			
		C=2	C=4	C=6	C=8
Small instances					
hamming6-4	64 / 704	(1.071, 1.231)	(1.000, 1.364)	(0.938, 1.250)	(0.938, 1.231)
johnson8-2-4	28 / 210	(0.833, 1.200)	(0.714, 0.857)	(0.625, 0.833)	(0.625, 0.750)
Medium instances					
c-fat500-5	500 / 23191	(1.504, 1.560)	(2.083, 2.457)	(2.735, 3.116)	(2.804, 3.398)
gen400_p0.9_55	400 / 71820	(1.523, 1.569)	(2.468, 2.646)	(3.118, 3.542)	(3.752, 4.157)
Large instances					
keller6	3361 / 4619898	(1.908, 1.983)	(3.535, 3.672)	(4.904, 5.096)	$\boxed{(6.111, 6.320)}$
hamming10-2	1024 / 518656	(1.805, 1.890)	(3.268, 3.382)	(4.473, 4.632)	(5.494, 5.703)

Outline

- Introduction
- Sequential and Parallel Framework of Ant-Based Optimization (ABO)
- An Example – The Maximum Clique Problem
- Experimental Results
- Conclusion

Conclusion

- **Ant-Based Optimization**
  - is different from traditional ACO
  - can be easily converted to parallelism to take advantage of *shared-memory* architecture
  - produces satisfactory run-time improvement while maintaining solution quality

- **Future work**
  - A hybrid shared and distributed memory framework for ABO