Automatic Invariant Generation and Program Repair

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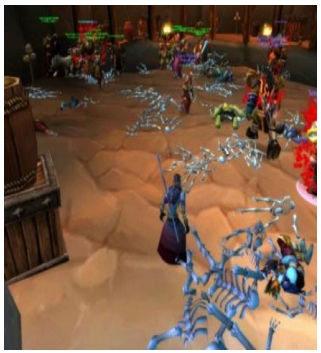
The Problem

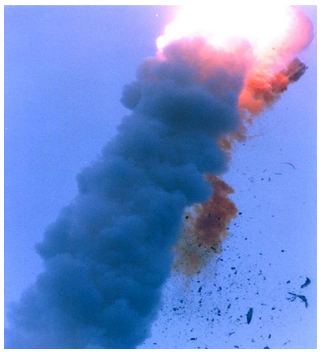
SOFTWARE BUGS



World of Warcraft bug

Therac-25 machines

X-rays overdose



Ariane-5 rocket self-destructs

North America blackout

The Cost

– Mozilla Developer

"Everyday, almost 300 bugs appear [..] far too many for only the Mozilla programmers to handle."

Software bugs annually cost 0.6% of the U.S GDP and $312 billion to the global economy

Average time to fix a security-critical error: 28 days

Program Analysis

Write Code

Check Code

Program Analysis

Write Code


Check Code


Automated program analysis techniques and tools can decrease debugging time by an average of 26% and $41 billion annually

Program Synthesis


Generate a program that meets a given specification

Program Verification


Check if a program satisfies a given specification

Invariant Generation and Template-based Synthesis

Invariant Generation

```
def intdiv(x, y):
  q = 0
  r = x
  while r ≥ y:
    a = 1
    b = y
    while [??] r ≥ 2b:
      a = 2a
      b = 2b
    r = r - b
    q = q + a
    [??]
    return q
```

- Discover invariant properties at certain program locations
- Answer the question *"what does this program do ?"*

Invariant Generation and Template-based Synthesis

Invariant Generation

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def intdiv(x, y):
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- Discover invariant properties at certain program locations
- Answer the question *"what does this program do ?"*

Template-based Synthesis

```
def intdiv(x, y):
  q = 0
  r = x
  while r [**] y:
    a = 1
    b = [**]
    while r ≥ 2b:
      a = [**]
      b = 2b
    r = r - b
    q = q + a

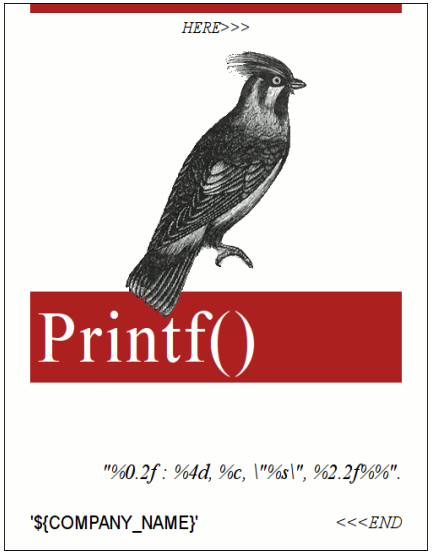
  return [**]
```

- Create code under specific templates from partially completed programs
- Can be used for automatic program repair

Outline

How We Analyze Programs

How We Analyze Programs




```
File  Edit  Options  Buffers  Tools  Help

def intdiv(x, y):

    q = 0
    r = x
    while r >= y:
        a = 1
        b = y

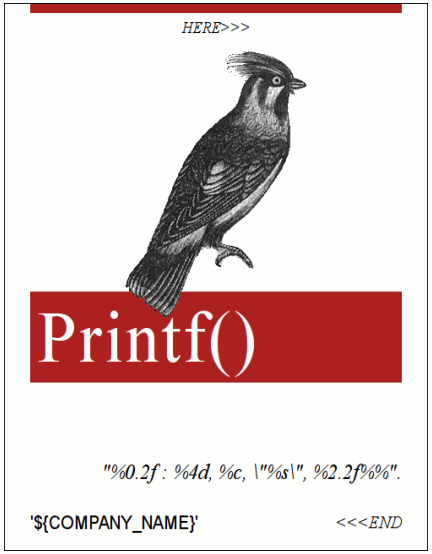
        while r >= 2*b:
            a = 2 * a
            b = 2 * b
        r = r - b
        q = q + a

    print "x %d, y %d, q %d, r %d" %(x,y,q,r)
    return q,r
```

```
x   0,  y   1,  q    0,  r  0
x   1,  y   1,  q    1,  r  0
x   1,  y   5,  q    0,  r  1
x   1,  y  10,  q    0,  r  1
x   3,  y   1,  q    3,  r  0
x   3,  y   4,  q    0,  r  3
x   3,  y   7,  q    0,  r  3
x   8,  y   1,  q    8,  r  0
x   8,  y   2,  q    4,  r  0
x   8,  y   9,  q    0,  r  8
x   8,  y  10,  q    0,  r  8
x  15,  y   1,  q   15,  r  0
x  15,  y   5,  q    3,  r  0
x  15,  y   7,  q    2,  r  1
x  20,  y   2,  q   10,  r  0
x  20,  y   7,  q    2,  r  6
x  20,  y  10,  q    2,  r  0
x 100,  y   1,  q  100,  r  0
x 100,  y   5,  q   20,  r  0
x 100,  y  10,  q   10,  r  0
```

```
-U:---  intdiv.py      All (18,0)     (Python)--5:
-U:---  intdiv.traces  All (21,0)
```

How We Analyze Programs




```
File  Edit  Options  Buffers  Tools  Help

def intdiv(x, y):

    q = 0
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            a = 2 * a
            b = 2 * b
        r = r - b
        q = q + a

    print "x %d, y %d, q %d, r %d" %(x,y,q,r)
    return q,r

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```

```
x   0,  y   1,  q    0,  r  0
x   1,  y   1,  q    1,  r  0
x   1,  y   5,  q    0,  r  1
x   1,  y  10,  q    0,  r  1
x   3,  y   1,  q    3,  r  0
x   3,  y   4,  q    0,  r  3
x   3,  y   7,  q    0,  r  3
x   8,  y   1,  q    8,  r  0
x   8,  y   2,  q    4,  r  0
x   8,  y   9,  q    0,  r  8
x   8,  y  10,  q    0,  r  8
x  15,  y   1,  q   15,  r  0
x  15,  y   5,  q    3,  r  0
x  15,  y   7,  q    2,  r  1
x  20,  y   2,  q   10,  r  0
x  20,  y   7,  q    2,  r  6
x  20,  y  10,  q    2,  r  0
x 100,  y   1,  q  100,  r  0
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x 100,  y  10,  q   10,  r  0

-U:---  intdiv.traces   All (21,0)
```

```
File  Edit  Options  Buffers  Tools  Python  Help

def intdiv(x, y):
    assert y != 0

    # .. compute result ..

    assert r >= 0
    assert x >= q

    return q,r

--:---  intdiv.py      All (11,0)
```

UDACITY – Software Testing course

"GCC: 9000 assertions,
LLVM: 13,000 assertions [..]
1 assertion per 110 loc"

Program Invariants

"invariants are asserted properties, such as relations among variables, at certain locations in a program"


```
assert(x == 2*y);
assert(0 <= idx < |arr|);
```

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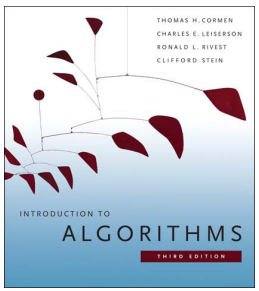
```

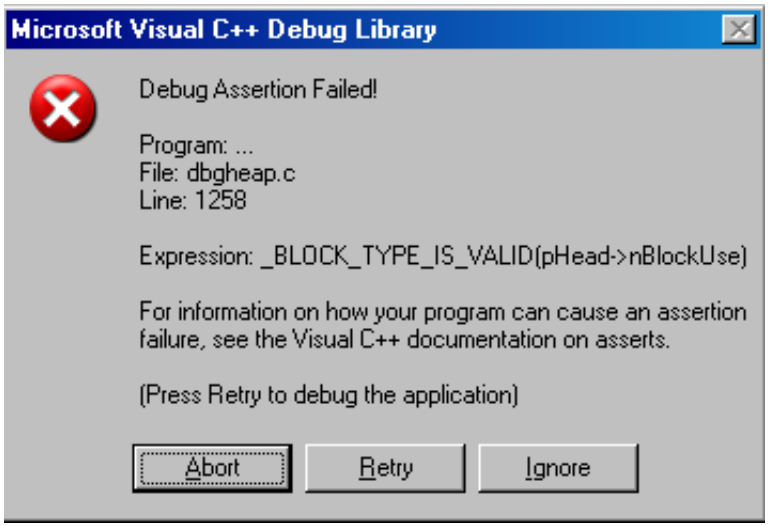

```
int getDateOfMonth(int m){
    /*pre: 1 <= m <= 12*/
    ..

    /*post: 0 <= result <= 31*/
}
```



"a loop invariant is a condition that is true on entry into a loop and is guaranteed to remain true on every iteration of the loop [..]"

Uses of Invariants



- Understand and verify programs
- Formal proofs
- Debug (locate errors)
- Documentations

Approaches to Finding Invariants

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```
def intdiv(x,y):
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    r = x
    while r ≥ y:
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        r = r - b
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    return q,r
```

Static Analysis

- Analyze source code directly
- Pros: results guaranteed on any input, proofs of correctness or errors
- Cons: computationally intensive, deduce simple invariants

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```

x	y	q	r
0	1	0	0
1	1	1	0
3	4	0	3
8	1	8	0
15	5	3	0
20	2	10	0
100	1	100	0
	⋮	⋮	

Static Analysis

- Analyze source code directly
- Pros: results guaranteed on any input, proofs of correctness or errors
- Cons: computationally intensive, deduce simple invariants

Dynamic Analysis

- Analyze program traces
- Pros: fast, source code not required
- Cons: results depend on traces, might not hold for all runs

Three Challenging Classes of Invariants

Three Challenging Classes of Invariants

1. Polynomials
   - Relations over numerical variables
     $x = 3.5,\ x = 2y,\ x = qy + r,\ x^2 \geq y + z^3, |\texttt{arr}| \geq \texttt{idx} \geq 0,\ \ldots$
   - *Nonlinear* polynomials: required in scientific and engineering applications, implemented in Astrée analyzer for Airbus systems

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   - *Nonlinear* polynomials: required in scientific and engineering applications, implemented in Astrée analyzer for Airbus systems
2. Disjunctions
   - Represent branching behaviors in programs, e.g., search, sort
     $a \vee b\ (i = \texttt{even}) \Rightarrow (A[i] = B[i]),\ \texttt{if}\ (a = b)\ \texttt{then}\ (c = 5)\ \texttt{else}\ (c = d + 7)$
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   - Many loops in OpenSSH require disjunctive invariants
3. Arrays
   - Relations among (multi-dimensional) array variables
     $A[i] = B[i],\ A[i][j] = B[i] + 3C[i] - 1.2,\ A[i] = B[C[i]][D[2i]],\ r = f(g(x), h(y, z))$
   - Popular data structure to implement strings, vectors, matrices, memory, ..

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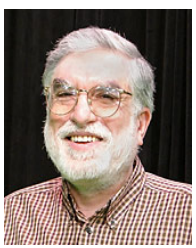
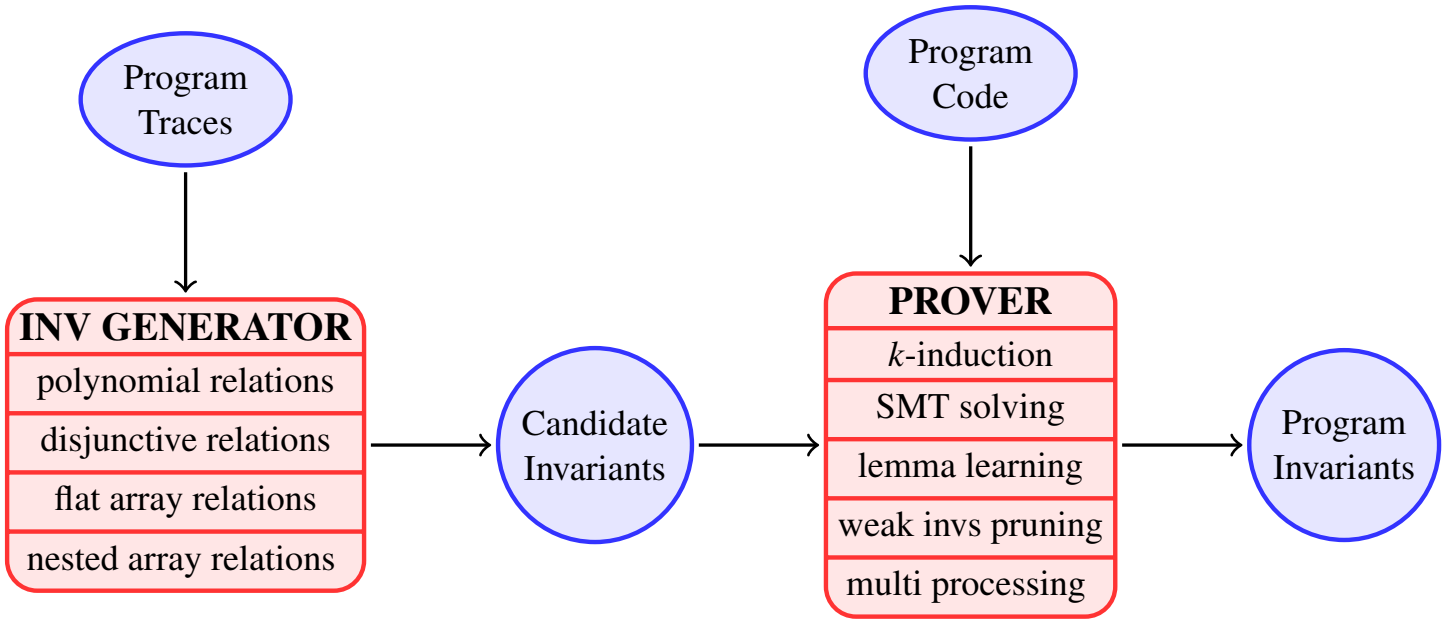
DIG: Dynamic Invariant Generation (TOSEM '14, ICSE '14, ICSE '12)





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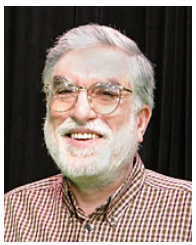

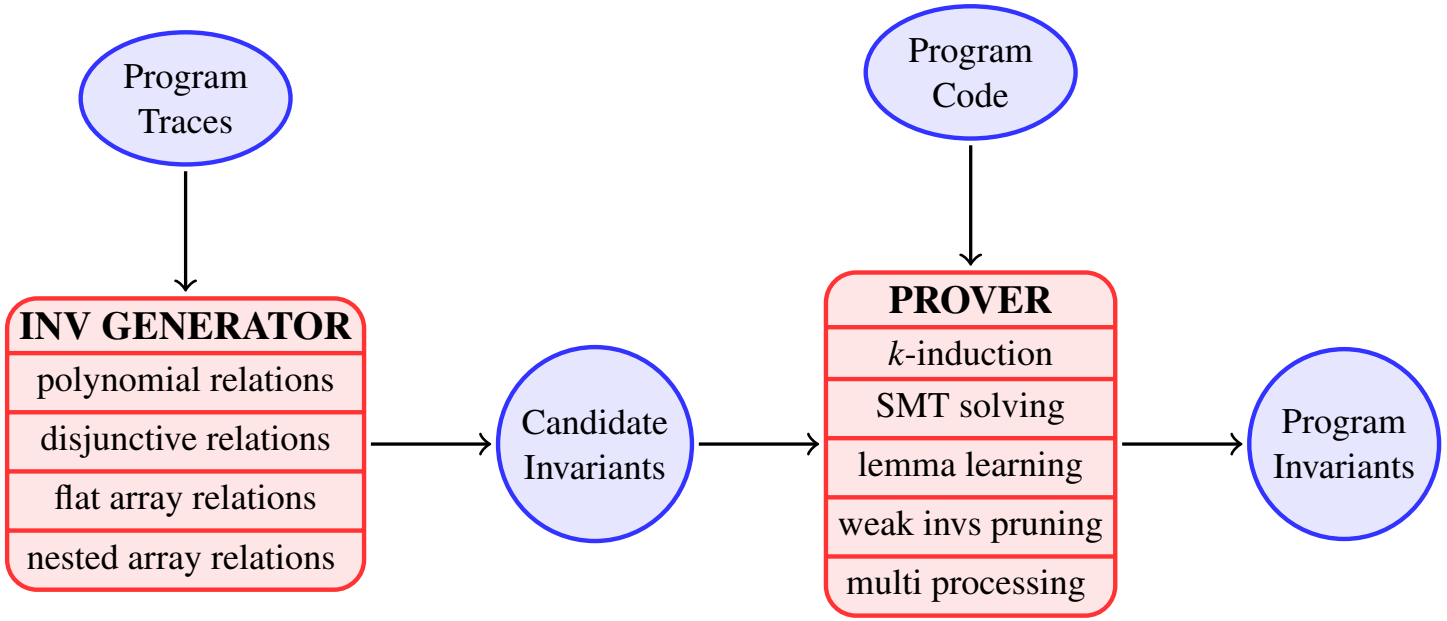

Goal: developing efficient methods to capture precise and provably correct program invariants

- Efficient: reformulate and solve using techniques such as equation solving and polyhedral construction
- Precise: employ expressive templates and infer invariants *directly* from traces
- Sound: integrate theorem proving to formally verify results

Outline

Example: Cohen Integer Division

```
def intdiv(x, y):
    q = 0
    r = x
    while r ≥ y:
        a = 1
        b = y
        while r ≥ 2b:
            [L]
            a = 2a
            b = 2b
        r = r - b
        q = q + a
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```

x	y	a	b	q	r
15	2	1	2	0	15
15	2	2	4	0	15
15	2	1	2	4	7
4	1	1	1	0	4
4	1	2	2	0	4

Invariants at L: $b = ya,\ x = qy + r,\ r \geq 2ya$

Polynomial Relations (ICSE '12)

DIG discovers polynomial relations of the forms

Equalities $c_0 + c_1x_1 + c_2x_n + c_3x_1x_2 + \cdots + c_mx_1^{d_1} \ldots x_n^{d_n} = 0$

Inequalities $c_0 + c_1x_1 + c_2x_n + c_3x_1x_2 + \cdots + c_mx_1^{d_1} \ldots x_n^{d_n} \geq 0, \quad c_i \in \mathbb{R}$

Examples

cubic $z - 6n = 6,\ \frac{1}{12}z^2 - y - \frac{1}{2}z = -1$

extended gcd $\gcd(a, b) = ia + jb$

sqrt $x + \varepsilon \geq y^2 \geq x - \varepsilon$

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Method

- Equalities: solve equations
- Inequalities: construct polyhedra

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4	1	2	2	0	4

Invariants at L: $b = ya,\ x = qy + r,\ r \geq 2ya$

Finding Nonlinear Equations using Equation Solver

x	y	a	b	q	r
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4	1	1	1	0	4
4	1	2	2	0	4

Finding Nonlinear Equations using Equation Solver

- Terms and degrees

$$V = \{r, y, a\};\ \texttt{deg} = 2$$

$$\downarrow$$

$$T = \{1, r, y, a, ry, ra, ya, r^2, y^2, a^2\}$$

x	y	a	b	q	r
15	2	1	2	0	15
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$$T = \{\ldots, \log(r), a^y, \sin(y), \ldots\}$$

x	y	a	b	q	r
15	2	1	2	0	15
15	2	2	4	0	15
15	2	1	2	4	7
4	1	1	1	0	4
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- Nonlinear equation template

$$c_1 + c_2 r + c_3 y + c_4 a + c_5 ry + c_6 ra + c_7 ya + c_8 r^2 + c_9 y^2 + c_{10} a^2 = 0$$

Finding Nonlinear Equations using Equation Solver

- Terms and degrees

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$$\downarrow$$
$$T = \{1, r, y, a, ry, ra, ya, r^2, y^2, a^2\}$$

x	y	a	b	q	r
15	**2**	**1**	2	0	**15**
15	2	2	4	0	15
15	2	1	2	4	7
4	1	1	1	0	4
4	1	2	2	0	4

- Nonlinear equation template

$$c_1+c_2r+c_3y+c_4a+c_5ry+c_6ra+c_7ya+c_8r^2+c_9y^2+c_{10}a^2 = 0$$

- System of *linear* equations

$$\begin{aligned} \text{trace 1} &: \{r = 15, y = 2, a = 1\} \\ \text{eq 1} &: c_1 + 15c_2 + 2c_3 + c_4 + 30c_5 + 15c_6 + 2c_7 + 225c_8 + 4c_9 + c_{10} = 0 \\ &\vdots \end{aligned}$$

Finding Nonlinear Equations using Equation Solver

- Terms and degrees

$$V = \{r, y, a\};\ \texttt{deg} = 2$$
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$$T = \{1, r, y, a, ry, ra, ya, r^2, y^2, a^2\}$$

x	y	a	b	q	r
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- Nonlinear equation template

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- Solve for coefficients c_i

$V = \{x, y, a, b, q, r\};\ \texttt{deg} = 2 \quad \longrightarrow \quad b = ya, x = qy+r$

Geometric Invariant Inference (TOSEM '14)

- Treat trace values as points in multi-dimensional space
- Build a convex hull (polyhedron) over the points
- Representation of a polyhedron: a conjunction of inequalities

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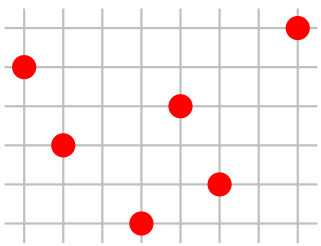
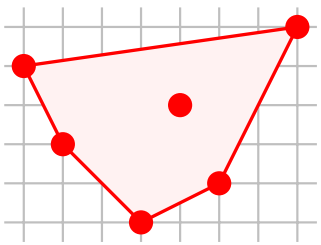
x	y
-2	1
-1	-1
1	-3
2	0
3	-2
5	2

program traces



trace pts in 2D



polygon

$c_1x + c_2y \geq c$

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x	y
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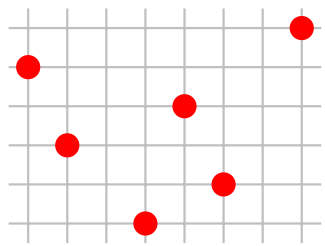
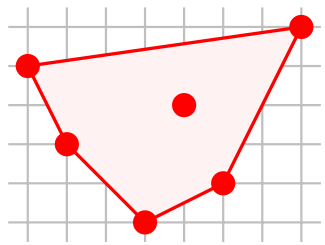
-1	-1
1	-3
2	0
3	-2
5	2

program traces



trace pts

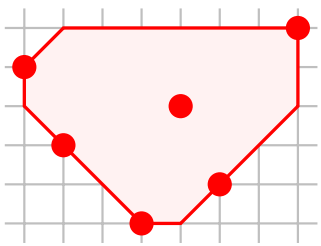
in 2D



polygon

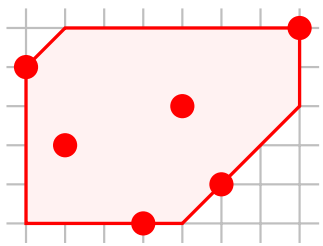
$c_1x + c_2y \geq c$

- Support simpler shapes (decreasing precision, increasing efficiency)



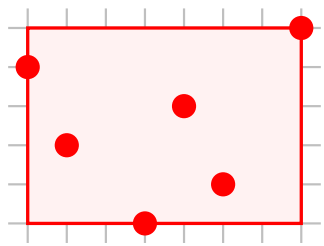
octagon

$\pm x \pm y \geq c$



zone

$x - y \geq c$



box

$\pm x, y \geq c$

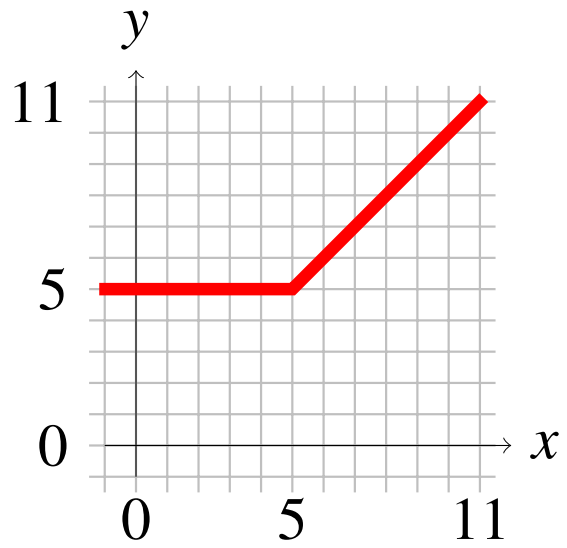
Outline

Example: Disjunctive Invariants

```
def ex(x):
  y = 5
  if x > y: x = y
  while[L] x ≤ 10:
    if x ≥ 5:
      y = y + 1
    x = x + 1

  assert y ≡ 11
```

x	y
-1	5
⋮	⋮
5	5
6	6
⋮	⋮
11	11



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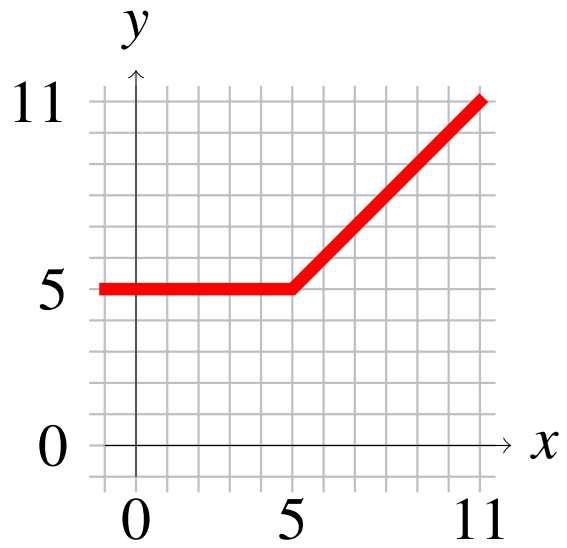

$$L : (x < 5 \wedge 5 = y) \ \vee \ (x \geq 5 \wedge x = y), 11 \geq x$$

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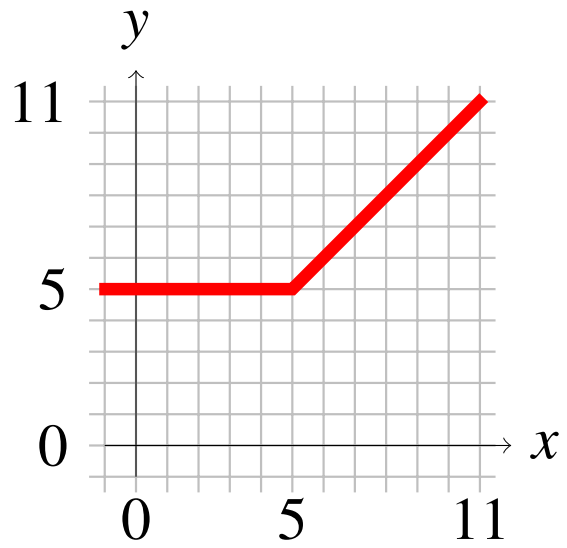

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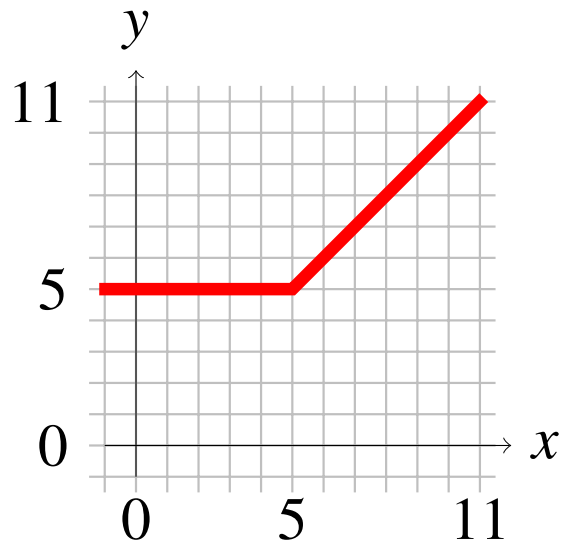

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$\longleftrightarrow$ if $0 > x - 5$ then $0 = y - 5$ else $x - 5 = y - 5$

$\max(0, x - 5) = y - 5$

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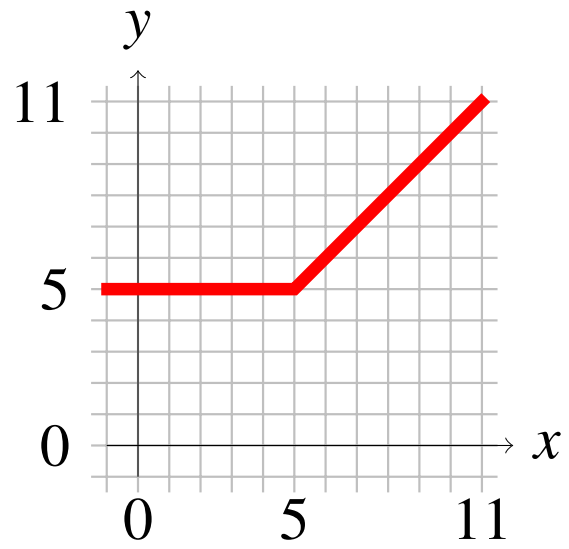

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a linear relation .. in max-plus algebra

Max-plus Algebra

Max-plus Algebra

	Linear	**Max-plus**
Domain	$\mathbb{R}$	$\mathbb{R} \cup \{-\infty\}$
Addition	$+$	max
Multiplication	$\times$	$+$
Zero elem	0	$-\infty$
Unit elem	1	0
Relation form	$c_0 + c_1 t_1 + \cdots + c_n t_n \geq 0$	$\max(c_0, c_1 + t_1, \ldots, c_n + t_n) \geq \max(d_0, d_1 + t_1, \ldots, d_n + t_n)$

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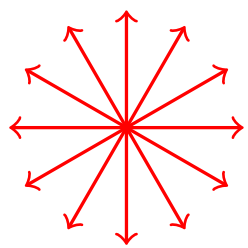
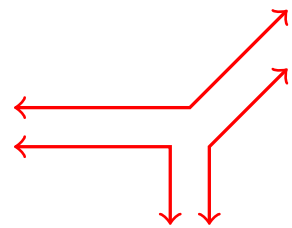
Line shapes		

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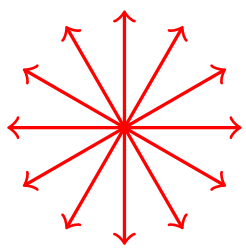
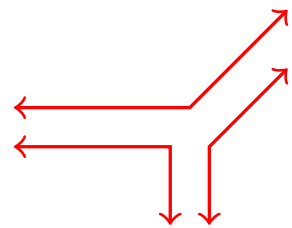
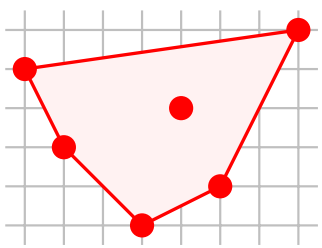
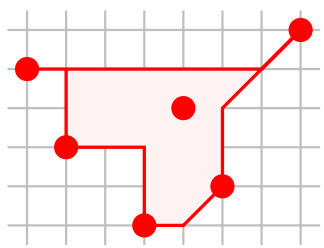
Line shapes		
Convex hull		

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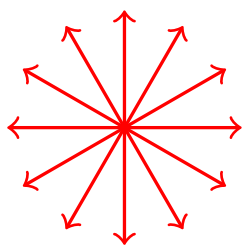
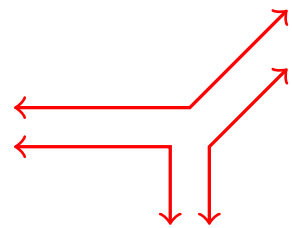
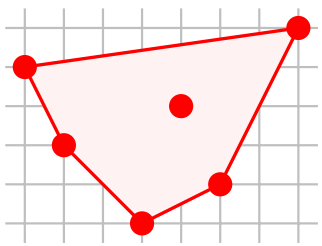
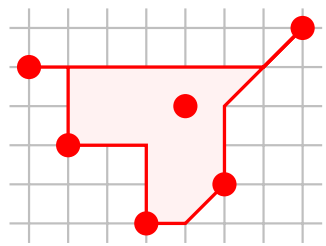
Line shapes		
Convex hull		

Not convex in classical sense!

Disjunctive Invariants (ICSE '14)

DIG discovers disjunctive relations of the max-plus form

$$\max(c_0, c_1 + t_1, \ldots, c_n + t_n) \geq \max(d_0, d_1 + t_1, \ldots, d_n + t_n)$$

Examples

$$\begin{array}{lcccc} z = \max(x, y) & \equiv & (x < y \wedge z = x) & \vee & (x \geq y \wedge z = y) \\ \text{strncpy}(s, d, n) & \equiv & (n \geq |s| \wedge |d| = |s|) & \vee & (n < |s| \wedge |d| \geq n) \end{array}$$

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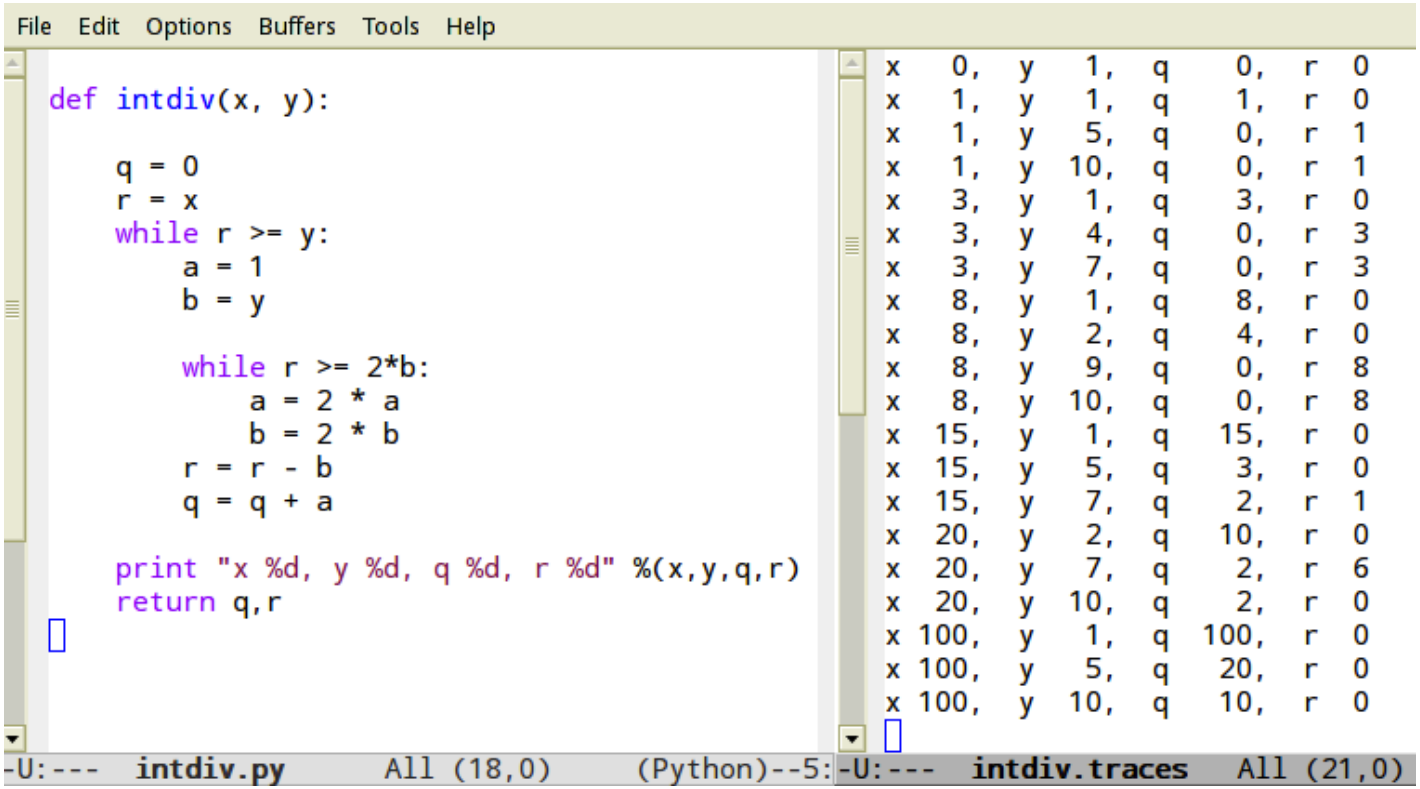
Method

- Use terms to express variables
- Build a max-plus convex polyhedron and extract facets
- Introduce simpler max-plus shapes for lower computational complexity

Outline

Spurious Invariants

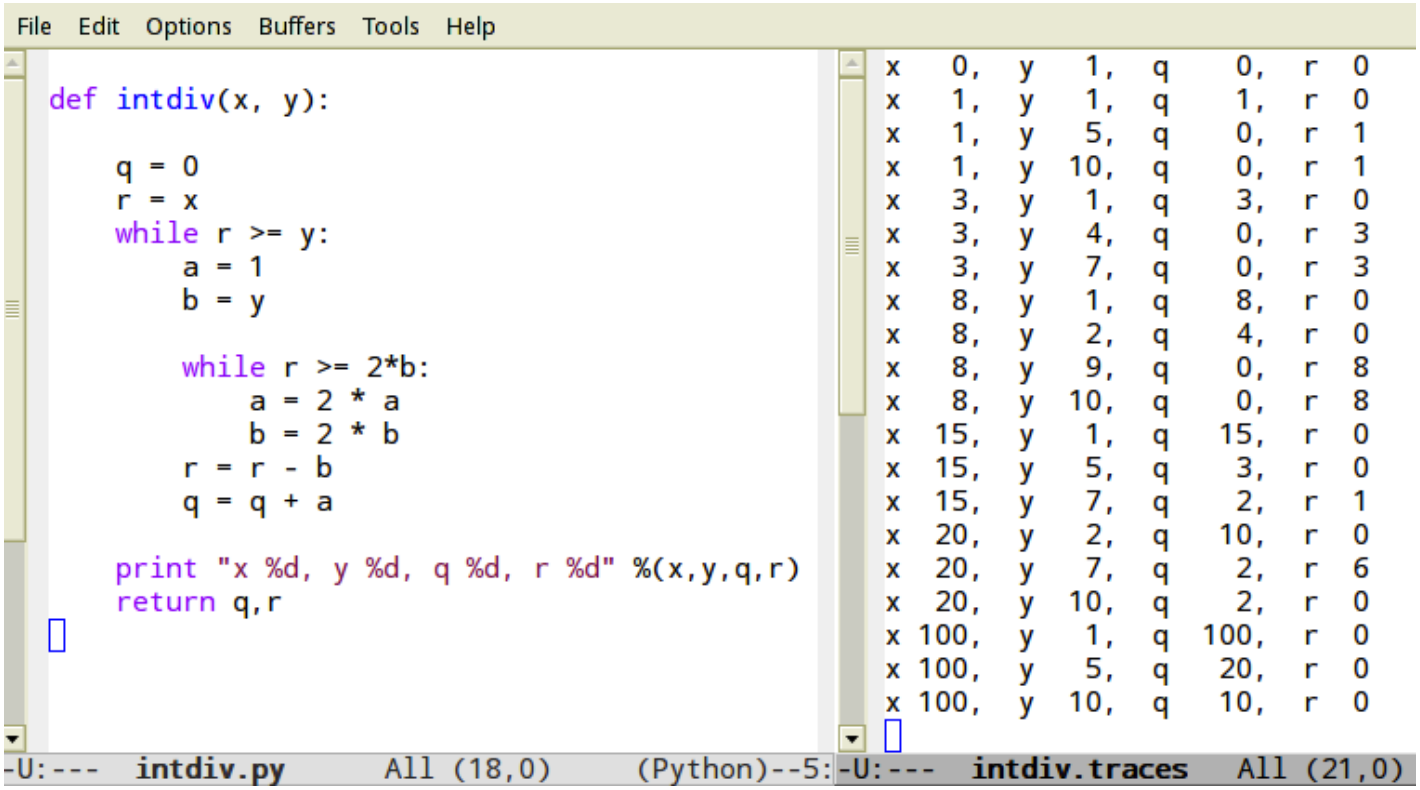


Valid results

- x, y, q, r are integers
- $r \geq 0$
- $x = q * y + r$

$\vdots$

Spurious Invariants



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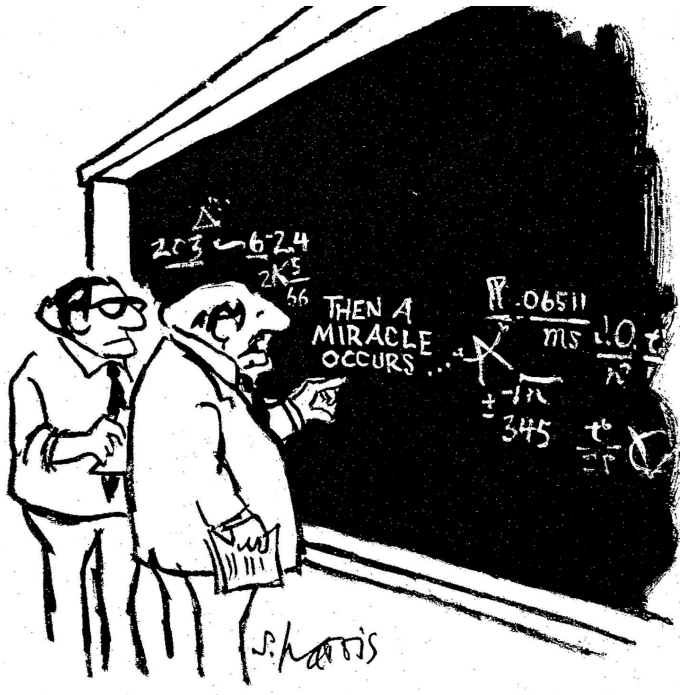
Spurious results

- $100 \geq x \geq 0$
- $10 \geq y \geq 1$
- $100 \geq q - r \geq -8$

$\vdots$

KIP: k-Induction Prover (ICSE '14)



KIP, an automatic theorem prover, for verifying invariants

1. Implement *k*-induction
2. Employ powerful constraint solving
3. Learn new lemmas
4. Use multi-processing
5. Identify strongest invariants

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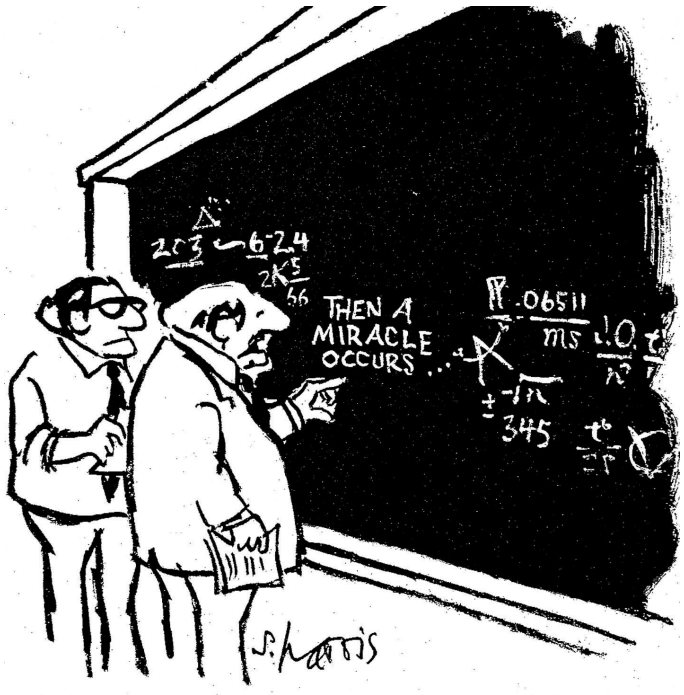

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DIG + KIP ≡ hybridization of dynamic and static analysis

Idea: generating (or solving) is typically harder than checking

- Dynamically infer invariants from program traces
- Statically prove candidate results using program source

An efficient and sound technique to generate complex program invariants

Experimental Results

Benchmarks

- Nonlinear test suite: 27 programs require nonlinear invariants
- Disjunctive testsuite: 14 programs require disjunctive invariants

Setup

- Implemented in SAGE/Python (with Z3 backend solver)
- Invariants obtained at loop entrances and program exits

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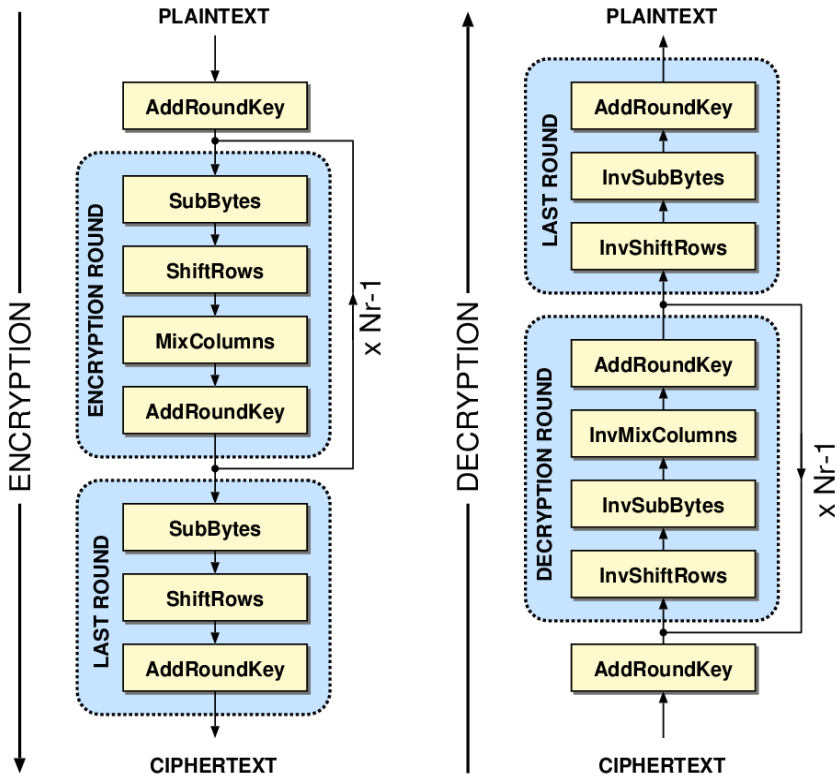
Results

- All generated equalities are valid and most inequalities are spurious (and removed by KIP)
- Invariants generated are sufficiently strong to explain program behavior
  - Proved correctness of 36/41 programs, 2 mins per program
  - No spurious results
- Current dynamic analysis cannot find any of these invariants

Outline

A Case Study: Advanced Encryption Standard (AES)



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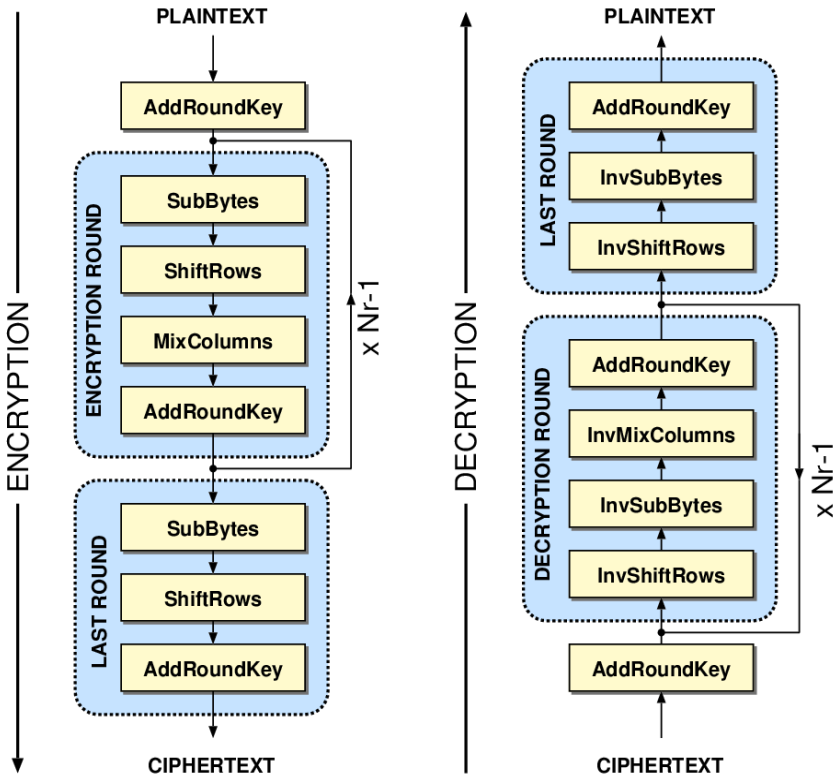

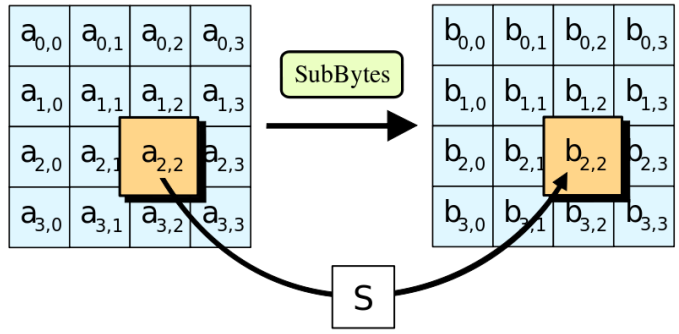


```
def SubBytes(S,a):
  #S is 1D array, a is 2D array
  b =
    [[S[a[0][0]],S[a[0][1]],
      S[a[0][2]],S[a[0][3]]],
     [S[a[1][0]],S[a[1][1]],
      S[a[1][2]],S[a[1][3]]],
     [S[a[2][0]],S[a[2][1]],
      S[a[2][2]],S[a[2][3]]],
     [S[a[3][0]],S[a[3][1]],
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  [L]
  return b
```

[L]: b[i][j] = S[a[i][j]]

Nested Array Problem (TOSEM '14, ICSE '12)

The Array Nesting (AN) problem

Given an n-dimensional array a and set B of single dimensional arrays, does there exist a *nesting* $(b_1, \ldots, b_l)$ from B such that

$$a[i_1] \ldots [i_n] = b_1[\ldots[b_l[c_0 + c_1 i_1 + \cdots + c_n i_n]] \ldots] \; ?$$

Example: $a[i] = b_1[i + 3j + 5]$, $a[i][j][k] = b_1[b_2[i + 2j + 3k]]$

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Complexity (**m**: number of array variables, **n**: size of the largest array variable)

- AN is strongly NP-Complete in **m** (reduction from *Exact Covering*)
- AN can be solved in polynomial time in **n** using reachability analysis
- Same complexity for the generalized version with multi-dimensional and repeating arrays, e.g., $\mathsf{a}[i][j] = 2\mathsf{b}_1[\mathsf{b}_2[2i + 3]][[\mathsf{b}_3[i][b_2[j]]]]$

Experimental Results

Nested Array Relations (functions are treated as a special type of arrays)

xor2Word	$R[i] = \mathsf{xor}(A[i], B[i]),\ A[i] = \mathsf{xor}(R[i], B[i]), B[i] = \mathsf{xor}(R[i], A[i])$
addRoundKey	$R[i][j] = \mathsf{xor}(T[i][j], H[i][j])$
multWord	$R[i] = T[\mathsf{mod}(L[A[i]] + L[B[i]], 255)]$

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Flat Array Relations (flattening array elements and solving equations)

block2State	$R[i][j] = T[4i + j]$
RotWord	$R = [W[1], W[2], W[3], W[0]]$
keySetupEnc8	$R[i][j] = \mathsf{cipherKey}[4i + j]$ for $i = 0, \ldots, 7;\ j = 0, \ldots, 3$

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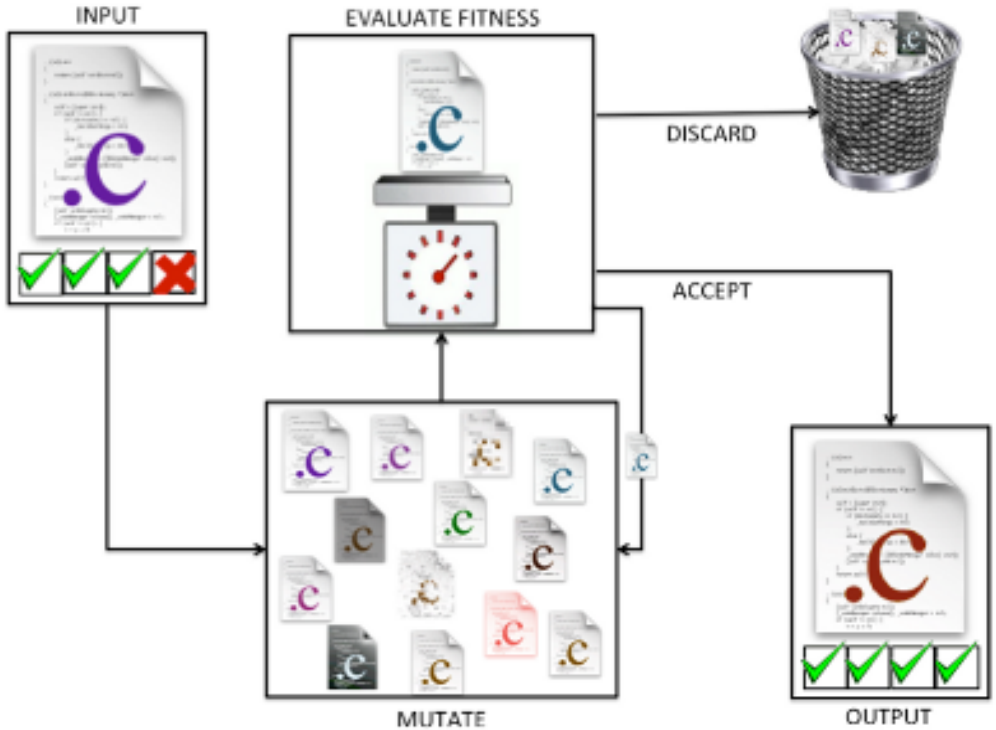
DIG found 60% of the documented array relations in AES under 15 minutes

Outline

GenProg (TSE '12, CACM '10, ICSE '09, GECCO '09)



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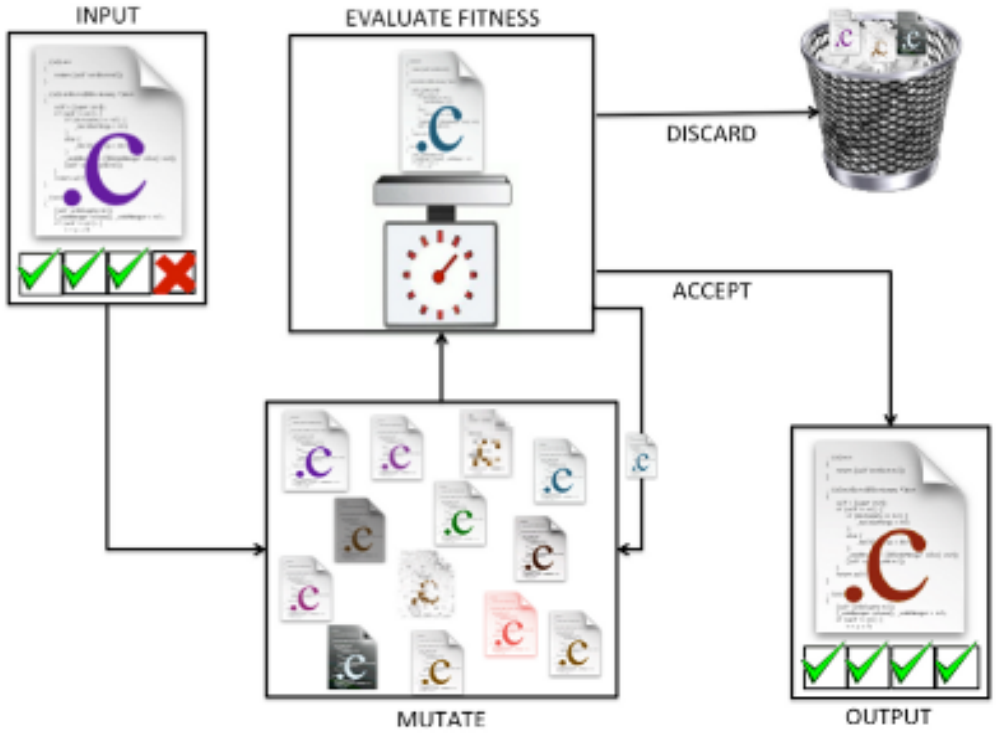

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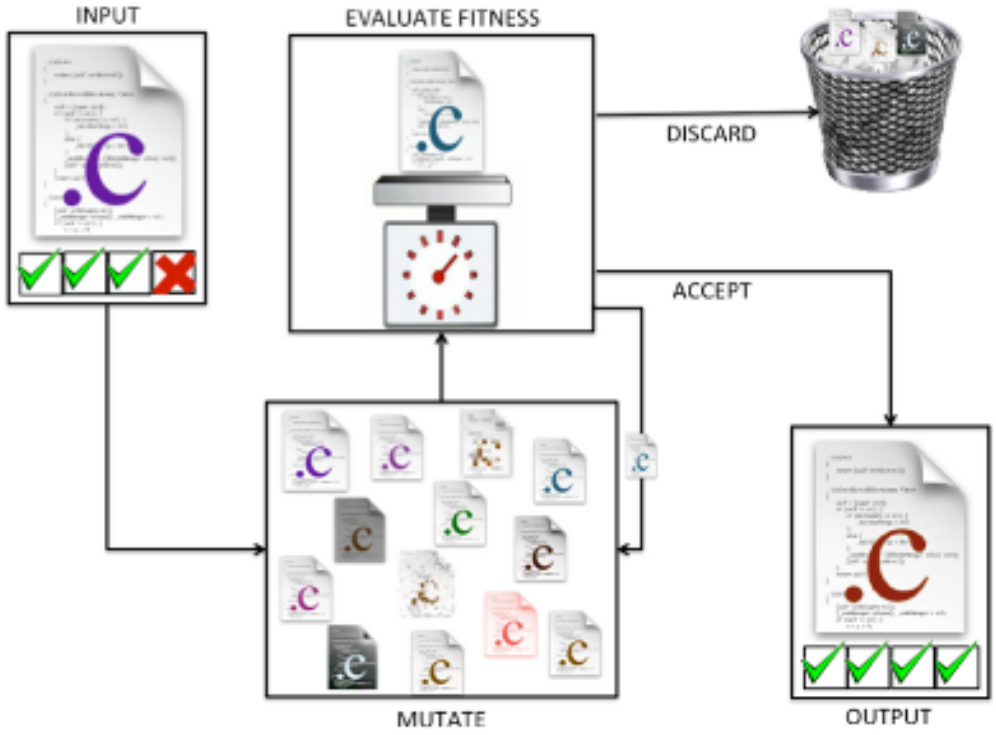

1. Isolate faults
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Results: demonstrated on bugs in real-world software (repair 16 programs over 1.25 MLocs, 2 mins avg)

Using Reachability to Synthesize Repairs

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Verification as a Reachability problem

- Show a program state violating a given specification is not reachable
- Test-input generation: finds inputs that reach a program location

```
def P(x, y):
 if 2 × x == y:
  if x > y + 10:
     [L] #reachable, e.g., x = -20, y = -40

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Template-based Synthesis

- A practical form of synthesis that creates code under specific templates

```
def Q(i, u, d):
  if i:
    b = c0 + c1 × u + c2 × d # linear exp template
  else:
    b = u
  if (b > d): r = 1
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```

Test suite

$$
\begin{array}{lcl}
Q(1, 0, 100) & = & 0 \\
Q(1, 11, 110) & = & 1 \\
Q(0, 100, 50) & = & 1 \\
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Goal: connecting verification to synthesis to leverage existing verification techniques and tools to synthesize programs

From Reachability to Synthesis

Theorem: Template-based Synthesis is reducible to Reachability

Given a general instance of synthesis, create a specific instance of reachability consisting of a special location reachable iff synthesis has a solution

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Input: a synthesis instance

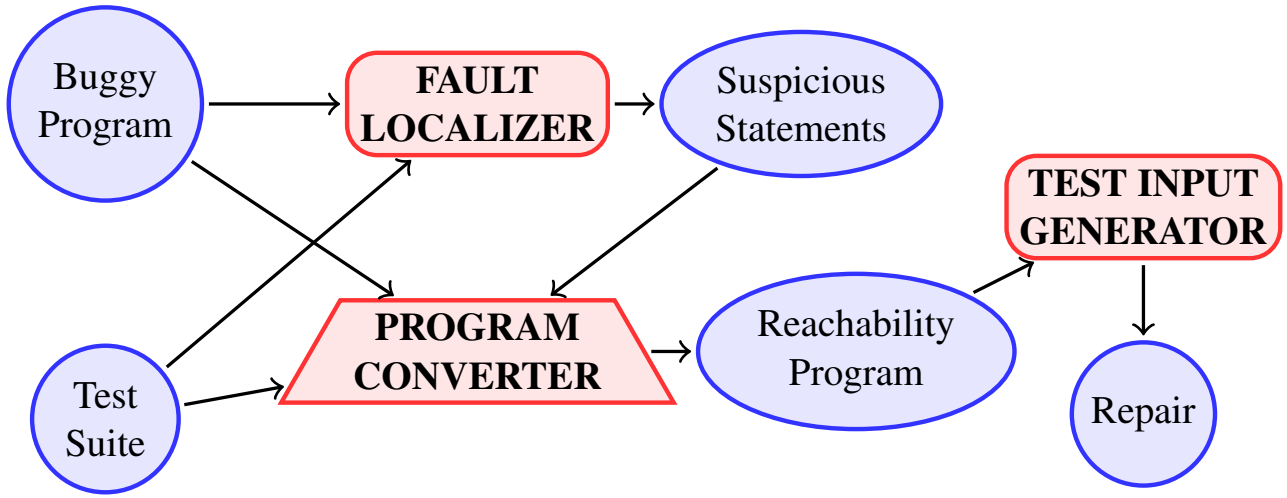
```
def pQ(i, u, d, c0, c1, c2):
  if i:
    b = c0 + c1 × u + c2 × d
  else: b = u
  if b > d: r = 1
  else: r = 0
  return r

def pmain(c0, c1, c2):
   e = pQ(1, 0,100, c0,c1,c2) == 0 and
       pQ(1,11,110, c0,c1,c2) == 1 and
       pQ(0,100,50, c0,c1,c2) == 1 and
       pQ(1,-20,60, c0,c1,c2) == 1 and
       pQ(0, 0, 10, c0,c1,c2) == 0 and
       pQ(0, 0,-10, c0,c1,c2) == 1

   if e:
      [L]  #pass the given test suite
   return 0
```

Output: a reachability instance, solvable using a test-input generation tool

CETI: Correcting Errors using Test Inputs (in submission)



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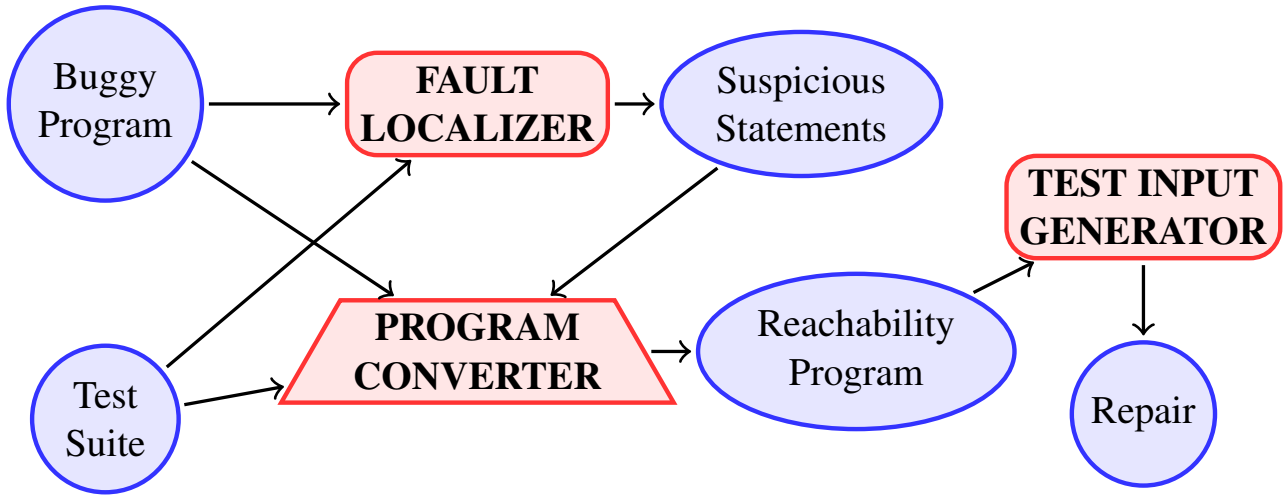

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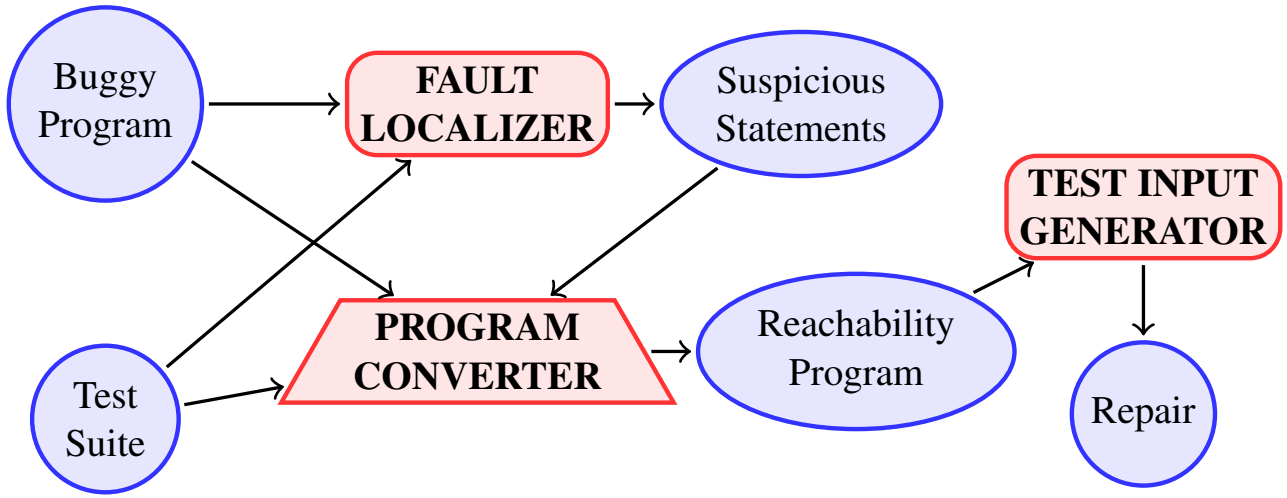

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- Can fix errors that other repair techniques cannot

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Analyzing Configurable Software (in submission)

- Modern software is highly configurable
- Different configurations can lead to different program behaviors

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#options: s,t,u,v,x,y,z ∈ {0...4}
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if x and y:
  #L0
  if not (0 < z < max_z):
    #L1

if s or t:
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c_2	0	0	1	1	1	1	0	$L0, L3$
c_3	0	1	1	1	1	0	3	$L2$
	⋮							

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- interactions (reaching conditions):

interaction	covering
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$x \wedge y \wedge (z \in \{0, 3, 4\})$	$L1$
$s \vee t$	$L2$
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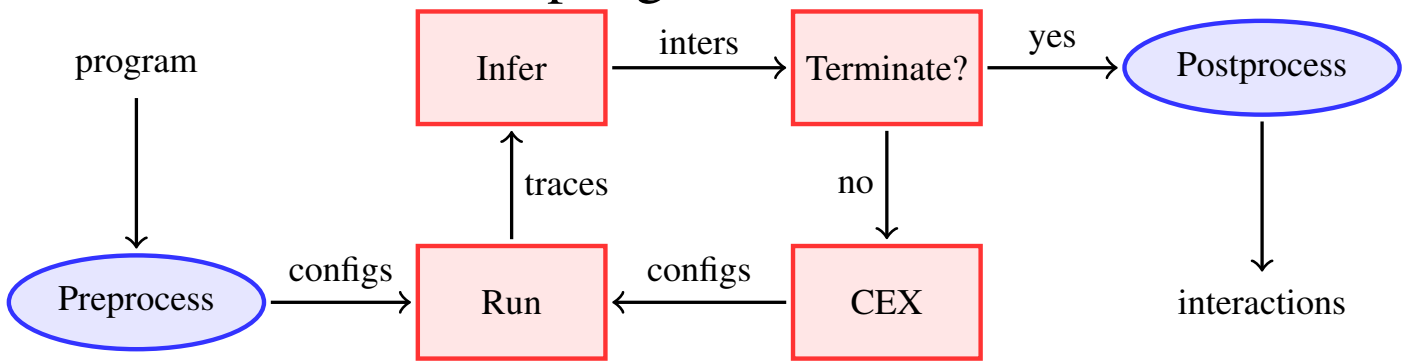
- Existing approaches do not scale and are restricted to simple interactions

iGen: a lightweight approach to finding interactions

- Apply dynamic invariant analysis
- CEGIR: counter-example guided interaction refinement



- Analyze *negative* traces to compute more accurate interactions

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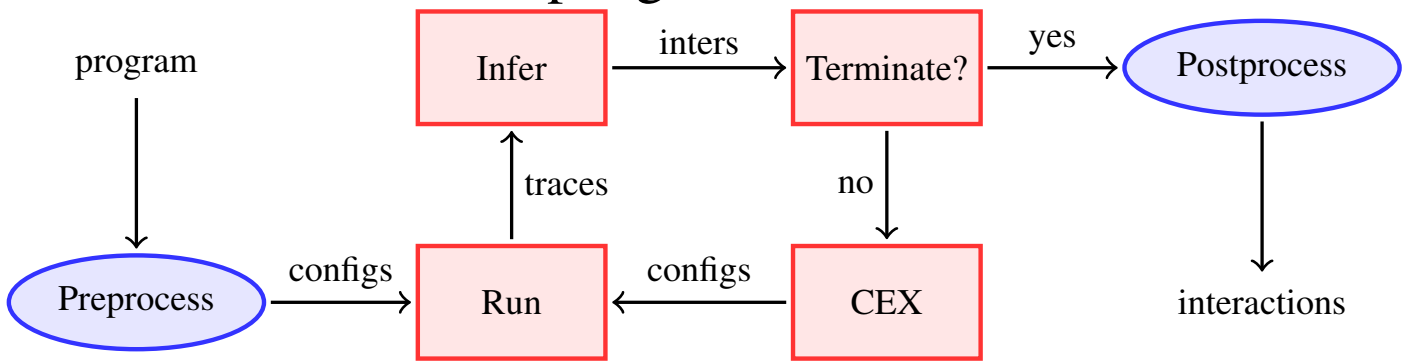

- Analyze *negative* traces to compute more accurate interactions

Results

- 25 open source software (GNU `coreutils`, Apache `httpd`, `unison`, `vsftpd`, etc) in different languages
- Generate accurate and useful interactions
  - Found bugs in the Perl implementation of GNU `coreutils`
  - Most interactions are small, but some are very large and complex
- More efficient than existing approaches (minutes vs weeks)

What's Next ?

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1. Invariant Generation
   - Apply invariant generations to analyze configurable software, feature-based interactions
   - DIG2: supports data structures including trees and lists and uses CEGIR for iterative invariant refinment
2. Dependent and Gradual Typing
   - Represent invariants as dependent types and apply current work to infer and check types
   - Use verification techniques (e.g., *k*-induction) to statically check dynamic languages (such as Python and Ruby)
3. Program Synthesis/Repair
   - Apply techniques in program synthesis to reachability, e.g., using repair tools to generate high quality test inputs
   - Combine CETI and other repair techniques, e.g., random search, to handle a wider range of errors

Conclusion

- Invariant Generation
  - **DIG**: treat invariants as set of equations and constraints and solve them to identify nonlinear polynomial relations, disjunctive invariants, and array properties
  - **KIP**: an automatic theorem prover for verifying candidate invariants
  - **iGen**: a lighweight dynamic analysis approach to finding program interactions
- Program Synthesis/Repair
  - **GenProg**: modify and repair programs using GA
  - **CETI**: apply equivalence theorem to reduce repair task to rechability problem, solvable using off-the-shelf test-input generators

References

- Source code, benchmarks, etc: http://www.cs.umd.edu/~tnguyen
- Invariant Generation
  - Nguyen, Kapur, Weimer, and Forrest. DIG: A dynamic invariant generator for polynomial and array invariants. TOSEM, 2014
  - Nguyen, Kapur, Weimer, and Forrest. Using dynamic analysis to generate disjunctive invariants. ICSE, 2014
  - Kapur, Zhang, Horbach, Zhao, Lu, and Nguyen. Geometric quantifier elimination heuristics for automatically generating octagonal and max-plus invariants. Automated Reasoning and Mathematics, 2013
  - Nguyen, Kapur, Weimer, and Forrest. Using dynamic analysis to discover polynomial and array invariants. ICSE, 2012 (Distinguished Paper)
- Program Repair
  - Geoues, Nguyen, Forrest, and Weimer. GenProg: A generic method for automated software repair. TSE, 2012
  - Weimer, Forrest, Geoues, and Nguyen. Automatic program repair with evolutionary computation. CACM, 2010 (Research Highlight)
  - Forrest, Weimer, Nguyen, and Geoues. A genetic programming approach to automated software repair. GECCO, 2009 (Best Paper)
  - Weimer, Nguyen, Geoues, and Forrest. Automatically finding patches using genetic programming. ICSE, 2012 (Distinguished Paper and Manfred Paul Award for Excellence in Software: Theory and Practice)
  - Nguyen, Weimer, Geoues, and Forrest. Using execution paths to evolve software patches. ICST, 2009 (Best Paper)