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CS 54

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Master Theorom

1} $T(n) = 3T(n/2) + n^2$

Apply master's Theorom

$a=3 \quad b=2 \quad k=2 \quad p=0$

$a=3 \;\&\; b^k = 2^2 = 4$

$b^k > a$

As $p=0$

$T(n) = \Theta(n^4)$

2} $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Apply Master's Theorom

$a=4 \quad b=2 \quad k=2 \quad p=0$

$a=4 \;\&\; b^k = 2^2 = 4$

$a = b^k$

As $p=0$

$T(n) = \Theta(n^2 \log^2 n)$

3} $T(n) = T\left(\frac{n}{2}\right) + n^2$

Apply Master's Theorom

$a=1 \quad b=2 \quad k=2 \quad p=0$

$a=1 \;\&\; b^k = 2^2 = 4$

$b^k > a$

As $p=0$

$T(n) = \Theta(n^2)$

4} $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

Apply Master's Theorom

As $a = 2^n$ which itself is a function we cannot apply master's Theorom

5} $T(n) = 16\, T\left(\frac{n}{4}\right) + n$

Apply Master's Theorom

$a = 16 \quad b = 4 \quad k = 1 \quad p = 0$

$a = 16 < b^k = 4^1 = 4$

$\therefore b^k < a$

As $p = 0$

$T(n) = \theta(n^2)$

6} $T(n) = 2T\left(\frac{n}{2}\right) + n\log$

Apply Master's Theorom

$a = 2 \quad b = 2 \quad k = 1 \quad p = 1$

$a = 2 < b^k = 2^1 = 2$

$\therefore a = b^k$

As $p = 1$

$T(n) = \theta(n \log^2 n)$

7} $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

Apply Master's Theorom

$a=2$ $b=2$ $k=1$ $p=-1$

$a=2$ & $b^k = 2^1 = 2$

$\therefore b^k = a$

As $p=-1$

$\cancel{T(n) = \theta(n \log \log n)}$ $\cancel{(n \log_b^a}$

$T(n) = \theta(n^{\log_b a} \log \log n)$

~~8}~~ $\cancel{T(n) = 2T\left(\frac{n}{2}\right) + n/\log n}$

8} $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Apply Master's Theorom

$a=2$ $b=4$ $k=0.51$ $p=0$

$a=2$ & $b^k = 4^{0.51} = 2.0279$

$\therefore b^k > a$

As $p=0$

$T(n) = \theta(n^{0.51})$

9} $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

Apply Master's Theorom

$a=0.5$ $b=2$ $k=\cancel{1}\ -1$ $p=0$

$a=0.5$ & $b^k = 2^{-1}$

As a & k doesn't satisfy the requirement we cannot apply it here

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10} $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

Apply Master's Theorom

$a=6 \quad b=3 \quad k=2 \quad p=1$

$a=6 \;\&\; b^k = 3^2 = 9$

$\therefore b^k > a$

As $p=1$

$T(n) = \theta(n^2 \log n)$

11} $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log$

Not possible as it does not match the master's Theorom equation

12} $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Apply Master's Theorom

$a=7 \quad b=3 \quad k=2 \quad p=0$

$a=7 \;\&\; b^k = 9$

$\therefore b^k > a$

As $p=0$

$T(n) = \theta(n^2)$

13} $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Apply Master's Theorom

$a=4 \quad b=2 \quad k=0 \quad p=1$

$a=4 \;\&\; b^k = 1$

$\therefore b^k < a$

As $p=1$ $\quad T(n) = \theta(n^2)$

14} $T(n) = \sqrt{2}\, T\left(\frac{n}{2}\right) + \log n$

Apply Master's Theorom

$a = \sqrt{2}$ $b = 2$ $k = 0$ $p = 1$

$a = \sqrt{2}$ & $b^k = 1$

$\therefore b^k < a$

As $p = 1$

$T(n) = \theta(\sqrt{n})$

15} $T(n) = 2T\left(\frac{n}{2}\right) + \sqrt{n}$

Apply Master's Theorom

$a = 2$ $b = 2$ $k = 1/2$ $p = 0$

$a = 2$ & $b^k = \sqrt{2}$

$\therefore b^k < a$

As $p = 0$

$T(n) = \theta(n)$

16} $T(n) = 3T\left(\frac{n}{2}\right) + n$

Apply Master's Theorom

$a = 3$ $b = 2$ $k = 1$ $p = 0$

$a = 3$ & $b^k = 2$

$\therefore b^k < a$

As $p = 0$

$T(n) = (n^{1.585})$

17} $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

Apply Master's Theorom

$a=3 \quad b=3 \quad k=1/2 \quad p=0$

$a=3$ & $b^k = 3^{1/2} = \sqrt{3}$

$\therefore b^k < a$

As $p=0$

$T(n) = \Theta(n)$

18} $T(n) = 4T\left(\frac{n}{2}\right) + n$

Apply Master's Theorom

$a=4 \quad b=2 \quad k=1 \quad p=0$

$a=4$ & $b^k = 2$

$\therefore b^k < a$

As $p=0$

$T(n) = \Theta(n^2)$

19} $T(n) = 3T\left(\frac{n}{4}\right) + (n \log n)$

Apply Master's Theorom

$a=3 \quad b=4 \quad k=1 \quad p=1$

$a=3$ & $b^k = 4$

$\therefore b^k > a$

As $p=1$

$T(n) = \Theta(n \log n)$