



# Interdependence of Growth, Structure, Size and Resource Consumption During an Economic Growth Cycle

Carey W. King<sup>1</sup>

Received: 12 August 2021 / Revised: 14 October 2021 / Accepted: 26 October 2021  
© The Author(s) 2021

## Abstract

This paper explains how the Human and Resources with MONEY (HARMONEY) economic growth model exhibits realistic dynamic interdependencies relating resources consumption, growth, and structural change. We explore dynamics of three major structural metrics of an economy. First, we show that an economic transition to relative decoupling of gross domestic product (GDP) from resource consumption is an expected pattern that occurs because of physical limits to growth, not a response to avoid physical limits. While increasing operational resource efficiency does increase the level of relative decoupling, so does a change in pricing from one based on full costs to one based only on marginal costs that neglect depreciation and interest payments. Marginal cost pricing leads to higher debt ratios and a perception of higher levels of relative resource decoupling. Second, if assuming full labor bargaining power for wages, when a previously-growing economy reaches peak resource extraction and GDP, wages remain high but profits and debt decline to zero. By removing bargaining power, profits can remain positive at the expense of declining wages. Third, the internal structure of HARMONEY evolves in the same way the post-World War II U.S. economy. This is measured as the distribution of intermediate transactions within the input-output tables of both the model and U.S. economy.

**Keywords** Energy · Resources · Macroeconomics · Post-Keynesian · Dynamics · Structural change

## Introduction

Scientists and economists often seek to understand the linkages among natural resources consumption and the cost of resources in tandem with the growth rate, size, and structure of complex systems. These systems can be biological organisms, ecosystems, and national or global economies.

The size of organisms and ecosystems is measured by their mass, volume, and population. Their structure is measured by the flow of nutrients and energy in food webs and internal distributions systems (i.e., circulatory systems) as well as social networks, such as within eusocial insect colonies.

The size of economies is measured by many metrics such as gross domestic product (or net output), gross output, population, the quantity of physical capital (in money and physical units), and others. The structure of an economy can

be measured by functions and metrics that summarize the distribution of stocks (e.g., capital) and flows (e.g., income, power) among people and jobs, companies, economic sectors (Hidalgo et al. 2007; Hidalgo and Hausmann 2009; King 2016), or other categories through which money and natural resources flow.

The purpose of this paper is to discuss the dynamic interdependencies among growth, size, and structure of an economy using outputs from an updated version of the Human and Resources with MONEY (HARMONEY) model of King (2020). This new version is HARMONEY v1.1. Because of its structure, the HARMONEY model helps “narrow the differences” between economic and ecological viewpoints, which as the late Martin Weitzmann suggested (Nordhaus et al. 1992), provides value by creating enhanced understanding of economic dynamics. That is to say, because the model simultaneously tracks physical and monetary stocks and flows, by including physical resources and constraints along with macroeconomic accounting and debt, HARMONEY speaks the language of *both* economists and physical and natural scientists.

✉ Carey W. King  
careyking@mail.utexas.edu

<sup>1</sup> Energy Institute, The University of Texas at Austin, Austin, TX, USA

All models are simplified representations of real-world processes, yet stylized models are still useful for providing insight into real-world data. A model that can accurately represent the dynamic interdependence between growth, size, and structure has more explanatory power than those that cannot. While HARMONEY v1.1 is not calibrated to a real-world economy, it has critical features and structural assumptions that make it applicable and valuable for comparing its trends to long-term trends in real-world data.

## Energy and Growth in Biology and Economies

Macroeological and biological growth literature (Sibly et al. 2012; Brown et al. 2004) has accumulated an extensive number of studies seeking to explain the universality and robustness of the finding that adult individual organisms (West et al. 2001; West and Brown 2005; Banavar et al. 2010; West 2017; Ballesteros et al. 2018) and groups of organisms, such as eusocial insect colonies (Hou et al. 2010; Shik 2010; Waters et al. 2010, 2017; Shik et al. 2014; Fewell and Harrison 2016), follow sublinear scaling, or allometry, relating their metabolism to mass. That is to say, once an organism obtains a mature structure, basal metabolism ( $B$ ) increases more slowly than mass ( $M$ ), as  $B \propto M^b$ , where  $b < 1$ . Before reaching its mature structure, an organism can exhibit superlinear scaling ( $b > 1$ ) when basal metabolism increases faster than mass, such as in fish embryos (Clarke and Johnston 1999; Mueller et al. 2011) and tree saplings (Mori et al. 2010). In addition, when increasing in size from very small single-celled organisms (bacteria) to larger eukaryotic single-celled organisms (protists) to multicelled organisms, DeLong et al. (2010) indicate the scaling of metabolism to mass transitions from superlinear to linear to sublinear, respectively. Hatton et al. (2019) suggest that linear scaling more accurately relates metabolism to mass across all eukaryotes (neglecting bacteria), and that it is an organism's growth (mass/time) that scales sublinearly, near  $b = 3/4$ , with mass. The implication is that the growth rate (%/time) declines with size.

This brings us to contemporary questions regarding the rate of economic growth as the global economy continues to increase in size. Must the global economy necessarily slow its growth rate as it increases in size? If so, are the reasons similar to those of biological organisms?

Due to its theoretical limitations, neoclassical exogenous growth theory does not answer these questions very well, despite its prevalent use in most integrated assessment models that inform policy for a transition to low-carbon energy economy. There are several reasons, but the HARMONEY model overcomes three neoclassical limitations: the inadequate incorporation of natural resource consumption as required physical inputs to operate capital, become embodied in new capital investment, and keep people alive (Keen

et al. 2019); the lack of consideration of credit, or private debt, in a modern economy (McLeay et al. 2014); and the assumption that factors of production contribute to growth in relation to their cost share. Exogenous neoclassical growth posits declining returns to growth of output (e.g., GDP) with respect to capital and labor (Solow 1956). In mathematical terms, assuming the usual Cobb–Douglas aggregate production function  $Y = A(t)K^\alpha L^{1-\alpha}$ , with  $0 < \alpha < 1$  and  $A(t)$  as total factor productivity (TFP), if  $dA(t)/dt = 0$ , then output  $Y$  grows more slowly than capital ( $K$ ) or labor ( $L$ ) as the two factors of production. Since approximately half of GDP growth in major economies is allocated to TFP (Ayres 2008), a.k.a. the “Solow residual,” this growth framework has significant room for improvement that even Robert Solow, the originator of the growth model, recognized 50 years after its development (Solow 2007).

Critics of neoclassical growth theory indicate that the choice of the exponents, or output elasticities, of the factors of production is based on tautological macroeconomic accounting assumptions rather than fundamental features of the economy (Shaikh 1974; Felipe and Fisher 2003; Felipe and McCombie 2006). Ayres and Kümmel separately find that if one includes primary energy consumption (PEC) or useful work (U) as a third factor of production in a Cobb–Douglas aggregate production function, one can accurately represent historical GDP without assuming exogenous growth in TFP (Ayres and Warr 2005; Ayres 2008; Kümmel 2011). However, these studies indicate that one must also abandon the so-called cost-share theorem assumption of neoclassical theory that “... says that the economic weight of a production factor, which is called the *output elasticity* of that factor, should always be equal to the factor's share in total factor cost” (Kümmel and Lindenberger 2014). Two separate groups of authors have shown that the output elasticity of either primary energy or useful work is near 0.4–0.7, an order of magnitude larger than its cost share (Ayres and Warr 2005; Ayres 2008; Kümmel and Lindenberger Kümmel 2011; Kümmel and Lindenberger 2014).

However, a study of Portugal's economy suggests one can accurately describe GDP while including one physical concept of energy and maintaining capital and labor cost shares in a Cobb–Douglas production function (Santos et al. 2021). This study by Santos et al. (2021) indicates that TFP is fully explained by their calculated measure of aggregate exergy efficiency (= useful exergy / final exergy consumed) of the Portuguese economy. The finding is consistent with those of Warr et al. on the growth-inducing role of energy efficiency (Warr et al. 2010). Their methods are also consistent with some endogenous growth models, such as the unified growth theory of Galor and Weil (2000) and Galor (2005). First, Santos et al. (2021) determine that using quality-adjusted capital and labor (“schooling-corrected” labor services as increased quality of human capital, as opposed to only

labor hours) as factor inputs. This method follows Galor and Weil's model of growth as a function of increasing quantity and quality, or education, of human capital. Using quality-adjusted capital and labor also reduces the residual, or TFP, of the aggregate production function such that it is then more accurately characterized by exergy efficiency. Second, Santos et al. (2021) do not explicitly input PEC or useful work output into the production function. While resources are not fully absent from Galor and Weil's model (Galor and Weil 2000; Galor 2005), their "effective resources" combine land and education-enhanced technological progress such that they do not explore the independent influence of resource consumption (quantity) and quality.

Unlike neoclassical growth theory (exogenous or endogenous), the post-Keynesian and biophysical structure of the HARMONEY model does not assume an aggregate production function, TFP, or directly impose scaling of GDP to aggregate labor, capital, or natural resources consumption. Thus, the model enables a different exploration into the effects of resource efficiency and whether the economy has similar energy-GDP scaling as biological systems, and for the same reasons, throughout a growth cycle. Several authors have indicated the similar scaling relation of primary energy consumption to country and global GDP as exists for metabolism and mass for biological organisms (Odum 1971, 1997, 2007; Georgescu-Roegen 1971, 1975; Hagens 2020; King 2021). Jarvis and King summarize how global primary energy consumption (PEC) and gross world product (GWP) scale approximately linearly from 1900-1970, and since 1970 scale sublinearly at  $PEC \propto GWP^{\frac{2}{3}}$  (Jarvis and King 2020). Giraud and Kahraman (2014) confirm a similar finding. Brown et al. (2011) indicate how post-1980 trends show PEC of countries scales with their GDP nearly as  $PEC \propto GDP^{\frac{3}{4}}$ , the same as basal metabolism scales with mass in mammals. As stated in Brown et al. (2014): "Regardless of whether the approximately 3/4 power scaling is due to a deep causal relationship or an amazing coincidence, both relationships reflect similar underlying causes – the energy cost of maintaining the structure and function of a large, complex system."

A contribution of this paper is to address these "underlying causes" in the context of energy-size scaling using an economic growth model that, among other things, explicitly considers the "energy cost of maintaining the structure and function" of an economy as a complex system. This paper does not address the exact scaling (i.e., value of  $b$ ) between energy consumption and GDP, but it explains why we expect a transition from superlinear or linear scaling to sublinear scaling, just as observed in biological systems.

Thus, this paper also contributes to the discussion of *decoupling* of GDP from PEC via increases in energy efficiency. Sublinear scaling in the economy, often referred to as a state of declining energy intensity (= PEC/GDP), is

often seen as a consequence of increasing energy efficiency. The issue of decoupling is important because economy-wide rebound effects might erode more than half the reductions in engineering energy efficiency investments (Brockway et al. 2021). Further, because the mechanisms of the rebound effect are largely overlooked by integrated assessment models and global energy models that guide policy (King 2021; Brockway et al. 2021; Keyßer and Lenzen 2021), policy-makers and energy efficiency advocates are unaware that their efforts to reduce carbon emissions by increasing device efficiency are not nearly as effective as they assume. That is to say, proponents of energy efficiency measures claim that declining energy intensity is caused by specific actions to increase energy conversion efficiency in machines and distribution networks, and that this reduces energy consumption since less energy is needed for a unit of work or GDP (taking the notion from Warr and Ayres that useful work scales linearly with GDP) (Ayres and Warr 2009; Warr et al. 2010; Keen et al. 2019). However, animals, including each of our own bodies as *Homo sapiens*, exhibit sublinear scaling without any conscious action or choice to do so.

If we do not make decisions to reduce the "energy intensity" of our own bodies, then how can we be so sure that a declining economy-wide energy intensity is a consequence of our collective conscious actions? One way to address this question is via a macroeconomic system growth model that contains the appropriate and sufficient biophysical features, economic accounting, and constraints. Such is the HARMONEY model described in this paper.

## Methods

Here we summarize the formulation of HARMONEY v1.1 model by discussing features both similar to and different from v1.0 King (2020). We also summarize the information theoretic metrics, as in King (2016), used to characterize the structure of the modeled HARMONEY economy during simulated growth. Using these metrics we check if and how the features of the theoretical HARMONEY model are consistent with trends from U.S. data as in King (2016).

### Description of HARMONEY Model (same features as v1.0)

HARMONEY is an economic growth model that is stock and flow consistent in both money and physical variables King (2020). Conceptually it combines the Goodwin-Keen model (Keen 1995, 2013) (that adds private debt to the Goodwin business cycle model (Goodwin 1967)) to the Lotka-Volterra framework of the Human and Nature Dynamics (HANDY) model (Motesharrei et al. 2014) of a population that survives by extracting a single regenerative (e.g.,

**Table 1** Equations for elements of technical requirements matrix,  $\mathbf{A}$ 

$a_{ee}$	Resources to operate extraction capital	$\frac{\eta_e K_e C U_e}{\delta_y y K_e C U_e} = \frac{\eta_e}{\delta_y y}$
$a_{eg}$	Resources to operate goods capital and invest	$a_{eg} = a_{eg}^o + a_{eg}^l$
$a_{eg}^o$	Resources to operate goods capital	$\frac{\eta_g K_g C U_g}{(K_g C U_g / v_g)} = \eta_g v_g$
$a_{eg}^l$	Resources that become physical goods (incl. investment)	$\frac{y_{X_g} K_g C U_g}{(K_g C U_g / v_g)} = y_{X_g} v_g$
$a_{ge}$	Goods input for extraction	Constant
$a_{gg}$	Goods input to produce goods	Constant

forest) natural resource. To these base frameworks, HARMONEY separates economic production into two industrial sectors: resource extraction and capital goods production. The goods production sector makes capital for both sectors, and the extraction sector extracts resources required to operate capital in each sector. Each sector has its own capital ( $K_i$ ), labor ( $L_i$ ), price of output ( $P_i$ ), debt as loans ( $D_i$ ) from a private bank, and physical stock of inventory.

### Production and Natural Resource Extraction

The rate of change of natural resource in the environment,  $y$ , is equal to resource regeneration minus gross extraction (Eq. 1), where gross extraction,  $X_e$ , is a Leontief production function of extraction capital,  $K_e$ , and extraction labor,  $L_e$  (operating at labor productivity  $a_e$ ) as in Eq. 2. Any of labor, extraction capital, or resource consumption (to operate capital as fuel) can constrain output such that the equality in Eq. 2 holds and capacity utilization,  $CU_e$ , adjusts as needed. The extraction technology parameter  $\delta_y$  describes the rate of resource extraction at full capacity utilization per unit of extraction capital,  $K_e$ . For the regenerative natural resource, regeneration is a function of the maximum size of the resource,  $\lambda_y$ , the resource regeneration rate,  $\gamma$ , and the available stock of resource in the environment,  $y$ . The maximum regeneration rate occurs when  $y = \lambda_y/2$ .

$$\begin{aligned}\dot{y} &= \text{regeneration} - \text{extraction} \\ \dot{y} &= \gamma y (\lambda_y - y) - \delta_y y K_e C U_e\end{aligned}\quad (1)$$

$$X_e = \delta_y y K_e C U_e = L_e a_e \quad (2)$$

The gross output of capital goods,  $X_g$ , from the goods sector is as in Eq. 3. As with extraction, it is a Leontief production function of labor,  $L_g$ , and capital,  $K_g$ , that requires resource consumption to operate, where  $v_g$  is constant capital:output ratio, and  $a_g$  is a constant sector-specific labor productivity. As with the extraction sector, given capital and labor, capacity utilization,  $CU_g$ , adjusts to ensure equality in Eq. 3.

$$X_g = \frac{K_g C U_g}{v_g} = L_g a_g \quad (3)$$

### Intermediate Demands

The 2-sector model has four technical coefficients for its Leontief input-output matrix,  $\mathbf{A}$ , in Eq. 4 and Table 1. The technical coefficients are the same as in HARMONEY v1.0 of King (2020). We assume the technical coefficients  $a_{ge}$  and  $a_{gg}$  are constant. The coefficient  $a_{ee}$  indicates the amount of resource consumption required to extract a unit of resources where  $\eta_e$  characterizes the level of resources consumption required to operate a unit of capital at full capacity utilization. Coefficient  $a_{eg}$  has two components. Component  $a_{eg}^o$  serves the same role for the goods sector as coefficient  $a_{ee}$  serves for the extraction sector as it accounts for resource consumption to operate goods capital. The second component  $a_{eg}^l$  accounts for resources that become physically embodied in capital. The factor  $y_{X_g}$  is the amount of resources embodied in a unit of capital, and it is analogous to the finding in biology that a constant amount of energy is required per unit of animal mass including offspring (Brown et al. 2018), where offspring are the analog of new capital investment in the economy.

$$\mathbf{A} = \begin{bmatrix} a_{gg} & a_{ge} \\ a_{eg} & a_{ee} \end{bmatrix} = \begin{bmatrix} \frac{x_{gg}}{X_g} & \frac{x_{ge}}{X_e} \\ \frac{X_g}{x_{eg}} & \frac{x_{ee}}{x_e} \\ \frac{x_{eg}}{X_g} & \frac{x_{ee}}{X_e} \end{bmatrix} \quad (4)$$

### Inventories and Capacity Utilization

Equations 5 and 6 show the rate of change of the physical quantity of inventory for goods,  $g$ , and extracted resources as wealth,  $w_H$ , respectively. As in King (2020), the term “wealth” for the physical inventory of extracted resources maintains the nomenclature of the HANDY model (Mote-sharrei et al. 2014). The change in physical inventory for each sector is the difference between the reference ( $IC_{ref,i}$ ) and current inventory coverage ( $IC_i$ ) multiplied by the targeted physical consumption of each sector output. If the inventory coverage is higher than the set reference, then inventory decreases, and vice versa. In essence, the inventories scale up with demand.

$$\begin{aligned}\dot{w}_H &= (\text{reference inventory coverage} - \text{inventory coverage}) \\ &\quad \times (\text{targeted consumption of resources}) \\ \dot{w}_H &= (IC_{ref,e} - IC_e) \times (C_e/P_e + a_{eg}X_g + a_{ee}X_e)\end{aligned}\tag{5}$$

$$\begin{aligned}\dot{g} &= (\text{reference inventory coverage} - \text{inventory coverage}) \\ &\quad \times (\text{targeted consumption of goods}) \\ \dot{g} &= (IC_{ref,g} - IC_g) \times ((C_g + I_g + I_e)/P_g + a_{ge}X_e + a_{gg}X_g)\end{aligned}\tag{6}$$

Wealth and goods inventories can rise and fall with business cycles. We model capital capacity utilization ( $CU_i$ ) as a function of perceived inventory coverage, following Sterman (2000). Perceived inventory coverage for sector  $i$ ,  $IC_{i,perceived}$ , is defined as the quantity of physical inventory divided by a time lag,  $\tau$ , and the targeted consumption for that sector output. The higher the total consumption for a given output, the larger the inventory stock needed to buffer consumption over the period of the time lag. See Supplemental Section SI.3.3 for inventory equations describing inventory coverage and capacity utilization. Section “[Biophysical Constraining Thresholds](#)” summarizes how we determine capacity utilization under resource and participation rate constraints (also see Supplemental Fig. S.2 of King (2020)).

### Monetary Net Output and Consumption

The sectoral monetary gross output ( $P_i X_i$ ) is equal to intermediate sales plus total net output,  $Y_i$ , of the economy (See Supplementary Table S.1). Since we specify gross extraction and intermediate sales, we solve for monetary net output vector,  $Y$ , as in Eq. 7. In Eq. 7,  $X$  is a vector of sectoral gross output,  $\hat{P}$  is a diagonal matrix with sectoral prices on the diagonal, and  $\mathbf{1}$  is the identity matrix.

$$Y = \hat{P}X - \hat{P}\mathbf{A}X = \hat{P}(\mathbf{1} - \mathbf{A})X\tag{7}$$

The value of inventory in sector  $i$ ,  $INV_i$ , is equal to the current unit production cost,  $c_i$ , times the physical quantity of inventory (Eqs. 8 and 9). The change in the value of inventory,  $\Delta INV$ , is the current value of inventory minus the value from the previous time period. We model the dynamics of value of inventory using lagged equations (see “[Lagged Equations for Simulation](#)” section).

$$INV_g = c_g g\tag{8}$$

$$INV_e = c_e w_H\tag{9}$$

The present model is of a closed economy (no imports or exports) with no government. Then by convention, net monetary output is equal to final consumption plus investment plus change in value of inventories. We assume household consumption,  $C_i$ , is fully accommodating and is the residual

left from subtracting investment and change in value of inventories from net output (Eqs. 10 and 11). Since we assume only the goods sector produces investment goods, there is no investment goods output from the extraction sector, and extraction sector net output is equal to sector consumption minus change in the value of inventory.

$$C_g = Y_g - P_g(I_g^g + I_g^e) - \Delta INV_g\tag{10}$$

$$C_e = Y_e - \Delta INV_e\tag{11}$$

### Population

Population,  $N$ , changes via constant birth rate,  $\beta_N$ , and death rate,  $\alpha_N$ , as a function of per capita physical consumption of extracted resources,  $\frac{C_e}{NP_e}$ , where  $P_e$  is the unit price of extracted resources (Eq. 12). This death rate function,  $\alpha_N\left(\frac{C_e}{NP_e}\right)$  decreases from a maximum value at zero resource consumption to a minimum positive death rate at some specified per capita resource consumption (see Supplemental Equation S.3).

$$\dot{N} = \beta_N N - \alpha_N\left(\frac{C_e}{NP_e}\right)N\tag{12}$$

### Debt

Debt for each sector,  $D_i$ , increases (decreases) when total monetary investment for that sector,  $I_i$ , exceeds (falls below) depreciation and net profits,  $\Pi_i$ .

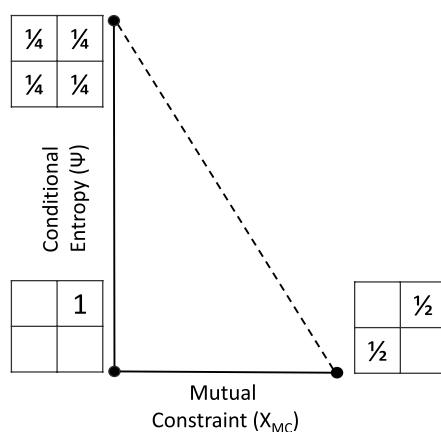
$$\dot{D}_i = I_i - P_g \delta K_i - \Pi_i\tag{13}$$

### Net Power Accounting of Power Return Ratios

King (2020) described the mathematics behind power return ratios (PRRs) as metrics of net power (or energy flow) accounting. These metrics, often referred to as “energy returned on (energy) invested”, or EROI, are the net external power ratio (NEPR) of the extraction sector and net power ratio at the economy-wide level (NPR). These biophysical metrics, indicating “how much energy consumption is required to extract energy,” provide an additional viewpoint to the cost of the energy system. Supplemental Section SI.3.6 describes the mathematics for NEPR and NPR.

### Lagged Equations for Simulation

In the real world, data are only available for decision making after some amount of time. For example, firms and



**Fig. 1** The conditional entropy versus mutual constraint phase space is used to interpret the proportion of all intermediate transactions that occur within any sector-to-sector transaction and in total for each sector. This figure indicates the 2-sector model's  $2 \times 2$  I-O table values, as fractions of the total, at the extreme points of the phase space

governments know profits and net output from the previous year, but they generally don't know those values for last month, yesterday, or the previous hour. To make certain variables available within the simulation code, we model their values from the "previous time period" as a first order lag (see Supplemental Section SI.3.5 and Equation S.8) (Sterman 2000). For each sector  $i$ , we update the following variables using a first order lag: capacity utilization ( $CU_i$ ), perceived inventory coverage ( $IC_{i,perceived}$ ), profit ( $\Pi_i$ ), value added ( $V_i$ ), and value of inventory ( $INV_i$ ). For further reference see the Appendix A.3 that lists the core differential equations of the model.

### Biophysical Constraining Thresholds

Resource extraction is allocated among the operation of each type of capital, household consumption, and resource embodied in investment (Fig. 1 of King (2020)). It is possible that the extraction rate of resources is insufficient to simultaneously fully satisfy minimum levels of household consumption, operational inputs for capital at the target capacity utilization, and the desired level of investment. To account for output constraints based on labor or resource flows, the model dynamics operate within one of eight possible modes based on three binary threshold criteria (i.e.,  $2^3 = 8$ ).

The first threshold criterion is the maximum participation rate ( $\lambda_{N,max}$ ). If there is not enough labor to operate capital at full utilization, then labor is at maximum participation rate and capacity utilization decreases to ensure the equality of Equations 2 and 3.

The second threshold criterion is a minimum household consumption of resources per person ( $\rho_e > 0$ ). If per capita

household resource consumption ( $= C_e/(P_e N)$ ) would otherwise be less than  $\rho_e$ , we set  $C_e/(P_e N) = \rho_e$  and reduce physical investment to match total resource consumption to extraction. A reduction in investment reduces gross output of goods which in turn reduces total resource consumption since resources are embodied in physical capital and the goods capital operates at a lower utilization.

The third and final threshold criterion is a minimum household consumption of goods per person that we set to zero ( $\rho_g \geq 0$ ). If this threshold is met, gross investment is reduced to reduce resource consumption. If gross investment declines to zero, intermediate demands account for all goods consumption.

### Description of HARMONEY Model (Differences in v.1.1 from v1.0)

This section describes HARMONEY v1.1 differences as compared to HARMONEY 1.0 in King (2020). These changes generally make the model more robust to parameter changes.

### Wages and Labor

The participation rate,  $\lambda_N$  (employment), is the labor of both sectors divided by population,  $N$ . We specify a maximum participation rate,  $\lambda_{N,max} \leq 1$ , (equal to 80% in this paper), to represent that some fraction of the population is too young, old, or otherwise unable to work.

Following Keen (2013) we model the rate of changes of wages ( $w$ ) as a function of participation rate and inflation:

$$\frac{\dot{w}}{w} = \phi(\lambda_N) + w_1 i + w_2 \frac{1}{\lambda_N} \frac{d\lambda_N}{dt} \quad (14)$$

where  $\phi(\lambda_N)$  is a short-run Phillips curve (see Supplemental Section SI.3.4),  $0 \leq w_1 \leq 1$  weights how much inflation ( $i$ ) affects the nominal wage, inflation is calculated as the consumption-weighted average change in prices (Equation 15), and  $w_2$  weights how much the rate of change of employment affects nominal wage. The difference from King (2020) is the addition of the second and third terms in Equation 14. When  $w_1 = 1$ , the participation rate can come to equilibrium to its nominal value  $\lambda_{N,o}$  as defined in the Phillips curve.

$$i = \frac{C_g}{C_g + C_e} \frac{\dot{P}_g}{P_g} + \frac{C_e}{C_g + C_e} \frac{\dot{P}_e}{P_e} \quad (15)$$

### Investment and Capital Accumulation

We model investment the same as in King (2020) but add an option to include what we term Ponzi investment. Ponzi

investment increases debt but does not contribute to new physical capital. It is unrealistic to think that firms will continue to invest in physical capital if that physical capital continues to accumulate but operate at declining capacity utilization. However, speculative and Ponzi-style investment does occur, as described by Hyman Minsky and by Keen (Minsky 1977; Keen 2013).

Total monetary investment in each sector is as in King (2020) and shown in Equation 16 where  $\kappa_{0,i}$  and  $\kappa_{1,i}$  describe investment as multipliers on depreciation and net profit,  $\Pi_i$ , respectively. The Ponzi fraction of investment in sector  $i$  is  $f_{P_i}$  ( $0 \leq f_{P_i} \leq 1$ ). The non-Ponzi fraction,  $(1 - f_{P_i})$ , of this total sectoral investment is allocated to new capital formation (Equation 17). The Ponzi fraction of monetary investment (Equation 18) does not increase the existing capital stock, but only increases debt since all monetary investment,  $I_i$ , can potentially increase debt (Equation 13). Our modeling of Ponzi investment is inspired by, but different from, that defined in Keen (2009) and Grasselli and Costa Lima (2012) who model Ponzi investment as debt that increases as a function of the real GDP growth rate (Keen 2009; Grasselli and Costa Lima 2012). We model the Ponzi fraction of investment as a function of capacity utilization as in Equation 19 where  $CU_{i,ref}$  is the reference, or target, capacity utilization which we set at 85%. The Ponzi parameter  $a_{Ponzi,i} \geq 0$  governs the magnitude of Ponzi investment, with larger values shifting more investment from physical capital to Ponzi.

$$I_i = \max\{0, \kappa_{0,i} P_g \delta K_i + \kappa_{1,i} \Pi_i\} \quad (16)$$

$$I_{newK,i} = (1 - f_{P_i}) I_i \quad (17)$$

$$I_{Ponzi,i} = f_{P_i} I_i \quad (18)$$

$$f_{P_i} = \min \left\{ 1, \max \left\{ 0, a_{Ponzi,i} \left( \frac{CU_{i,ref} - CU_i}{CU_{i,ref}} \right) \right\} \right\} \quad (19)$$

$I_e^g$  and  $I_g^g$  represent physical investment in  $K_e$  and  $K_g$ , respectively, where the superscript  $g$  indicates capital has units of goods. Physical investment in new capital for each sector is thus equal to the non-Ponzi monetary investment divided by the price of goods, or  $I_i^g = \frac{(1-f_{P_i})I_i}{P_g}$ . We use the perpetual inventory method for capital accumulation as physical investment minus physical depreciation occurring at constant rate,  $\delta$ , for each sector (Equation 20).

$$\dot{K}_i = I_i^g - \delta K_i = \frac{(1-f_{P_i})I_i}{P_g} - \delta K_i \quad (20)$$

## Prices and Costs

Similarly to King (2020) we calculate prices,  $P_i$ , based on a constant markup,  $\mu_i \geq 0$ , multiplied by the cost of production, or  $P_i = (1 + \mu_i)c_i$ . Thus, prices change based upon the difference between the marked-up cost and price as in Equations 21 and 22, and these equations are equivalent to those in King (2020). However, unlike King (2020), we no longer solve for all sector prices simultaneously (using a matrix inversion), but use Equation 22 to solve for the change in price for each sector.

Equations 23 and 24 define the *full cost* of production. In the results we explore differences that arise from the assumption that producers set prices on the full cost versus only the *marginal costs* that neglect the cost of both interest payments ( $r_L D_i$ , with  $r_L$  as the interest rate on loans, or debt, borrowed by each sector) and depreciation ( $P_g \delta K_i$ ). We compare these two pricing assumptions to explore differences in their long-term dynamic effects that represent different regulatory structures of a (capitalist) economy.

$$\frac{\dot{P}_i}{P_i} = \left( \frac{1}{\tau_{P_i}} \right) ((1 + \mu_i)(c_i/P_i) - 1) \quad (21)$$

$$\dot{P}_i = \left( \frac{1}{\tau_{P_i}} \right) ((1 + \mu_i)c_i - P_i) \quad (22)$$

$$c_g = P_g a_{gg} + P_e a_{eg} + (wL_g + r_L D_g + P_g \delta K_g)/X_g \quad (23)$$

$$c_e = P_e a_{ee} + P_g a_{ge} + (wL_e + r_L D_e + P_g \delta K_e)/X_e \quad (24)$$

## Information Theory and Self Organization

### Information Theory to Assess Economy Structure

Over the course of a few decades Robert Ulanowicz developed the use of information theoretic metrics to quantify the structure of food webs (Ulanowicz et al. 2009; Ulanowicz 2009). King (2016) applied those methods to the U.S. economy. We use these mathematics to quantify the internal structure of the HARMONY model economy. We are interested in structure because in network science, ecology, and economics, system structures that distribute flows more evenly are sometimes considered more resilient and complex. By internal structure, we refer to the proportional distribution of economic transactions within the model's  $2 \times 2$  intermediate transactions matrix,  $\mathbf{X}$  (Equation 25). By discussing structural metrics of information theory along with measures of size and growth (population, debt, resource

extraction rate, net output, etc.) we enable a more holistic description of economic evolution and dynamics.

In Results and Supplemental material we discuss three information theoretic metrics: information entropy ( $H$ ), conditional entropy ( $\Psi$ ), and mutual constraint ( $X_{MC}$ ).<sup>1</sup> Equations 26–29 show the mathematics for these metrics, and we summarize them here but refer the reader to King (2016) for full details. We use economic rather than network terminology where a network node is a sector, and network flow is the transaction between sectors. A monetary purchase (flow) within input-output (I-O) matrix  $\mathbf{X}$  from sector  $j$  to sector  $i$  is represented as  $\mathbf{X}_{ij}$  (Equation 25). The ‘dot’ subscript on  $\mathbf{X}$  in Equations 26–29 indicates the sum of items over that dimension. For example,  $\mathbf{X}_j$  is the sum of all purchases by sector  $j$ , and  $\mathbf{X}_i$  is the sum of all sales by sector  $i$ . In addition,  $\mathbf{X}_{..}$  is the total system throughput (TST), or the sum of all transactions in the I-O table  $\mathbf{X}$  (see Equation 29).

$$\mathbf{X} = \begin{bmatrix} x_{gg} & x_{ge} \\ x_{eg} & x_{ee} \end{bmatrix} \quad (25)$$

The economy information entropy, or indeterminacy  $H$ , is defined in Equation 26, and is equal to the sum of mutual constraint and conditional entropy (Equation 30). The economy mutual constraint,  $X_{MC}$  (Equation 27), measures the degree to which an economy efficiently distributes flows among its sectors or its average degrees of constraint. The conditional entropy,  $\Psi$  (Equation 28), is a measure of the average degrees of freedom of the economic network for all transactions  $X_{ij}$ , or the remaining choice of flow pathways for transactions going from sector  $i$  to sector  $j$ . Ulanowicz interprets  $X_{MC}$  as what is known about the network and  $\Psi$  as what is not known, but what is possible in terms of flows moving through the network (Ulanowicz 2009).

$$H = - \sum_{ij} \frac{\mathbf{X}_{ij}}{\mathbf{X}_{..}} \log_2 \left( \frac{\mathbf{X}_{ij}}{\mathbf{X}_{..}} \right) \quad (26)$$

$$X_{MC} = \sum_{i,j} \frac{\mathbf{X}_{ij}}{\mathbf{X}_{..}} \log_2 \left( \frac{\mathbf{X}_{ij} \mathbf{X}_{..}}{\mathbf{X}_i \mathbf{X}_j} \right) \quad (27)$$

$$\Psi = - \sum_{ij} \frac{\mathbf{X}_{ij}}{\mathbf{X}_{..}} \log_2 \left( \frac{\mathbf{X}_{ij}^2}{\mathbf{X}_i \mathbf{X}_j} \right) \quad (28)$$

<sup>1</sup> Here we use  $X_{MC}$  to represent mutual constraint, instead of  $X$  as in much of the literature because this paper already uses  $X$  to represent physical gross output of each sector in our model. Mutual constraint is mathematically equal to the terms mutual information and average mutual information (Shannon and Weaver 1962) that are used in the field of information theory.

$$\mathbf{X}_{..} = \sum_{ij} \mathbf{X}_{ij} \quad (29)$$

$$H = X_{MC} + \Psi \quad (30)$$

Fig. 1 helps interpret the metrics. The calculations of  $X_{MC}$  and  $\Psi$  are restricted to the triangular area, or phase space, encompassed by the solid and dashed lines. The maximum number for each metric increases with the number of nodes,  $n$ , of the network ( $H_{max} = \Psi_{max} = \log_2(n^2)$ ,  $X_{MC,max} = \log_2(n)$ ). The higher the conditional entropy, the more equal is each intersectoral transaction. At maximum conditional entropy (also maximum information entropy and zero mutual constraint) all intersectoral transactions are equal (upper boundary point in Fig. 1). At maximum mutual constraint each sector transacts with only one other sector, and each of these single transactions are equal (lower-right boundary point in Fig. 1). At zero conditional entropy and mutual constraint, there is only one intersectoral transaction (lower-left boundary point in Fig. 1). *Ceteris paribus*, an economy with higher information entropy is more resilient to changing conditions and has a more diverse economy because many sectors contribute a significant share of economic transactions. However, in general, physical constraints in the economy prevent achieving a state of maximum conditional entropy of a monetary I-O matrix (e.g., the “petroleum refining” sector inherently purchases more from the “oil and gas extraction” sector than the other way around).

## Endogenous and Exogenous Variables

Table 2 lists the endogenous and exogenous variables included in the model.

## Results

### Scenario Definitions

We run several scenarios to explore the influence of changing four major factors and assumptions (Table 3). The first is the assumption whether prices are based on full cost (FC) or marginal cost (MC) pricing. The second decreases the resource consumption to operate each sector’s capital,  $\eta_i$ , to observe the effects of increasing efficiency. Starting at time  $T = 0$ , we decrease  $\eta_i$  as a 3rd order function of investment (into physical capital) to approximate that improvements in capital stock occur via investing in new capital (i.e., learning by doing). See Fig. 3e, f and Supplemental Section SI.3.7 for description of  $\eta_i$  as a function of investment.

The third concept defining the scenarios relates to wages. We linearly decrease the wage function parameters  $w_1$  and  $w_2$  from 1 to 0, during the time span indicated in Table 3, to

**Table 2** A list of endogenous and exogenous variables within the model

Endogenous Variables	Exogenous Variables
Population via death rates ( $\alpha_N$ )	Birth rates ( $\beta_N$ )
Prices ( $P_e, P_g$ )	Natural resource definition
Wages ( $w$ )	Wage function parameters (Phillips curve, $w_1, w_2$ )
Debt of firms ( $D_e, D_g$ )	Interest rates ( $r_L, r_M$ )
Capacity utilization ( $CU_e, CU_g$ )	Sector-specific labor productivity ( $a_e, a_g$ )
Inventories ( $g, w_H$ )	Goods sector productivity (capital:output ratio, $v_g$ )
Economic output ( $Y_e, Y_g$ )	Resource requirements to make capital ( $y_{X_g}$ )
Physical output ( $X_e, X_g$ )	Resource requirement to operate capital ( $\eta_e, \eta_g$ )
Investment ( $I_e, I_g$ )	Time constants ( $\tau$ )
Capital ( $K_e, K_g$ )	
Labor ( $L_e, L_g$ )	
Household consumption ( $C_e, C_g$ )	
Power return ratios (e.g., NEPR, NPR)	

**Table 3** Definitions of simulated scenarios. The shorthand format to describe scenarios is FC-XYZ and MC-XYZ such that FC-000 and MC-000 represent “baseline” scenarios to which changes are made where X, Y, and Z switch to 1 as follows. FC=full cost pricing, MC=marginal cost pricing, X=0:  $\eta_i$  remain constant, X=1:  $\eta_i$  decrease (higher efficiency scenarios), Y=0: labor retains full bargaining power ( $w_1 = w_2 = 1$ ), Y=1: labor bargaining power reduces to zero ( $w_1 = w_2 = 0$ ), Z=0: no Ponzi investing, and Z=1: includes Ponzi investing

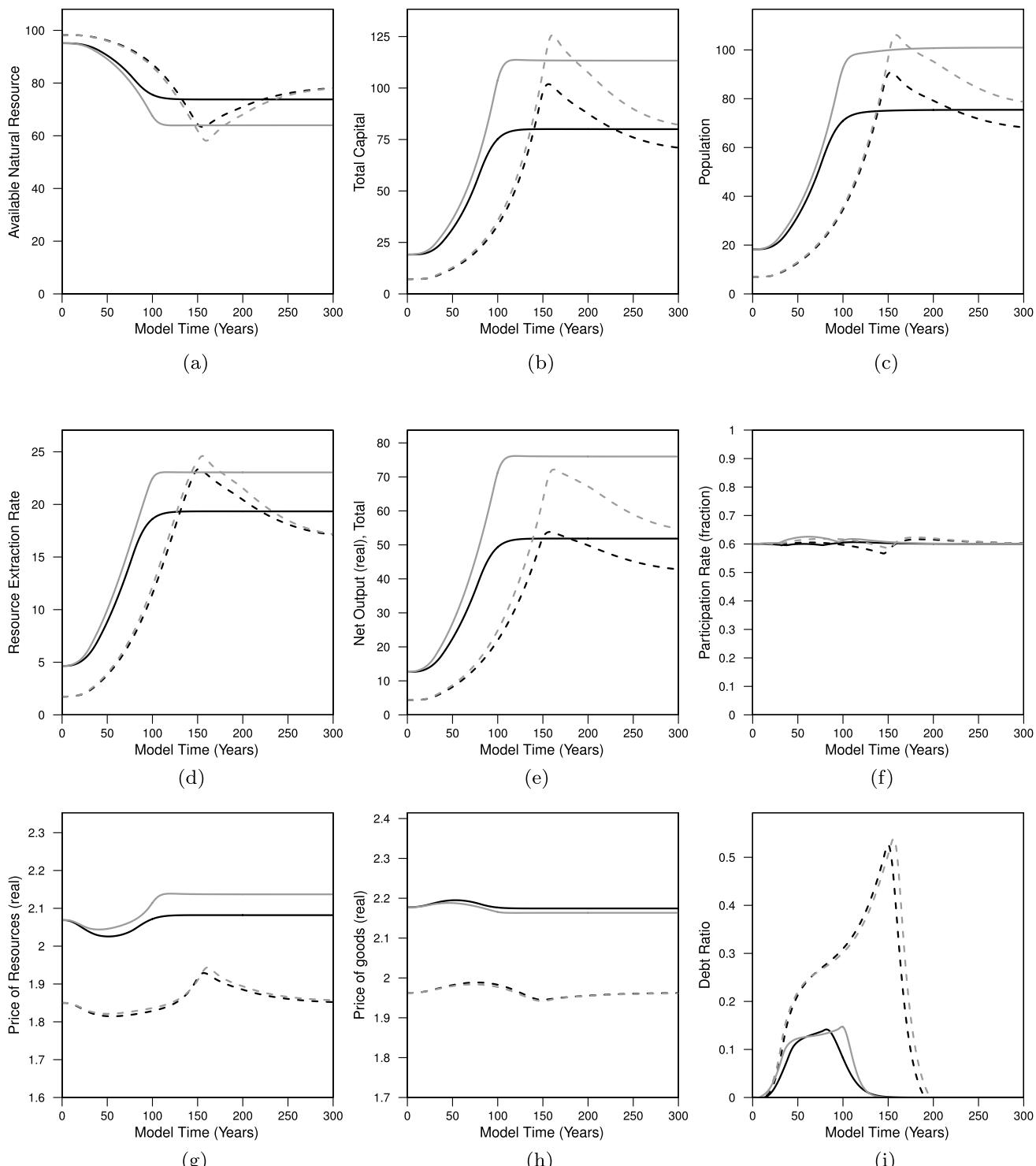
Scenario	$\eta_i$	Pricing: Full or Marginal Costs	$(w_1, w_2)$	Ponzi factor, $a_{Ponzi,i}$
		Table 1	Eq. 24, 23	Eq. 19
FC-000	Constant	Full	(1,1)	0
MC-000	Constant	Marginal	(1,1)	0
FC-010	Constant	Full	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 60$ to $t = 160$	0
MC-010	Constant	Marginal	(1, 1) $\rightarrow$ (0, 0)@ $t = 100$ to $t = 200$	0
FC-011	Constant	Full	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 60$ to $t = 160$	3
MC-011	Constant	Marginal	(1, 1) $\rightarrow$ (0, 0)@ $t = 100$ to $t = 200$	3
FC-100	Decreasing	Full	(1,1)	0
MC-100	Decreasing	Marginal	(1,1)	0
FC-110	Decreasing	Full	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 60$ to $t = 160$	0
MC-110	Decreasing	Marginal	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 100$ to $t = 200$	0
FC-111	Decreasing	Full	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 60$ to $t = 160$	3
MC-111	Decreasing	Marginal	(1, 1) $\rightarrow$ (0.0, 0.0)@ $t = 100$ to $t = 200$	3

simulate the loss of labor “bargaining power.” Finally, the fourth scenario factor is whether to include Ponzi investing (at all times) by changing Ponzi parameter  $a_{Ponzi,i}$  from 0 (no Ponzi investing) to 3 (with Ponzi investing).

### High Level Results: Full and Marginal Cost Assumptions

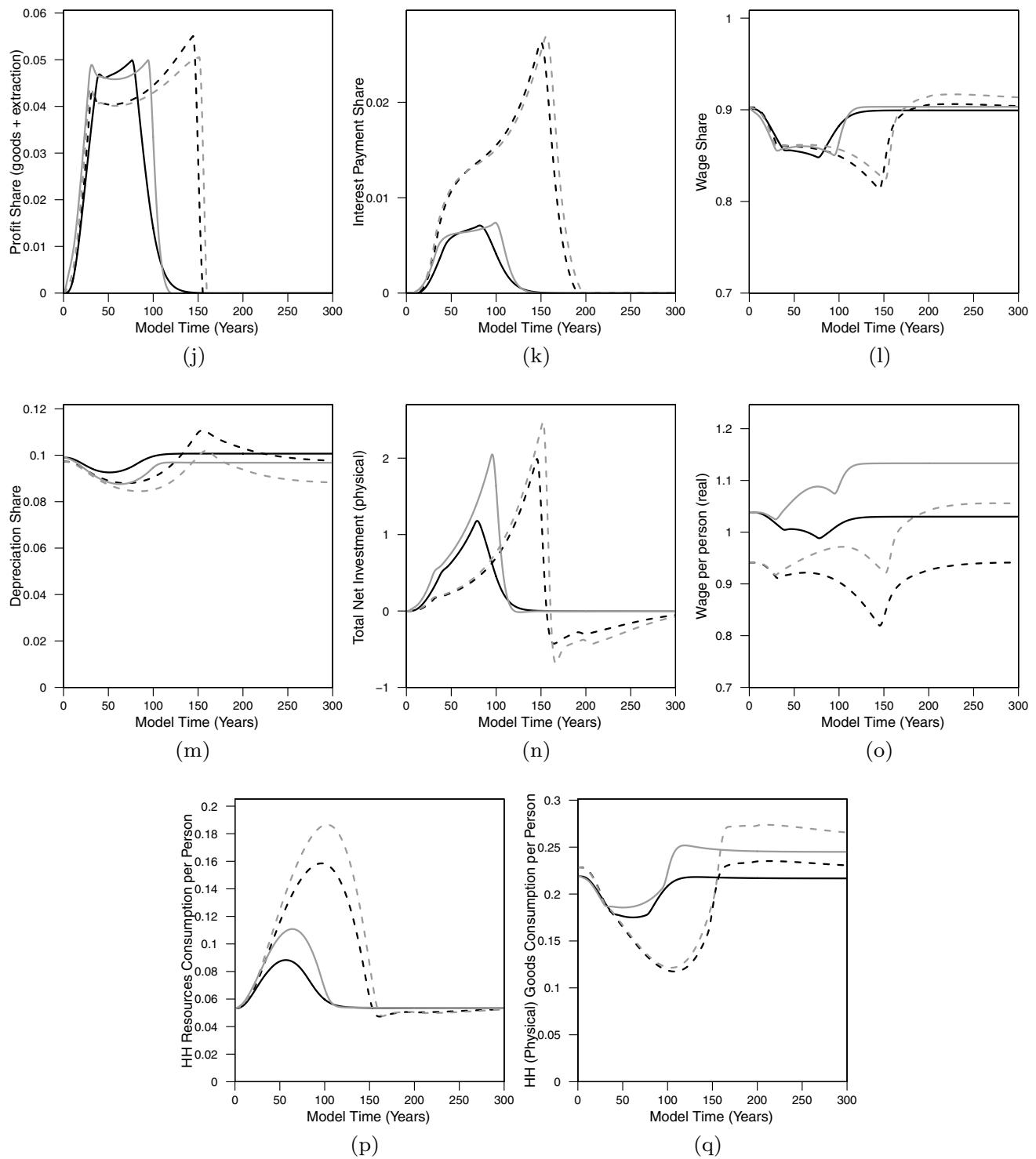
We first discuss some high level takeaways, and highlight differences from the first HARMONEY paper of King (2020). Fig. 2 compares scenarios with and without gains in capital operating efficiency for both full and marginal cost assumptions. Scenarios FC-000 and MC-000 assume no change in resource consumption per unit of capital output ( $\eta_i$ ) whereas scenarios FC-100 and MC-100 assume an increase in machine efficiency as a decrease in  $\eta_i$ . Each scenario begins from a steady state condition (e.g., constant

level of stocks). The equilibrium conditions are defined as the steady state values achieved by simulating each cost scenario at a relatively low value of the technological extraction parameter,  $\delta_y = 0.0072$ , such that very little resource is exploited (less than 5% of maximum level). They start from these initial conditions (including zero debt and profit share) such that population and capital increase by gradually increasing  $\delta_y$  to a value of  $\delta_y = 0.009$  to enable a larger quantity of the natural resource base to be profitably extracted (see Fig. 3). We increase  $\delta_y$  via a 3rd order time delay, reaching its halfway mark at  $T = 40$  and 99% of its maximum value at  $T = 84$ . This assumption of increasing the extraction parameter  $\delta_y$  mimics an improvement in technological capability for a known resource base, and drives an initial decrease in technical coefficient  $a_{ee}$ , the resource consumption required to operate extraction capital. Similar results and investigations of growth can be achieved in other



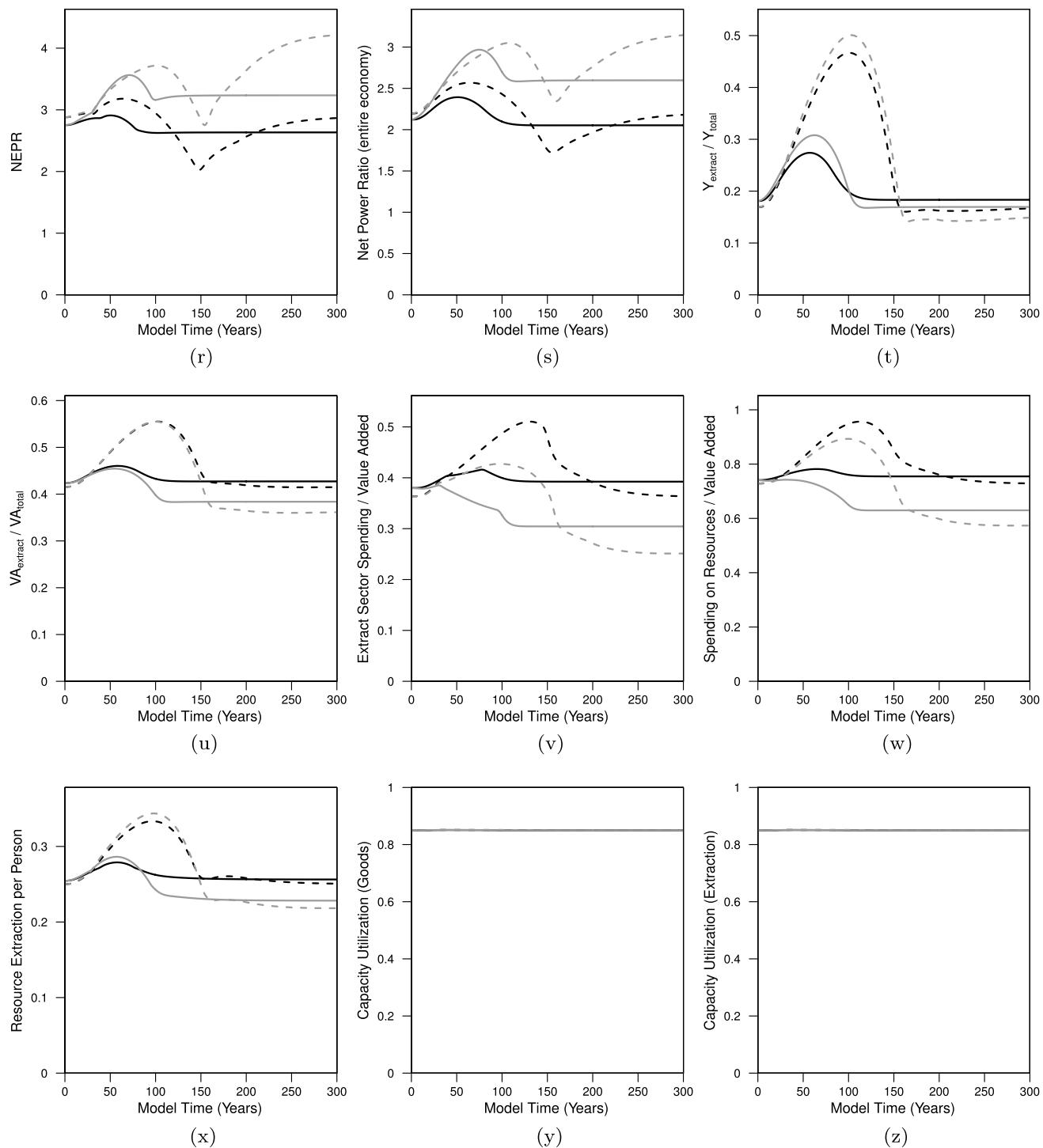
**Fig. 2** Scenarios FC-000, FC-100, MC-000, and MC-100 (black, gray, black dashed, gray dashed). **a** available resources (in the environment), **b** total capital, **c** population, **d** resource extraction rate, **e** total real net output, **f** participation rate, **g** real price of extracted resources, **h** real price of goods, **i** debt ratio, **j** profit share, **k** interest share, **l** wage share, **m** depreciation share, **n** physical net investment in new capital, **o** real wage per person, **p** household consumption of (physical) resources per person, **q** household consumption of (physi-

cal) goods per person, **r** net external power ratio (extraction sector, NEPR), **s** net power ratio (entire economy, NPR), **t** fraction of net output from extraction sector, **u** fraction of value added in extraction sector, **v** extraction sector spending per total value added (= per total net output), **w** spending on resources per total value added (= per total net output), **x** total resources extraction per person, **y** capacity utilization of goods capital, and **z** capacity utilization of extraction capital

**Fig. 2** (continued)

ways, such as by increasing the maximum size of the natural resource,  $\lambda_{y,max}$ , that mimics finding more resources at constant extraction technology (see Supplemental Section SI.4.1 and Fig. S.1 for a brief comparison), but we do not further explore these other assumptions to induce growth.

The results are significantly different from results in King (2020), and they stem from changes to the calculation of wages and price, investigation into the effects of increasing resource consumption efficiency in capital, changes in the

**Fig. 2** (continued)

definition of cost that informs pricing, and the consideration of several structural comparisons to global and U.S. data.

#### Full and Marginal Cost Pricing

Scenario FC-000 with full cost pricing (comparable to King (2020)) and full bargaining power (not comparable to King (2020)) reaches an equilibrium steady state at the nominal participation rate (60% in Fig. 2f) with zero debt and profits

(Fig. 2i, j). This steady state outcome was not attainable in HARMONEY v1.0 because wages could not increase with inflation.

From the beginning, the economy grows, accumulating capital, population, and debt while depleting the natural resource. During initial rapid growth profits and debt increase. They then level off before declining rapidly when peak resource extraction (Fig. 2d) inhibits further growth at which time all resources are needed to maintain existing capital and population, thus driving net capital investment to zero (Fig. 2n). During the entire simulation, there is no “overinvestment” such that capacity utilization remains at the target level of 85% (Fig. 2y, z).

With full bargaining power, after net investment declines to zero, wages and wage share increase at the expense of profits. Thus, HARMONEY puts the battle of labor versus capital into the context of resource consumption, and we expand on this later. The depreciation share of value added does not appreciably change because capital accumulation remains relatively low.

The power return ratios of net external power ratio (NEPR) and economy-wide net power ratio (NPR) (Fig. 2r, s) first increase, mostly driven by the exogenous assumption of an increase in extraction technology factor  $\delta_y$  that drives down  $a_{ee}$ . Eventually, once  $\delta_y$  reaches its final value and resource depletion starts to increase costs and thus prices (Fig. 2g, h), NEPR and NPR decrease due to the need to increasingly consume more resources to extract the next unit of resource. This rise and fall of NEPR generally mimics an expected trend that growth is first characterized by use of resources with increasing or high values of net energy, followed by some economic growth, perhaps slower than before, associated with declining net energy as the highest NEPR resources are used first. For example, the NEPR of U.S. oil and gas (often termed EROI = “energy return on energy invested”) increased from 16 in 1919 to 24 in 1954 before declining, with volatility, to 11 in 2007 (Guilford et al. 2011).

Population (Fig. 2c) levels off as per capita household resource consumption declines enough (Fig. 2p) to increase death rates to equal birth rates.

### Marginal Cost Pricing (Differences from Full Cost)

There are a few notable differences between the marginal and full cost pricing results, and we expand on this in later sections. First, marginal cost pricing enables an overshoot of steady state values for population and capital. While marginal cost pricing reaches higher peak levels of capital, population, net output, debt, resource extraction rates, and resource depletion, the steady state values are lower than in the full cost assumption. That is to say, with all other parameters the same, the marginal cost pricing reaches higher

peaks in growth, but they are only temporarily higher than if assuming full cost pricing. Another intuitive finding is that because costs are lower when assuming marginal cost pricing, the marginal cost prices are lower than full cost prices (Fig. 2g, h).

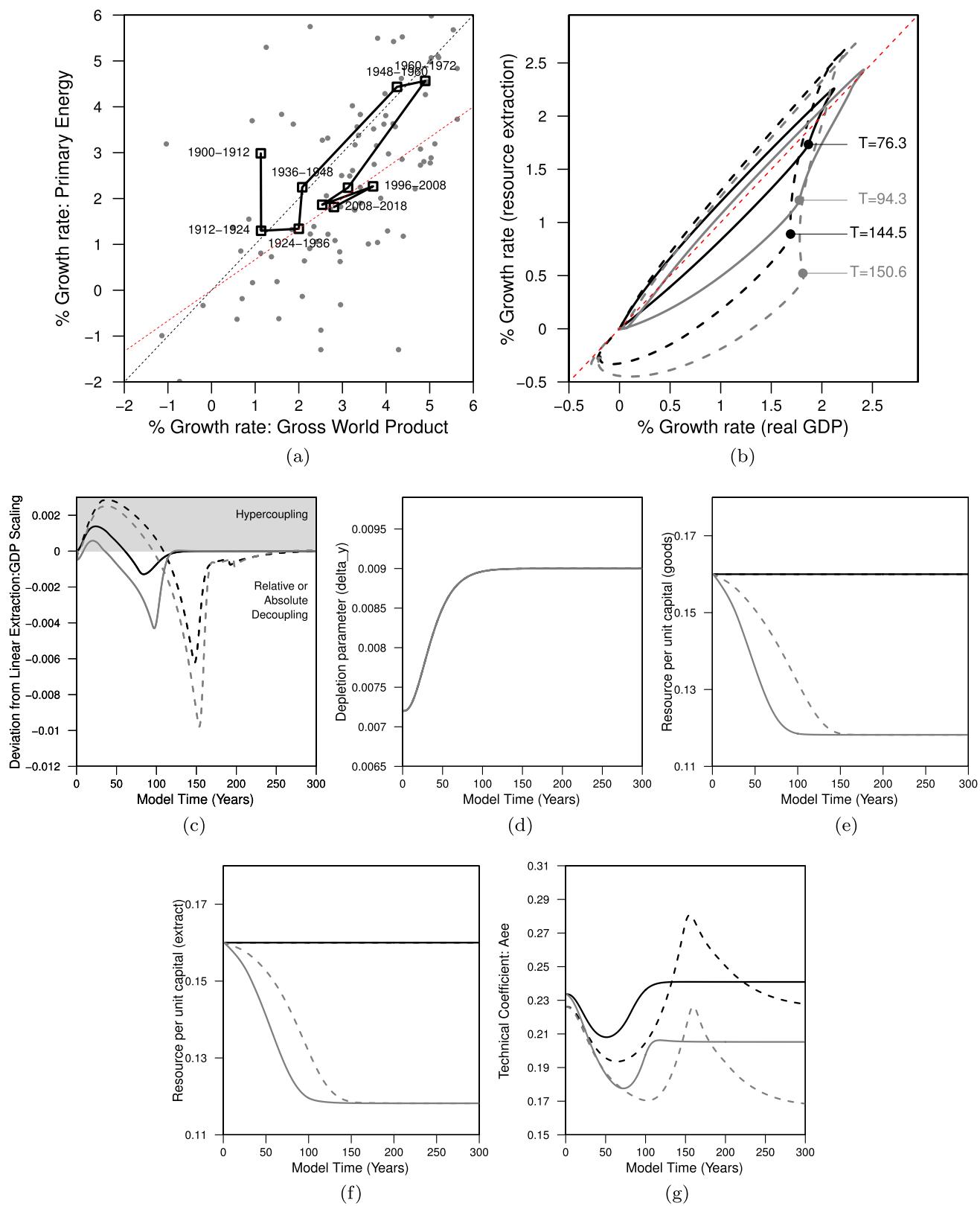
### Explanation of Patterns in Growth Rates: Extraction and GDP

Figure 3 compares global and model trends for growth rates in consumption of primary energy (global data) and natural resources (model) versus real GDP growth rates. The global data from 1900 to 2018 show a general clockwise trend (Jarvis and King 2020). From 1900 to the early 1970s the trajectory moves from lower-left to upper-right along a near 1:1 ratio in growth rates, in other words at near constant energy intensity. After the 1960s and early 1970s the data move to a region with a 2:3 ratio in growth rates, or declining energy intensity, centered near a 2% and 3% growth rate in primary energy consumption and gross world product (GWP), respectively. The HARMONEY model simulation results follow the same clockwise trajectory as seen in the global data in moving from an increasing or near constant resource intensity to a declining resource intensity. Thus, the HARMONEY v1.1 model endogenously recreates an important observed pattern in the real economy.

These HARMONEY model results inform the discussion of both relative and absolute decoupling. Here, relative decoupling refers as the situation in which growth of both GDP and energy (or resources) is positive, but GDP growth is higher. Absolute decoupling refers to positive growth in GDP but zero or declining growth of energy (or resource) consumption. The model patterns in Fig. 3b exhibit time periods of both relative and absolute decoupling. We measure the level of coupling or decoupling in Fig. 3c as the distance of the growth rate trajectories in Fig. 3b from the 1:1 slope line that would represent that resource extraction growth rates equal those of real GDP. Positive values indicate hypercoupling (or increasing resource intensity above the 1:1 line) and negative values indicate decoupling (or decreasing resource intensity below the 1:1 line).

We now discuss three concepts to explain the relationship between growth in resource extraction and GDP: biophysical constraints, resource consumption efficiency, and the level of private debt as influenced by the definition of cost of production.

First, there are biophysical constraints. Indefinite exponential growth (at a constant or increasing rate) from a finite resource (as a stock or flow) is not possible. HARMONEY v1.1 explicitly assumes a regenerative stock resource with finite size and limits of extraction, but early stages of growth do occur at an increasing exponential growth rate. The maximum growth rates (upper right extents in Fig. 3b) occur at



times of  $T = 40, 33, 32$ , and  $32$  for the FC-000, MC-000, FC-100, and MC-100 scenarios, respectively. Biophysical constraints govern the growth rates of resource extraction,

but the direct linkage to economic growth, as measured by GDP (or net output), is exhibited by the fact that after a peak in growth rates, both the historical data and the model results

**Fig. 3** **a** Global data (1900–2018) indicating annual growth rates of primary energy consumption versus annual growth rates of gross world product (data as in Jarvis and King (2020)). Squares indicate average growth rates for groups of 12-years. **b** The parallel figure of **a** for the HARMONEY scenarios showing the growth rates of natural resource extraction versus growth rates of real GDP (FC-000: black solid, MC-000: black dashed, FC-100: gray solid, and MC-100: gray dashed). **c** The distance of the curves in subfigure **b** from the 1:1 slope. Positive values indicate hypercoupling (above 1:1 line). Negative values indicate decoupling (below 1:1 line). **d** The exogenous change ( $t = 0$  to  $t = 100$ ) in depletion parameter,  $\delta_y$  is the same for all simulations. **e** and **f** Scenarios FC-100 and MC-100 decrease the resource consumption requirements for goods and extraction capital,  $\eta_g$  and  $\eta_e$  respectively, from 0.16 to 0.12 as a 3rd-order delay function of the rate of investment in each respective type of capital. **(g)** Technical coefficient  $a_{ee}$ , resource consumption of extraction sector per unit of extraction

move “down and to the left,” meaning that both growth rates decrease. This contrasts with an alternative trajectory of, for example “down and to the right,” as if GDP continues to increase at higher rates while resource extraction experiences a decreasing growth rate. At the peak growth rates, the economy has effectively grown to such a sufficient size relative to the size of its environment that it can no longer maintain existing capital and population while growing at an increasingly fast rate. This reasoning regarding the comparison of data in Fig. 3a and b implies that the decreasing GWP growth rate can also be interpreted as the global economy becoming “large” relative to its environment from which it extracts energy.

Each curve of Fig. 3b exhibits a “knee” in the sublinear scaling regime where the trajectory changes from downward (near vertical) to a near 45 degree angle down and to the left. For each scenario, this “knee” occurs when the sum of firm profits and interest payments (as a share of GDP) is at its maximum (at  $T = 76, 145, 94$ , and 151 for scenarios FC-000, MC-000, FC-100, and MC-100, respectively). In addition, very soon after this time, total physical net investment peaks ( $T = 78, 145, 95$ , and 152, respectively, Fig. 2n) and debt ratios peak ( $T = 82, 150, 99$ , and 156, respectively, Fig. 2i) before the rapid repayment of debt and population death rates increase from their minimum value (death rates increase from the minimum value when household per person resource consumption is below  $s = 0.08$  in Fig. 2p). Importantly, these events occur nearly simultaneously but before resource extraction actually peaks (i.e., growth rate of extraction remains positive at these peaks). Nonetheless, there is no longer enough physical flow of natural resources to invest in new capital at the previous rates while maintaining the existing levels of population and capital. Something must give, and eventually everything does in the process of reaching a steady state without overshoot (full cost scenarios) or with overshoot (marginal cost scenarios). Importantly, the assumption that wages fully adjust to inflation

( $w_1 = 1$ ) means that as profit and interest shares drop to zero, wages (Fig. 2o) and the wage share (Fig. 2l) rise.

Second, we now discuss resource consumption efficiency as a driver of decoupling. A common interpretation of relative decoupling is that the economy becomes more efficient in its consumption of energy in producing the goods and services of which GDP is composed (e.g., UN Sustainable Development Goal 7: Affordable and Clean Energy as in United Nations (2020), climate mitigation as in Edenhofer et al. (2014)). Because studies have shown that useful work scales proportionally to GDP, then increasing efficiency of converting primary fuels to useful work could also explain relative decoupling (Warr et al. 2010; Jarvis and King 2020). However, scenarios FC-000 and MC-000 (black solid and dashed lines) assume no exogenous increase in sector-specific labor productivity ( $a_g = a_e = \text{constant}$ ), resource efficiency in operating capital ( $\eta_g = \eta_e = \text{constant}$ ), or resource efficiency as embodied in new capital ( $y_{X_g} = \text{constant}$ ), yet the results still show that such an economy can reside, for some time period, in the relative decoupling regime. For the marginal cost scenario, there is a short time period in the absolute decoupling regime (more explanation later). The explanation does not reside in the fact that all scenarios in Figs. 2 and 3 assume an initial increase in the extraction parameter,  $\delta_y$ , that represents an exogenous increase in the amount of resources extracted per unit of extraction capital. The scenarios show no relative decoupling during initial growth when  $\delta_y$  increases, and they reside in the regime of relative decoupling well after  $\delta_y$  reaches a constant level.

The technical coefficient  $a_{ee} = \frac{\eta_e}{\delta_y y}$  (see Fig. 3g) is another parameter with which to interpret efficiency: a higher value represents a less efficient resource sector. It initially decreases due to our exogenous initial increase in technological parameter  $\delta_y$ . However, it eventually increases as the resource,  $y$ , depletes and does so during the time when the economy appears most relatively decoupled (Fig. 3c). This is again more evidence that resource efficiency is not the full explanation of apparent relative decoupling. Note that because of their definition as a function of  $a_{ee}$ , the power return ratios, NEPR and NPR (Fig. 2r and s), tend to have opposite trends as  $a_{ee}$ .

The model does provide justification, however, for the view that increasing energy efficiency can move the economy to a more decoupled state compared to an economy with no energy efficiency. Scenarios FC-100 and MC-100 (gray lines) are identical to FC-000 and MC-000 (black lines), respectively, except that they assume a decrease in resource consumption to operate capital  $\eta_g$  and  $\eta_e$  (see Fig. 3e and f). The increase in resource consumption efficiency of capital (i.e., fuel consumption of machines) indeed causes a shift further into the relative decoupled zone compared to no increase.

However, given the assumption that higher profits translate to increased investment, increasing energy efficiency clearly enables increased natural resource depletion (Fig. 2a) while enabling the economy to achieve higher resource extraction rates (Fig. 2d), population (Fig. 2c), capital accumulation (Fig. 2b), and net output (Fig. 2e). In other words, the HARMONEY model supports the Jevons Paradox, or backfire effect (Jenkins and Nordhaus 2011), in that higher fuel efficiency in operating capital *increases* overall economic size and consumption (Jevons 1866). This theoretical finding is consistent with studies supporting the evidence for a strong rebound effect (Brockway et al. 2021), as well as the general observation that over the course of industrialization to date, the human economy has indeed invented and employed more efficient processes while at the same time consumed more energy decade to decade.

Finally, we discuss decoupling in the context of the definition of cost that sets prices. The maximum level of decoupling occurs at almost the same times ( $T = 84, 148, 97$ , and  $154$ , for FC-000, MC-000, FC-100, and MC-100 scenarios, respectively) as the peaks in debt ratios. Compared to full cost price scenario with no change in resource consumption efficiency, no Ponzi investing, and full bargaining power (FC-000), the comparable marginal cost price scenario MC-000 reaches 4.8 times higher decoupling and 3.7 times higher peak debt ratio. For the scenarios with increasing resource efficiency, compared to full cost price scenario FC-100, marginal cost price scenario MC-100 reaches 2.3 times higher decoupling and 3.7 times higher peak debt ratio.

In short, the more decoupled scenarios are those that assume marginal costs of production inform prices. They also lead to higher debt ratios than the full cost pricing scenarios. In addition, marginal cost pricing scenarios reach states of “absolute decoupling”, whereas the full cost scenarios do not. It is important to note this apparent absolute decoupling occurs transiently for a very short time from about  $T = 149$  to  $T = 158$  for the MC-000 scenario and  $T = 156$  to  $T = 162$  for the MC-100 scenario. After these time intervals the economy again approaches full coupling as resource extraction and GDP decline, along with declines in both capital and population.

We conclude that while “decoupling” does increase with energy efficiency of machines, there are other factors that are at least as relevant, if not more so. Decoupling (as it appears) can come from producers using marginal cost pricing if they lack ability to, or choose not to, pass depreciation costs and interest payments into prices. The exclusion of depreciation in cost, much more so than excluding interest payments, has the majority affect on apparent increases in decoupling.

Further, the HARMONEY model supports the conclusion that a decoupled state is a natural expected stage during a growth cycle that follows the inability to increase growth

rates. That is to say, not only does a stage of relative decoupling occur during periods with no perceivable increase in device energy efficiency, this stage is evidence *for* limits to (increasingly fast) growth, not evidence against limits to growth.

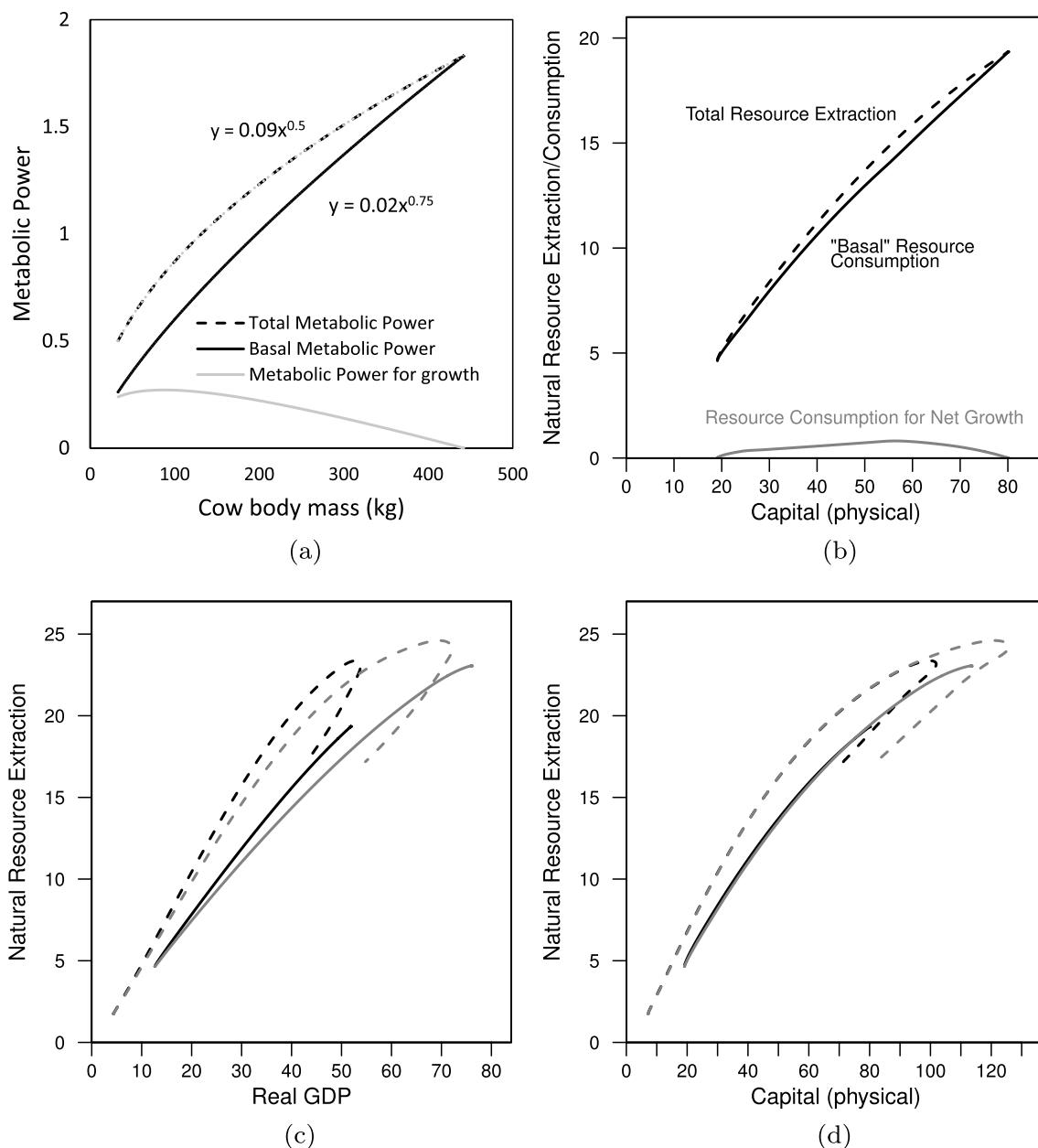
## Comparison of Biological to Economic Growth in HARMONEY

Here we provide an additional explanation of the relationship between natural resources consumption and growth by making an explicit comparison to biological growth. Figure 4 compares a typical metabolism versus mass trend (for a cow starting from birth per West et al. 2001) to corresponding results from the HARMONEY model as total resource extraction versus GDP and capital, the latter of which is a more appropriate analog to animal mass. Figure 4a displays curves for total metabolic power and basal metabolic power, scaling with mass to the 0.5 and 0.75 power, respectively. The difference between these is the metabolic power allocated to growth of new mass. One important point is that most organisms grow only to a certain size, with the trajectory of Fig. 4a moving up and to the right, eventually stopping at some point. The full cost HARMONEY scenarios show a similar trajectory.

In Fig. 4c and d the full cost pricing scenarios grow until a point at which they stop and the economy remains at a steady state value of resource extraction, GDP, and capital. In contrast the marginal cost pricing scenarios move up and to the right before looping downward and to the left, eventually also resting at non-zero steady-state values of extraction, GDP, and capital. Perhaps more clearly than in Fig. 3, Fig. 4 shows how the modeled marginal cost economy exists in an apparent state of absolute decoupling (increasing capital with decreasing resource consumption) for only a very short time near the maximum accumulation of capital during overshoot. It is as if biological organisms, such as mammals, are forced to adhere to “full cost pricing” of their allocation of energy to mass because of a lack of cultural or economic choice among their cells to use “marginal cost pricing.” In our human economy, however, we have the option to define rules based upon “marginal cost” accounting that appear to, but do not, thwart the necessity of resource consumption for growth and maintenance.

## Impacts on Loss of Wage Bargaining Power and Ponzi Investing

The results in Fig. 2 assume that wages are fully indexed to inflation by assuming the “wage bargaining” factors as  $w_1 = 1$  and  $w_2 = 1$  in Equation 14. We test how a reduction in these wage bargaining factors affects the real wage and wage share, as many posit a loss of bargaining power as a major



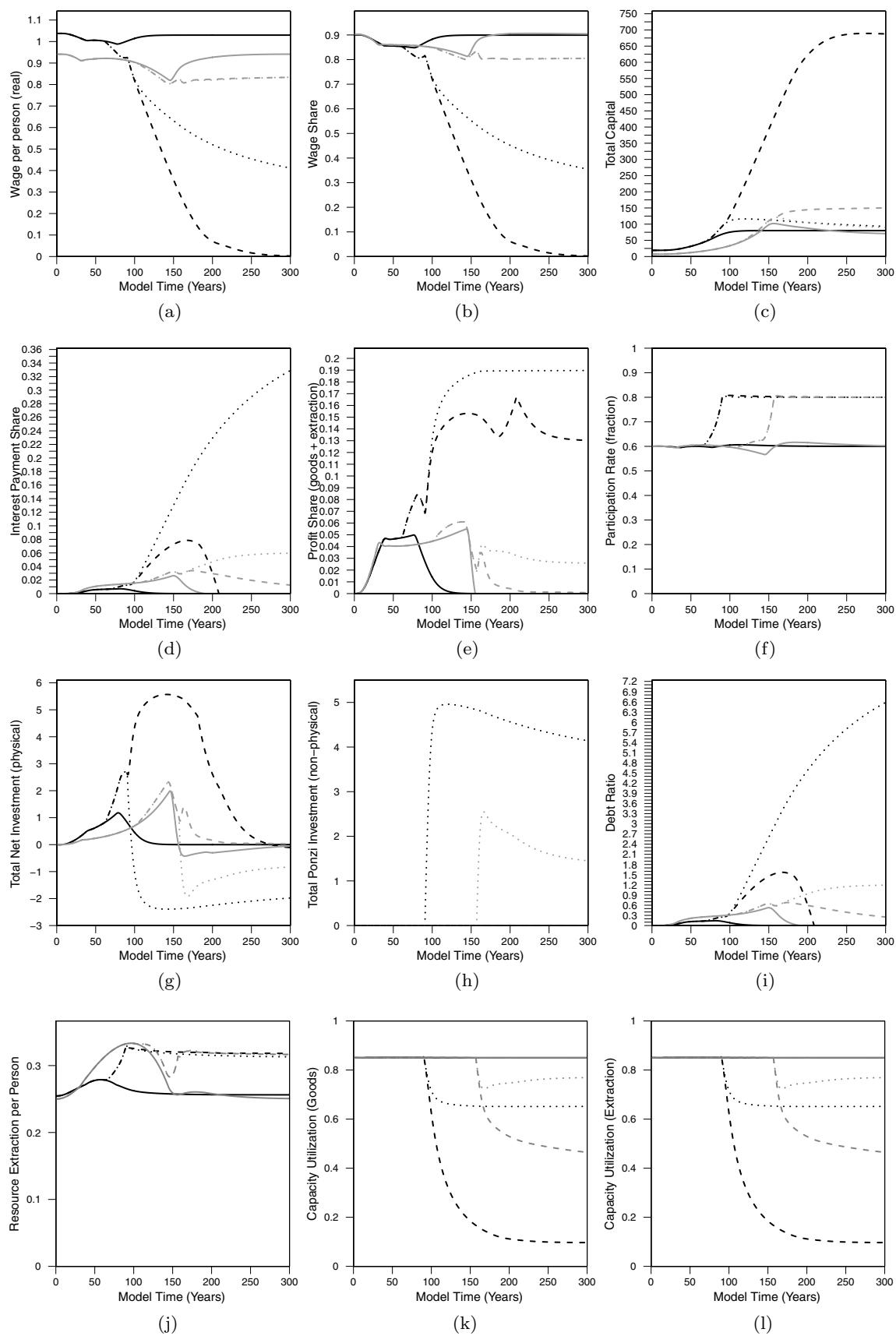
**Fig. 4** Comparison of **a** biological ontogenetic metabolic scaling (relation of energy consumption to mass), including power for growth, from West et al. (2001) with **b** a parallel plot from HARMONEY full cost pricing result with no change in efficiency. HARMONEY plots of the same scenarios as in Fig. 2 of natural resource

extraction versus **c** real GDP and **d** versus total physical capital ( $K_e + K_g$ ). **c** and **d** show data for full cost scenarios FC-000 (black solid) and FC-100 (gray solid) with marginal cost scenarios MC-000 (black-dashed) and MC-100 (gray-dashed)

explanation for stagnant U.S. wages since the 1970s (Bivens and Mishel 2015). We choose the timing for lowering the wage bargaining factors to mimic what occurred in the U.S. The data indicate that, starting in the early 1970s once the U.S. reached peak per capita energy consumption, real wages stopped increasing and wage share began to decline (King 2020, 2021; Bivens and Mishel 2015). U.S. data indicate wage share of GDP resided between 49 and 51.6% from

1948 to 1974 declining to a range of 42–45% between 2003 and 2020<sup>2</sup>. Thus, we begin reducing  $w_1 = 1$  and  $w_2 = 1$  once

<sup>2</sup> For example, see data at the Federal Reserve Bank of St. Louis website “Shares of gross domestic income: Compensation of employees, paid: Wage and salary accruals: Disbursements: to persons (W270RE1A156NBEA)”, <https://fred.stlouisfed.org/series/W270RE1A156NBEA>.



**◀Fig. 5** Scenarios FC-000, FC-010, FC-011, MC-000, MC-010, and MC-011 (black, black dashed, black dotted, gray, gray dashed, gray dotted) **a** wage per person (real), **b** wage share, **c** total capital, **d** interest share, **e** profit share, **f** participation rate, **g** total net investment (physical capital), **h** Ponzi investment, **i** debt ratio, **j** total resources extraction per person, **k** capacity utilization of goods capital, and **l** capacity utilization of extraction capital

the simulation reaches peak per capita resource extraction (as indicated in Table 3). Figure 5 (dashed lines) shows the impact of gradually reducing  $w_1$  and  $w_2$  to zero (see Supplemental Figs. S.2 and S.4 for the full suite of Table 3 scenarios exploring bargaining power reductions).

In all cases, the removal of bargaining power leads to participation rate increasing to the maximum level of 80% (Fig. 5f). When considering full cost pricing without increasing capital operating resource efficiency (scenario FC-010), the loss of bargaining power allocates a higher share of value added to profits (Fig. 5e) and therefore increases investment (Fig. 5g). Capital then accumulates to a much higher level (Fig. 5c) while operating at declining capacity utilization (Fig. 5k, l). Both real wage (Fig. 5a) and wage share (Fig. 5b) decrease to zero as depreciation (Fig. S.2(m)) accounts for the dominant share of value added.

The result from loss of bargaining power is quite different when using the marginal cost pricing assumption (MC-010, gray-dashed line). While real wage and wage share no longer rise as occurs when full bargaining power remains, they also do not decline but stay approximately level once bargaining power is removed (Fig. 5a, b). While investment increases with respect to maintaining full labor bargaining power, the inability to pass through depreciation costs limits net investment to similar levels as with full bargaining power (Fig. 5g).

### Ponzi Investing with Loss of Bargaining Power

As defined in this paper, Ponzi investing diverts private investment away from investment in physical capital. If full bargaining power remains, capacity utilization remains at its target level, and Ponzi investment is zero (not shown).

If wage bargaining power is removed, Ponzi investing (Fig. 5h) significantly increases private debt ratios (Fig. 5i). For full cost pricing (scenario FC-011), Ponzi investing translates to a slower decline in wages (Fig. 5a) compared with all investment going to physical capital. Thus, perhaps unintuitively from a wage perspective, if bargaining power is removed, Ponzi investing appears good for labor. In such a case, Ponzi investing also appears good for firms since profit share is also higher than without Ponzi investing (Fig. 5e). However, the full cost pricing assumption still drives capacity utilization to low levels (near 65%) that might not remain in a real-world economy (Fig. 5k, l).

If using marginal cost pricing with loss of bargaining power and Ponzi investing (scenario MC-011), there is no meaningful change in wages and wage share as they remain constant at their lowest values. Capacity utilization declines to near 75% compared to the targeted 85% (Fig. 5k, l). Private debt ratio continues to rise but to much lower levels than under the full cost pricing assumption (Fig. 5i).

### U.S. Structural Trend Comparison

Here, we compare structural trends of the HARMONEY results to those of the U.S. since World War II.

#### Capacity Utilization

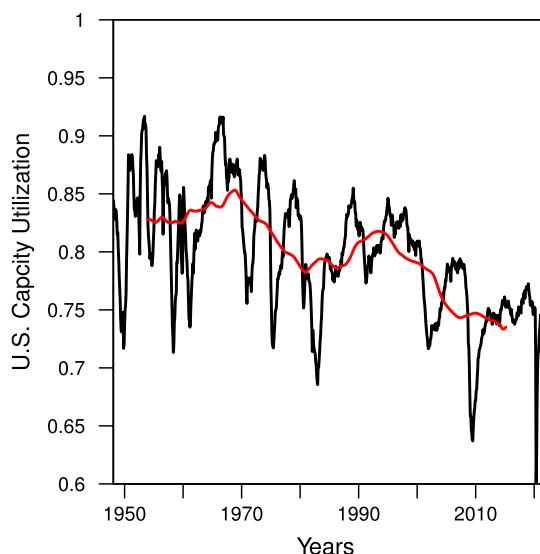
Figure 6 shows the capacity utilization of U.S. manufacturing from 1948–2020. It was highest during the post-World War II decades through the early 1970s, with a 12-year running average between 0.83–0.85. U.S. capacity utilization declined after the 1970s with the running average remaining below 0.75 since 2005.

HARMONEY goods sector (i.e., manufacturing) capacity utilization shows a parallel with that of U.S. manufacturing (see Fig. 5k) in that the scenarios with the loss of wage bargaining power also show a decline in capacity utilization. Thus, independent of concerns about the offshoring of U.S. manufacturing jobs, since HARMONEY assumes a closed economy (i.e., no imports or exports), the model results provide justification for interdependence among overinvestment in capital (leading to lower capacity utilization), loss of wage bargaining power, and a slow-down (or stagnation) in per capita energy consumption.

#### Labor Productivity and Real Wages

The effect of resource consumption on economy-wide labor productivity and wages is evident in Fig. 7 which is meant to compare to the trend noted in Bivens and Mishel (2015). They note that U.S. hourly compensation rises with net productivity (= growth of output of goods and services minus depreciation per hour worked) from 1948 to 1973, but afterwards hourly compensation is relatively constant while net productivity continues to increase. In Fig. 7a, b, gross labor productivity is real GDP divided by total labor, and net labor productivity in Fig. 7c, d subtracts depreciation from GDP.

The trends are roughly the same between gross and net productivity, except for the effect when labor bargaining power is removed. Aside from the initial 20–30 years of the simulation, real wages increase with increased productivity and decrease with decreasing productivity. In addition, both increase significantly more when assuming an increasing efficiency of resource consumption to operate capital. This matches exactly with the U.S. data that show the rise in U.S.



**Fig. 6** Monthly capacity utilization of U.S. manufacturing (SIC) (black: monthly data; red: 144 month = 12 year rolling average). Data from Federal Reserve Bank of St. Louis data set CUMFNS

real wages from 1945 to the early 1970s coincided with the U.S.'s largest increase in conversion efficiency of primary energy to useful work from 6% to 11% (Warr et al. 2010). From 1900–1940 and 1970–2000, this conversion efficiency was nearly stagnant.

The loss of bargaining power (starting at peak resource extraction per person per Table 3) does cause a divergence in trend between wages and productivity. When removing labor bargaining power, gross labor productivity slightly increases with full cost pricing (Fig. 7a), and it follows a similar trend as with bargaining power for the marginal cost pricing scenario (Fig. 7b). On the other hand, with the loss of bargaining power, net labor productivity decreases dramatically with full cost pricing (Fig. 7c) but only asymptotically declines with marginal cost pricing (Fig. 7d). In short, when removing wage bargaining power, gross and net labor productivity move in opposite directions.

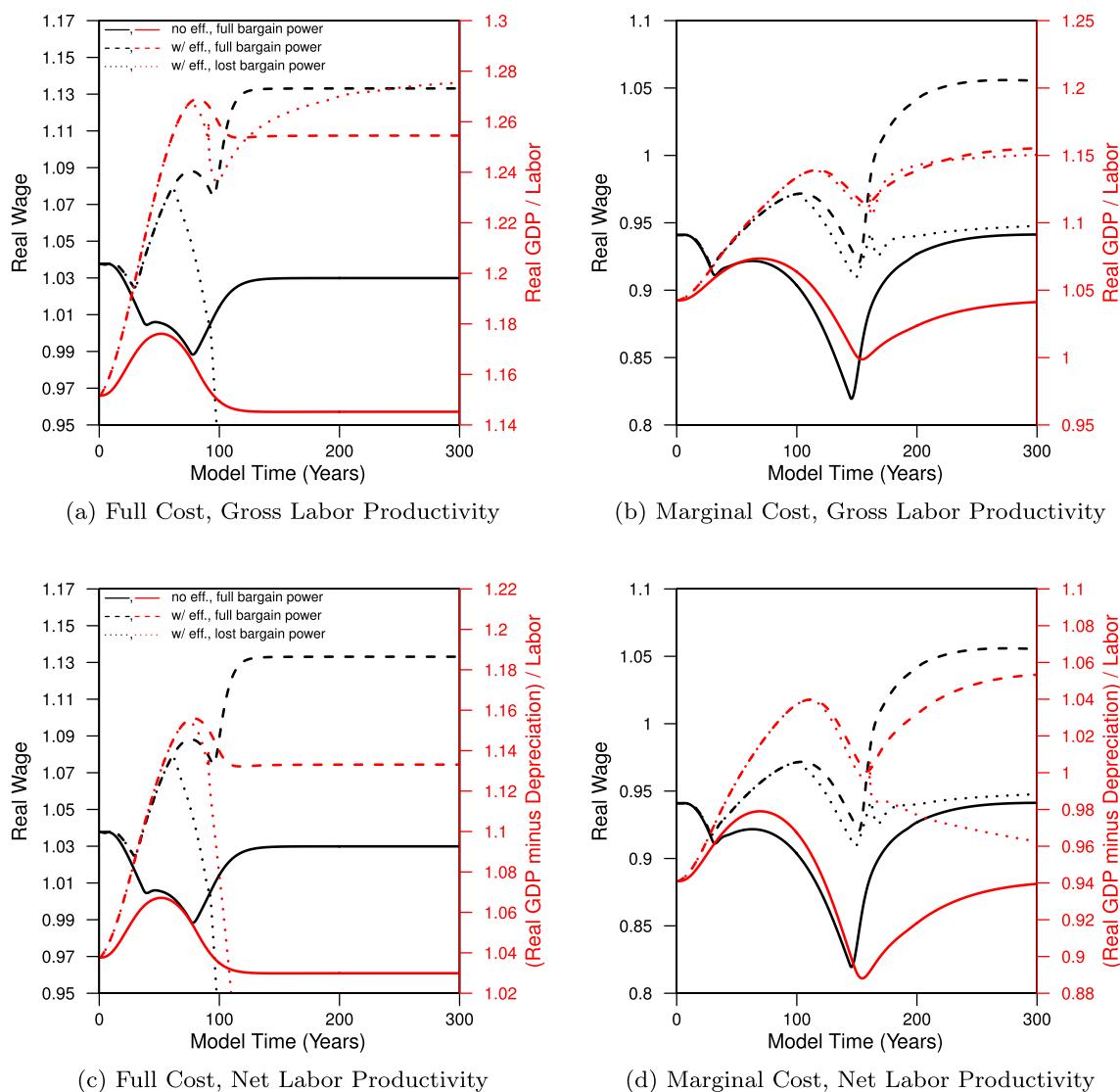
None of the scenarios translates accurately to the U.S. trend from Bivens and Mishel (2015), as the real world U.S. situation is more complicated (e.g., imports and exports, offshore investment). The HARMONEY results do, however, point to the need to consider resource consumption in tandem with policies and labor laws.

## Internal Structure of U.S. and HARMONEY I-O Tables

Both the model and the post-World War II U.S. economy show similar temporal patterns in internal structural change as measured via information theoretic metrics applied to the respective input-output (I-O) tables. Even though the HARMONEY model is neither calibrated to the U.S. economy nor explicitly intended to mimic U.S. patterns, we overlay the U.S. economy calculations on the simulation data to enable qualitative comparison of economic structural dynamics. Figure 8 shows information theoretic calculations for the U.S. from King (2016) and results for six marginal cost pricing simulations. Supplemental Fig. S.3 shows the corresponding results for the full cost pricing simulations.

Considering the information theory phase space in Fig. 8a both the model and the U.S. data move temporally in a counter-clockwise direction with the initial direction increasing both mutual constraint and conditional entropy. The counter-clockwise pattern from model simulations is not specific to parameter choices, but comes generally from its structure and growth from initial conditions. The marginal cost simulations show higher correlation to the U.S. data than the full cost simulations, and we expand our comparison of those results.

The U.S. information theoretic metrics from 1947–2012 align well with those of the marginal cost HARMONEY model from  $T = 30$  to  $T = 180$ . For both the model and U.S. data *information entropy* rises, remains nearly constant for a significant time, and finally declines. An interesting similarity between the U.S. data and HARMONEY simulations is that the peaks in per capita energy consumption (U.S. in the 1970s) and resource extraction (model, near  $T = 100$  per Fig. 2x) correspond to the approximate times of maximum information entropy that occur when the phase space plots of Figs. 8a and S.3a are moving up and to the left. Further, as pointed out by Atlan (1974), the rise and fall of information entropy is a natural consequence of a self-organizing system with potential to grow. He discusses system self-organization in the context of being “induced by the environment,” via a pattern of rising and then falling information entropy. Atlan states the further a system starts from maximum information entropy (for the  $2 \times 2$  HARMONEY I-O matrix this maximum is  $= -\log_2(1/4) = 2$ ), the more the potential for self-organization. Both the full cost (Fig. S.3) and marginal cost (Fig. 8) results show that increasing



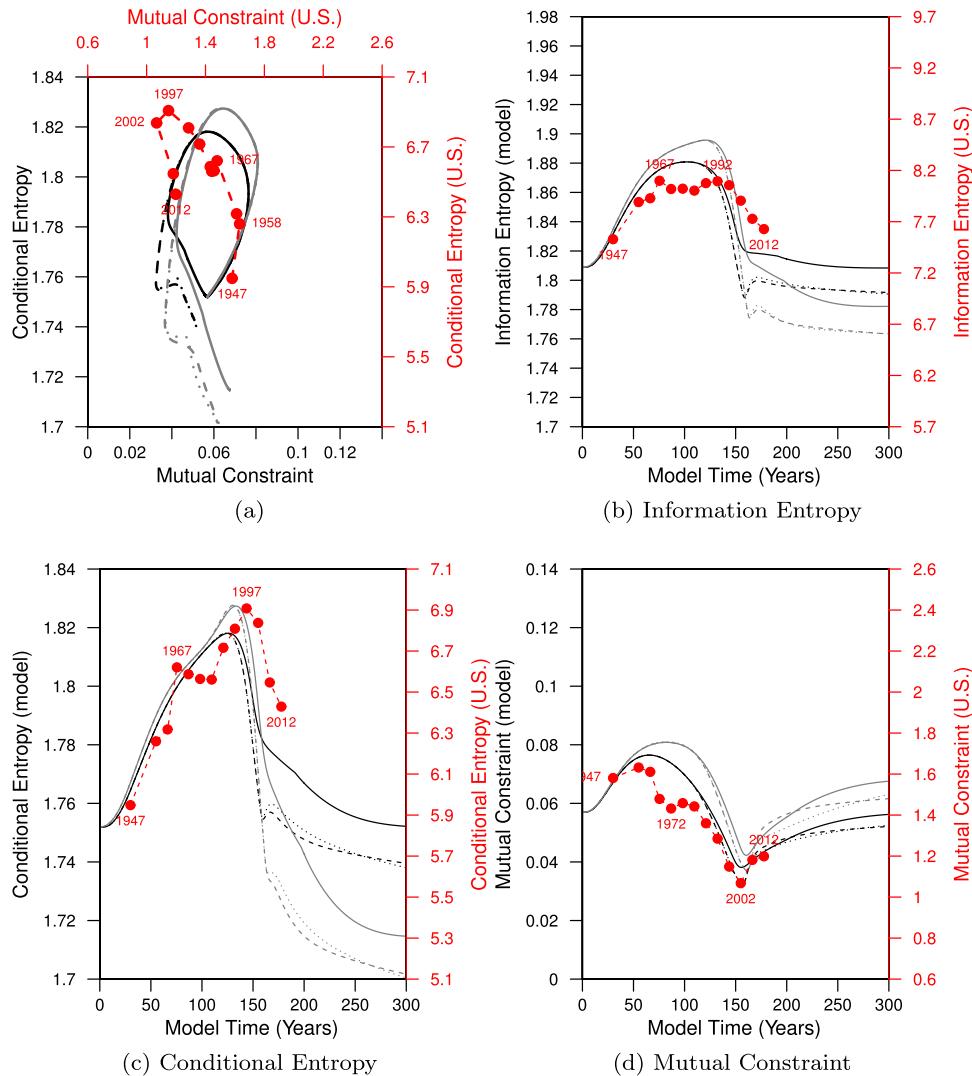
**Fig. 7** Real wages (left axis, black) plotted with total economy labor productivity (right axis, red) for the full cost (FC-000, FC-100, and FC-110 in **a** and **c**) and marginal cost (MC-000, MC-100, and MC-110 in **b** and **d**) pricing scenarios. Subfigures **a** and **b** show “full” labor productivity, and subfigures **c** and **d** show net labor productivity that subtracts depreciation from GDP. Per Table 3—Solid

lines: no efficiency increase, labor has full bargaining power (FC-000 and MC-000). Dashed lines: with efficiency increase, labor has full bargaining power (FC-100 and MC-100). Dotted lines: with efficiency increase, labor bargaining power declines starting at peak resource extraction per person (FC-110 and MC-110) (Color figure online)

resource consumption efficiency of capital (via decreasing  $\eta_g$  and  $\eta_e$ ) enables a higher peak in information entropy as this represents increased capability to use environmental resources.

*Conditional entropy* generally rises to a maximum point before falling even more quickly (Fig. 8c). In both the U.S. and model calculations, maximum conditional entropy occurs soon before maximum energy consumption for the

**Fig. 8** Information theoretic metrics of the marginal cost scenarios compared to the 37-sector aggregation of the U.S. Use tables from 1947–2012 from King (2016) shown as red dashed lines with filled circles that refer to the right and top axes. Here, information theoretic metrics for both the HARMONEY model and U.S. are calculated using base 2 logarithm instead of natural logarithm in King (2016). MC-000 (black solid), MC-010 (black dashed), MC-011 (black dotted), MC-100 (gray solid), MC-110 (gray dashed), MC-111 (gray dotted). **a** Conditional entropy versus mutual constraint, **b** information entropy vs time, **c** conditional entropy vs time, and **d** mutual constraint vs time (Color figure online)



U.S. (in 1997) and maximum resource extraction for the model (near  $T = 145$  in the marginal cost results).

*Mutual constraint* remains relatively constant during the early growth period, declines until the time period of maximum energy and resource extraction rates, and then rises after that point with the HARMONEY simulations exhibiting a gradual leveling off toward a steady state (Fig. 8d). The important times of comparison are 2002 in the U.S. data and approximately  $T = 160$  in the simulations when mutual constraint reaches its lowest value. In the U.S., the year 2002 represents the year with cheapest “food+energy” costs relative to GDP (King 2016). Further, total U.S.

primary energy consumption has not appreciably increased since the year 2000, remaining relatively constant since that time (at between 99 and 107 EJ/year, see Table 1.1 of EIA (2020)). In the same way, near  $T = 160$ , the marginal cost pricing scenarios show both a peak in resource extraction rates (Figs. 2d and S.4d) and a low point in energy cost measured as the share of GDP associated with the extraction sector ( $Y_{\text{extract}}/Y_{\text{total}}$  in Figs. 2(u) and S.4u).

One important conclusion is that, for both the HARMONEY model and U.S. I-O tables, the turning points in information theoretic metric trends occur at similar critical times of transition from increasing to stagnant per capita

or total resource consumption rates. That is to say, the HARMONEY model captures several important linkages between the structure of the economy and its ability (or lack thereof) to extract increasing rates of energy and other natural resources from its environment.

## Conclusion

The purpose of this paper was to explore the coupled growth and structural dynamic patterns of the HARMONEY model (v1.1) as updated from King (2020). The differences in the simulation results in this paper versus King (2020) derive from the more robust method in solving for prices and the explicit inclusion of wage bargaining power that augments a short-run Phillips Curve. Despite the assumption of a single regenerative natural resource (akin to a forest) to support the modeled economy, HARMONEY v1.1 exhibits several important high-level structural, biophysical, and economic patterns that compare well with global and U.S. data, and thus provide insight into long-term trends.

The HARMONEY model provides a consistent biophysical and monetary basis for explaining the progression in global and country-level data from an increasing or near constant energy intensity (energy consumption/GDP) to one of decreasing energy intensity. That is to say, both HARMONEY and global data first show a period of increasing growth rates, when the growth rate of natural resource consumption exceeds or is nearly equal to the growth rate of GDP, followed by a period of decreasing growth rates when the growth rate of resource consumption is lower than that of GDP. Thus, given this latter condition referred to as a state of relative decoupling, we conclude that it occurs due to a natural progression of self-organized growth, and not

necessarily from independent conscious choice by actors within the economy to pursue resource efficiency.

While we show that explicit choices to increase resource consumption efficiency in capital (e.g., machines) do increase the level of relative decoupling, we also show the choice of price formation affects apparent decoupling just as much. When basing prices on only marginal costs the economy appears more decoupled than if prices are based on full costs that include depreciation and debt interest payments. Further, marginal cost pricing generates higher debt ratios than full cost pricing, implying higher debt levels might provide only a perception of a more decoupled economy. Thus, relative decoupling of GDP from resource consumption represents an expected stage of growth, still similarly dependent on resource consumption, rather than a stage during which an economy is less constrained by resource consumption.

When assuming full labor bargaining power for wages, such that wages increase with inflation, once resource consumption stagnates, profit shares decline to zero and wage share increases. An explicit reduction in labor bargaining power at peak resource consumption enables some profits to remain. Thus, the HARMONEY model provides a basis for arguing that because profits decline to zero once resource consumption peaks under a full bargaining power situation, a new pressure emerges to reduce wage bargaining power of labor to ensure some level of profits at the expense of labor. This reasoning helps explain the wage stagnation and declining wage share experienced in the U.S. since the 1970s.

## A.1 Nomenclature

See Table 4.

**Table 4** Definition and nomenclature of symbols used in the HARMONEY model equations

Symbol	Description	Units
$a_e$	Labor productivity of resource extraction sector	Resources/person/time
$a_{ee}$	Resources input required per unit of gross nature extraction	(Resources/time)/(resources/time)
$a_{eg}$	Resources input required per unit of gross goods production	(Resources/time)/(goods/time)
$a_g$	Labor productivity of goods sector	Goods/person/time
$a_{ge}$	Goods input required per unit of gross resource extraction	(Goods/time)/(resources/time)
$a_{gg}$	Goods input required per unit of gross goods production	(Goods/time)/(goods/time)
$c_e$	Unit cost of production of extraction sector	Money/resource
$c_g$	Unit cost of production of goods sector	Money/good
$C_e$	Consumption of extraction sector output by households	Money/time
$C_g$	Consumption of goods sector output by households	Money/time
$CU_e, CU_g$	Capacity utilization of extraction and goods capital	—
$CU_{e,ref}, CU_{g,ref}$	Reference (target) Capacity utilization of each capital	—
$D$	Total private debt (of firms)	Money
$D_e, D_g$	Debts of extraction and goods sectors	Money
$g$	Physical inventory of goods	Goods
$I$	Total private investment (by firms)	Money/time
$I_e, I_g$	Investment by extraction and goods sector (by firms)	Money/time
$I_e^g, I_g^g$	Investment by extraction and goods sector (by firms)	Goods/time
$IC_e, IC_g$	Inventory coverage of extraction and goods capital	—
$INV_e, INV_g$	Value of inventory of extraction and goods sector	Money
$\Delta INV_e, \Delta INV_g$	Change in value of inventory of extraction and goods sector	Money/time
$K$	Total capital	Goods
$K_e, K_g$	Capital for resources extraction and goods production	Goods
$L$	Total labor	Person
$L_e, L_g$	Labor for resources extraction and goods production	Person
$M$	Total deposits (of money) in banks	Money
$M^h$	Household deposits (of money)	Money
$N$	Population	Person
$NEPR$	Net external power ratio	(Resources/time)/(resources/time)
$NPR_{economy}$	Net power ratio	(Resources/time)/(resources/time)
$P_e$	Price of extraction output	Money/resource
$P_g$	Price of goods output	Money/good
$r_D, r_M$	Interest rate (D: on private debt; M: for household deposits)	1/time
$s$	Threshold household consumption of resources per person	Resources/person/time
$S^b$	Saving of banks	Money/time
$S^h$	Saving of households	Money/time
$V$	Total value added of the economy	Money/time
$V_e, V_g$	Value added of extraction and goods sectors	Money/time
$w_H$	Physical inventory of resources, wealth (per Motesharrei et al. 2014)	Resource
$w$	Wage per person	Money/person/time
$W_e, W_g$	Wages for labor in extraction and goods sector ( $W = wL$ )	Money/time
$X_e$	Gross output of extraction sector (= resources extraction)	Resources/time
$X_g$	Gross output of goods sector (= goods production)	Goods/time
$X^b$	Net worth of banks	Money
$X^h$	Net worth of households	Money
$X_e^f, X_g^f$	Net worth of extraction and goods firms	Money
$X^{tot}$	Net worth of entire economy	Money
$y$	Resources in the environment	Resource
$y_{X_g}$	Resource input embodied per unit of capital as goods sector output	(Resources/time)/good
$Y$	Total net economic output of the economy	Money/time
$Y_e, Y_g$	Net economic output of extraction and goods sectors	Money/time
$\alpha_N$	Death rate of population	1/time

**Table 4** (continued)

Symbol	Description	Units
$\beta_N$	Birth rate of population	1/time
$\gamma$	Regeneration rate of resources	1/time
$\delta$	Depreciation rate of capital	1/time
$\delta_y$	Technology parameter affecting resources extraction rate	1/(time × good)
$\eta_e, \eta_g$	resources input to operate extraction and goods capital	(Resources/time)/good
$\kappa_0$	Investment function parameter (Equation 16)	–
$\kappa_1$	Investment function parameter (Equation 16)	–
$\lambda_y$	Maximum size of natural resources with no depletion	Resource
$\lambda_N$	Participation (employment) rate	–
$\lambda_{N,o}$	Wage function parameter (Equation S.7), equilibrium employment	–
$\mu_e, \mu_g$	Cost markup for extraction and goods sector (price Equations 22 & 21)	–
$v_e$	Capital to output ratio for extraction output	Goods/(resources/time)
$v_g$	Capital to output ratio for goods output	Goods/(goods/time)
$\Pi_b$	Bank dividends	Money/time
$\Pi_e, \Pi_g$	Net profit of extraction and goods sectors	Money/time
$\pi_e, \pi_g$	Profit share of extraction and goods sectors	–
$\pi_{r,e}, \pi_{r,g}$	Profit rate of extraction and goods sectors	1/time
$\rho_e$	Minimum per capita household consumption of extraction sector output	Resources/(person/time)
$\rho_g$	Minimum per capita household consumption of goods sector output	Goods/(person/time)
$\tau_{CU,e}, \tau_{CU,g}$	Time constant for extraction and goods CU differential (lag) equation	Time
$\tau_{IC,e}, \tau_{IC,g}$	Time constant for extraction and goods IC differential (lag) equation	Time
$\tau_{P,e}, \tau_{P,g}$	Time constant for extraction and goods price differential (lag) equation	Time
$\tau_{V,e}, \tau_{V,g}$	Time constant for extraction and goods value added differential (lag) equation	Time
$\tau_{Y,e}, \tau_{Y,g}$	Time constant for extraction and goods net output differential (lag) equation	Time
$\tau_{\Pi,e}, \tau_{\Pi,g}$	Time constant for extraction and goods profit differential (lag) equation	Time
$\phi_{min}$	Wage function parameter (Equation S.7)	1/time
$\phi_o$	Wage function parameter (Equation S.7)	1/time
$\phi_s$	Wage function parameter (Equation S.7)	1/time
$\omega$	Wage share (of value added)	–

## A.2 Parameter Values Used in for Simulations

See Table 5.

**Table 5** Parameter values used in the base run of the HARMONEY model

Symbol	Description	Value
$a_{ge}$	Goods input required per unit of gross resources extraction	0.2
$a_{gg}$	Goods input required per unit of gross goods production	0.1
$IC_{ref,g}$	Reference (target) inventory coverage for goods sector	1
$IC_{ref,e}$	Reference (target) inventory coverage for extraction sector	1
$r_L$	Interest rate (L: on loans)	0.05
$r_M$	Interest rate (M: for household deposits)	0.0
$s$	Threshold household consumption of resources per person (Equation S.3)	0.08
$y_{X_g}$	Resource input embodied per unit of capital as goods sector output	0.1
$\alpha_m$	Minimum death rate of population (Equation S.3)	0.01
$\alpha_M$	Maximum death rate of population (Equation S.3)	0.07
$\beta_N$	Birth rate of population	0.03
$\delta_y$	Resources extraction parameter	0.0072 to 0.009
$\gamma$	Regeneration rate of resources (renewable scenarios)	0.01
$\delta$	Depreciation rate of capital	0.03
$\eta_e, \eta_g$	resources input to operate extraction and goods capital	0.16
$\kappa_0$	Investment function parameter (Equation 16)	1.0
$\kappa_1$	Investment function parameter (Equation 16)	1.5
$\lambda_y$	Maximum size natural resources stock (renewable scenarios)	100
$\lambda_{N,o}$	Wage function parameter (Equation S.7), equilibrium participation rate	0.6
$v_g$	Capital to output ratio for goods sector	1.5
$\rho_e$	Minimum household consumption of extraction sector output	0.01
$\rho_g$	Minimum household consumption of goods sector output	0
$\tau_{C,e}$	Time constant for extracted resource consumption differential (lag) equation	0.05
$\tau_{C,g}$	Time constant for goods consumption differential (lag) equation	0.05
$\tau_{CU,e}$	Time constant for extraction capacity utilization differential (lag) equation	0.25
$\tau_{CU,g}$	Time constant for goods capacity utilization differential (lag) equation	0.25
$\tau_{IC,e}$	Time constant for extraction inventory coverage differential (lag) equation	0.25
$\tau_{IC,g}$	Time constant for goods inventory coverage differential (lag) equation	0.25
$\tau_{P,e}$	Time constant for extraction price differential (lag) equation	1
$\tau_{P,g}$	Time constant for goods price differential (lag) equation	1
$\tau_{V,e}$	Time constant for extraction value added differential (lag) equation	1
$\tau_{V,g}$	Time constant for goods net value added differential (lag) equation	1
$\tau_{Y,e}$	Time constant for extraction net output differential (lag) equation	1
$\tau_{Y,g}$	Time constant for goods net output differential (lag) equation	1
$\tau_{\Pi,e}$	Time constant for extraction profit differential (lag) equation	1
$\tau_{\Pi,g}$	Time constant for goods profit differential (lag) equation	1
$\phi_{min}$	Wage function parameter (Equation S.7)	-0.05
$\phi_o$	Wage function parameter (Equation S.7)	0
$\phi_s$	Wage function parameter (Equation S.7)	0.05

### A.3 List of Differential Equations for Model

See Table 6.

**Table 6** The differential equations of the merged biophysical and economic model

State Variable	Equation	Equation No.
<i>I. Core equations</i>		
Available resources	$\dot{y} = \gamma y(\lambda_y - y) - \delta_y y K_e C U_e$	1
Population	$\dot{N} = \beta_N N - \alpha_N \left( \frac{C_e}{P_e} \right) N$	12
Wealth	$\dot{w}_H = (IC_{ref,e} - IC_{perceived,e}) \times (C_e/P_e + a_{eg}X_g + a_{ee}X_e)$	5
Goods	$\dot{g} = (IC_{ref,g} - IC_{perceived,g}) \times ((C_g + I_g + I_e)/P_g + a_{ge}X_e + a_{gg}X_g)$	6
Extraction capital	$\dot{K}_e = I_e/P_g - \delta K_e$	20
Goods capital	$\dot{K}_g = I_g/P_g - \delta K_g$	20
Debt for extraction sector	$\dot{D}_e = I_e - P_g \delta K_e - \Pi_e$	13
Debt for goods sector	$\dot{D}_g = I_g - P_g \delta K_g - \Pi_g$	13
Wage per worker	$\frac{\dot{w}}{w} = \phi(\lambda_N) + w_1 i + w_2 \frac{1}{\lambda_N} \frac{d\lambda_N}{dt}$	14
Price, extraction sector output	$\dot{P}_e = \left( \frac{1}{\tau_{P_e}} \right) ((1 + \mu_e) c_e - P_e)$	22 (or 21)
Price, goods sector output	$\dot{P}_g = \left( \frac{1}{\tau_{P_g}} \right) ((1 + \mu_g) c_g - P_g)$	22 (or 21)
<i>II. Lag equations for accessing ‘past’ states needed to solve core equations</i>		
Capacity utilization for extraction sector	$\dot{C}U_e = \frac{CU_{e,indicated} - CU_e}{\tau_{CU,e}}$	Of form of S.8
Capacity utilization for goods sector	$\dot{C}U_g = \frac{CU_{g,indicated} - CU_g}{\tau_{CU,g}}$	Of form of S.8
Perceived inventory coverage for extraction sector	$\dot{IC}_{e,perceived} = \frac{IC_e - IC_{e,perceived}}{\tau_{IC,e}}$	Of form of S.8
Perceived inventory coverage for goods sector	$\dot{IC}_{g,perceived} = \frac{IC_g - IC_{g,perceived}}{\tau_{IC,g}}$	Of form of S.8
Value added for extraction sector	$\dot{V}_{e,lagged} = \frac{V_e - V_{e,lagged}}{\tau_{V,e}}$	Of form of S.8
Value added for goods sector	$\dot{V}_{g,lagged} = \frac{V_g - V_{g,lagged}}{\tau_{V,g}}$	Of form of S.8
Profit from extraction sector	$\dot{\Pi}_{e,lagged} = \frac{\Pi_e - \Pi_{e,lagged}}{\tau_{\Pi,e}}$	Of form of S.8
Profit from goods sector	$\dot{\Pi}_{g,lagged} = \frac{\Pi_g - \Pi_{g,lagged}}{\tau_{\Pi,g}}$	Of form of S.8
Value of inventory, extraction sector	$\dot{INV}_{e,lagged} = \frac{INV_e - INV_{e,lagged}}{\tau_{P,e}}$	Of form of S.8
Value of inventory, goods sector	$\dot{INV}_{g,lagged} = \frac{INV_g - INV_{g,lagged}}{\tau_{P,g}}$	Of form of S.8
Labor of extraction sector	$\dot{L}_{e,lagged} = \frac{L_e - L_{e,lagged}}{\tau_{P,e}}$	Of form of S.8
Labor of goods sector	$\dot{L}_{g,lagged} = \frac{L_g - L_{g,lagged}}{\tau_{P,g}}$	Of form of S.8

## A.4 Initial Conditions for Base Run of Model

See Table 7.

**Table 7** Initial values for state equations as used in the base run of our merged biophysical and economic model

Symbol	Description	Full cost pricing	Marginal cost pricing
<i>I. Initial conditions for core differential equations</i>			
$D_{e,o}$	Debt for extraction sector	0.001	0.001
$D_{g,o}$	Debt for goods sector	0.001	0.001
$g_o$	Goods (initial inventory of goods)	1.531	0.596
$K_{e,o}$	Initial extraction capital	7.980	2.883
$K_{g,o}$	Initial goods capital	11.116	4.276
$w_o$	Wage per worker	1.038	0.941
$w_{H,o}$	Wealth (initial inventory of resources)	1.129	0.427
$x_o$	Population	18.240	6.928
$y_o$	Available resources	95.117	98.235
$P_{e,o}$	Price of extraction sector output	2.069	1.85
$P_{g,o}$	Price of goods sector output	2.178	1.963
<i>II. Initial conditions for (first order) lag differential equations</i>			
$CU_{e,o}$	Capacity utilization for extraction sector	0.85	0.85
$CU_{g,o}$	Capacity utilization for goods sector	0.85	0.85
$IC_{e,perceived,o}$	Perceived inventory coverage for extraction sector	1	1
$IC_{g,perceived,o}$	Perceived inventory coverage for goods sector	1	1
$V_{e,o}$	Value added for extraction sector	4.727	1.683
$V_{g,o}$	Value added for goods sector	6.427	2.366
$\Pi_{e,o}$	Profit from extraction sector	0.0002	0.0002
$\Pi_{g,o}$	Profit from goods sector	0.0003	0.0002
$INV_{e,o}$	Value of inventory, extraction sector	2.067	0.699
$INV_{g,o}$	Value of inventory, goods sector	2.950	1.036
$L_{e,o}$	Labor of extraction sector	4.645	1.733
$L_{g,o}$	Labor of goods sector	6.299	2.423

**Supplementary Information** The online version contains supplementary material available at <https://doi.org/10.1007/s41247-021-00093-8>.

**Funding** This work was supported in part by the V. Kann Rasmussen Foundation via the “Carbon Clarity in the Global Petrochemical Supply Chain” project.

**Code availability** The HARMONY model used to simulate the results in this paper is available on the author’s website (<http://careyking.com/data-downloads>) and upon request.

## Declarations

**Conflict of interest** The author declares no competing interests or financial conflicts of interest.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

## References

- Atlan H (1974) On a formal definition of organization. *J Theor Biol* 45:295–304
- Ayres RU (2008) Sustainability economics: where do we stand? *Ecol Econom* 67(2):281–310
- Ayres RU, Warr B (2005) Accounting for growth: the role of physical work. *Struct Chang Econ Dyn* 16:181–209
- Ayres RU, Warr B (2009) The economic growth engine: how energy and work drive material prosperity. Edward Elgar, Cheltenham
- Ballesteros Fernando J, Martínez Vicent J, Luque Bartolo, Lucas, Valor Enric, Moya Andrés (2018) On the thermodynamic origin of metabolic scaling. *Sci Rep* 8(1448)
- Banavar JR, Moses ME, Brown JH, Damuth J, Rinaldo A, Sibly RM, Maritan A (2010) A general basis for quarter-power scaling in animals. *Proc Natl Acad Sci* 107(36):15816–15820
- Bivens J, Mishel L (2015) Understanding the historic divergence between productivity and a typical worker’s pay: Why it matters and why it’s real. <https://www.epi.org/files/2015/understanding-productivity-pay-divergence-final.pdf>, September. Online; accessed March 13 2018
- Brockway PE, Sorrell S, Semieniuk G, Heun MK, Court V (2021) Energy efficiency and economy-wide rebound effects: a review of the evidence and its implications. *Renew Sustain Energy Rev* 141:110781
- Brown JH, Gillooly JF, Allen AP, Savage VM, West GB (2004) Toward a metabolic theory of ecology. *Ecology* 85(7):1771–1789
- Brown JH, Burnside WR, Davidson AD, Delong JR, Dunn WC, Hamilton MJ, Mercado-Silva N, Nekola JC, Okie JG, William HW, Wenyun Z (2011) Energetic limits to economic growth. *BioScience* 61:19–26
- Brown JH, Burger JR, Burnside WR, Chang M, Davidson AD, Fristoe TS, Hamilton MJ, Hammond ST, Kodric-Brown A, Mercado-Silva N, Nekola JC, Okie JG (2014) Macroecology meets macroeconomics: resource scarcity and global sustainability. *Ecol Eng* 65:24–32
- Brown JH, Hall CAS, Sibly RM (2018) Equal fitness paradigm explained by a trade-off between generation time and energy production rate. *Nat Ecol Evol* 2:262–268
- Clarke A, Johnston NM (1999) Scaling of metabolic rate with body mass and temperature in teleost fish. *J Anim Ecol* 68(5):893–905
- DeLong JP, Okie JG, Moses ME, Sibly RM, Brown JH (2010) Shifts in metabolic scaling, production, and efficiency across major evolutionary transitions of life. *Proc Natl Acad Sci* 107(29):12941–12945
- Edenhofer O, Pichs-Madruga R, Sokona Y, Kadner S, Minx JC, Brunner S, Agrawala S, Baiocchi G, Bashmakov IA, Blanco G, Broome J, Bruckner T, Bustamante M, Clarke L, Conte Grand M, Creutzig F, Cruz-Núñez X, Dhalak S, Dubash NK, Eickemeier P, Farahani E, Fischedick M, Fleurbaey M, Gerlagh R, Gómez-Echeverri L, Gupta S, Harnisch J, Jiang K, Jotzo F, Kartha S, Klasen S, Kolstad C, Krey V, Kunreuther H, Lucon O, Masera O, Mulugetta Y, Norgaard RB, Patt A, Ravindranath NH, Riahi K, Roy J, Sagar A, Schaeffer R, Schröder S, Seto KC, Seyboth K, Sims R, Smith P, Somanathan E, Stavins R, von Stechow C, Sterner T, Sugiyama T, Suh S, Urge-Vorsatz D, Urama K, Venables A, Victor DG, Weber E, Zhou D, Zou J, Zwickel T (2014) Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, chapter Technical Summary. Cambridge University Press, Cambridge
- EIA (2020) Monthly energy review
- Felipe J, Fisher FM (2003) Aggregation in production functions: what applied economists should know. *Metroeconomica* 54(2–3):208–262
- Felipe J, McCombie J (2006) The tyranny of the identity: growth accounting revisited. *Int Rev Appl Econ* 20(3):283–299
- Fewell JH, Harrison JF (2016) Scaling of work and energy use in social insect colonies. *Behav Ecol Sociobiol* 70(7):1047–1061
- Galor O, Weil DN (2000) Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond. *Am Econ Rev* 90(4):806–828
- Geoffrey W (2017) Scale the universal laws of growth, innovation, sustainability, and the pace of life in organisms, cities, economies, and companies. Penguin Press, New York
- Georgescu-Roegen N (1971) The entropy law and the economic process. Harvard University Press, Cambridge
- Georgescu-Roegen N (1975) Energy and economic myths. *South Econ J* 41(3):347–381
- Giraud G, Kahraman Z (2014) How dependent is growth from primary energy? the dependency ratio of energy in 33 countries (1970–2011). Université Panthéon-Sorbonne, Documents de travail du centre d’économie de la sorbonne
- Goodwin RM (1967) A growth cycle. Socialism, Capitalism and Economic Growth, pp 54–58
- Grasselli MR, Costa Lima B (2012) An analysis of the keen model for credit expansion, asset price bubbles and financial fragility. *Math Financ Econ* 6:190–210
- Guilford MC, Hall CAS, Connor PO’, Cleveland CJ (2011) A new long term assessment of energy return on investment (eroi) for U.S. oil and gas discovery and production. *Sustainability* 3(10):1866–1887
- Hagens NJ (2020) Economics for the future—beyond the superorganism. *Ecol Econ* 169:106520
- Hatton IA, Dobson AP, Storch D, Galbraith ED, Loreau M (2019) Linking scaling laws across eukaryotes. *Proc Natl Acad Sci* 116(43):21616–21622
- Hidalgo CA, Hausmann R (2009) The building blocks of economic complexity. *Proc Natl Acad Sci* 106(26):10570–10575

- Hidalgo CA, Klinger B, Barabasi A-L, Hausmann R (2007) The product space conditions the development of nations. *Science* 317(5837):482–487
- Hou C, Kaspari M, Vander Zanden HB, Gillooly JF (2010) Energetic basis of colonial living in social insects. *Proc Natl Acad Sci* 107(8):3634–3638
- Jarvis A, King C (2020) Energetic regimes of the global economy—past, present and future. *Earth Syst Dyn Discuss* 1–17:2020
- Jenkins J, Nordhaus T, Michael S (2011) Rebound & backfire as emergent phenomena, energy emergence
- Jevons WS (1866) The coal question: an inquiry concerning the progress of the nation, and the probable exhaustion of our coal mines. Macmillan and Co., London, 2nd edn, revised edition. Kessinger Legacy Reprints
- Keen S (2009) Household debt: The final stage in an artificially extended Ponzi bubble. *Aust Econ Rev* 42(3):347–357
- Keen S (2013) A monetary Minsky model of the great moderation and the great recession. *J Econ Behav Organ* 86:221–235
- Keen S, Ayres RU, Standish R (2019) A note on the role of energy in production. *Ecol Econ* 157:40–46
- Keyßer Lorenz T, Lenzen Manfred (2021) 1.5FIXMEc degrowth scenarios suggest the need for new mitigation pathways. *Nat Commun* 12(1):2676
- King CW (2016) Information theory to assess relations between energy and structure of the U.S. economy over time. *Biophys Econ Resour Quality* 1(2):10
- King CW (2020) An integrated biophysical and economic modeling framework for long-term sustainability analysis: the Harmony model. *Ecol Econ* 169:106464
- King CW (2021) The economic superorganism: beyond the competing narratives on energy, growth, and policy. Springer, New York
- Kümmel R (2011) The second law of economics: energy, entropy, and the origins of wealth. Springer, Berlin
- Kümmel R, Lindenberger D (2014) How energy conversion drives economic growth far from the equilibrium of neoclassical economics. *New J Phys* 16(12):125008
- McLeay M, Radia A, Thomas R (2014) Money creation in the modern economy. *Q Bull Bank Engl* (Q1)
- Minsky Hyman P (1977) The financial instability hypothesis: an interpretation of Keynes and an alternative to standard. *Challenge* 20(1):20–27
- Motesharrei S, Rivas J, Kalnay E (2014) Human and nature dynamics (handy): modeling inequality and use of resources in the collapse or sustainability of societies. *Ecol Econ* 101:90–102
- Mueller Casey A, Joss Jean MP, Seymour Roger S (2011) The energy cost of embryonic development in fishes and amphibians, with emphasis on new data from the Australian lungfish, *Neoceratodus forsteri*. *J Compar Physiol B* 181:43–52
- Nordhaus WD, Stavins RN, Weitzman ML (1992) Lethal model 2: the limits to growth revisited. *Brook Pap Econ Act* 1992(2):1–59
- Oded G (2005) Handbook of economic growth, vol 1A, chapter 4: from stagnation to growth: unified growth theory. Elsevier, New York 171–293
- Odum HT (1971) Environment, power, and society. Wiley, New York
- Odum HT (1997) The ecosystem, energy, and human values. *Zygon* 12(2):109–133
- Odum HT (2007) Environment, power, and society for the twenty-first century. The Hierarchy of Energy. Columbia University Press, New York
- Santos J, Borges AS, Domingos T (2021) Exploring the links between total factor productivity and energy efficiency: Portugal, 1960–2014. *Energy Econ* 105407
- Shaikh A (1974) Laws of production and laws of algebra: the humbug production function. *Rev Econ Stat* 56(1):115–120
- Shannon C, Weaver W (1962) The mathematical theory of communication. University of Illinois Press, Urbana
- Shigeta M, Keiko Y, Atsushi I, Prokushkin Stanislav G, Masyagina Oxana V, Akio H, Rafiqul Hoque ATM, Rempei S, Akira O, Tomohiro N, Tatsushiro U, Masaru K, Tsuyoshi M, Takuya K, Takayoshi K, Yojiro M, Takeshi T, Zyryanova Olga A, Abaimov Anatoly P, Yoshio A, Araki Masatake G, Tatsuro K, Yukihiko C, Marjnah U (2010) Mixed-power scaling of whole-plant respiration from seedlings to giant trees. *Proc Natl Acad Sci* 107(4):1447–1451
- Shik JZ (2010) The metabolic costs of building ant colonies from variably sized subunits. *Behav Ecol Sociobiol* 64(12):1981–1990
- Shik JZ, Santos JC, Seal JN, Kay A, Mueller UG, Kaspari M (2014) Metabolism and the rise of fungus cultivation by ants. *Am Nat* 184(3):364–373
- Sibly RM, Brown JH, Kodric-Brown A (2012) Metabolic ecology: a scaling approach. Wiley-Blackwell, Hoboken
- Solow RM (1956) A contribution to the theory of economic-growth. *Q J Econ* 70(1):65–94
- Solow RM (2007) The last 50 years in growth theory and the next 10. *Oxf Rev Econ Policy* 23(1):3–14
- Sterman JD (2000) Business dynamics: systems thinking and modeling for a complex world. McGraw Hill Higher Education, USA
- Steve K (1995) Finance and economic breakdown: modeling Minsky's financial instability hypothesis. *J Post Keynesian Econ* 17(4):607–635
- Ulanowicz RE (2008) The dual nature of ecosystem dynamics. *Ecological Modelling*, 220(16):1886–1892, (2009) Selected Papers from the Workshop on ‘Emergence of Novelties’, 9–16 October. Pacina, Siena, Italy
- Ulanowicz RE, Goerner SJ, Lietaer B, Gomez R (2009) Quantifying sustainability: resilience, efficiency and the return of information theory. *Ecol Complex* 6(1):27–36
- United Nations (2020) The sustainable development goals report
- Warr B, Ayres R, Eisenmenger N, Krausmann F, Schandl H (2010) Energy use and economic development: a comparative analysis of useful work supply in Austria, Japan, the United Kingdom and the US during 100 years of economic growth. *Ecol Econ* 69(10):1904–1917
- Waters James S, Tate HC, Fewell Jennifer H, Harrison Jon F (2010) Allometric scaling of metabolism, growth, and activity in whole colonies of the seed-harvester ant *pogonomyrmex californicus*. *Am Nat* 176(4):501–510
- Waters JS, Ochs A, Fewell JH, Harrison JF (2017) Differentiating causality and correlation in allometric scaling: ant colony size drives metabolic hypometry. *Proc R Soc B* 284(1849):20162582
- West GB, Brown JH, Enquist BJ (2001) A general model for ontogenetic growth. *Nature* 413(6856):628–31
- West GB, Brown JH (2005) The origin of allometric scaling laws in biology from genomes to ecosystems: towards a quantitative unifying theory of biological structure and organization. *J Exp Biol* 208(9):1575–1592

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.