END system('convert -density 600 -background white -flatten output.pdf pages.png');

1 Distance and Redshift Integrals

Here $E(z) = H(z)/H_0$, $\mathcal{D}$ is angular diameter distance. χ is comoving radial distance

$$\begin{aligned}
\int_0^{z_s} f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} dz &= \int_0^{z_s} f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} dz && (1)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a^2} da && (2)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a^2} \frac{da}{dt} dt && (3)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} \frac{\dot{a}}{a} dt && (4)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} H(z) dt && (5)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} \frac{cH(z)}{c} dt && (6)\\
&= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{a} \frac{cH_0}{c} dt && (7)\\
&= \frac{H_0}{c} \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{cdt}{a} && (8)\\
&= \frac{H_0}{c} \int f\left(z, \frac{\ell a}{\mathcal{D} a}\right) \frac{cdt}{a} && (9)\\
&= \frac{H_0}{c} \int_0^{\chi_s} f\left(z, \frac{\ell}{\chi a}\right) d\chi && (10)\\
&= \frac{H_0}{c} \int_0^{\chi_s} f\left(z, \frac{\ell(1+z)}{\chi}\right) d\chi && (11)
\end{aligned}$$