

# 1 Distance and Redshift Integrals

Here  $E(z) = H(z)/H_0$ ,  $\mathcal{D}$  is angular diameter distance.  $\chi$  is comoving radial distance

$$\int_0^{z_s} f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} dz = \int_0^{z_s} f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} dz \quad (1)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a^2} da \quad (2)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a^2} \frac{da}{dt} dt \quad (3)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} \frac{\dot{a}}{a} dt \quad (4)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} H(z) dt \quad (5)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{E(z)} \frac{1}{a} \frac{cH(z)}{c} dt \quad (6)$$

$$= \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{1}{a} \frac{cH_0}{c} dt \quad (7)$$

$$= \frac{H_0}{c} \int f\left(z, \frac{\ell}{\mathcal{D}}\right) \frac{cdt}{a} \quad (8)$$

$$= \frac{H_0}{c} \int f\left(z, \frac{\ell a}{\mathcal{D}a}\right) \frac{cdt}{a} \quad (9)$$

$$= \frac{H_0}{c} \int_0^{\chi_s} f\left(z, \frac{\ell}{\chi a}\right) d\chi \quad (10)$$

$$= \frac{H_0}{c} \int_0^{\chi_s} f\left(z, \frac{\ell(1+z)}{\chi}\right) d\chi \quad (11)$$