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PROTON-NUCLEUS TOTAL INELASTIC CROSS SECTIONS: AN EMPIRICAL FORMULA FOR $E > 10$ MeV

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ABSTRACT

An empirical formula for the total inelastic cross section of protons on nuclei with charge greater than 1 is presented. The formula is valid with a varying degree of accuracy down to proton energies of 10 MeV. At high energies (≥ 2 GeV) the formula reproduces experimental data to within reported errors (~ 2%).

Subject heading: nuclear reactions

I. INTRODUCTION

Proton-nucleus total inelastic cross sections are of fundamental importance in various ion transport problems of astrophysical interest. These include propagation of cosmic rays through interstellar hydrogen, excitation of matter (e.g., lunar soil or meteors) by cosmic protons, and transport of protons through the atmosphere. In addition, these cross sections find practical application in the estimation of radiation effects on materials. In spite of the general usefulness of these cross sections, there are no detailed (energy-dependent) empirical fits of the data for proton energies above 10 MeV. A fit of this sort is presented here.

Much literature on these reactions exists (Johansson, Svanberg, and Sundberg 1961; Goloskie and Strauch 1962; Bellettini *et al.* 1966; Kirkby and Link 1966; Menet *et al.* 1971; Renberg *et al.* 1972; Montague *et al.* 1973; Gorin *et al.* 1974; McGill *et al.* 1974; Carlson *et al.* 1975; Davison *et al.* 1977; Westfall *et al.* 1979; Abegg *et al.* 1979; Bobchenko *et al.* 1979), although data are scarce in the region 500 MeV to 5 GeV. Recently published data at high energies (Bobchenko *et al.* 1979) make possible an empirical formula with better than 2% accuracy at energies greater than 2 GeV. At lower energies the accuracy decreases for two reasons: (1) the variations in cross section are less regular than at high energies, and (2) the cross sections are known with much less accuracy than at high energies. Thus, at energies down to 100 MeV the data are reproduced by our formula to better than 10%, while down to 10 MeV it is accurate only within a factor of 2. For many uses the exact cross section at low energies is unimportant because loss of energy by ionization of atomic electrons dominates flux attenuation.

Several fits to the experimental data at high energies have previously been reported, of which we will consider only two in detail: Tsao and Silberberg (1975) and Hagen (1976). Karol (1975) has discussed a theoretical model applicable to all nucleus-nucleus collisions. This model incorporates some energy dependence but, again, is only valid at relatively high energies. None of these formulae has achieved general acceptance.

a) *High Energy Formula*

Data on the asymptotic value of the proton-nucleus total inelastic cross section at high energies are found in Bobchenko *et al.* (1979), Gorin *et al.* (1974), Westfall *et al.* (1979), and Bellettini *et al.* (1966). These data are in excellent agreement, except for those of Bellettini *et al.* which are systematically high, and in reasonably good agreement with the neutron data of Schimmerling *et al.* (1973). Our fit at high energy is based on a condensation of data from Bobchenko *et al.* (1979) in the energy range 5–9 GeV. The data points for each material were consistent with a constant cross section in this energy range and were averaged to produce the entries in Table 1. Statistical errors in Table 1 are our estimates based on the reported errors and the standard deviation of the average. These errors are generally less than 2%.

Simple fits of the form $kA^{2/3}$ and kA^{β} were attempted. The best fit of the first type was found with $k = 50.7$ (corresponding to a nucleon radius of 1.27×10^{-13} cm). This fit deviates significantly from the data

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TABLE 1

CONDENSATION OF THE TOTAL INELASTIC CROSS SECTION DATA (from Bobchenko *et al.* 1979)

Element	Mass	Cross Section (mb)
Be	9.0	209 ± 3
B	10.8	235 ± 4
C	12.0	251 ± 2
F	19.0	351 ± 5
Mg	24.3	422 ± 5
Al	27.0	456 ± 7
S	32.1	514 ± 6
Ca	40.1	603 ± 6
Ti	47.9	683 ± 8
V	50.9	714 ± 8
Fe	55.8	760 ± 8
Cu	63.5	831 ± 8
Nb	92.9	1070 ± 11
Cd	112.4	1215 ± 12
Sn	118.7	1255 ± 13
Ta	180.9	1666 ± 19
Pb	207.2	1859 ± 16
U	238	2090 ± 45

(see Table 2). The best fit of the second type is

$$44.9A^{0.7}. \tag{1}$$

This fit is much better than the first and is generally of sufficient accuracy for cosmic ray work.

For some of the data in Table 1, equation (1) is not within the experimental error. For this reason, a refinement of equation (1) was devised to lie within the experimental errors for all points. This high energy formula is

$$\sigma(\text{h.e}) = 45\,A^{0.7}[1 + 0.016 \sin(5.3 - 2.63 \ln A)] \text{ mb}. \tag{2}$$

Details of the differences between equations (1) and (2) are shown in Figure 1. Overall error in equation (2) is clearly less than in equation (1). Emphasis in equation (2) was put on closely fitting light elements (such as carbon) at the expense of slightly greater errors in the heavy elements (such as uranium). Maximum errors in equation (1) are about uniform over the mass range.

A comparison of equation (2) with two previous formulae, as well as the simple fits mentioned above, is given in Table 2. The formula (in slightly modified form) which has been used previously at the Naval Research Laboratory (Tsao and Silberberg 1975) is

$$\sigma = 10\pi(1.37\,A^{1/3})^2(1 - 0.47/A^{0.4}) \text{ mb}. \tag{3}$$

Except for beryllium, this formula predicts consistently higher cross sections than the data in Table 1. The formula of Hagen (1976) as reported by Protheroe, Ormes, and Comstock (1981),

$$\sigma = 10\pi(1.29)^2(A^{1/3} - 0.126)^2 \text{ mb}, \tag{4}$$

is consistently lower than the data.

b) Energy Dependence at Low Energies

At energies below 2 GeV the total inelastic cross section is not independent of energy. In general terms, it decreases to a minimum (~ 15% below the high energy value) at 200 MeV. It then sharply increases to a maximum at ~ 20 MeV (60% above the high energy value). Below this energy resonance effects become dominant, and the cross section fluctuates rapidly. Equation (5) is our energy-dependent cross section formula:

$$\sigma(E) = \sigma(\text{h.e.})\left[1 - 0.62\,e^{-E/200} \sin(10.9E^{-0.28})\right]. \tag{5}$$

This function as applied to carbon and aluminum is shown in Figures 2 and 3. We have made no attempt to

TABLE 2

PERCENT ERRORS IN VARIOUS FITS TO THE TOTAL INELASTIC CROSS SECTION DATA AT HIGH ENERGY[a]

Source	Fe and Lighter		All Elements	
	Mean	Maximum	Mean	Maximum
This paper				
(eq. [2])	0.4	0.6	0.5	1.3
$50.7A^{2/3}$	3.3	6.0	3.5	6.7
$44.9A^{0.7}$	1.2	1.9	1.2	2.5
Tsao and Silberberg (1975)				
(eq. [3])	2.2	2.8	3.5	6.7
Hagen (1976)				
(eq. [4])	5.2	6.4	5.5	8.1

[a] Errors are for the fit only and do not incorporate experimental error which is ~ 1%.

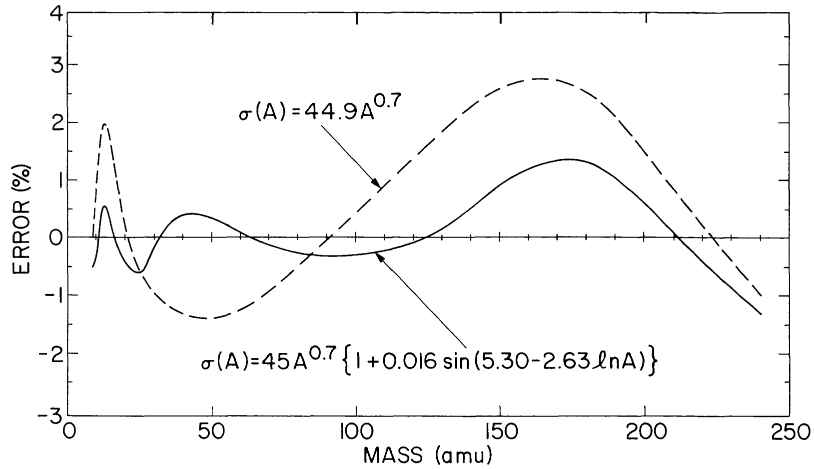

FIG. 1.—Percent error as a function of mass in the two best fits, equations (1) and (2), to proton-nucleus total inelastic cross section data

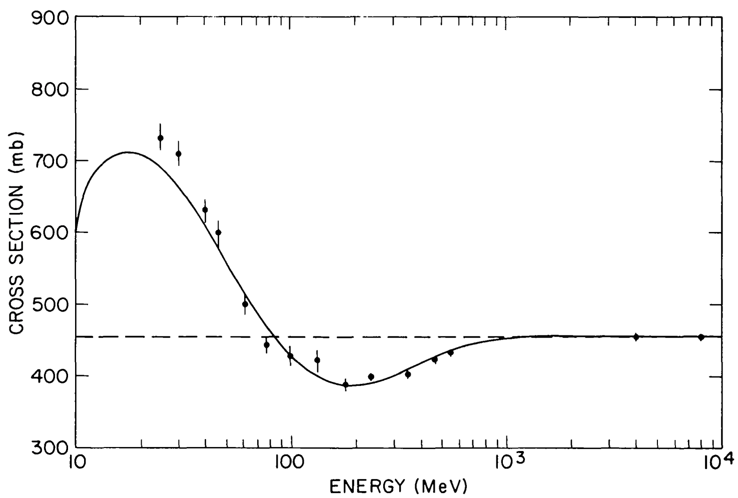

FIG. 2.—Carbon-proton total inelastic cross section data with the energy-dependent fit of this paper (*solid line*). Dotted line indicates the asymptotic value of our formula at high energy.

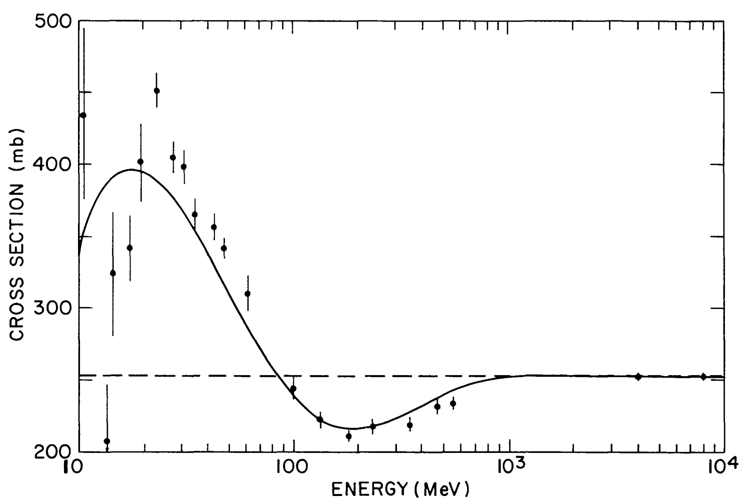

FIG. 3.—Aluminum-proton total inelastic cross section data with the energy-dependent fit of this paper (*solid line*). Dotted line indicates the asymptotic value of our formula at high energy.

TABLE 3

ERRORS IN ENERGY-DEPENDENT FORMULA (eq. [5])[a]

Energy Range	Estimated Maximum Error
$E > 1$ GeV..........	2%
300 MeV $< E \leq 1$ GeV	5%
100 MeV $< E \leq 300$ MeV ...	10%
40 MeV $< E \leq 100$ MeV ...	20%
10 Mev $< E \leq 40$ MeV	a factor of 2

[a]Mean errors are ~ 0.5 times maximum errors.

model the resonance features at low energies. In addition, a possible peak ($\leq 3\%$ above the high energy value) at ~ 1 GeV per nucleon, which might be expected from *p-p* scattering data (Karol 1975), was not modeled because of insufficient experimental evidence. At very high energies ($\geqslant 100$ GeV) one has some evidence for a slow increase in the cross sections (Grigorov 1976). If it is necessary, such an increase can be modeled by a multiplicative factor of the form $1+\alpha E$ without affecting our results.

Equation (5) is based on a selection of ~ 200 points from the available data. Our estimates of the errors associated with this fit are shown in Table 3. Mean errors are about half the maximum errors reported in the table. Equation (5) tends to overestimate the cross sections of heavy nuclei and underestimate those of light nuclei at energies less than 100 MeV. Between 100 MeV and 1 GeV cross sections on light nuclei are overestimated, while for heavy nuclei they are underestimated. Further refinements of equation (5) are, of course, possible but were not deemed necessary for two reasons. First, there is no real need for higher accuracy. Energy loss due to ionization dominates fragmentation loss at low energies. For example, at 50 MeV the range of an iron nucleus in hydrogen is ~ 0.1 g cm^{-2}, while the mean free path between nuclear interactions is greater than 2 g cm^{-2}. Cross section errors are therefore substantially diluted. Second, the error in a typical experimental data point at low energies is relatively great compared to that at higher energies. This is, to some extent, reflected in the errors in equation (5).

c) *Modifications for Light Nuclei*

The cross section formula presented here is based exclusively on data for beryllium and heavier elements. One finds that the fit for beryllium degrades badly at low energies. A detailed examination of the results for light nuclei ($Z < 6$) is therefore indicated.

A compilation of the experimental data on *p-p* total inelastic cross sections is given in Barashenkov (1968). At high energies the cross section is ~ 30 mb, substantially less than our prediction of 44.4 mb. This decreases monotonically to zero at the pion threshold. Equation (5) therefore does not even qualitatively reproduce the cross section data for hydrogen.

Comparison with available helium data (Barashenkov and Toneev 1972) shows that equation (5) overestimates the cross section at high energies by ~ 25%. A relatively good fit at all energies is obtained if our formula is multiplied by 0.8 for helium.

Data on lithium were presented in papers by Bellettini *et al.* (1966), Gorin *et al.* (1974), and Johansson, Svanberg, and Sundberg (1961). Excellent agreement is found with data of the latter two authors: 149 ± 3 mb at 180 MeV and 175 ± 2 mb at several energies between 20 and 60 GeV. The data point of Bellettini *et al.* (1966) at 20 GeV is $\sim 270 \pm 12$ mb, 15% to 20% higher than that of Gorin *et al.*

The substantial amount of beryllium data is better fitted by including the multiplicative correction

$$1 + 0.75 e^{-E/75}, \tag{6}$$

where E is in MeV.

This brings it well within the errors cited in Table 3. The "over reactivity" of beryllium is perhaps due to a relatively low barrier between it and $n + 2\alpha$.

Boron is represented in our data set by only one point at high energy. Our equation predicts exactly the value of 235 mb given by Bobchenko *et al.* (1979).

II. SUMMARY

An empirical formula for the total inelastic cross section of protons on nuclei of charge 2 and greater has been presented. The high energy form of this formula is equation (2), modified by a factor of 0.8 for helium. The correction for energy dependence at low energies is given by equation (5), modified for beryllium by equation (6). Errors in the high energy formula fall within experimental limits. Errors in the low energy correction gradually increase to a factor of 2 below 40 MeV but represent a substantial improvement over the assumption of constant cross section at all energies.

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