Generalized upper bound for inelastic diffraction

S.M. Troshin and N.E. Tyurin

SRC IHEP of NRC "Kurchatov Institute"

Protvino, 142281, Russian Federation

Abstract

For the inelastic diffraction, we obtain an upper bound valid in the whole range of the elastic scattering amplitude variation allowed by unitarity. We discuss the energy dependence of the inelastic diffractive cross-section on the base of this bound and recent LHC data.

Introduction

The recent experimental measurements of the global observables at the LHC in proton–proton collisions have confirmed the trends observed at lower energies, namely, continuous increase of the total, elastic and inelastic cross–sections in the new energy region. Those experiments have brought us closer to clarification of an elusive asymptotic regime of strong interactions at fixed, low values of the momentum transferred and increasing energy of collision. This regime belongs to the soft interaction region where perturbative QCD does not work. Nonperturbative QCD is still not able to provide calculations of these global observables despite that lattice QCD has successfully predicted various static characteristics of hadrons[1].

However, all hadron interactions are under constraints which follow from the general principles of theory such as unitarity and analyticity. Those principles are valid for any quantum field theory including QCD. The known for a long time properties of analyticity and unitarity of the scattering matrix could lead to assumption that strong interactions are "as strong as possible" [1, 2] and therefore the Froissart-Martin bound [3, 4] for the total cross-sections should be saturated at asymptotical energies. Then, the explicit functional energy behavior of the total cross-sections is often taken to follow $\ln^2 s$-dependence but the value of the factor in front of $\ln^2 s$ is still a remaining problem. The magnitude of this factor is an important issue since it is correlated with a particular choice for the upper limit for the elastic partial amplitude. Namely, this limit (it is $1/2$ in the case of the pure imaginary elastic scattering amplitude) may correspond to the maximum of the inelastic channel contribution to the elastic unitarity and leads to the ratio

$$\sigma_{el}(s)/\sigma_{tot}(s) \to 1/2, \tag{1}$$

or it can correspond to a maximal value of the partial amplitude allowed by unitarity (i.e. unity) resulting in the asymptotical limit

$$\sigma_{el}(s)/\sigma_{tot}(s) \to 1. \tag{2}$$

In the above equations $\sigma_{el}(s)$ denotes integral cross–section of the elastic scattering and $\sigma_{tot}(s) = \sigma_{el}(s) + \sigma_{inel}(s)$, where $\sigma_{inel}(s)$ is the total cross–section of inelastic interactions. Note, that Eq. (2) does not preclude asymptotical growth of $\sigma_{inel}(s)$, it just means that the inelastic cross-sections growth $\sigma_{inel}(s) \sim \ln s$ is slower than the growth of the elastic cross-section, i.e. $\sigma_{el}(s) \sim \ln^2 s$. The experimental values for total cross-section $\sigma_{tot}(s)$ are obtained from the differential

---

[1]We are grateful to R.N. Rogalev for discussion on the present status and problems in the lattice QCD calculations of soft hadron interactions.

cross-section of elastic scattering extrapolated to the point $t = 0$ through optical theorem.

In the impact parameter representation Eq. (1) corresponds to the limiting case of the so called BEL picture. The acronym BEL has the meaning that the interaction region of protons becomes Blacker, Edgier and Larger with increasing energy. This kind of energy evolution was proposed and discussed in [5]. Such behaviour corresponds to the presupposed absorptive nature of the scattering when the elastic scattering partial amplitude would never exceed the black disc limit of $1/2$. With this assumption of the absorptive scattering domination the original Froissart-Martin bound for the total cross-sections has been improved by factor of $1/2$. An origin for appearance of this factor is an upper bound for the total inelastic cross-section reduced by factor of 4 (cf. [6]). Thus, the related ratio of the cross–sections $\sigma_{el}(s)/\sigma_{tot}(s)$ is an important function of energy, e.g. it stands in front of $\ln^2 s$ in the asymptotical bound on the total cross-section [7, 8]:

$$\sigma_{tot}(s) \leq \frac{4\pi}{t_0} \left(\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \right) \left[\ln \left(\frac{s}{\sigma_{el}(s)} \right) \right]^2 \left[1 + \left(\frac{\mathrm{Re} F(s, t=0)}{\mathrm{Im} F(s, t=0)} \right)^2 \right]^{-1}, \quad (3)$$

where $F(s, t)$ is the amplitude of the elastic scattering process $pp \to pp$. The elastic scattering amplitude $F(s, t)$ is related to the amplitude in the impact parameter representation $f(s, b)$ through the Fourier-Bessel transformation.

Since our consideration is a qualitative one, we will adopt for simplicity that the scale of s in the above bound is determined by the energy-independent value $s_0 = 1$ GeV^2 (i.e. s/s_0 is replaced by s and the latter is considered to be dimensionless, despite that this scale is an energy-dependent one and is determined by $\sigma_{el}(s)$) in the above bound for the total cross–section. In Eq. (3) $\sqrt{t_0}$ is the mass of the lowest state in the t channel in this bound[2].

In this note, an upper bound for the cross–section of inelastic diffraction is discussed. We point out a problem with an existing upper bound for the cross–section of inelastic diffraction. In fact, the existing, Pumplin bound [9], has been derived under assumption of the absorptive nature of diffraction at all the energies. A generalized bound which is free from such restriction and valid in the whole range of the elastic scattering amplitude variation allowed by unitarity is obtained. We also discuss its implications on the model ground.

---

[2]For most processes, the factor in Eq. (3) is determined by a pion mass, i.e. $t_0 = 4m_\pi^2$. Assumption of saturation of this bound leads to necessity of a significant shift of t_0 into the region of larger values, and it corresponds to replacement of the pion mass by a mass of a heavy particle in order to fit the experimental data.

1   The absorptive and reflective scattering modes

A distinctive feature of the impact parameter representation is a diagonal form of the unitarity equation for the elastic scattering amplitude $f(s, b)$, i.e.

$$\mathrm{Im} f(s, b) = |f(s, b)|^2 + h_{inel}(s, b). \tag{4}$$

Eq. (4) is valid at high energies with $\mathcal{O}(1/s)$ precision [10]. The $|f(s, b)|^2$ is the elastic channel contribution, while the inelastic overlap function $h_{inel}(s, b)$ includes all contributions from the intermediate inelastic channels into unitarity relation. The elastic scattering S-matrix element in the impact parameter representation is related to the elastic scattering amplitude $f(s, b)$ by the equation $S(s, b) = 1 + 2if(s, b)^3$ and it can be represented in the form

$$S(s, b) = \kappa(s, b) \exp[2i\delta(s, b)]$$

which includes the two real functions $\kappa(s, b)$ and $\delta(s, b)$. The function κ (it varies in the region $0 \leq \kappa \leq 1$) is called an absorption factor: its value $\kappa = 0$ corresponds to a complete absorption of the initial state.

As it was already mentioned, we assume the pure imaginary scattering amplitude[4] and perform the replacement $f \rightarrow if$ at high energies. We should also mention that the Pumplin bound discussed below has been derived with the approximation of pure imaginary amplitudes of the elastic and diffractive scattering. The elastic scattering amplitude $f(s, b)$ is related then to the elastic scattering matrix element by the relation $S(s, b) = 1 - 2f(s, b)$, i.e. the function $S(s, b)$ is real, but it should not have a definite sign in the whole range of the amplitude variation allowed by unitarity. On the base of the existing experimental trends a monotonic, without oscillations over s, increase of the elastic scattering amplitude $f(s, b)$ with the energy is assumed.

In fact, the choice of the particular elastic scattering mode, namely, absorptive or reflective one [13], depends on the sign of the function $S(s, b)$, in another words, on the value of the phase $\delta(s, b)$ [14]. The widely used picture is based on the limit $S(s, b) \rightarrow 0$ at fixed impact parameter b and $s \rightarrow \infty$. This is known as a black-disc limit. The elastic scattering is a completely absorptive one in this mode and the function $S(s, b)$ is always non-negative and the limitation $f(s, b) \leq 1/2$ is implied.

There is another mode (reflective scattering) when the function S becomes negative with energy growth. It has the limiting behavior $S(s, b) \rightarrow -1$ at fixed b

---

[3]This relation determines normalization of the elastic scattering amplitude $f(s, b)$

[4]It should be noted that saturation of the black–disc limit or the unitarity limit leads to a vanishing real part of the scattering amplitude, $\mathrm{Re} f \rightarrow 0$ in the region where the both limits are saturated [11]. The recent data [12] on the precise measurements of the ratio of the real to the imaginary part of the forward amplitude are consistent with decreasing energy dependence of this ratio.

and $s \to \infty$, i.e. $\kappa \to 1$ and $\delta = \pi/2$ and the amplitude varies in the range $1/2 < f(s,b) \le 1$ since the function S is negative. The non-zero phase $\delta = \pi/2$ can be interpreted as the geometric phase and its appearance is related to the presence of a dynamical singularity [13, 15].

Which mode will be realized at the asymptotical energies is unclear now. The claims on the equipartition of the elastic and inelastic collision probabilities at $s \to \infty$ are the model-dependent ones. Moreover, conclusions on such equipartition are ambiguous without impact parameter analysis performed. The respective conclusions [16] are based on the forward scattering data only and are not trustworthy therefore. The base for such statement is grounded on the fact that the forward scattering observables are represented by definite integrals over impact parameter. It is obvious, that an integration does not allow one to reconstruct an integrand since different functions can lead to the same results after integration (cf. e.g. [17] where the equipartition $\sigma_{el}(s)/\sigma_{tot}(s) = \sigma_{inel}(s)/\sigma_{tot}(s) = 1/2$ at $s \to \infty$ has been obtained just for a gaussian exponential dependence of the profile function saturating unitarity limit. Such function has nothing to do with the black disc model).

2   Sub-leading role of inelastic diffraction in the absorptive mode

It is known that soft hadron interaction dynamics is determined by the nonperturbative QCD. Since it does not provide calculation recipe at the moment, the essential role under discussion of soft processes belongs to general principles of the theory such as unitarity and analyticity, as it was mentioned in the Introduction. We start with discussion of an upper bound in the impact parameter representation. The assumption on absorptive scattering domination at all the energies including asymptotics was an essential point under derivation of the Pumplin bound [9] for the inelastic diffraction:

$$\sigma_{diff}(s,b) \le \frac{1}{2}\sigma_{tot}(s,b) - \sigma_{el}(s,b), \tag{5}$$

where

$$\sigma_{diff}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{diff}}{db^2}$$

is the total cross–section of all the inelastic diffractive processes in the impact parameter representation and

$$\sigma_{tot}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{tot}}{db^2} \ , \ \ \sigma_{el}(s,b) \equiv \frac{1}{4\pi}\frac{d\sigma_{el}}{db^2}.$$

The bound Eq. (5) was obtained in the framework of the Good–Walker formalism for the inelastic diffraction [18] and is based on the eikonal form of the diffractive amplitudes. This form can be considered as one of the realizations of the absorptive scattering mode. Eq. (5) is valid for each value of the impact parameter of the collision b, i.e. it is diagonal in b-space and integration over impact parameter provides:

$$\sigma_{diff}(s) \leq \frac{1}{2}\sigma_{tot}(s) - \sigma_{el}(s). \tag{6}$$

It should be emphasized again that eqs. (1) and (6) are to be fulfilled if the hadron scattering picture corresponding to the black-disc limit is valid at asymptotical energies, i.e.

$$\sigma_{inel}(s)/\sigma_{tot}(s) \to 1/2 \tag{7}$$

while

$$\sigma_{diff}(s)/\sigma_{tot}(s) \to 0 \tag{8}$$

and

$$\sigma_{diff}(s)/\sigma_{inel}(s) \to 0 \tag{9}$$

at $s \to \infty$. It is difficult to reconcile those limits. Indeed, $\sigma_{diff}(s)$ is, by definition[5], a leading part of the inelastic cross–section $\sigma_{inel}(s)$ and the LHC experimental data demonstrate approximate energy–independence of ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$ [20, 21].

In contrast to its definition and the available data, one should conclude then, that the inelastic diffraction corresponds, in fact, to a sub-leading mechanism in the inelastic cross-section and the leading role in the growth of $\sigma_{inel}(s)$ belongs to the nondiffractive inelastic processes. Such a statement is not easy to reconcile with the definition of the diffractive processes and their energy dependence, i.e. the eq. (9) is not in favor of the black-disc limit saturation by the partial scattering amplitude at $s \to \infty$.

We also note that Eq. (5) can be rewritten in terms of $S(s, b)$ in the form

$$\sigma_{diff}(s, b) \leq \frac{1}{4}S(s, b)(1 - S(s, b)) \tag{10}$$

and this inequality indicates that the Pumplin bound on $\sigma_{diff}(s, b)$ cannot be applied in the region where $S(s, b)$ becomes negative.

---

[5]Generally, any type of inelastic diffraction is associated with one or several Pomeron exchanges. Cf. for discussion [19].

3 Generalized upper bound for inelastic diffraction

Apparently, there is no embarrassment in the approach which allows saturation of the unitarity limit. The limiting dependence Eq. (2) assumes an alternative option which corresponds to saturation of the unitarity limit for the elastic partial amplitude and can be interpreted as a reflective scattering [13]. Saturation of unitarity can be associated with the strong coherent parton interactions in QCD relevant to confinement dynamics.

The inelastic overlap function at the asymptotical energies will acquire a peripheral form in the impact parameter representation [22]. This peripherality was treated as a manifestation of an emerging transparency in the central hadron collisions (or in vicinity of the impact parameter $b = 0$ of the colliding particles) at very high energies. Later on, this interpretation has been generalized and specified in papers [23, 24, 25, 26] where such a phenomenon was related to antishadowing or reflection in the hadron interactions. It should be noted that the concept of the on-shell optical potential also lead up to conclusion on the central grayness in the inelastic overlap function [27]. The peripheral form of the inelastic overlap function appears at high energies due to acquiring negative values by $S(s, b)$ with the collision energy increase:

$$\frac{\partial h_{inel}(s, b)}{\partial b} = S(s, b)\frac{\partial f(s, b)}{\partial b} \qquad (11)$$

Thus, the central profile of $f(s, b)$ transforms with the energy growth into a peripheral profile of the function $h_{inel}(s, b)$ due to negative values of $S(s, b)$. The appearing of peripheral form of the inelastic overlap function is an energy-dependent effect. It happens at the values of energy $s > s_r$, where s_r is solution of the equation

$$S(s_r, b = 0) = 0 \qquad (12)$$

A recent analysis [28] of the elastic scattering data obtained by the TOTEM Collaboration at $\sqrt{s} = 7$ TeV [29] confirmed an existence of this novel feature in strong interaction dynamics revealing that way transition to such scattering mode (also referred as a resonant scattering [30]). A gradual transition to the REL picture, acronym REL means that the interaction region becomes Reflective (the term reflective means that the elastic scattering matrix element acquires negative values) close to the center ($b = 0$) and simultaneously becomes Edgier, Larger and completely black in the ring at periphery (at $b \neq 0$), seems to be observed by the TOTEM under the measurements of the $d\sigma/dt$ in elastic pp–scattering [29]. The acronym REL was formed similar to the above acronym BEL. In this REL picture elastic partial amplitude exceeds black-disc limit at small values of b. Several phenomenological models are able to reproduce such transition and among them the

one based on the rational unitarization of the leading vacuum Regge–pole contribution with the intercept greater than unity [22] and similar models known under the generic name of the unitarized supercritical Pomeron (cf. [30] for a recent discussion and the references).

The assumption that the unitarity limit instead of the black-disc limit is to be saturated asymptotically leads to a relatively slower increase of the inelastic cross-section

$$\sigma_{inel}(s)/\sigma_{tot}(s) \to 0 \tag{13}$$

which allows one to keep considering inelastic diffraction as a leading mechanism of the inelastic cross–sections growth. It should be noted that the available experimental data are consistent with decreasing dependence of the ratio $\sigma_{inel}(s)/\sigma_{tot}(s)$ with energy.

The possibility that the elastic amplitude exceeds the value of $1/2$ (which corresponds to the black-disc limit) was discussed earlier in the framework of the rational unitarization on the base of the CDF data obtained at Tevatron [24]. It should be noted that the value of $\mathrm{Im}f(s, b = 0)$ has increased from 0.36 (CERN ISR) to 0.492 ± 0.008 (Tevatron) and it is just on the edge of the black-disc limit in the Tevatron energy domain[31].

The exceeding of the black-disc limit of $1/2$ turns the Pumplin bound to be groundless [23]. But, this conclusion deserves to be more specified. In fact, the Pumplin bound is not valid only in the particular region of the small and moderate values of the impact parameter where the absorptive approach becomes not applicable. Such region of impact parameter values appears at very high energy.

Namely, the model-independent reconstruction of the impact–parameter dependent quantities from the TOTEM data demonstrates that the black-disc limit has been crossed in elastic scattering at small values of b [28]. In fact, the elastic scattering S-matrix element $S(s, b)$ is negative at $0 < b < 0.2$ fm and crosses the zero at $b = 0.2$ fm at $\sqrt{s} = 7$ TeV. These estimates are consistent with the result of the Tevatron data analysis [31]. It should be noted here that the region of the negative values of $S(s, b)$ is determined by the interval $0 < b < r(s)$. The function $r(s)$ is the solution of the following equation:

$$S(s, b = r(s)) = 0,$$

$r(s) = 0$ at $s = s_r$.

The qualitative impact parameter dependencies of the functions $f(s, b)$ and $h_{inel}(s, b)$ at fixed energy $s_2 > s_r$ are depicted in Figs. 1 and 2 and the schematic energy evolution of the function $S(s, b)$ is depicted in Fig. 3 at three different energy values. In the impact parameter range $0 < b \leq r(s)$ only a trivial bound

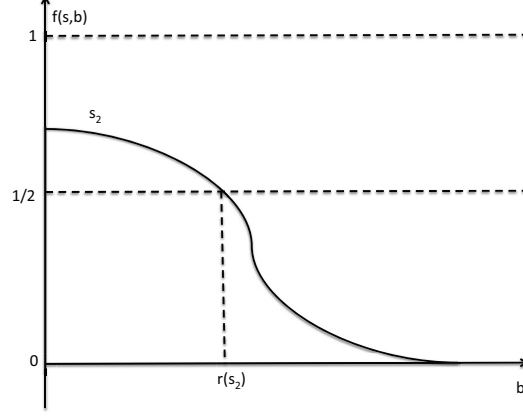$$\sigma_{diff}(s, b) \leq \sigma_{inel}(s, b) \tag{14}$$

8

Figure 1: Qualitative impact-parameter dependence of $f(s, b)$ at $s = s_2$ $(s_2 > s_r)$.

can be applied. But, at $b \geq r(s)$ the scattering is absorptive and therefore the original Pumplin bound should be restored.

However, the integrated bound is modified. Namely, in this case it should be written in the form

$$\bar{\sigma}_{diff}(s) \leq \frac{1}{2}\bar{\sigma}_{tot}(s) - \bar{\sigma}_{el}(s), \tag{15}$$

where $\bar{\sigma}_i(s)$ are the "reduced" cross-sections:

$$\bar{\sigma}_i(s) \equiv \sigma_i(s) - 8\pi \int_0^{r(s)} bdb\sigma_i(s, b), \tag{16}$$

for $i \equiv diff, tot, el$, respectively. Combining Eqs. (14) and (15), the following inequalities relevant for the LHC energies, can be easily obtained:

$$\sigma_{diff}(s) \leq \sigma_{inel}(s) - 2\pi \int_{r(s)}^{\infty} bdb[1 - S(s, b)] \tag{17}$$

and

$$\sigma_{ndiff}(s) \geq 2\pi \int_{r(s)}^{\infty} bdb[1 - S(s, b)], \tag{18}$$

where $\sigma_{ndiff}(s)$ is the total cross-section of the nondiffractive procsses, i.e.

$$\sigma_{ndiff}(s) \equiv \sigma_{inel}(s) - \sigma_{diff}(s).$$

The function $S(s, b)$ can be reconstructed from the experimental data on $d\sigma/dt$ in elastic pp-scattering. Using the TOTEM data at $\sqrt{s} = 7$ TeV and the value of $r(s) = 0.2$ fm extracted from the analysis [28], one obtains the magnitude of the
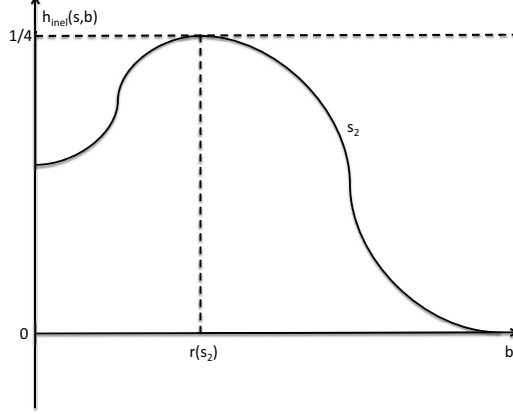
Figure 2: Qualitative impact-parameter dependence of $h_{inel}(s,b)$ at $s = s_2$ ($s_2 > s_r$).

upper bound on the value of $\sigma_{diff}(s)$ to be equal to 25.6 mb at this energy. The positive contribution of reflective scattering to the right hand side of Eq. (17) at this energy is about 5%. Extrapolating data to the energy $\sqrt{s} = 13$ TeV one can get an estimate for the bound on $\sigma_{diff}(s)$ and the reflective scattering contribution to it at the level of 28.2 mb and $(6-8)\%$, respectively[6]. It would be interesting to confront these estimates to the experimental data for $\sigma_{diff}(s)$ at the above energy. The data at $\sqrt{s} = 7$ TeV [21] are in agreement with this bound. It is useful to consider an inverted energy evolution, i.e. the case of the decreasing energy, also. Due to the supposed monotonous energy dependence, the function $r(s)$ will be moving to zero with the energy decrease and it can only be approximated by this zero value at lower energies ($s < s_r$) because the negative values of the impact parameter have no sense (cf. Fig. 4). Thus, the function $2\pi \int_{r(s)}^{\infty} bdb[1 - S(s,b)]$ will be transformed into $4\pi \int_{0}^{\infty} bdbf(s,b)$ at $s \leq s_r$, (note that $1 - S = 2f$). The latter is just $\sigma_{tot}(s)/2$ and, therefore, the bound Eq.(17) is being converted into the standard Pumplin bound. This demonstrates selfconsistency of the above considerations and universality of Eq. (17), i.e. it is valid at low and high energies.

Thus, the Eq.(17) should be considered as a generalization of the Pumplin bound for the inelastic diffraction cross-section. Eq.(17) is valid in the whole range of the elastic amplitude variation allowed by unitarity and, in particular, in the energy region where the black-disc limit is exceeded. The above generalization has a practical meaning since there is a definite indication that the black-disc limit has been exceeded in the central hadron collisions at the LHC energy $\sqrt{s} = 7$ TeV [28]. Despite that the asymptotic regime is not reached at the LHC energies,

---

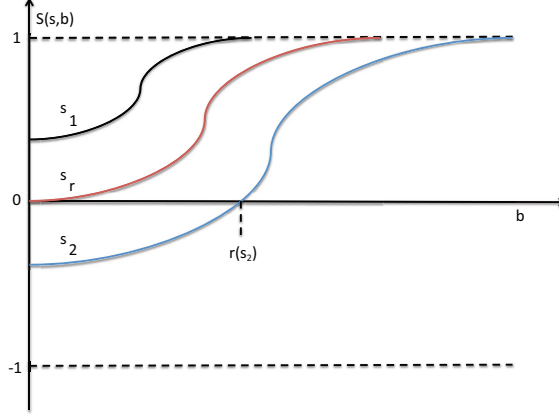[6]The extrapolated value of $r(s)$ at this energy is about 0.3 fm.

10

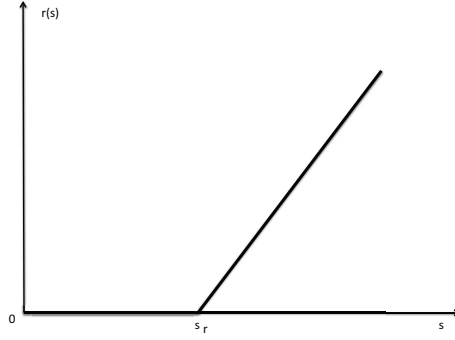Figure 3: Schematic energy evolution of the impact-parameter dependence $S(s,b)$.

Figure 4: Schematic energy dependence of the function $b = r(s)$.

there are certain indications on the turn-on of the transitional regime at these energies where elastic scattering starts to dominate at small impact parameters. Thus, the measurements at $\sqrt{s} = 13$ TeV are in the energy range where the generalized upper bound for the inelastic diffractive cross-section should be applied. The reflective scattering mode could also be responsible for the interesting phenomena observed uder cosmic ray studies [32].

4   The model estimates

The unitary model for the $S(s,b)$ can also be used to estimate qualitatively the dependencies of the cross-sections $\sigma_{diff}(s)$ and $\sigma_{ndiff}(s)$. The reflective scattering is a characteristic picture of the model presented below. It is based on the rational

11

form of the unitarization and represents the function $S(s, b)$ in the form:

$$S(s, b) = [1 - U(s, b)]/[1 + U(s, b)], \qquad (19)$$

The $U(s, b)$ is the generalized reaction matrix element, which is considered to be an input dynamical quantity and it is taken to be a real function. The various dynamical models can be used for the function $U(s, b)$. To get the qualitative estimates we are using the simplified form of this function which conforms to rising total cross-section and analytical properties over the transferred momentum, i.e.

$$U(s, b) = g(s) \exp(-\mu b), \qquad (20)$$

where $g(s) \sim s^\lambda$, λ and μ are some constants. Eq. (20) resembles form used by Heisenberg in his model for the total cross–sections of the inelastic processes [33]. However, the model is relevant for the black-disc limit saturation only (cf. e.g. [34] and references therein) and it does not include elastic scattering and effects of self-damping of the inelastic channels [35].

Then the following asymptotical dependencies will take place[7]:

$$\sigma_{tot}(s) \sim \ln^2 s, \ \sigma_{el}(s) \sim \ln^2 s, \ \sigma_{inel}(s) \sim \ln s \text{ and } r(s) \sim \ln s. \qquad (21)$$

From Eq. (17) it follows that for the ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$ the inequality takes place

$$\frac{\sigma_{diff}(s)}{\sigma_{inel}(s)} \le 1 - \frac{2\pi}{\sigma_{inel}(s)} \int_{r(s)}^\infty b db [1 - S(s, b)]. \qquad (22)$$

From Eqs. (18) and (21) it follows that $\sigma_{ndiff}(s) \sim \ln s$ and the second term in Eq. (22) tends to $1/2$ at $s \to \infty$. Thus, in this approach both parts of $\sigma_{inel}(s)$ would have similar asymptotical energy dependencies, which are proportional to $\ln s$.

Eq. (22) can be further simplified if one notes that at large values of b

$$1 - S(s, b) \simeq 2h_{inel}(s, b),$$

$h_{inel}(s, b)$ has its maximal value at $b = r(s)$ and

$$\sigma_{inel} \simeq \frac{8\pi}{\mu^2} \ln g(s)$$

---

[7]The explicit expressions for $r(s)$ and $\sigma_{inel}(s)$ are the following

$$r(s) = \frac{1}{\mu} \ln g(s) \text{ and } \sigma_{inel}(s) = \frac{8\pi}{\mu^2} \ln(1 + g(s)).$$

12

at $s \to \infty$:

$$\frac{\sigma_{diff}(s)}{\sigma_{inel}(s)} \leq 1 - \frac{\mu}{2} \int_{r(s)}^{\infty} db h_{inel}(s, b). \tag{23}$$

The limiting value of the inelastic overlap function integral over b

$$\int_{r(s)}^{\infty} db h_{inel}(s, b)$$

in the model is $1/\mu$ and bound on inelastic diffractive cross takes the simplest form

$$\frac{\sigma_{diff}(s)}{\sigma_{inel}(s)} \leq 1/2. \tag{24}$$

Finally, one can assume saturation of the bound Eq. (24) and arrive that way to the asymptotic equipartition of the inelastic cross–section into diffractive and non-diffractive parts.

Conclusion

The generalized upper bound Eq.(17) for the inelastic diffraction has been derived. It has also been shown that there is no inconsistency between saturation of the unitarity limit leading to Eq. (2) and this bound on the inelastic diffractive cross–section. The obtained energy-independent ratio $\sigma_{diff}(s)/\sigma_{inel}(s)$ conforms to the definition of the inelastic diffraction as a result of the Pomeron exchanges as well as to the recent experimental trends observed at the LHC. This allows one to reconcile the results of s- and t-channel approaches to the inelastic diffraction cross-section.

If one assumes any mechanism resulting in saturation of the black-disc limit at the asymptotic energies, it is difficult to ensure such reconcilement already in the LHC energy range. The LHC experiments at the new higher energies would be definitely helpful for resolving picture of the inelastic diffraction and elastic scattering at $s \to \infty$.

References

[1] G.F. Chew and S.C. Frautschi, Phys. Rev. Lett. **5**, 580 (1960).

[2] G.F. Chew and S.C. Frautschi, Phys. Rev. Lett. **7**, 394 (1961).

[3] M. Froissart, Phys. Rev. **123**, 1053 (1961).

[4] A. Martin, Nuovo Cimento **42**, 930 (1966).

[5] R. Henzi and P. Valin, Phys.Lett. B **164**, 411 (1985).

[6] A. Martin, Phys. Rev. D **80**, 065013 (2009).

[7] V. Singh and S.M. Roy, Ann. of Phys. **57**, 461 (1970).

[8] S.M. Roy, Phys. Rep. **5**, 128 (1972).

[9] J. Pumplin, Phys. Rev. D **8**, 2899 (1973).

[10] M.L. Goldberger and K.M. Watson, Collision Theory, John Wiley and Sons, Inc., New-York—London—Sydney 1964.

[11] S.M. Troshin, Phys. Lett. B **682** , 40 (2009).

[12] G. Antchev et al. CERN-PH-EP-2015-325.

[13] S.M. Troshin and N.E. Tyurin, Int. J. Mod. Phys. A. **22**, 4437 (2007).

[14] S.M. Troshin and N.E. Tyurin, Phys. Rev. D **88**, 077502 (2013).

[15] S.M. Troshin and N.E. Tyurin, arXiv:1305.6153.

[16] M.M. Block, L. Durand, P. Ha, F. Halzen, Phys.Rev. D **92**, 114021 (2015).

[17] I.M. Dremin, S.N. White, arXiv:1604.0369.

[18] M.L. Good and W.D. Walker, Phys. Rev. **120**, 1857 (1960).

[19] E. Predazzi, Proc. of the Interanational Workshop on Diffraction in High–Energy Physics, Cetraro, Italy, 2-7 September 2000, Eds. R. Fiore, M.I. Kotsky, A. Papa, E. Predazzi, G. Susinno, Nucl. Phys. B (Proc. Suppl.) **99A**, 3 (2001).

[20] B. Abelev et al. (ALICE Collaboration), Eur. Phys. J. C **73**, 2456 (2013).

[21] P. Lipari and M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013).

[22] V.F. Edneral, O.A. Khrustalev, S.M. Troshin and N.E. Tyurin, Preprint CERN-TH-2126, 1976.

[23] S.M. Troshin and N.E. Tyurin, Phys. Lett. B **208**, 517 (1988).

[24] S.M. Troshin and N.E. Tyurin, Phys. Lett. B **316**, 175 (1993).

[25] S. M. Troshin and N. E. Tyurin, Int. J. Mod. Phys. A **22**, 4437 (2007).

[26] P. Desgrolard, L.L. Jenkovszky and B.V. Struminsky, Phys. Atom. Nucl. **63**, 891 (2000).

[27] E.R. Arriola and W. Broniowski, arXiv: 1602.00288.

[28] A. Alkin, E. Martynov, O. Kovalenko and S.M. Troshin, Phys. Rev. D **89**, 091501(R) (2014).

[29] G. Antchev et al. Europhys. Lett. **101**, 21002 (2013).

[30] V.V. Anisovich, V.A. Nikonov and J. Nyiri, Phys. Rev. D **90**, 074005 (2014).

[31] P. Giromini, Proc. of the Vth BLOIS Workshop, Elastic and Diffractive Scattering, Eds. H.M. Fried, K. Kang, C-I Tan, World Scientific, Singapore, 1994, p. 30.

[32] S.M. Troshin and N.E. Tyurin, arXiv: 1504.05893.

[33] W. Heisenberg, Z. Phys. **133**, 65 (1952).

[34] H.G. Dosch, P. Gauron and B. Nicolescu, Phys. Rev. D. **67**, 077501 (2003).

[35] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).