Proton–Nucleus Cross Section at High Energies

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Cross sections for proton inelastic collision with different nuclei are described within the Glauber and multiple scattering approximations. A significant difference between approximate "Glauber" formula and exact calculations with a geometrical scaling assumption for very high-energy cross section is shown. Experimental values of proton–proton cross sections obtained using extensive air shower data are based on the relationship of proton–proton and respective proton–air absorption cross sections. According to obtained results values reported by the Akeno and Fly's Eye experimental groups are about 10% overestimated. The proper energy dependence of absorption cross section for collisions with air nuclei is of a great importance for studies of high energy cosmic rays using the Monte Carlo technique.

I. INTRODUCTION

The rise of the proton-proton cross section (total, inelastic) as the interaction energy increases is an important feature of the strong interaction picture. The growth itself is established quite well both from theoretical and experimental point of view. However the question how fast do cross sections rise is discussed permanently. A definite answer is still lacking. Theoretical predictions agree well with one another and with accelerator data in the region where data exist ($\sqrt{s} \sim 20 \div 2000$ GeV) but they differ above taht level. Before the large hadron Collider LHC [1] shifts the direct measurements limit to $10 \sim 14$ TeV the only existing information comes from the cosmic ray extensive air shower (EAS) data. Two EAS experiments, Akeno [2] and Fly's Eye [3], gave estimations of proton–proton total cross section at about $\sqrt{s} \approx 10^4$ GeV.

The important difference between the collider and EAS proton–proton cross section measurements is that in fact the proton–air interactions are involved in the EAS development. Thus the value which is real measured is the cross section for the interactions with air nuclei. The value of proton–proton cross section is then obtained using a theory for nuclei interactions. In many recent papers concerning this subject one finds only a brief reference such as "calculations have been made in the standard Glauber formalism" or something very similar [2,3].

The original Glauber paper [4] was published over 40 years ago and it appears that since then some misunderstandings have arisen. Rather complicated equations for scattering cross sections can be simplified significantly applying some additional assumptions — of limited validity. It was pointed out in 1970 [5] that some expressions which are most frequently identified with the nuclear optical model should not be used at least for light nuclei, yet this still sometimes happens nowadays.

In this paper we compare results of calculations with and without mentioned simplifications. We will shown that there is quite significant difference between them. The exact Glauber formalism will be used to evaluate the proton–proton cross section values from the Akeno and Fly's Eye data.

The paper is organized as follows. In the next section a detail description of proton–proton scattering, the basis of further nuclei cross section calculations, is given. In section 3 the proton-nucleus cross section evaluation for some commonly used methods is given, and in section 4 the quantitative results are presented and discussed.

II. PROTON–PROTON CROSS SECTIONS

Introducing the impact parameter formalism cross sections can be described using one, in general complex, function χ in the form

$$\begin{aligned}
\sigma_{\rm tot} &= 2 \int \left[1 - {\rm Re} \left({\rm e}^{i\chi(b)} \right) \right] d^2\mathbf{b} \quad , \\
\sigma_{\rm el} &= \int \left| 1 - {\rm e}^{i\chi(b)} \right|^2 d^2\mathbf{b} \quad , \\
\sigma_{\rm inel} &= \int 1 - \left| {\rm e}^{i\chi(b)} \right|^2 d^2\mathbf{b} \quad . \qquad (1)
\end{aligned}$$

The phase shift χ is related to the scattering amplitude by the two-dimensional Fourier transform

$$
\begin{aligned}
1 \,-\, \mathrm{e}^{i\chi(\mathbf{b})} \;&=\; \frac{1}{2\,\pi\,i}\int \mathrm{e}^{-i\mathbf{b}\,\mathbf{t}}S(\mathbf{t})d^2\mathbf{t};\\
S(t) \;&=\; \frac{i}{2\,\pi}\int \mathrm{e}^{i\mathbf{b}\,\mathbf{t}}\left(1\,-\,\mathrm{e}^{i\chi(\mathbf{b})}\right)d^2\mathbf{b}.
\end{aligned}
\tag{2}
$$

Using the optical analogy one can interpret the $1\,-\,\mathrm{e}^{i\chi(\mathbf{b})}$ function as a transmission coefficient for a given impact parameter. Considering two colliding object we can assume (for pure absorptive potential)

$$
\chi(b) \;=\; i\,\omega(b) \;=\; i\,K_{ab}\int d^2\mathbf{b}'\,\rho_a(\mathbf{b})\rho_b(\mathbf{b}\,+\,\mathbf{b}'),
\tag{3}
$$

where ρ_h is a particle's "opaqueness" (the matter density integrated along the collision axis). To some extend the hadronic matter density could be identified with the charge density measured precisely in the leptonic-scattering experiments. However, for high-energy hadron–hadron collisions the real part of the phase shift χ can be considerable. Also, the simple interpretation given in Eq.(3) ought to be modified — at least, by introducing some dependence on the interaction energy (s). Thus the general phase shift χ is a two-variable complex function. Two possible ways of simplifying the situation have been proposed in the literature: the factorization hypothesis (FH) and geometrical scaling (GS). They can be expressed as

$$
\begin{aligned}
\chi(s,\,b) \;&=\; i\,\omega(b)\,f(s) \qquad \text{(FH)}\\
\chi(s,\,b) \;&=\; i\,\omega\,(b\,/b_0(s)) \qquad \text{(GS)}.
\end{aligned}
\tag{4}
$$

From the optical point of view, the FH means that the hadron is getting blacker as the energy increases while the GS means that it is getting bigger.

The bulk of information about the hadron phase-shift function χ comes from elastic scattering experiments: more precisely, from the measured differential elastic cross section. The value of the total cross section can be obtained from the imaginary part of the forward scattering amplitude using the optical theorem. Analysis of the elastic data above $\sqrt{s}\sim 20$ GeV shows that neither assumption given in Eq.(4) is realized exactly (see e.g. Refs. [6,7]). However, a combination of the two can reproduce the data quite well.

In this paper the form of χ is assumed after [8] in the form

$$
\chi(s,b) \;=\; (\lambda(s)+i)\,\omega(b,s)
\tag{5}
$$

[with ω defined by Eq.(3)]. This follows the original GS idea [9] and differs from the known Martin formula where the ratio of real to imaginary part of the scattering amplitude depends on the momentum transfer. A full discussion and some recent references can be found in [8]. In any case, due to a lack of both theoretical and experimental information about the phase shift, Eq.(5) can be treated as a first approximation. In this paper the parametrization of λ used is

$$
\lambda(s) \;=\; \frac{0.077\,\ln(s/s_0)}{1\,+\,0.18\,\ln(s/s_0)\,+\,.015\,\ln^2(s/s_0)} \quad,\quad s_0 \;=\; 500\ \mathrm{GeV}^2\ .
\tag{6}
$$

For the ω energy dependence in Eq.(4) the GS is assumed, thus

$$
\omega(b,s) \;=\; \omega(\widetilde{b}) \qquad \text{with} \quad \widetilde{b} \;=\; b\left[\frac{\sigma_{\mathrm{inel}}(s_0)}{\sigma_{\mathrm{inel}}(s)}\right]^{\frac{1}{2}},
\tag{7}
$$

where s_0 is the center of mass energy for which the detail shape of ω has been originally determined. The accurate data description can be found in Ref. [10] with the "hadronic matter" distribution of the form

$$
\rho_h(\mathbf{b}) \;=\; \int dz\frac{m_h}{8\pi}\mathrm{e}^{-m_h\mathbf{r}}
\tag{8}
$$

with the coefficients m_h [and K_{ab} in Eq.(3)] adjusted to the hadron–proton elastic scattering data at $\sqrt{s}\sim 20$ GeV. The energy dependence of the phase shift is thus introduced by using the elastic to total cross sections ratio change [$\lambda(s)$ function] and by the scaling of the interaction impact parameter which reproduces the increase of cross section as the interaction energy increases [$\sigma_{\mathrm{inel}}(s)$].

The parametrization of $\sigma_{\mathrm{inel}}(s)$ has been made in the $\ln^2(s)$ form

$$
\sigma_{\mathrm{inel}}(s) = 32.4\;-\;1.2\ln(s)\;+\;0.21\ln^2(s)
\tag{9}
$$

which gives the cross section in mb when s is given in GeV^2. The deviation from the Pomeron-type power-law fit (see e.g. [11]) is negligible in the region where the fit can be compared with existing data. The advantage of using the $\ln^2$ form is that we do not need to concern oruselves about the violation of the Froissart unitarity bound when using our formula at very high energies.

Using Eqs.(8, 6, and 9) we can calculate hadron–nucleon cross sections at any energy of interest from the point of view of EAS physics.

The quality of proposed parametrizations is presented in Figs.1 and 2.

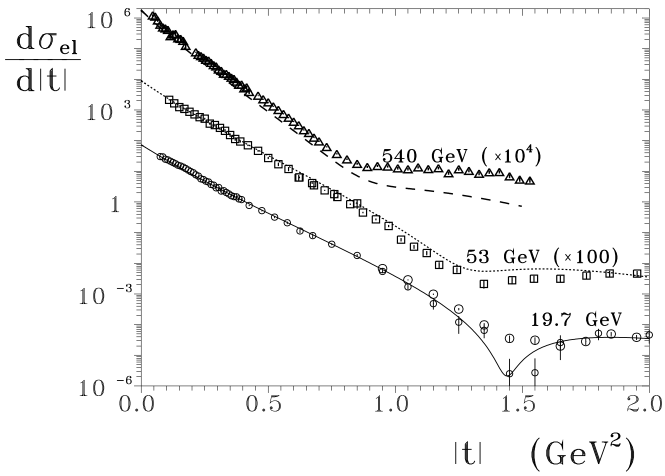

FIG. 1. Differential p–p elastic cross sections obtained using the proposed parametrization of χ for different energies compared with experimental data from FNAL, ISR and SPS [12].

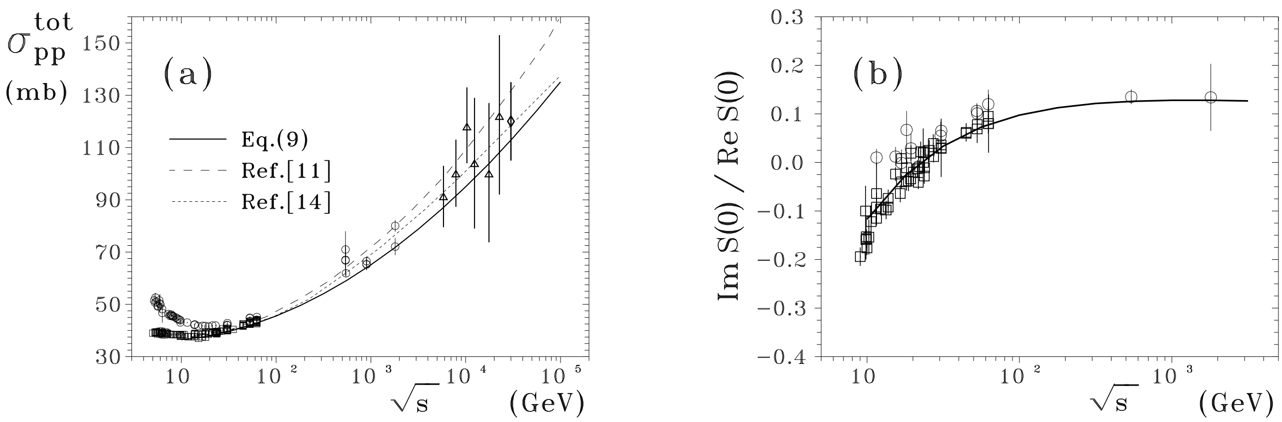

FIG. 2. The energy dependence of p–p total cross sections (a) and imaginary to real part of elastic forward amplitude (b) calculated using the proposed parametrization of λ compared with experimental data from [13]. The broken curve shows Block and Cahn fit [11] and the dotted corve Durand and Pi [14] calculation result.

III. NUCLEON–NUCLEUS CROSS SECTION CALCULATIONS

There are two different approaches to nuclei cross section calculations. Both give similar results, but foundations are quite different. Broadly, it can be said that the Glauber method is based on the superposition of scattering potentials, while the other method (which we shall call hereafter the multiple scattering method) works on probabilities. The multiple scattering approximation is widely presented in [15], mainly for the elastic scattering of different particles on nuclei, but the probabilistic formalism can be used to obtain cross sections of other specific processes.

Of course we do not want to judge which method describes reality better. The comparison with experimental data is possible in principle, however, as will be shown, expected differences between these two approaches are comparable with other effects which has to be known with appropriate precision (e.g. quasi-elastic scattering).

The description given below is performed for the proton–nucleus interactions, but the extension to the nucleus–nucleus case is straightforward.

A. Glauber approximation

For the scattering of particle on the close many-particle system (nucleus), if each interaction is treated as a two-particle one, the overall phase shift for incoming wave is a sum of all the two-particle phase shifts.

$$\chi_A(b,\,\{\mathbf{d}\}) \;=\; \sum_{j=1}^{A} \chi_j(\mathbf{b} \,-\, \mathbf{d}_j) \tag{10}$$

where $\{\mathbf{d}\}$ is a set of nucleon positions in the nucleus ($\mathbf{d}_j$ is a position of the jth nucleon in the plane perpendicular to the interaction axis). Eq. (10) is the essence of the Ref. [4] and in fact defines the Glauber approximation (at least in this paper).

The scattering amplitude is thus given by

$$S(t) \;=\; \frac{i}{2\pi}\int \mathrm{e}^{i\mathbf{tb}} d^2\mathbf{b} \int |\psi(\{\mathbf{d}\})|^2 \,\left\{ 1 \,-\, \mathrm{e}^{i\chi_A(b,\,\{\mathbf{d}\})} \right\} \prod_{j=1}^{A} d^2\mathbf{d}_j \quad , \tag{11}$$

where ψ describes the wavefunction of the nucleus with nucleons distributed according to $\{\mathbf{d}\}$. If one neglect position correlations of the nucleons and denotes by ϱ_j each single nucleon density we have

$$|\psi(\{\mathbf{d}\})|^2 \;=\; \prod_{j=1}^{A} \varrho_j(\mathbf{d}_j) \quad \text{with} \quad \int \varrho_j(\mathbf{r}_j) d^3\mathbf{r} \;=\; 1 \quad . \tag{12}$$

If all interactions can be described by the same phase-shift function χ then

$$\begin{aligned} S(t) = \frac{i}{2\pi}\int \mathrm{e}^{i\mathbf{tb}} d^2\mathbf{b} \int \prod_{j=1}^{A} \varrho_j(\mathbf{d}_j) \left\{ 1 - \mathrm{e}^{i\sum_{j=1}^{A} \chi(\mathbf{b}-\mathbf{d}_j)} \right\} d^2\mathbf{d}_j \; = \\ = \frac{i}{2\pi}\int \mathrm{e}^{i\mathbf{tb}} d^2\mathbf{b} \left\{ 1 - \int \prod_{j=1}^{A} \varrho_j(\mathbf{d}_j) \mathrm{e}^{i\chi(\mathbf{b}-\mathbf{d}_j)} d^2\mathbf{d}_j \right\} \quad . \end{aligned} \tag{13}$$

On the other hand, the scattering process can be treated as the single collision process with its own nuclear phase shift $\chi_{\mathrm{opt}}(b)$

$$S(t) \;=\; \frac{i}{2\pi}\int \mathrm{e}^{i\mathbf{tb}} \left\{ 1 \,-\, \mathrm{e}^{i\chi_{\mathrm{opt}}(b)} \right\} d^2\mathbf{b} \quad . \tag{14}$$

The comparison with Eq.(11) gives

$$\mathrm{e}^{i\chi_{\mathrm{opt}}(b)} \;=\; \int |\psi(\{\mathbf{d}\})|^2 \, \mathrm{e}^{i\sum_{j=1}^{A} \chi_j(\mathbf{b}\,-\,\mathbf{d}_j)} \prod_{j=1}^{A} d^2\mathbf{d}_j \;=\; \left\langle \mathrm{e}^{i\chi(b,\,\{\mathbf{d}\})} \right\rangle \quad , \tag{15}$$

where the $\langle\ \rangle$ means the averaging over all possible configurations of nucleons $\{\mathbf{d}\}$. To go further with the calculations of χ_{opt} a commonly used assumption has to be made. If we assume that the number of scattering centers (A) is large and the transparency of the nucleus as a whole remains constant then

$$\chi_{\mathrm{opt}}(b) \;=\; i \int d^2\mathbf{d}\rho_A(\mathbf{d}) \, \left[1 - \mathrm{e}^{i\chi(\mathbf{b}-\mathbf{d})} \right] \quad . \tag{16}$$

where ρ_A is the distribution of scattering center (nucleon) positions in the nucleus ($\sum \varrho_j$).

When the individual nucleon opacity $|1 - \mathrm{e}^{i\chi(b)}|$ is a very sharply peaked compared with ρ_A then with the help of the optical theorem the simple formula can be found

$$\chi_{\rm opt}(b) \ = \ \frac{1}{2}\,\sigma_{pp}^{\rm tot}\left[\left(\frac{{\rm Re}S(0)}{{\rm Im}S(0)}\right) \ + \ i\right]\rho_A(b) \quad . \tag{17}$$

Substituting (17) into (1) the proton nucleus inelastic cross section is

$$\sigma_{pA}^{\rm inel} \ = \ \int d^2{\bf b}\left[1-{\rm e}^{-\sigma_{pp}^{\rm tot}\rho_A(b)}\right] \ = \ \int d^2{\bf b}\left\{1-\left[1-\sigma_{pp}^{\rm tot}\frac{\rho_A}{A}\right]^A\right\} \tag{18}$$

where the last equality holds in the large A limit [Eq.(18) cannot be used for $A=1$ to compare results for $\sigma_{pp}^{\rm inel}$] This result is often but not quite correctly called "the Glauber approximation". As has been shown, the original Glauber assumption given in Eq.(10) has to be supported by small nucleon sizes and a large value of A.

B. Multiple scattering approach

The $\sigma_{\rm inel}$ given in Eq.(1) can be interpreted in the probabilistic way by identifying the $[1-|{\rm e}^{i\chi({\bf b})}|^2]$ term as the probability of inelastic scattering at impact parameter **b**. This can be extended to the interaction with nucleus in a straightforward way. If we denote this probability by $P({\bf b})$ and nucleons in a nucleus A are distributed according to ρ_A then the averaged probability of inelastic interaction with one of the nucleons is

$$\overline{P}_A({\bf b}) \ = \ \int d^2{\bf d}\ \frac{\rho_A({\bf b})}{A}\ P({\bf b}-{\bf d}). \tag{19}$$

The inelastic cross section with the whole nucleus is then

$$\sigma_{pA}^{\rm inel} \ = \ \int d^2{\bf b}\left\{1-\left[1-\overline{P_A({\bf b})}\right]^A\right\} \quad . \tag{20}$$

In the multiple scattering picture the point-nucleon approximation can be also introduced simplifying the cross section formula. If one puts $P({\bf b}) \ = \ \delta^2({\bf b})\,\sigma_{pp}^{\rm inel}$ then

$$\overline{P}_A({\bf b}) \ = \ \frac{\rho_A({\bf b})}{A}\sigma_{pp}^{\rm inel}, \tag{21}$$

what leads to

$$\sigma_{pA}^{\rm inel} \ = \ \int d^2{\bf b}\left\{1-\left[1-\sigma_{pp}^{\rm inel}\frac{\rho_A({\bf b})}{A}\right]^A\right\} \tag{22}$$

The above equation has a very similar form to Eq.(18) but the difference is also quite clear.
It is interesting to note that Eq.(22) is often called the "Glauber approximation" too (see e.g. [16]).

C. Comparison with the exact Glauber formula

Both approaches discussed above lead to slightly different formulae for an inelastic proton–nucleus cross section. To see how big the difference is we have calculated respective cross sections using exactly the same procedures, nucleus shapes and proton–proton cross sections. It is interesting to compare results for different nucleus masses and incoming proton energies.

The energy dependence in the discussed approximation is introduced only via the proton-proton cross sections change. Respective formulae have been given in section 2

The distributions of the nucleon position in the nuclei of the form given in the FRITIOF interaction model [17] have been used

$$\varrho_j \ \sim \ \begin{cases} \left\{\ 1+\exp\left[\left(r-r_0A^{1/3}\right)/C\right]\ \right\}^{-1} & \text{for} \quad A>16 \\ \left[1+\frac{A-4}{6}\left(\frac{r}{d}\right)^2\right]\exp\left(-\frac{r}{d}^2\right) & \qquad\ \ A\le 16 \end{cases} \tag{23}$$

with the parameters given there, except for $A=4$ where we have used the so-called "parabolic Fermi distribution" [16] and for lighter ($A=2,3$) nuclei, where the simple Gaussian was used. The minimum allowable distance between two nucleons was introduced (0.8 fm) which modify the "initial" (uncorrelated) distribution rather strongly, especially for very light nuclei, so the detail shape of the light nuclei uncorrelated density distribution is not necessary for our purposes.

The validity of the nucleus description used is shown in Fig.3 where the proton–nucleus inelastic cross section data measured at low energies are compared with our calculations. The interaction energy is about the same that it was used as the reference energy s_0 for the proton "hadronic matter density distribution" (ω) estimation, so the energy dependence of all proton–proton cross section parameters are not involved here.

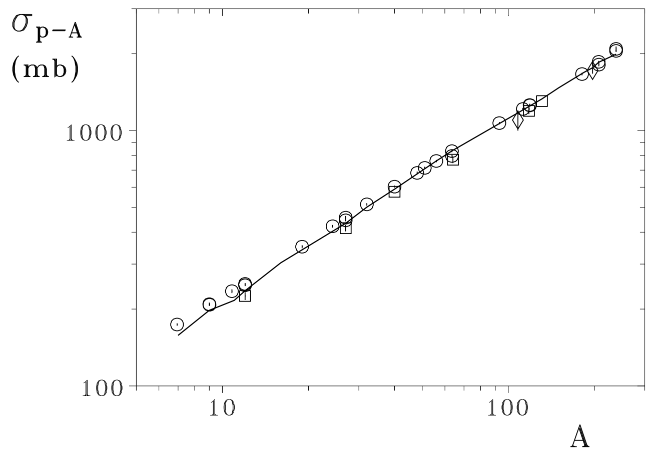

FIG. 3. Inelastic cross sections measured at the laboratory proton momentum of $\sim$ 200 GeV/c compared with our calculation results as a function of target atomic mass. Data from Ref.[18].

The comparison between the simplified Glauber [Eq.(18)] and multiple scattering [Eq.(22)] results is given in Fig. 4 for proton–nucleus as well as for nucleus–nucleus interactions. We have chosen here the "air nucleus" as a target and five different projectiles which represent main components of primary cosmic ray mass spectrum. The difference is not very significant. The energy dependence is similar and the systematic shift is rather constant as maight be expected. It should be remembered that for light nuclei (especially for protons) Eq.(18) looses its physical basis.

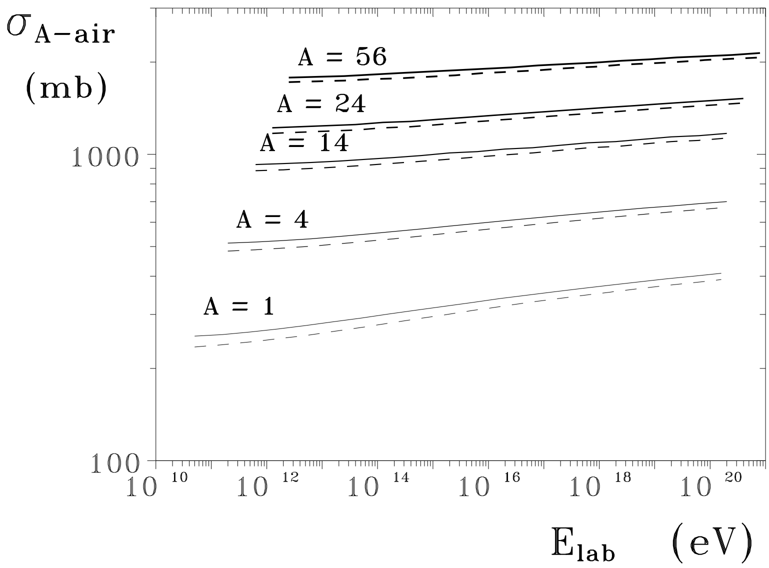

FIG. 4. Cross section of collisions of different nuclei with the "air nucleus" calculated with simplified Glauber (full lines) and multiple scattering (broken lines) approximations for different interaction energies.

The important point of this paper is to show how the point-nucleon approximation changes the results. Both formulae given in Eqs.(22) and (18) are in fact in agreement with the factorization hypothesis for individual nucleon–nucleon χ function [Eq.(4)], according to which nucleons get blacker as the interaction energy increases. Our analysis, presented in section 2, strongly favoured the geometrical scaling which treats nucleons as getting bigger. Nucleus profiles obtained using exact Glauber formula [Eq.(16)] differ from the $A\sigma_{pp}^{\rm inel}\frac{\rho_A}{A}$ suggested by Eq.(22) [$A\sigma_{pp}^{\rm tot}\frac{\rho_A}{A}$ in Eq.(18)]. The difference can be seen in Fig.5.

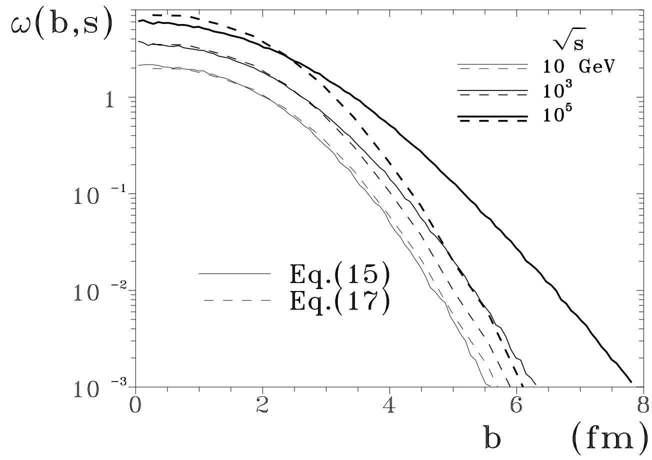

FIG. 5. Nitrogen nucleus profile functions ($\chi_{p-{\rm N}}$) obtained using the exact Glauber formula with geometrical scaling (full curves) and factorization hypothesis (simplified Glauber and multiple scattering) (broken curves) for different interaction energy (per proton–nucleon collision).

A significant change of the nucleus size have to influence the value of inelastic cross section. Fig.6 shows the change in inelastic cross section of proton–nucleus with the interaction energy calculated using geometrically scaled and factorized nucleus profiles.

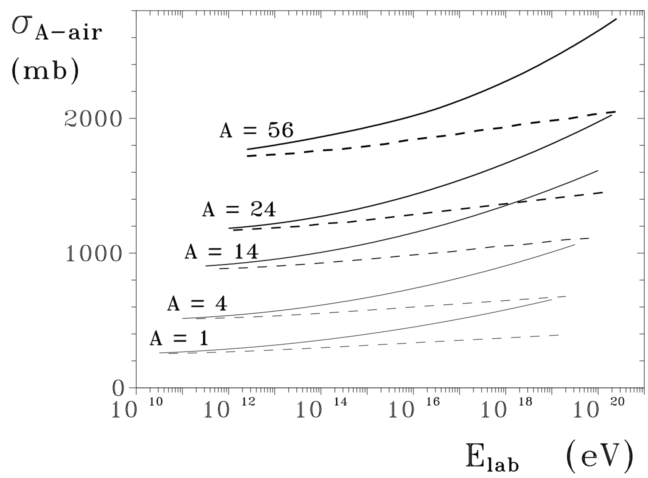

FIG. 6. Cross section of collisions of different nuclei with the "air nucleus" calculated using the exact Glauber formula with geometrical scaling (full curves) and simplified formulas (broken curves) as a function of interaction energy.

As can be seen, the difference at very high energies is remarkable.

IV. PROTON–PROTON CROSS SECTION FROM COSMIC RAY DATA.

Results presented in the previous section indicate the importance of re-examination of the proton–proton cross section estimation based on proton–air data measured in EAS experiment.

The conversion from proton–air to proton–proton cross section presented in Fig.7 is obtained using the exact Glauber formalism.

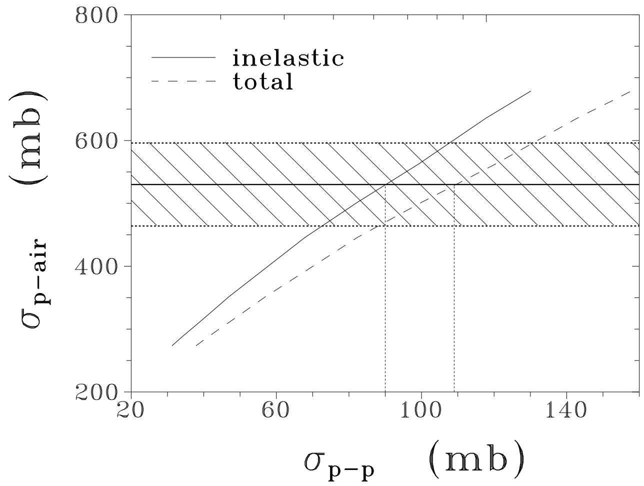

FIG. 7. The relationship between inelastic proton–air cross section and the value of proton–proton cross section (inelastic - full curve and total - broken curve) calculated using exact Glauber formula with geometrical scaling. Full horizontal line represents the value measured in Fly's Eye [3] experiment (dashed area shows 1σ bounds).

The original Fly's Eye estimation of proton–proton total cross section given in [3] is 120 mb, yet according to results given in Fig.7 it is 109 mb. The same procedure was applied to the Akeno data and all their proton–proton cross sections also appear to be about 10% overestimated.

High-energy points plotted in Fig.2(a) which are often reproduced in the literature are taken from the original works. They agree quite well with the two phenomenological descriptions plotted there. The approximation of the cross section rise used in this paper [Eq.(9)] agrees with the Block and Cahn and with the Durand and Pi parametrizations in the low-energy region, but falls below the original EAS proton–proton data points.

In Fig.8 the calculated proton–air cross section energy dependence is given. The full curve represents result obtained using the exact Glauber formula [Eq.(15)] and proton–proton phase shift χ function described in section 2 [with the proton–proton cross section rise defined by Eq.(9)and shown in Fig.2 by the full curve]. The outcome of the "simplified Glauber" approach [Eq.(17)] is given for a comparison by the broken curve.

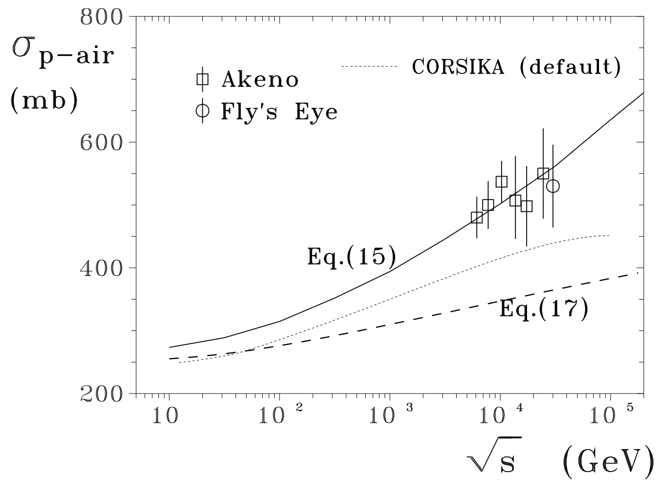

FIG. 8. Inelastic proton–air cross sections calculated using exact Glauber formula with geometrical scaling as a function of interaction energy (per nucleus–nucleus interaction). Experimental points are from Akeno and Fly's Eye experiments (squares and the circle, respectively). Results of calculations with the "simplified Glauber" formula with the same proton–proton cross section energy dependence are given by the broken curve. The default CORSIKA proton–air cross section is shown for a comparison by the dotted curve.

As can be seen, the proposed cross section rise gives very good agreement with EAS measurements at about $\sqrt{s} \sim 10^4$ GeV.

Comparison of our results with outcomes from some of the most popular shower development codes (see e.g. [19] with different models used there) shows that energy dependence of nucleus–air cross sections used there are rather flat, which perhaps suggests that a "simplified Glauber" have been used in cross section calculations (see the dotted curve in Fig.6). The relatively correct, fast, rise of the SIBYLL [20] model proton–air cross section is obtained with "simplified Glauber" formalism, but with a quite extraordinary rise of the total proton–proton cross section at high energies (reaching about 150 mb at $\sqrt{s} \sim 3 \cdot 10^4$ GeV).

V. SUMMARY

We have shown that the geometrical scaling hypothesis with the exact Glauber formalism gives the value of proton–proton total cross section at about 30 TeV slightly (10%) smaller than that reported in original Fly's Eye and Akeno papers. The fit for $\sigma_{pp}^{\rm inel}$ given in Eq.(9) leads to $\sigma_{pp}^{\rm tot}$ shown in Fig.2(a) by the full curve and correctly reproduces σ_{p-air} cross section measured by EAS experiments as it is shown in Fig.8.

The rise of the σ_{A-air} predicted by the geometrical scaling hypothesis with the exact Glauber formalism is significantly faster than that which can be obtained using simplified formulas (Fig.6). This can change the physical conclusions based on Monte Carlo simulations of the EAS development at very high energies.

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[1] M. Buenerd (TOTEM Collaboration), in Frontiers in Strong Interactions, VII Blois Workshop on Elastic and Diffractive Scattering, Chateau de Blois, France, June 1995, edited by P Chiappetta, M. Haguenauer and J. Tran Thanh Van (Editions Frontières), 437 (1996).

[2] M. Honda *et al.*, Phys. Rev. Lett. **70**, 525 (1993).

[3] R. M. Baltrusatis *et al.*, Phys. Rev. Lett. bf 52, 1380 (1984).

[4] R. Glauber, in *Lectures in Theoretical Physics*, edited by A. O. Barut and W. E. Brittin (Interscience, New York, 1956).

[5] R. J. Glauber and G. Matthiae, Nucl. Phys. **B21** 135, (1970).

[6] T. T. Chou and C. N. Yang, Phys. Lett B **244**, 113 (1990).

[7] U. Amaldi and K. R. Schubert, Nucl. Phys. B **166**, 301 (1980).

[8] A. F. Martini and M. J. Menon, Phys. Rev. **D56**, 4338 (1997).

[9] A. J. Buras and J. Dias de Deus, Nucl. Phys. B **71**, 481 (1974).

[10] T. Wibig and D. Sobczyńska, Phys. Rev. **D49**, 2268 (1994).

[11] M. M. Block and R. N. Cahn, Phys. Lett. B **188**, 143 (1987).

[12] C. W. Akerlof *et al.*, Phys. Rev. D **14**, 2864 (1976); R. Rubinstein *et al.*, Phys. Rev. D **30**, 1413 (1984); A. Breakstone *et al.*, Nucl. Phys. B**248**, 253 (1984); A. Breakstone *et al.*, Phys. Rev. Lett. **54**, 2180 (1985); G. Arnison *et al.*, Phys. Lett. **128B**, 336 (1983); M. Bozzo *et al.*, Phys. Lett. **155B**, 197 (1985).

[13] R. M. Barnell *et al.*, Phys. Rev. D**54**, 1 (1996); S. Hadjitheodoridis, Ph.D. thesis, Brown University, Providence, USA (1989).

[14] L. Durand and H. Pi, Phys. Rev. Lett. **58**, 303 (1987).

[15] W. Czyż and L. C. Maximon, Phys. Lett. B **27**, 354 (1968); Ann. Phys. **52**, 59 (1969); A. Białas, M Błeszy nski and W. Czyż, Nucl. Phys. **B111**, 461 (1976); A. Białas, M Błeszy nski and W. Czyż, Acta Phys. Pol. **B8**, 389 (1977); A. Białas and W. Czyż, Acta Phys. Pol. **B12**, 459 (1981);

[16] J. Engel, T. K. Gaisser, P. Lipari and T. Stanev, Phys. Rev. **D46**, 5013 (1992).

[17] B. Andersson, G. Gustafson and H. Pi, Z. Phys. C**50**, 405 (1991).

[18] S. P. Denisov *et al.*, Nucl. Phys. B **61**, 62 (1973); B. M. Bobchenko *et al.*, Sov. J. Nucl. Phys. **30**, 805 (1979); S. Fredriksson, G. Eilam, G. Berlad and L. Bergström, Phys. Rep. **144**, 187 (1987).

[19] D. Heck, J. Knapp, J. N. Capdevielle, G. Schatz and T. Thouw, Forschungszentrum Karlsruhe Report No. FZKA 6019, Karlsruhe, (1998).

[20] J. Engel, T. K. Gaisser, P. Lipari and T Stanev, Phys. Rev. D **46**, 5013 (1992).