SAR Image Filtering Based on the Heavy-Tailed Rayleigh Model

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***Abstract*—Synthetic aperture radar (SAR) images are inherently affected by a signal dependent noise known as speckle, which is due to the radar wave coherence. In this paper, we propose a novel adaptive despeckling filter and derive a *maximum a posteriori* (MAP) estimator for the radar cross section (RCS). We first employ a logarithmic transformation to change the multiplicative speckle into additive noise. We model the RCS using the recently introduced *heavy-tailed Rayleigh* density function, which was derived based on the assumption that the real and imaginary parts of the received complex signal are best described using the alpha-stable family of distribution. We estimate model parameters from noisy observations by means of second-kind statistics theory, which relies on the Mellin transform. Finally, we compare the proposed algorithm with several classical speckle filters applied on actual SAR images. Experimental results show that the homomorphic MAP filter based on the heavy-tailed Rayleigh prior for the RCS is among the best for speckle removal.**

***Index Terms*—α-stable distributions, heavy-tailed Rayleigh model, maximum *a posteriori* (MAP) estimation, Mellin transform, synthetic aperture radar (SAR).**

I. Introduction

OVER the last couple of decades, there has been a growing interest in synthetic aperture radar (SAR) imaging on account of its importance in a variety of applications such as remote sensing for mapping, search-and-rescue, mine detection, and target recognition. Modern airborne and satellite-borne SAR systems are capable of producing high-quality pictures of the earth's surface while avoiding some of the shortcomings of other forms of remote imaging systems [1]. Specifically, SAR imaging systems overcome the night-time limitations of optical cameras and the cloud-cover limitations of infrared imagers.

A major issue in SAR imagery is that basic textures are generally affected by multiplicative speckle noise [2]. Speckle noise is a consequence of image formation under coherent radiation. It is not truly a noise in the typical engineering sense, since it often carries useful information about the scene being imaged. However, the presence of speckle is generally considered undesirable since it damages radiometric resolution and it affects the tasks of human interpretation and scene analysis. Thus, it appears sensible to reduce speckle in SAR images, provided that the structural features and textural information are not lost.

Many adaptive filters for SAR image denoising have been proposed in the past. The simplest approaches to speckle reduction are based on temporal averaging [2], median filtering, and Wiener filtering. The classical Wiener filter, which utilizes the second-order statistics of the Fourier decomposition, is not adequate for removing speckle since it is designed mainly for additive noise suppression. To address the multiplicative nature of speckle noise, Jain developed a homomorphic approach, which, by taking the logarithm of the image, converts the multiplicative into additive noise, and consequently applies the Wiener filter [3]. The Frost filter was designed as an adaptive Wiener filter that assumed an autoregressive (AR) exponential model for the scene reflectivity [4]. Kuan considered a multiplicative speckle model and designed a linear filter based on the minimum mean-square error (MMSE) criterion, optimal when both the scene and the detected intensities are Gaussian distributed [5]. The Lee MMSE filter was a particular case of the Kuan filter based on a linear approximation made for the multiplicative noise model [6]. A two-dimensional (2-D) Kalman filter was developed by Sadjadi and Bannour under the modeling of the image as a Markov field satisfying a causal AR model [7]. The MBD technique [8] is essentially a MAP filter, which was developed based on modeling textured areas of SAR images by a Gaussian Markov random field. For the case of co-registered SAR images, Bruniquel and Lopes [9] developed a speckle reduction method based on pixel-to-pixel summation. The Gamma MAP filter was based on a Bayesian analysis of the image statistics where both radar cross section (RCS) and speckle noise follow a Gamma distribution [10]. Finally, a number of recently developed filters were based on wavelet transform [11], [12].

In this paper, we propose the use of a new RCS model for designing a speckle removal filter. Thus, we employ the heavy-tailed Rayleigh distribution [13] that was shown to be well justified by the physics of the radar wave scattering. Specifically, the model was developed based on the observation that the real and imaginary parts of the received complex signal can be accurately modelled using the symmetric alpha-stable ($S\alpha S$) family of distribution [14], [15]. The generalized Gaussian Rayleigh model [16], [17] arising from the assumption that the real and imaginary parts of the backscattered SAR signal are distributed according to a generalized Gaussian distribution

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could have been an equally appropriate choice. Under the assumption of a multiplicative speckle noise model, we first employ a logarithmic transformation in order to change the noise into an additive one and to differentiate its characteristics from the signal characteristics. Then, the general MAP solution for the resulting model is derived and the model parameter estimation is presented. The proposed estimation method is based on the second-kind statistic theory employing Mellin's transform [18] as recently proposed by Nicolas *et al.* [19], [20].

The paper is organized as follows. In Section II, we discuss the statistical properties of SAR images, as well as those of log-transformed images. In Section III, we present the design of a MAP estimator based on the heavy-tailed Rayleigh signal model, which includes a novel parameter estimation method based on Mellin transform. In Section IV, we evaluate the performance of the proposed filter and we compare it with existing speckle removal methods. Finally, in Section V, we conclude the paper.

II. Statistical Modeling of SAR Images

Parametric Bayesian processing presupposes proper modeling for the prior probability density function (pdf) of both the RCS and speckle noise. In this section, we briefly review the statistical properties of speckle and we describe the model used for the RCS.

A. Statistics of Log-Transformed Speckle

The statistical properties of speckle noise have been first studied by Arsenault and April [21] and by Goodman [2]. Realistic speckle noise models have been developed in the past and include the K-distribution [22], log-normal distribution [12], [23], and correlated speckle pattern [22]. A general model for speckle noise proposed by Jain [3] is constantly employed when one is concerned with the implementation of a homomorphic filter. Specifically, if we denote by $y(u,v)$ a noisy observation (i.e., the recorded SAR image envelope) of the 2-D function $x(u,v)$ (i.e., the noise-free SAR image that has to be recovered) and by $\eta(u,v)$ and $\xi(u,v)$ the corrupting multiplicative speckle noise and some additive noise, respectively, one can write

$$y(u,v) = x(u,v) \cdot \eta(u,v) + \xi(u,v), \quad (u,v) \in \mathbf{Z}^2. \tag{1}$$

Generally, the effect of the additive noise in SAR images is less significant than the effect of speckle noise. Thus, ignoring the term $\xi(u,v)$, one can rewrite (1) as

$$y(u,v) = x(u,v) \cdot \eta(u,v). \tag{2}$$

To transform the multiplicative noise model into an additive one, we apply the logarithmic function on both sides of (2)

$$Y(u,v) = X(u,v) + N(u,v) \tag{3}$$

where $Y(\cdot)$, $X(\cdot)$, and $N(\cdot)$ are the logarithms of $y(\cdot)$, $x(\cdot)$, and $\eta(\cdot)$, respectively. In Sections II-A1 and II-A2, we briefly describe the statistical properties of speckle noise in both the original and the logarithmic transform domain.

1) Intensity Image: Closed-form analytical expressions can be obtained for the logarithmically transformed speckle noise starting from the physics of wave scattering [24]. For a SAR image representing an average of L looks in intensity format, the speckle noise random variable η in (2) follows a Gamma distribution with unit mean and variance $1/L$. Its pdf can be written as

$$p_I(\eta) = \frac{L^L \eta^{L-1} e^{-L\eta}}{\Gamma(L)}. \tag{4}$$

Having in mind that $p(\eta)d\eta = p(N)dN$, one can readily obtain the pdf of the random variable $N = \log \eta$

$$p_I(N) = \frac{L^L e^{NL} e^{-Le^N}}{\Gamma(L)}. \tag{5}$$

2) Amplitude Image: For an amplitude image, the pdf can be obtained from the one corresponding to the intensity image by recalling that [19]

$$p_A(x) = 2xp_I(x^2). \tag{6}$$

Consequently, using the above expression with (4), one obtains

$$p_A(\eta) = \frac{2L^L \eta^{2L-1} e^{-L\eta^2}}{\Gamma(L)} \tag{7}$$

which is basically the Nakagami distribution. Finally, the pdf of the random variable $N = \log(\eta)$ for amplitude images is given by

$$p_A(N) = \frac{2L^L e^{2NL} e^{-Le^{2N}}}{\Gamma(L)}. \tag{8}$$

B. Generalized Rayleigh Model

The SAR image formation theory has been long time dominated by the assumption of Gaussianity for the real and imaginary parts of the received complex signals. Based on this assumption, the detected amplitude SAR images can be modeled by a Rayleigh distribution. However, as we will show in this section, invoking a generalized version of the central limit theorem, the assumption of Gaussianity can be replaced by an assumption of "alpha-stability" resulting in a more powerful model for the detected amplitude pdf. In brief, the appeal of $S\alpha S$ distributions as a statistical model for signals derives from two main theoretical reasons [14]. First, stable random variables satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. Second, stable processes arise as limiting processes of sums of independent identically distributed (i.i.d.) random variables via the generalized central limit theorem. Actually, the *only* possible nontrivial limit of normalized sums of i.i.d. terms is stable.

Kuruoglu and Zerubia [13] assumed that the real and imaginary parts of the received SAR signal are jointly $S\alpha S$. Consequently, they derived the following integral equation for the amplitude pdf of SAR images, which they called the *heavy-tailed Rayleigh distribution*

$$p_A(x) = x \int_0^\infty u \exp(-\gamma u^\alpha) J_0(ux) du \tag{9}$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, γ ($\gamma > 0$) is the *dispersion* of the distribution, and J_0 is the zeroth order Bessel function of the first kind.

It is important to note at this point that, by considering the special case $\alpha = 2$, we obtain

$$p_A(x) = \frac{x}{2\gamma} \exp\left(-\frac{x^2}{4\gamma}\right) \tag{10}$$

which is basically the classical Rayleigh distribution as expected since, for $\alpha = 2$, the $S\alpha S$ distribution reduces to Gaussian. In fact, in [13], the authors have shown that the heavy-tailed Rayleigh distribution can be expressed as a mixture of Rayleighs. Also, by taking $\alpha = 1$ in (9), one obtains the following pdf, which we will refer to as the Cauchy–Rayleigh model

$$p_A(x) = \frac{x\gamma}{(x^2 + \gamma^2)^{3/2}}. \tag{11}$$

In Fig. 1, we show the tail behavior of several heavy-tailed Rayleigh densities, including the particular cases corresponding to the Cauchy and the Gaussian distributions. For the case of images in intensity format, the pdf can be readily obtained from (9) recalling that $p_A(x) = 2xp_I(x^2)$. Thus, one obtains

$$p_I(x) = \frac{1}{2} \int_0^\infty u \exp(-\gamma u^\alpha) J_0(u\sqrt{x}) du. \tag{12}$$

In our further developments, we will employ a homomorphic transformation in order to transform the multiplicative speckle noise into an additive one. Consequently, let us also provide here the logarithmic domain pdfs corresponding to the heavy-tailed Rayleigh model in both amplitude and intensity formats, respectively

$$p_A(X) = e^{2X} \int_0^\infty u \exp(-\gamma u^\alpha) J_0(ue^X) du \tag{13}$$
$$p_I(X) = \frac{e^X}{2} \int_0^\infty u \exp(-\gamma u^\alpha) J_0(ue^{X/2}) du \tag{14}$$

where $X = \ln x$.

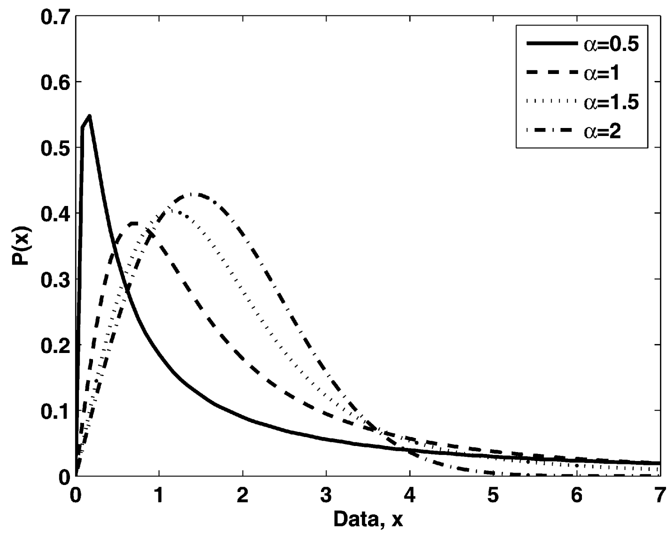

Fig. 1. Generalized Rayleigh probability density functions for different values of the characteristic exponent α. The dispersion parameter is kept constant at $\gamma = 1$.

III. Adaptive MAP Filtering of Speckle Noise

After applying a logarithmic transformation to the original data, we get an image represented as the sum of the transformations of the signal and of the noise

$$Y = X + N. \tag{15}$$

The MAP estimator of X given the noisy observation Y is

$$\hat{X}(Y) = \arg\max_X P_{X|Y}(X|Y). \tag{16}$$

Using Bayes' theorem, this equation can be written as

$$\begin{aligned} \hat{X}(Y) &= \arg\max_X P_{Y|X}(Y|X)P_X(X) \\ &= \arg\max_X P_N(Y - X)P_X(X) \\ &= \arg\max_X P_N(N)P_X(X). \end{aligned} \tag{17}$$

In (17), we use a heavy-tailed Rayleigh model for the signal component, while we use a Gamma or Nakagami model for the noise component depending on whether the image to be filtered is in intensity or in amplitude format, respectively. Naturally, in order for the processor in (17) to be of any practical use, one should be able to estimate the parameters α_X and γ_X of the signal from the observed data. In Section III-A, we derive parameter estimation methods for the generalized Rayleigh pdf based on second-kind cumulants.

A. Parameter Estimation Using Mellin Transform

In order to develop a robust MAP filter for SAR images, the heavy-tailed Rayleigh model parameters characterizing the RCS need to be accurately estimated from noisy observations. Following the arguments in [25] and [26], Nicolas has recently proposed the use of Mellin transform as a powerful tool for deriving novel parameter estimation methods based on log-cumulants for the case of multiplicative noise contamination [19], [20]. In the following, we briefly review the Mellin transform and its main properties that we used in our derivations.

1) Mellin Transform: Let f be a function defined over $\Re^+$. The Mellin transform of f is defined as

$$\Phi(s) = \mathbf{M}[f(u)](s) = \int_0^{+\infty} u^{s-1} f(u) du \tag{18}$$

where s is the complex variable of the transform. The inverse transform is given by

$$f(u) = \mathbf{M}^{-1}[\Phi(s)](u) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} u^{-s} \Phi(s) ds. \tag{19}$$

The transform $\Phi(s)$ exists if the integral $\int_0^{+\infty} |f(x)| x^{k-1} dx$ is bounded for some $k > 0$, in which case the inverse $f(u)$ exists with $c > k$. The functions $\Phi(s)$ and $f(u)$ are called a Mellin transform pair, and either can be computed if the other is known.

By analogy with the way in which common statistics are deducted based on Fourier transform, the following second-kind statistic functions can be defined, based on Mellin transform [19], [20].

- Second-kind first characteristic function

$$\Phi(s) = \int_0^{+\infty} x^{s-1} p(x) dx. \tag{20}$$

- Second-kind second characteristic function

$$\Psi(s) = \log(\Phi(s)). \tag{21}$$

- rth order second-kind cumulants

$$\tilde{k}_r = \frac{d^r \Psi(s)}{ds^r}\Big|_{s=1}. \tag{22}$$

The first two second-kind cumulants can be estimated empirically from N samples y_i, as follows:

$$\begin{aligned} \hat{\tilde{k}}_1 &= \frac{1}{N} \sum_{i=1}^{N} [\log(y_i)] \\ \hat{\tilde{k}}_2 &= \frac{1}{N} \sum_{i=1}^{N} \left[\left(\log(y_i) - \hat{\tilde{k}}_1 \right)^2 \right]. \end{aligned} \tag{23}$$

- Finally, for two functions f and g, Mellin's convolution is defined over the interval $[0, \infty]$ as

$$(f \hat{*} g)(y) = \int_0^{+\infty} f(x) g\left(\frac{y}{x}\right) \frac{dx}{x} = \int_0^{+\infty} f\left(\frac{y}{x}\right) g(x) \frac{dx}{x}. \tag{24}$$

2) Log-Moment Estimation of the Generalized Rayleigh Model: By plugging the expression of the heavy-tailed Rayleigh pdf given by (9) into (20) and after some straightforward manipulations, details of which can be found in [13], one gets

$$\Phi(s) = \frac{2^s \Gamma\left(\frac{s+1}{2}\right) \gamma^{((s-1)/\alpha)} \Gamma\left(\frac{1-s}{\alpha}\right)}{\Gamma\left(\frac{1-s}{2}\right) \alpha} \tag{25}$$

which is the second-kind first characteristic function of the heavy-tailed Rayleigh density. Kuruoglu and Zerubia [13] used this expression for two different values of s and subsequently solved the resulting system in order to get estimates of the parameters α and γ. However, here, we are interested in deriving estimates of the model parameters in the case of multiplicative noise contamination. Consequently, we settled by plugging the above expression in (21) and subsequently in (22), thus obtaining the following results for the second-kind cumulants of the model:

$$\begin{aligned} \tilde{k}_{A(1)} &= -\psi(1) \frac{1-\alpha}{\alpha} + \log(2\gamma^{(1/\alpha)}) \\ \tilde{k}_{A(2)} &= \frac{\psi(1,1)}{\alpha^2} \end{aligned} \tag{26}$$

where ψ is the Digamma function and $\psi(r, \cdot)$ is the Polygamma function, i.e., the rth derivative of the Digamma function. Using the above systems of equations, one can readily solve for the parameters α and γ of the heavy-tailed Rayleigh distribution. Remember, however, that our measurement is a mixture of heavy-tailed Rayleigh signal and Nakagami (or Gamma) distributed speckle noise. The first and second orders log-cumulants of a Nakagami distribution are

$$\begin{aligned} \tilde{k}_{A(1)} &= \frac{1}{2}(\psi(L) - \log(L)) \\ \tilde{k}_{A(2)} &= \frac{1}{4}\psi(1, L) \end{aligned} \tag{27}$$

while those corresponding to a Gamma distribution are given by

$$\begin{aligned} \tilde{k}_{I(1)} &= \psi(L) - \log(L) \\ \tilde{k}_{I(2)} &= \psi(1, L). \end{aligned} \tag{28}$$

Under the multiplicative speckle noise model (2), if we denote by $p_y(y)$, $p_x(x)$, and $p_\eta(\eta)$, the pdfs of y, x, η, respectively, it

TABLE I
IMAGE ENHANCEMENT MEASURES OBTAINED BY FIVE DENOISING METHODS APPLIED ON THE "AERIAL" IMAGE. FOUR LEVELS OF NOISE ARE CONSIDERED CORRESPONDING TO $L=1$, 3, 9, AND 12. THE S/MSE OF EACH DESPECKLED IMAGE IS GIVEN IN DECIBELS

	L = 1		L = 3		L = 9		L = 12	
Method	S/MSE	β	S/MSE	β	S/MSE	β	S/MSE	β
Median	14.53	0.2421	15.52	0.3309	15.85	0.3639	15.88	0.3677
Wiener	14.23	0.3187	15.27	0.3812	15.56	0.4097	15.60	0.4150
Lee	14.69	0.2118	15.35	0.3540	16.64	0.5417	17.28	0.6186
ΓMAP	14.68	0.2512	15.36	0.3525	16.74	0.5246	17.45	0.6022
proposed	15.86	0.2556	16.83	0.3541	18.00	0.5833	18.68	0.6471

Fig. 2. Results of various despeckling methods. (a) Original aerial image. (b) Image degraded with simulated speckle noise ($L = 3$, amplitude format). (c) Lee filter. (d) Proposed MAP filter based on the heavy-tailed Rayleigh model.

can be shown that the pdf of y is given in fact by the Mellin convolution between the pdfs of x and η, since

$$\begin{aligned} p_y(y) &= \int_0^{+\infty} p_{y|x}(y|x)p_x(x)dx \\ &= \int_0^{+\infty} p_\eta\left(\frac{y}{x}\right)p_x(x)\frac{dx}{x} = p_\eta \hat{*} p_x. \end{aligned} \tag{29}$$

Consequently, the second-kind cumulant of any order of y is given by the sum of the second-kind cumulants of the same order of x and η [19]

$$\tilde{k}_{y(r)} = \tilde{k}_{x(r)} + \tilde{k}_{\eta(r)}. \tag{30}$$

Using expressions (26) and (27) in the above equation together with the empirical log-cumulants in (23), we obtain the following estimates for the parameters of the heavy-tailed Rayleigh model mixed with Nakagami distributed speckle noise

$$\begin{aligned} \hat{\alpha} &= \sqrt{\frac{\psi(1,1)}{\hat{\tilde{k}}_{(2)} - \frac{1}{4}\cdot\psi(1,L)}} \\ \hat{\gamma} &= \left[\frac{\exp\left(\hat{\tilde{k}}_{(1)} + \psi(1)\frac{1-\alpha}{\alpha} - \frac{1}{2}(\psi(L) - \log(L))\right)}{2}\right]^{\alpha}. \end{aligned} \tag{31}$$

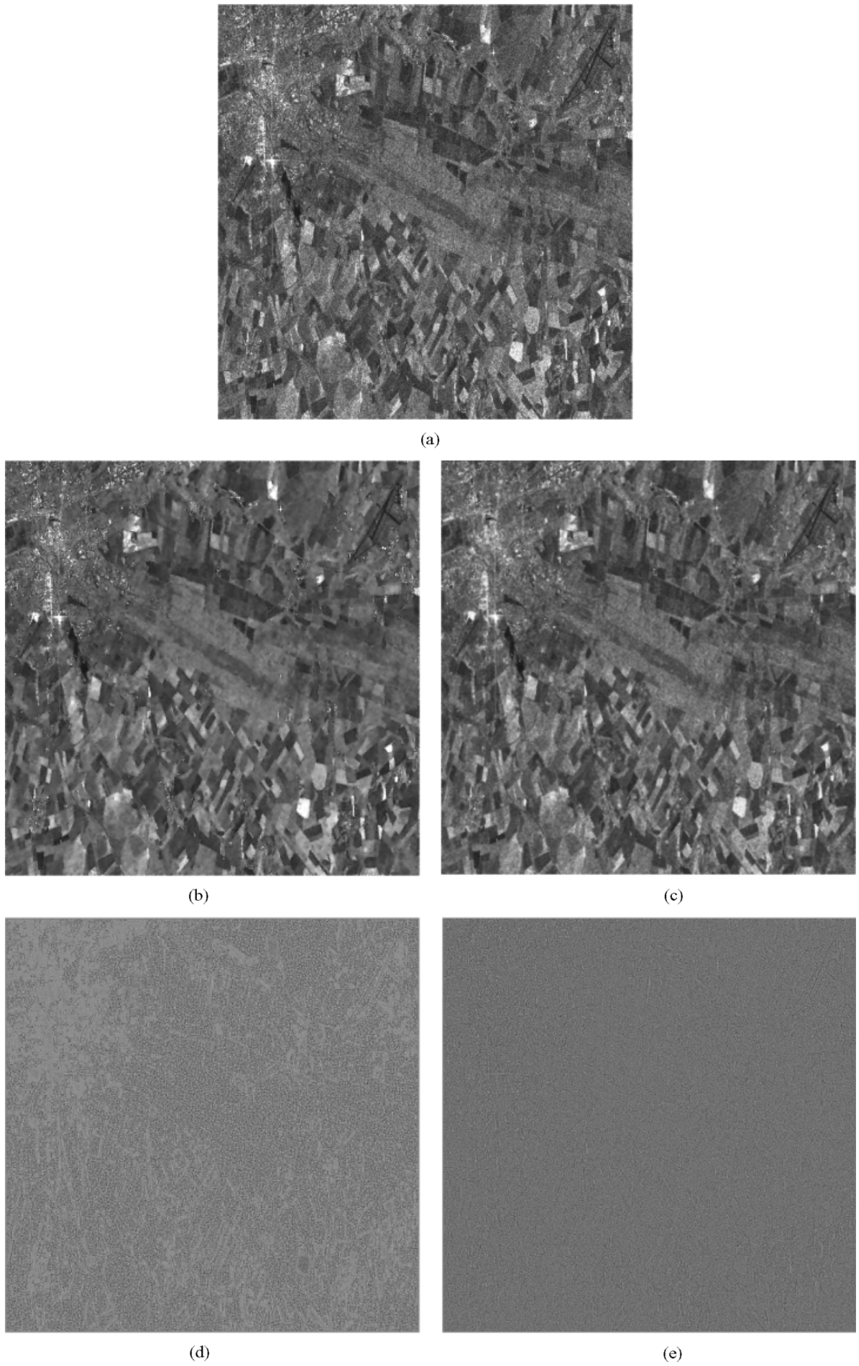

Fig. 3. (a) Original 4-look "Bourges" image. (b) $\Gamma\mathrm{MAP}$ filtered. (c) Proposed filter. (d) Ratio image after $\Gamma\mathrm{MAP}$ filtering. (e) Ratio image after applying the proposed filter.

For the case of intensity images, estimates of α and γ for the model in (12) can be easily obtained using the same methodology as above.

IV. Experimental Results

In this section, we present simulation results obtained by processing several test SAR images using the proposed MAP speckle filter based on the heavy-tailed Rayleigh prior. We compared the results of our approach with those obtained using other classical speckle filters including the median, the homomorphic Wiener, the Lee, and the Γ-MAP filter.

A. Synthetic Data Examples

We started by first degrading an original "speckleless" image with synthetic speckle in amplitude format. For this purpose, an aerial image was chosen due to its identical content with real SAR images. This image was obtained by cropping the *"westaerialconcorde"* image that can be found in Matlab's Image Processing Toolbox. In the experiments, we considered four different levels of simulated speckle noise corresponding to $L = 1$, 3, 9, and 12 [cf. (7)].

In order to assess the quality of the proposed filter, we computed two different measures based on the original and the denoised data. A common way to evaluate the noise suppression in case of multiplicative contamination is to calculate the signal-to-mean squared error (S/MSE) ratio, defined as [23]

$$S/\mathrm{MSE} = 10\log_{10}\left(\frac{\sum_{i=1}^{K} S_i^2}{\sum_{i=1}^{K}(\hat{S}_i - S_i)^2}\right) \tag{32}$$

where S is the original image, $\hat{S}$ is the denoised image, and K is the image size. This measure corresponds to the classical SNR in the case of additive noise.

In addition to the above quantitative performance measure, we also consider a qualitative measure for edge preservation in the form of a parameter β, originally defined in [27]. More exactly, β represents a correlation measure, which should be close to unity for an optimal effect of edge preservation. For this experiment, we chose to compare the proposed filter with the median, the homomorphic Wiener, Lee, and $\Gamma\mathrm{MAP}$ filters. All these filters were implemented adaptively using a square shaped sliding window of size 7×7. The obtained values of S/MSE, and β for all methods applied to our test image are given in Table I. From the table, it can be seen that, in most situations, the proposed filter exhibits the best performance according to both metrics. Fig. 2 shows a representative result from the processing of the aerial test image. The image in Fig. 2(b) was obtained by degrading the original test Fig. 2(a) with Nakagami distributed speckle noise [cf. (7)] with $L = 3$ looks. From the figure, it can be seen that all the tested filters achieved a good speckle surpressing performance. However, clearly, the homomorphic MAP filter based on the heavy-tailed Rayleigh signal prior did the best job in preserving the structural features that can be observed in the original image.

B. Real SAR Imagery Examples

In order to further study the merit of the proposed generalized Rayleigh-based MAP processor, we also chose noisy SAR images, we applied the algorithm without artificially adding noise, and we visually evaluated the denoised images. The test image, shown in Fig. 3(a), depicts mainly a rural scene, but one can also see the city of Bourges and some other villages, forests, an airport, a river, and many roads. The image is in 4-looks amplitude format and was acquired in April 1993 by ERS.

For visually assessing the quality of filtered images, we show results obtained using the $\Gamma\mathrm{MAP}$ filter [Fig. 3(b)] and the generalized Rayleigh-based MAP [Fig. 3(c)]. Although qualitative evaluation in this case is highly subjective, i.e., no universal quality measure for filtered SAR data exists, we also chose to study the ratio image [8], which is the ratio of the original image (speckled) by the denoised image. Ideally, in areas of the image where speckle is fully developed, this ratio should have the characteristics of pure speckle. Thus, the best filter is the one for which the ratio image has the mean value closer to unity and the equivalent number of looks (ENL) closer to 4. The results of this experiment are shown in Table II together with the results obtained after processing the original image by means of the median, homomorphic Wiener, $\Gamma\mathrm{MAP}$, Lee, and MBD filters.

TABLE II
QUANTITATIVE FILTER EVALUATION FOR "BOURGES" TEST IMAGE

Filter	Median	Wiener	ΓMAP	Lee	MBD	proposed
Mean	1.03	1.03	1.05	0.98	0.98	0.99
ENL	2.27	3.70	7.02	3.54	4.23	3.83

The results of the above experiment seem to be consistent with the simulation results in the previous section, the best performance being again achieved by the proposed algorithm. Altough conceptually similar with the $\Gamma\mathrm{MAP}$ filter, we attribute the better performance of the proposed filter to the better performance of the generalized Rayleigh model in capturing the heavy-tailed nature of SAR data.

V. CONCLUSION

We presented a novel homomorphic statistical filter for speckle noise removal in SAR images, which is based on the recently introduced heavy-tailed Rayleigh model for the amplitude of the RCS. We have applied the developed filter to a number of simulated, as well as real speckle images and we compared the results with those obtained by means of classical speckle filters. Simulation results showed that the homomorphic MAP filter based on the heavy-tailed Rayleigh model is among the best for speckle removal.

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