Q11. Distinguish between a) Probability distribution and sampling distribution b) Estimation and testing c) Critical value and p-value.

Solution:

a) Probability Distribution vs. Sampling Distribution:

- Probability Distribution:

  - Definition: It describes the likelihood of obtaining the possible values that a random variable can take.

  - Characteristics: It is fixed and associated with a population.

  - Example: In a fair six-sided die, the probability distribution of outcomes is {1/6, 1/6, 1/6, 1/6, 1/6, 1/6}.

- Sampling Distribution:

  - Definition: It describes the distribution of a statistic (e.g., mean or standard deviation) computed from multiple samples of the same size drawn from the same population.

  - Characteristics: It varies depending on the samples taken but centers around the true population parameter.

  - Example: If you repeatedly draw samples of size n from a population and calculate the sample mean each time, the distribution of those sample means is a sampling distribution.

b) Estimation vs. Testing:

- Estimation:

  - Objective: To estimate a population parameter (e.g., mean, proportion) based on a sample.

  - Methods:Point estimation involves providing a single value as an estimate, while interval estimation gives a range of values (confidence interval) within which the true parameter is likely to fall.

  - Example: Estimating the average height of students in a school based on a sample.

- Testing:

- Objective: To assess a hypothesis about a population parameter.

- Methods: Hypothesis testing involves setting up a hypothesis, collecting data, and assessing whether the data provide enough evidence to reject or fail to reject the null hypothesis.

- Example: Testing whether a new drug has a significant effect compared to a placebo.

c) Critical Value vs. \(p\)-value:

- Critical Value:

- Definition: A value that separates the region of acceptance from the region of rejection in a hypothesis test.

- Usage: If the test statistic falls in the rejection region (beyond the critical value), you reject the null hypothesis.

- Example: In a Z-test with a significance level of 0.05, the critical value might be ±1.96 for a two-tailed test.

- \(p\)-value:

- Definition: The probability of observing a test statistic as extreme as, or more extreme than, the one observed, assuming the null hypothesis is true.

- Usage: If the \(p\)-value is less than the chosen significance level (e.g., 0.05), you reject the null hypothesis.

- Example: If the \(p\)-value is 0.03, you would reject the null hypothesis at the 0.05 significance level.

Q11. Distinguish between (a) Two sample and paired data situations (b) Parameter and statistic (c) Critical value and p-value.

(a) Two-sample and paired data situations:

- Two-sample data:

- In a two-sample situation, you are comparing the means, variances, or other characteristics of two independent groups.

- Each group is treated as a separate entity, and the measurements in one group are not related to the measurements in the other group.

- Example: Comparing the average test scores of students from two different schools.

- Paired data:

- In a paired data situation, you are dealing with pairs of observations that are somehow related or matched.

- Each pair typically represents measurements taken on the same subject or under similar conditions.

- Example: Comparing the blood pressure of individuals before and after a treatment; each individual serves as their own pair.

(b) Parameter and statistic:

- Parameter:

- A parameter is a numerical summary of a population characteristic.

- It is a fixed value, often unknown, that describes a certain aspect of the population.

- Example: The population mean, population standard deviation.

- Statistic:

- A statistic is a numerical summary of a sample characteristic.

- It is a computed value based on the data collected from a sample and is used to estimate the corresponding population parameter.

- Example: The sample mean, sample standard deviation.

(c) Critical value and p-value:

- Critical value:

- A critical value is a point on the scale of a test statistic beyond which we reject the null hypothesis.

- It is compared to the test statistic to determine whether to reject the null hypothesis in hypothesis testing.

- Example: In a Z-test, the critical values might be the Z-scores corresponding to a 5% level of significance.

- P-value:

- The p-value is the probability of obtaining a test statistic as extreme as, or more extreme than, the one observed in the sample data, assuming that the null hypothesis is true.

- A smaller p-value indicates stronger evidence against the null hypothesis.

- Example: If the p-value is 0.03, it means there is a 3% chance of observing the data (or more extreme) if the null hypothesis is true. If this is smaller than the significance level (e.g., 0.05), you might reject the null hypothesis.