CHAPTER 2

The Challenges of Asset Pricing: A Road Map

Chapter Outline

2.1 The Main Question of Financial Theory

Valuing risky cash flows or, equivalently, pricing risky assets is at the heart of financial theory.

Our discussion thus far has been conducted from the perspective of society as a whole, and it argues that a progressively more complete set of financial markets will generally enhance societal welfare by making it easier for economic agents to transfer income across future dates and states via the sale or purchase of individually tailored portfolios of securities. The desire of agents to construct such portfolios will depend as much upon the market prices of the relevant securities as on their strict availability, and this leads us to the main topic of the text.

Indeed, the major practical question in finance is, "How do we value a risky cash flow?" and the main objective of this text is to provide a complete and up-to-date treatment of how it can be answered. For the most part, this book is thus a text on asset pricing. Indeed, an asset is nothing else than the right to future cash flows, whether these future cash flows are the result of interest payments, dividend payments, insurance payments, or the resale value of the asset. Furthermore, when we compute a project's risk-adjusted present value (PV), we are, in effect, asking the question: If this project's cash flow were traded as though it were a security, at what price would it sell given that it should pay the prevailing rate on other securities with the same risk level? We compare its fair market value, estimated in this way, with its cost, P_0. Evaluating a project is thus a special case of evaluating a security.

Viewed in this way and abstracting from risk for the moment, the key object of our attention, be it an asset or an investment project, can be summarized as in Table 2.1.

In Table 2.1, $t = 0,1,2,\ldots\tau,\ldots,T$ represents future dates. The duration of each period, the length of time between $\tau - 1$ and τ, is arbitrary and can be viewed as 1 day, 1 month, 1 quarter, or 1 year. The expression $\tilde{C}F_\tau$ stands for the possibly uncertain cash flows in period τ (whenever useful, we will identify random variables with a tilde), r_τ^f is the risk-free, per-period interest rate prevailing between date 0 and τ, and P_0 denotes the to-be-determined current price or valuation of the future cash flow. If the future cash flows will be available for sure, valuing the flow of future payments is easy. It requires adding the future cash flows after discounting them by the risk-free rate of interest, i.e., adding the cells in the last line of the table. The discounting procedure is indeed at the heart of our problem: it clearly serves to translate future payments into current dollars (those that are to be used to purchase the right to these future cash flows or in terms of which the current value of the future cash flow is to be expressed). In other words, the discounting procedure is what makes it possible to compare future dollars (i.e., dollars that will be available in the future) with current dollars.

If, however, the future cash flows will not be available for certain but are subject to random events—the interest payments depend on the debtor remaining solvent, the dividend payments depend on the financial strength of the equity issuer, the returns to the investment project depend on its commercial success—then the valuation question becomes trickier, so much so that there does not exist a universal way of proceeding that dominates all others.

In the same way that one dollar for sure tomorrow does not generally have the same value as one current dollar, one dollar tomorrow under a set of more or less narrowly defined

Table 2.1: Valuing a risk-free cash flow

$t=0$	$t=1$	$t=2$	$\ldots\tau\ldots$	$t=T$
P_0**?**	$\tilde{C}F_1$	$\tilde{C}F_2$	$\tilde{C}F_\tau$	$\tilde{C}F_T$
	$\frac{CF_1}{(1+r_1^f)}$	$\frac{CF_2}{(1+r_2^f)^2}$	$\frac{CF_\tau}{(1+r_\tau^f)^\tau}$	$\frac{CF_T}{(1+r_T^f)^T}$

circumstances, i.e., in a subset of all possible states of nature, is also not worth even one current dollar discounted at the risk-free rate. Assume the risk-free rate of return is 5% per year, then discounting one dollar available in 1 year at the risk-free rate yields $(1\$/1.05) \cong \$0.95$. This equality is exactly that: it states that \$1 tomorrow will have a market price of \$0.95 today when 1-year risk-free securities earn 5%. It is a market assessment to the extent that the 5% risk-free rate is an equilibrium market rate. Now if \$1 for sure tomorrow is worth \$0.95, it seems likely that \$1 tomorrow "possibly," i.e., in a restricted subset of the states of nature, should certainly be worth less than \$0.95. One can speculate, for instance, that if the probability of \$1 in a year is about $\frac{1}{2}$, then one should not be willing to pay more than $\frac{1}{2} \times \$0.95$ for that future cash flow. But we have to be more precise than this. To that end, several lines of attack will be pursued. Let us outline them.

2.2 Discounting Risky Cash Flows: Various Lines of Attack

First, as in the certainty case, it is plausible to argue (and it can be formally demonstrated) that the valuation process is additive: the value of a sum of future cash flows will take the form of the sum of the values of each of these future cash flows. Second, as already anticipated, we will work with probabilities, so that the random cash flow occurring at a future date τ will be represented by a random variable: $\tilde{C}F_\tau$, for which a natural reference value is its expectation $E\tilde{C}F_\tau$. Another reference value would be this expected future cash flow discounted at the risk-free rate: $E\tilde{C}F_\tau/((1+r_\tau^f)^\tau)$. Now the latter expression cannot generally be the solution to our problem, although it is intuitively understandable that it will be when the risk issue does not matter—i.e., when market participants can be assumed to be risk neutral. In the general case where risk must be taken into account, which typically means that risk-bearing behavior needs to be remunerated, alterations to that reference formula are necessary. These alterations may take any of the following forms:

1. The most common strategy consists of discounting at a rate that is higher than the risk-free rate, i.e., to discount at a rate that is the risk-free rate increased by a certain amount π (a risk premium) as in

$$\frac{E\tilde{C}F_\tau}{(1+r_\tau^f+\pi_\tau)^\tau}$$

   The underlying logic is straightforward: To price an asset equal to the present value of its expected future cash flows discounted at a particular rate is to price the asset in a manner such that, at its present value price, it is expected to earn that discount rate. The appropriate rate, in turn, must be the analyst's estimate of the expected rate of return on other financial assets that represent title to cash flows similar in risk and timing to that of the asset in question. This strategy has the consequence of pricing the asset to pay the prevailing competitive rate for its risk class. When we follow this approach, the key issue is to compute the appropriate risk premium.[1]

2. Another approach, in the same spirit, consists of correcting the expected cash flow itself in such a way that one can continue discounting at the risk-free rate. The standard way of doing this is to decrease the expected future cash flow by a factor Π that once again will reflect some form of risk or insurance premium as in

$$\frac{E\tilde{C}F_\tau - \Pi_\tau}{(1+r^f_\tau)^\tau}$$

3. The same idea can take the form, it turns out quite fruitfully, of distorting the probability distribution over which the expectations operator is applied so that taking the expected cash flow with this modified probability distribution justifies once again discounting at the risk-free rate:

$$\frac{\hat{E}\tilde{C}F_\tau}{(1+r^f_\tau)^\tau}$$

Here $\hat{E}$ denotes the expectation taken with respect to the modified probability distribution.

4. Finally, one can think of decomposing the future cash flow $\tilde{C}F_\tau$ into its state-by-state elements. Denote $(CF(\theta_\tau))$ the actual payment that will occur in the specific possible state of nature θ_τ. If one is able to find the price today of \$1 in period τ conditional on that particular state θ_τ being realized, say $q(\theta_\tau)$, then surely the appropriate current valuation of $\tilde{C}F_\tau$ is

$$\sum_{\theta_\tau \in \Theta_\tau} q(\theta_\tau) CF(\theta_\tau)$$

where the summation takes place over all the possible future states θ_τ. The quantity $q(\theta_t)$ is often referred to as a "state price," and in important applications it will resemble the more traditional discount factor.

The procedures described above are alternative ways of attacking the difficult valuation problem we have outlined, but they can only be given content in conjunction with theories explaining how to compute the risk premia (cases 1 or 2), to identify the distorted

[1] Let us be sure we understand exactly what this expression says: the r^f_τ denotes the period by period rate of return on a default-free security which pays an amount of money τ periods in the future while π_τ is the return risk premium expected to prevail over this same time horizon. We thus discount each future cash flow "individually." In a typical calculation where there are cash flows in many future periods, we frequently assume $r^f_\tau = r_f$ and $\pi_\tau = \pi$ for all τ; i.e., the risk-free rate and risk premium are constant looking forward. All future cash flows thus end up being discounted at the same rate.

probability distribution (case 3) or to price future dollars state by state (case 4). For strategies 1 and 2, this can be done using the capital asset pricing model (CAPM), the consumption capital asset pricing model (CCAPM), or the arbitrage pricing theory (APT); strategy 3 is characteristic of the Martingale approach; strategy 4 describes the perspective of Arrow–Debreu (AD) pricing.

2.3 Two Main Perspectives: Equilibrium versus Arbitrage

There is another, even more fundamental way of classifying alternative valuation theories. All the known valuation theories cited above employ one of two main methodologies: the equilibrium approach or the arbitrage approach.

The traditional equilibrium approach consists of an analysis of the factors determining the supply and demand for the cash flow (asset) in question. The arbitrage approach attempts to value a cash flow on the basis of observations made on the values of the various elements making up that cash flow.

Let us illustrate this distinction with an analogy. You are interested in pricing a bicycle. There are two ways to approach the question. If you follow the equilibrium approach, you will want to study the determinants of supply and demand. Who are the producers? How many bicycles are they able to produce? What are the substitutes, including probably the existing stock of old bicycles potentially appearing on the second-hand market? After dealing with supply, turn to demand: Who are the buyers? What are the forecasts of the demand for bicycles? And so on. Finally, you will turn to the market structure. Is the market for bicycles competitive? If so, we know how the equilibrium price will emerge as a result of the matching process between demanders and suppliers. The equilibrium perspective is a sophisticated, complex approach, with a long tradition in economics, one that has also been applied in finance, at least since the 1950s. We will follow it in the first part of this book, adopting standard assumptions that simplify, without undue cost, the supply and demand analysis for financial objects: the supply of financial assets at any point in time is assumed to be fixed, and financial markets are viewed as competitive. Our analysis can thus focus on the determinants of the demand for financial assets.

This requires that we first spend some time discussing the preferences and attitudes toward risk of investors, those who demand the assets (Chapters 3 and 4), before modeling the investment process, i.e., how the relative demands for the various financial assets are determined (Chapters 5–7). Armed with these tools, we will review the three main equilibrium theories, the CAPM in Chapter 8, AD pricing in Chapter 9, and the CCAPM in Chapter 10.[2]

[2] In Web Chapter A we generalize the CCAPM by making more explicit the macroeconomic setting that underlies the CCAPM. An understanding of these macrofinancial linkages has become more important given the recent financial crisis and the advent of the "Great Recession."

The arbitrage approach to valuing bicycles starts from observing that a bicycle is not (much) more than the sum of its parts. Accordingly, if you know the price of all the necessary components—frame, handlebar, wheel, tire, saddle, brake, and gearshift—you can determine relatively easily the market value of the bicycle. The knowledge of how to assemble the bicycle and the time required to do so, however, are not in infinite supply. These considerations suggest that the arbitrage approach may hold only as an approximation, one that may be rather imprecise in circumstances where the time and intellectual ability required to "assemble the bicycle" from the necessary spare parts are nontrivial; i.e., when the remuneration of the necessary "engineers" matters.[3]

The arbitrage approach is, in a sense, much more straightforward than the equilibrium approach. It is also more robust: if the arbitrage relationship between the price of the bicycle and the price of its parts does not hold, anyone with a little time could become a bicycle manufacturer and make good money. If too many people exploit that idea, however, the prices of parts and the prices of bicycles will start adjusting and be forced into line. This very idea is especially powerful for the object at hand, financial assets, because if markets are complete in the sense discussed in Section 1.6, then it can easily be shown that *all* the component prices necessary to value *any* arbitrary cash flow are available. Furthermore, little time and few resources (relative to the global scale of product markets) are needed to exploit arbitrage opportunities in financial markets.

There is, however, an obvious limitation to the arbitrage approach. Where do we get the price of the parts if not through an equilibrium approach? That is, the arbitrage approach is much less ambitious and more partial than the equilibrium approach. Even though it may be more practically useful in the domains where the price of the parts is readily available, it does not make up for a general theory of valuation and, in that sense, has to be viewed as a complement to the equilibrium approach. In addition, the equilibrium approach, by forcing us to rationalize investors' demand for financial assets, provides useful lessons for the practice of asset management. The foundations of this inquiry will be put in place in Chapters 3−7—which together make up Part II of the book—while Chapter 16 will extend the treatment of this topic beyond the case of the traditional one-period static portfolio analysis and focus on the specificities of long-run portfolio management.

Finally, the arbitrage and equilibrium approaches can be combined. In particular, one fundamental insight that we will develop in Chapter 11 is that any cash flow can be viewed

[3] In a similar vein, "financial engineers" seek to create new securities by cleverly packaging existing ones, or seek to design arbitrage portfolios which increase in value as the relative prices of their constituent securities (by analogy, the bicycle, and its independently traded constituent parts) come into better alignment. While a pure arbitrage portfolio is strictly risk free, most arbitrage portfolios arising from financial engineering will have positive payoffs provided certain low probability ("Black Swan") events do not occur. At the start of the financial crisis, it was these very low probability events that came to pass with disastrous consequences for the institutions themselves (e.g., AIG).

Table 2.2: The road map

	Equilibrium	Arbitrage
Preliminaries	Utility theory—Chapters 3–4 Investment demand—Chapters 5–7	
Computing risk premia	CAPM—Chapter 8 CCAPM—Chapter 10	APT—Chapter 14
Identifying distorted probabilities		Martingale measure—Chapters 12–13
Pricing future dollars state by state	AD pricing I—Chapter 9	AD pricing II—Chapter 11

as a portfolio of AD securities, i.e., it can be replicated with AD securities. This makes it very useful to start using the arbitrage approach with AD securities as the main building blocks for pricing assets or valuing cash flows. Conversely, the same chapter will show that options can be very useful in completing the markets and thus in obtaining a full set of prices for "the parts that will then be available to price the bicycles." In other words, the AD equilibrium pricing theory is a good platform for arbitrage valuation. The link between the two approaches is indeed so tight that we will use our acquired knowledge of equilibrium models—reviewed in Part III—to understand one of the major arbitrage approaches, the Martingale pricing theory (Chapters 12 and 13).[4] We will then propose an overview of the APT in Chapter 14. Chapters 11 through 15 together make up Part IV of this book. This outline is summarized in Table 2.2.

Part V will focus on three extensions. As already mentioned, Chapter 16 deals with long-run asset management. Chapter 17 focuses on some implications of incomplete markets whose consequences are illustrated from the twin viewpoints of the equilibrium and arbitrage approaches. We will use it as a pretext to review the Modigliani–Miller theorem and, in particular, to understand why it depends on the hypothesis of complete markets. Finally, in Chapter 18, we will open up, just a little, the Pandora's box of heterogeneous beliefs. Our goal is to understand a number of issues that are largely swept under the rug in standard asset management and pricing theories and, in the process, restate the efficient market hypothesis.[5]

2.4 Decomposing Risk Premia

Method 1 in Section 2.2 represents the standard methodology for real investment project evaluation and, if only for this reason, deserves a bit more attention. Generalizing

4 Web Chapter B presents additional illustrations and applications of Martingale pricing theory.

5 Web Chapter D concludes with a discussion of the recent "financial crisis." While we describe the causes (in our view) and consequences of the crisis, our principal objective is to relate the event to the concepts introduced in this text.

the present value expression to a many period cash flow timing typical of real projects yields

$$V_{\text{project}} = \sum_{t=1}^{T} \frac{E\tilde{C}F_t}{(1+r_{\text{f}}+\pi_{\text{p}})^t}$$

where we simplify the discussion below by assuming a constant risk-free rate of interest yields ($r_t^f \equiv r_{\text{f}}$ for all t) and a constant project risk premium π_{p}. Recall that an objective, market-based valuation of the project requires that π_{p} represents the risk premium on a stock (or portfolio of stocks) whose cash-flow timing and risk characteristics resemble (in a manner to be made formal in later chapters) those of the project; i.e.

$\pi_{\text{p}} = \pi_i = E\tilde{r}_i - r_{\text{f}}$, where $E\tilde{r}_i$ is the expected return to the "approximating stock or stock portfolio i."

Work in empirical asset pricing suggests is that the return premium $\tilde{r}_i - r_{\text{f}}$ on a stock i may be intertemporarily decomposed as a linear combination of fundamental stochastic factors $\tilde{F}^j$, $j = 1, 2,\ldots, J$:

$$\tilde{r}_{i,t} - r_{\text{f}} = \alpha_i + \beta_i^1 \tilde{F}_t^1 + \beta_i^2 \tilde{F}_t^2 + \cdots + \beta_i^J \tilde{F}_t^J + \tilde{\varepsilon}_{i,t} \tag{2.1}$$

where the β_i^j measures the sensitivity of stock i's return to the specific underlying factor j, and $\tilde{\varepsilon}_{i,t}$ is an i.i.d. mean zero random component. Under representation (2.1), all stock returns are thus determined by the same J factors. These factors affect different stocks to differing degrees as measured by a stock's specific factor sensitivities. The impact of these factors subsumes the entire risk premium except for a "white noise" residual.

What are these factors? Some factors define macroeconomic conditions such as the inflation rate or the state of the business cycle as measured by the GDP growth rate. All firms are affected, to varying degrees, by the business cycle. Chen et al. (1986) consider factors such as industrial production, inflation expectations, and oil prices. Other factors are believed to measure various permanent psychological biases on the part of investors, biases that appear, in some cases, to have a permanent influence on equity return patterns.

Why the focus is on estimating risk premia, when our ultimate objective is asset pricing? Given cash-flow estimates, prices and returns are, of course, "dual" to one another in the sense that knowing a project's price determines the implied risk premium and vice versa via the present value relationship. Another motivation is that most "investors" are not undertaking real investment projects but are buying portfolios of individual securities. Rather than thinking of next period's price of a security relative to its price today, it is more natural for investors to think in terms of what the security is expected to earn above r_{f}—its risk premium—over their chosen time horizon.

2.5 Models and Stylized Facts

Financial economics has as its goal the understanding of financial market behavior. While this statement may seem obvious, it has no real content until we clarify what an "understanding" would mean. For asset pricing phenomena, in particular, it means that the event under study can be explained in a model economy where self-interested economic agents determine their demands for various securities based on certain first principles/axioms of economic behavior.[6] The model may also go on to specify how these demands interact with the supplies of the various securities to determine equilibrium prices and returns. Chapters 3–10 of this text essentially construct the basic model paradigms of finance.

A model is necessarily an abstraction or (dramatic) simplification of reality. Many economic mechanisms are ignored with only the most critical retained. As a result, there will inevitably be some aspects of reality which it will be unable to explain. What characteristics, then, describe a good in contrast to a poor economic model? We propose the three criteria listed below:

i. A good model must be simple enough to enrich our intuition. In other words, the principal economic mechanisms within the model that allow it to explain the phenomenon under study must be readily apparent.
ii. The abstraction which the model represents must be tailored to (or rich enough to address meaningfully) the questions being asked of it. Researchers would not seek to understand observed patterns in the US distributions of income and wealth, for example, in a representative (single) agent model.[7]
iii. The model should be able to give precise answers to questions we pose concerning the behavior of the real economy.

How does one acquire confidence in a model that ostensibly satisfies the above criteria? The answer is straightforward: the more historical phenomena the model is able to explain successfully, the more confidence researchers have in its ability to provide answers to current and future real-world questions. It is here that the financial "stylized facts" enter the scene. These "stylized facts" are simply well-documented price, quantity, or return patterns that have been present in financial market data over long periods of time. In the latter sense, they are said to be "secular."[8] At a minimum, it is imperative that a good and trustworthy asset pricing model be able to replicate (reproduce) them.

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6 As such economic models differ from scientific ones (e.g., of the atom). Scientific models describe the laws of nature, which are invariant to human activity. In economic models, everything is the result of human activity.

7 Requirement (ii) is not intended to suggest that radically different models should be proposed to explain different data regularities. The underlying principals must be the same and the results mutually consistent in areas of overlap.

8 That is, "secular" as in "existing or continuing through ages and centuries" (Webster's Collegiate Dictionary, Ninth Edition).

Although his focus is on the mechanisms underlying business cycles, we adopt the general modeling perspective of R.E. Lucas, Jr. as expressed in Lucas (1980):

> *One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost. To serve this function well, it is essential that the artificial "model" economy be distinguished as sharply as possible in discussion from actual economies. Insofar as there is confusion between statements of opinion as to the way we believe actual economies would react to particular policies and statements of verifiable fact as to how the model will react, the theory is not being effectively used to help us to see which opinions about the behavior of actual economies are accurate and which are not. This is the sense in which insistence on the "realism" of an economic model subverts its potential usefulness in thinking about reality. Any model that is well enough articulated to give clear answers to the questions we put to it will necessarily be artificial, abstract, patently "unreal."*
>
> *At the same time, not all well-articulated models will be equally useful. Though we are interested in models because we believe they may help us to understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions. This is the sense in which more "realism" in a model is clearly preferred to less.*
>
> *On this general view of the nature of economic theory then, a "theory" is not a collection of assertions about the behavior of the actual economy but rather an explicit set of instructions for building a parallel or analogue system — a mechanical, imitation economy. A "good" model, from this point of view, will not be exactly more "real" than a poor one, but will provide better imitations. Of course, what one means by a "better imitation" will depend on the particular questions to which one wishes answers.*

Accordingly we next highlight a few quantitative and qualitative properties of financial markets that serve as basic stylized facts against which the financial models we will propose in the remainder of this text should be measured. We focus exclusively on capital market phenomena rather than empirical regularities related to the firm's corporate financing activities.

2.5.1 The Equity Premium

A broadly diversified portfolio of stocks (e.g., the $S\&P_{500}$, the DAX, the CAC) consistently earns average returns substantially in excess of the risk-free rate (normally proxied by the return on short-term debt securities issued by a nation's national treasury authority). Tables 2.3 and 2.4 give some ideas as to the magnitudes involved.

Table 2.3: US returns: 1889–2010[a]

Time period	Real Return on a Market Index[b]	Real Return on a Relatively Riskless Security	% Risk Premium
	Mean	Mean	Mean
1889–2010	7.5%	1.1%	6.4%
1889–1978	7.0%	0.8%	6.2%
1926–2010	8.0%	0.8%	7.2%
1946–2010	7.5%	0.8%	6.7%

[a]Data from Mehra (2012); annualized returns.
[b]The $S\&P_{500}$ and its antecedents.

Table 2.4: The equity premium: the principal capital markets[a]

Country	Time Period	% Risk Premium	Country	Time Period	% Risk Premium
Belgium	1900–2010	5.5%	Sweden	1900–2010	6.6%
Holland	1900–2010	6.5%	UK	1900–2010	6.0%
France	1900–2010	8.7%	Australia	1900–2010	8.3%
Germany	1900–2010	9.8%	Canada	1900–2010	5.6%
Ireland	1900–2010	5.3%	India	1991–2004	11.3%
Italy	1900–2010	9.8%	Japan	1900–2010	9.0%

[a]*Source and details*: Dimson et al. (2010); annualized returns.

Note that in the US statistics (Table 2.3), the premium for long horizons never falls below 6%. With minor exceptions, the same results usually carry over to 10-year horizons (i.e., the pattern is secular). Similar if not stronger results carry over to international data (Table 2.4), despite two wars which led to substantial capital destruction and, in some cases, the cessation of organized competitive stock trading (e.g., Germany, France). The experiences of the United Kingdom and Canada compare most closely with the United States in that there was no interruption to trading and the respective governments did not seek to control this form of financial market activity. India's stock market experience is more recent and Japan's stellar performance is largely a post-World War II phenomenon prior to 1990. Across all markets, a robust equity premium over long horizons is an empirical fact.[9]

Within the current class of "rational economic models," those that we are about to describe in the chapters to follow, it is extremely difficult to replicate these statistics, so much so

[9] A prominent exception is the experience of recent history. For the US stock market, the average annual return for the period 2000–2010 was −0.39%, while US Treasury bonds of time to maturity exceeding 10 years paid on average 7.18% over the same period.

Table 2.5: Average annualized excess returns for 10 portfolios sorted on BE/ME[a]

Lowest		→ Increasing (BE/ME) →						Highest	
Port 1	Port 2	Port 3	Port 4	Port 5	Port 6	Port 7	Port 8	Port 9	Port 10
6.76	7.64	7.89	7.65	8.43	8.92	9.02	10.88	11.65	12.75

[a]Based on monthly data for the period 1963.1 through 2011.7. These (value weighted) portfolios are reconstructed (i.e., all the Compustat stocks are reassigned to one of the 10 portfolios) at the end of June of each year based on the end of the previous year's BE and ME values. We thank Tano Santos for making this data available to us.

that they are often described as constituting the "equity premium puzzle."[10] Furthermore, despite the substantial risk premia evident in Tables 2.3 and 2.4, most of the US population owns very little stock: the 2007 Survey of Consumer Finance reports that about one-half of US households own no stock at all; for high-income households 23% own no stock. This fact constitutes a second "puzzle," at least as regards the modern theory of portfolio composition (Chapters 6 and 7).

The equity premium is a secular time series property of stock returns.

2.5.2 The Value Premium

The value premium is a statement about the cross section of stock returns. It is the empirically robust observation that stocks with a higher (book value of equity)/(market value of equity) (BE/ME) ratio have, on average, higher excess returns than stocks with low (BE/ME) values. Consider Table 2.5, which describes the annualized average excess returns on 10 portfolios ($Er_{port\ i} - r_f$, $i = 1, 2, \ldots, 10$) of Compustat stocks sorted on the basis of their (BE/ME) ratio.[11]

Notice that the highest (BE/ME) portfolio has nearly twice the average excess returns of the lowest (BE/ME) portfolio. The return pattern observed in Table 2.5 is known as the "value premium." It is characteristic of international stock markets and all historical time periods. The value premium becomes the "value premium puzzle" because financial theory has no all-encompassing explanation as to why it should be observed. Traditional risk-based theories—the idea that investors dislike risky returns and must therefore be compensated with higher average returns in order to hold, willingly, higher risk assets—cannot explain the pattern of Table 2.5. In particular, the CAPM, which we discuss in

[10] In the models we will consider the supply of equities and risk-free bonds is typically assumed to be constant. This is generally an innocuous assumption: the supply of IBM equity shares outstanding, for example, does not change from year to year. The focus of these models is thus principally to characterize security demands by investors, from which follow (given fixed supplies) equilibrium prices and rates of return. The resulting equity premium usually falls far short of what is manifest in the data.

[11] Compustat is a large publicly accessible database of historical stock returns (and much other information).

Chapter 8, and which is the most widely cited risk-based theory of returns, cannot explain Table 2.5.

The value premium is a secular cross-sectional property of stock returns.

2.5.3 The Term Structure

In what follows we will also explore various features of the bond market and, in particular, the market for default free government securities (e.g., US Treasury securities). In this specific market, the fundamental notion is that of the term structure of nominal interest rates: the family of interest rates on zero-coupon, default-free nominal bonds of progressively greater maturity.[12] More specifically, at any time t, it is the collection of interest rates $\{r_{t,\ 1}, r_{t,\ 2},\ldots, r_{t,\ J}\}$, where $r_{t,j}$ is the period t interest rate on a default-free security with cash-flow pattern:

T	$t+1$	$t+2\cdots$	$t+j-1\cdots$	$t+j$	$t+j+1\cdots$	$t+J$
$-q^b_{t,j}$	0	0	0	\$1000	0	0

where $q^b_{t,j} = \$1000/((1+\mathrm{r}_{t,j})^j)$ with $q^b_{t,j}$ denoting the security's period t market price. With no uncertainty in the payments, these rates reflect the pure time value of money and, as such, constitute one of the building blocks of any discounting procedure (recall Section 2.2). In this sense, the term structure is a fundamental concept. Accordingly, the basic features of the bond market are usually expressed as properties of the term structure. In particular, we will seek to explain the following:

i. The term structure of interest rates is typically upward sloping: default free discount securities with longer times to maturity typically command higher rates. We want to understand not only why this is true but also what drives the exceptions.
ii. The term structure of interest rates generally moves up or down for all maturities simultaneously. This means that an increase in the short rate is accompanied by an increase in rates for bonds of all maturities. For US Treasury securities, 99% of the variation in returns at any maturity is related to shifts in the entire term structure. This fact stands in stark contrast to its analogous relationship for stocks: roughly 80−90% of the variation in returns to any particular stock is generally *unrelated* to aggregate market movements.

    This observation suggests that there may be one macroeconomic quantity (factor) affecting all default-free rates similarly. Can this factor be identified?

[12] By a "nominal" bond we mean one that pays prefixed dollar (CHF, Euro) amounts, that are not adjusted for inflation.

iii. Lastly, there is the phenomenon of relative volatility: properly assessed, returns on long term US Treasury securities are more volatile than the returns on short-term bonds, even when normalized by their higher expected returns (see topic (i)). Such a result is puzzling in that our immediate intuition would suggest otherwise: long-term bond prices and returns should not be as sensitive to business cycle or other macroeconomic events, as these are generally of shorter duration (than the bond's time to maturity) and thus their consequences for bond returns should tend to "average out" over the long bond's time to maturity. Shiller (1979) refers to this phenomenon as the "bond volatility puzzle."[13]

Taken together, the equity premium (a quantitative assessment) and the three qualitative properties of the term structure detailed above illustrate important stylized facts that good models should be able to replicate.

2.6 Asset Pricing Is Not All of Finance!

2.6.1 Corporate Finance

Intermediate financial theory focuses on the valuation of risky cash flows. Pricing a future (risky) dollar is the dominant ingredient in most financial problems. But it is not all of finance! Our capital markets perspective in particular sidesteps many of the issues surrounding how the firm generates and protects the cash-flow streams to be priced. It is this concern that is at the core of corporate financial theory or simply *corporate finance*.

In a broad sense, corporate finance is concerned with decision making at the firm level whenever it has a financial dimension, has implications for the financial situation of the firm, or is influenced by financial considerations. In particular, it is a field concerned, first and foremost, with the investment decision (what projects should be accepted), the financing decision (what mix of securities should be issued and sold to finance the chosen investment projects), the payout decision (how should investors in the firm, and in particular the equity investors, be compensated), and risk management (how corporate resources should be protected against adverse outcomes). Corporate finance also explores

[13] Strictly speaking, these are statements about default-free coupon bonds. In fact, the US Treasury, for instance, does not issue zero coupon bonds of more than 6 months time to maturity. It is possible, however, to extract the implied default-free zero coupon bond prices from the prices and cash flows associated with default-free coupon bonds, if a sufficient number of distinct types (different cash flows) are issued. The set of IRRs (internal rates of return) on default free coupon bonds of successively greater maturity is referred to as the "yield curve." Its qualitative properties do not differ significantly from those of the term structure (e.g., both move in tandem). Typically there is little quantitative difference as well, and the expressions "term structure" and "yield curve" are often used (incorrectly) interchangeably. See Chapter 11 for a more detailed discussion. The percentages are from Ang (2012).

issues related to the size and the scope of the firm, e.g., mergers and acquisitions and the pricing of conglomerates, the internal organization of the firm, the principles of corporate governance, and the forms of remuneration of the various stakeholders.[14]

All of these decisions individually and collectively influence the firm's free cash-flow stream and, as such, have asset pricing implications. The decision to increase the proportion of debt in the firm's capital structure, for example, increases the riskiness of its equity cash-flow stream and the standard deviation of the equilibrium return on equity.

Of course, when we think of the investment decision itself, the solution to the valuation problem is of the essence. Indeed, many of the issues typically grouped under the heading of capital budgeting are intimately related to the focus of the present text. We will be silent, however, on most of the other issues listed above, which are better viewed as arising in the context of bilateral (rather than market) relations and, as we will see, in situations where asymmetries of information play a dominant role.

The goal of this section is to illustrate the difference in perspectives by reviewing, selectively, the corporate finance literature, particularly as regards the capital structure of the firm and contrasting it with the capital markets perspective that we will be adopting throughout this text. In so doing, we also attempt to give the flavor of an important research area while reminding the reader of the many important topics this text elects not to address.

2.6.2 Capital Structure

We focus on the capital structure issue in Chapter 17 where we explore the assumption underlying the famous Modigliani–Miller irrelevance result: in the absence of taxes, subsidies, and contracting costs, the value of a firm is independent of its capital structure if the firm's investment policy is fixed and financial markets are complete. Our emphasis will concern how this result fundamentally rests on the complete markets assumption.

The corporate finance literature has not ignored the completeness issue but rather has chosen to explore its underlying causes, most specifically information asymmetries between the various agents concerned, managers, shareholders, and so forth.[15] While we touch on

[14] The recent scandals (Hewlett-Packard, AIG) in the United States, Europe (gigantic trading losses in JPM Chase, UBS, and Société Générale, due to rogue or unsupervised trading) and in Japan (Olympus Optical) place in stark light the responsibilities of boards of directors for ultimate firm oversight as well as their frequent failure to provide it. The large question here is what sort of board structure is consistent with superior long-run firm performance?

[15] Tax issues have tended to dominate the corporate finance capital structure debate until recently, and we will review this arena shortly. The relevance of taxes is not a distinguishing feature of the corporate finance perspective alone. Taxes also matter when we think of valuing risky cash flows, although we will have very little to say about it except that all the cash flows we consider are to be thought of as after-tax cash flows.

the issue of heterogeneity of information in a market context, we do so only in Chapter 18, emphasizing there that heterogeneity raises a number of tough modeling difficulties. These difficulties justify the fact that most of capital market theory either is silent on the issue of heterogeneity (in particular, when it adopts the arbitrage approach) or explicitly assumes homogeneous information on the part of capital market participants.

In contrast, the bulk of corporate finance builds on asymmetries of information and explores the various problems they raise. These are typically classified as leading to situations of "moral hazard" or "adverse selection." An instance of the former is when managers are tempted to take advantage of their superior information to implement investment plans that may serve their own interests at the expense of those of shareholders or debtholders. An important branch of the literature concerns the design of contracts, which take moral hazard into account. The choice of capital structure, in particular, will be seen potentially to assist in their management (see, for example, Zwiebel, 1996).

A typical situation of adverse selection occurs when information asymmetries between firms and investors make firms with "good" investment projects indistinguishable to outside investors from firms with poor projects. This suggests a tendency for all firms to receive the same financing terms (a so-called pooling equilibrium where firms with less favorable prospects may receive better than deserved financing arrangements). Firms with good projects must somehow indirectly distinguish themselves in order to receive the more favorable financing terms they merit. For instance, they may want to attach more collateral to their debt securities, an action that firms with poor projects may find too costly to replicate (see, for example, Stein, 1992). Again, the capital structure decision may sometimes help in providing a resolution of the "adverse selection" problem. Below we review the principal capital structure perspectives.

2.6.3 Taxes and Capital Structure

Understanding the determinants of a firm's capital structure (the proportion of debt and equity securities it has outstanding in value terms) is the classical problem in corporate finance. Its intellectual foundations lie in the seminal work of Modigliani and Miller (1958), who argue for capital structure irrelevance in a world without taxes and with complete markets (an hypothesis that excludes information asymmetries).

The corporate finance literature has also emphasized the fact that when one security type receives favored tax treatment (typically, this is debt via the tax deductibility of interest), then the firm's securities become more valuable in the aggregate if more of that security is issued, since to do so is to reduce the firm's overall tax bill and thus enhance the free cash flow to the security holders. Since the bondholders receive the same interest and principal payments, regardless of the tax status of these payments from the firm's perspective, any tax-based cash-flow enhancement is captured by equity holders. Under a number of further specialized assumptions (including the hypothesis that the firm's debt is risk-free), these considerations lead to the classical relationship

$$V_{\mathrm{L}} = V_{\mathrm{U}} + \tau D$$

The value of a firm's securities under partial debt financing (V_{L}, where L denotes leverage in the capital structure) equals its value under all equity financing (V_{U}, where U denotes unlevered or an all-equity capital structure) plus the present value of the interest tax subsidies. This latter quantity takes the form of the corporate tax rate (τ) times the value of debt outstanding (D) when debt is assumed to be perpetual (unchanging capital structure).

In return terms, this value relationship can be transformed into a relationship between levered and unlevered equity returns:

$$r_{\mathrm{L}}^{e} = r_{\mathrm{U}}^{e} + (1 - \tau)(D/\mathrm{E})(r_{\mathrm{U}}^{e} - r_{\mathrm{f}})$$

i.e., the return on levered equity, r_{L}^{e}, is equal to the return on unlevered equity, r_{U}^{e}, plus a risk premium due to the inherently riskier equity cash flow that the presence of the fixed payments to debt creates. This premium, as indicated, is related to the tax rate, the firm's debt/equity ratio (D/E), a measure of the degree of leverage, and the difference between the unlevered equity rate and the risk-free rate, r_{f}. Immediately we observe that capital structure considerations influence not only expected equilibrium equity returns via

$$Er_{\mathrm{L}}^{e} = Er_{\mathrm{U}}^{e} + (1 - \tau)D/E(Er_{\mathrm{U}}^{e} - r_{\mathrm{f}})$$

where E denotes the expectations operator, but also the variance of returns since

$$\sigma_{r_{\mathrm{L}}^{e}}^{2} = (1-(1+\tau)D/E)^{2}\sigma_{r_{\mathrm{U}}^{e}}^{2} > \sigma_{r_{\mathrm{U}}^{e}}^{2}$$

under the mild assumption that r_f is constant in the very short run. These relationships illustrate but one instance of corporate financial considerations affecting the patterns of equilibrium returns as observed in the capital markets.

The principal drawback to this tax-based theory of capital structure is the natural implication that if one security type receives favorable tax treatment (usually debt), then if the equity share price is to be maximized the firm's capital structure should be composed exclusively of that security type—i.e., all debt, which is not observed. More recent research in corporate finance has sought to avoid these extreme tax-based conclusions by balancing the tax benefits of debt with various costs of debt, including bankruptcy and agency costs.[16] Our discussion broadly follows Harris and Raviv (1991).

2.6.4 Capital Structure and Agency Costs

This important segment of the literature seeks to explain financial decisions by examining the conflicts of interests among claimholders within the firm. Although agency conflicts can take a variety of forms, most of the literature has focused on manager's incentives to increase investment risk—the asset substitution problem—or to reject positive Net Present Value (NPV) projects—the underinvestment problem. Both of these conflicts increase the cost of debt and thus reduce the firm's value-maximizing debt ratio.

Another commonly discussed determinant of capital structure arises from manager−stockholder conflicts. Managers and shareholders have different objectives. In particular, managers tend to value investment more than shareholders do. Although there are a number of potentially powerful internal mechanisms to control managers, the control technology normally does not permit the costless resolution of this conflict between managers and investors. Nonetheless, the cash-flow identity implies that constraining financing, hedging, and payout policy places indirect restrictions on investment policy. Hence, even though investment policy is not contractible, by restricting the firm in other dimensions, it is possible to limit the manager's choice of an investment policy. For instance, Jensen (1986) argues that debt financing can increase firm value by reducing the free cash flow. This idea is formalized in more recent papers by Stulz (1990) and Zwiebel (1996). Also, by reducing the likelihood of both high and low cash flows, risk management can control not only shareholders' underinvestment incentives but managers' ability to overinvest as well.

More recently, the corporate finance literature has put some emphasis on the cost that arises from conflicts of interest between controlling and minority shareholders. In most countries, publicly traded companies are not widely held but rather have controlling shareholders. Moreover, these controlling shareholders have the power to pursue private benefits at the expense of minority shareholders, within the limits imposed by investor protection. The recent "law and finance" literature following Shleifer and Vishny (1997) and La Porta et al. (1998) argues that the expropriation of minority shareholders by the controlling shareholder

[16] There are many other proposed capital structure theories. Lee (2014), for example, proposes that firms eschew present and future available tax benefits to debt financing and rather maintain large cash balances in order to finance unexpected future investments and as a precaution against bad times (periods of low cash-flow generation). Such firms are unable to raise cash at times of critical need by selling existing assets because of severe capital adjustment costs.

is at the core of agency conflicts in most countries. While these conflicts have been widely discussed in qualitative terms, the literature has largely been silent on the magnitude of their effects.

2.6.5 The Pecking Order Theory of Investment Financing

The seminal reference here is Myers and Majluf (1984) who again base their work on the assumption that investors are generally less well informed (asymmetric information) than insider-managers vis-à-vis the firm's investment opportunities. As a result, new equity issues to finance new investments may be so underpriced (reflecting average project quality) that NPV positive projects from a societal perspective may have a negative NPV from the perspective of existing shareholders and thus not be financed. Myers and Majluf (1984) argue that this underpricing can be avoided if firms finance projects with securities that have more assured payout patterns and thus are less susceptible to undervaluation: internal funds and, to a slightly lesser extent, debt securities, especially risk-free debt. It is thus in the interests of shareholders to finance projects first with retained earnings, then with debt, and lastly with equity. An implication of this qualitative theory is that the announcement of a new equity issuance is likely to be accompanied by a fall in the issuing firm's stock price since it indicates that the firm's prospects are too poor for the preferred financing alternatives to be accessible.

The pecking order theory has led to a large literature on the importance of security design. For example, Stein (1992) argues that companies may use convertible bonds to get equity into their capital structures "through the backdoor" in situations where informational asymmetries make conventional equity issues unattractive. In other words, convertible bonds represent an indirect mechanism for implementing equity financing that mitigates the adverse selection costs associated with direct equity sales. This explanation for the use of convertibles emphasizes the role of the call feature—that will allow good firms to convert the bond into common equity—and costs of financial distress—that will prevent bad firms from mimicking good ones. Thus, the announcement of a convertible bond issue should be greeted with a less negative—and perhaps even positive—stock price response than an equity issue of the same size by the same company.

2.7 Banks

Banks merit our attention at the present juncture because the concepts we have been discussing (e.g., firm leverage) apply as well to banks but to a "unique" degree. To organize our banking discussion, let us first compare the (simplified) balance sheets of a typical industrial firm and a commercial bank.

Balance sheets (all entries measured in value terms)

Industrial Firm		Bank	
Assets	**Liabilities**	**Assets**	**Liabilities**
• Tangible assets (factories, machinery, inventories) • Intangible assets (patents, process technology, trademarks, etc.)	• Debt issued by the firm • Equity of shareholders	• Loans • Cash • Default-free government securities • Other securities (mortgage backed securities, for example)	• Deposits taken by the bank • Debt securities issued by the bank • Equity of shareholders

Note that the deposits of the banks constitute a (large) portion of its liabilities. Pure investment banks, as distinct from commercial banks, are financial companies that extend loans and make equity investments, but they cannot take deposits as a source of funding. The great general-purpose international banks (e.g., Deutsche, UBS, Société Générale, Citi) are both; i.e., they are holding companies with investment banking and commercial banking divisions. In periods of financial distress, either division effectively becomes responsible for the debts of the other since the overall bank is a single legal entity.

For nonfinancial Eurozone corporations, the average (D/E) ratio prior to the financial crisis (2008) was about 0.8; for banks in the Eurozone, the average (D/E) ratio was about 30. Banks were, and continue to be, much more highly leveraged than other firms. In the past, pure investment banks were even more highly leveraged than bank-holding companies in general. Lehman Brothers, for example, was leveraged to a (D/E) ratio exceeding 50. A high leverage ratio is the first unique feature of bank corporations.

A second distinction, already alluded to, concerns the funding banks receive in the form of deposits. Nonbank corporations that take out bank loans or issue long-term bonds know exactly the timing and magnitude of the future interest and principal payments they must make. To the extent they borrow in the form of taking deposits, banks, however, may be required, suddenly and unexpectedly, to repay the "loans" if depositors collectively initiate large aggregate withdrawals. Such an event may occur if depositors suspect that the bank's investments (its loan and securities portfolio) have not maintained their value, leading to the possibility of bankruptcy, and a delayed return of deposits or potential deposit losses. Economists call these circumstances "bank runs."[17] The phenomenon by which some of a bank's funding may suddenly disappear is the second special feature of banks.[18]

[17] To forestall bank runs, governments now provide deposit insurance against deposit losses at least up to a maximum. In the United States, the relevant entity is the Federal Deposit Insurance Corporation (FDIC), which is funded by a small tax on banks.

[18] Anticipating our discussions of portfolio theory, banks can be viewed as very highly leveraged equity portfolios: a large long position in long-term assets financed by a short position in short-term assets.

Lastly, banks are the principle providers of credit to smaller businesses which find it costly to issue their own bonds directly in the capital markets. This notion of credit is very broad: banks provide loans for investment projects, loans for inventories and wage payments, temporary trade financing, etc. When these services become deficient because certain prominent banks are in financial difficulty (a so-called credit crunch), the macroeconomic effects are both negatively severe and long lasting. The ongoing Great Recession is only the most recent case in point. While the bankruptcy of a nonfinancial firm can have devastating effects on the local economy in which it operates, the consequences of bankruptcies or near-bankruptcies among the large players in the international banking system can have adverse national and international consequences. This feature of the banking system constitutes its third unique characteristic. It is an aspect of the "too big to fail" debate.

We return to the unique features of the banking system in web chapter D.

2.8 Conclusions

We have presented four general approaches and two main perspectives on the valuation of risky cash flows. This discussion was meant to provide an organizing principle and a road map for the extended treatment of a large variety of topics on which we are now embarking. We then went on to present some stylized facts of the financial markets and used these to define what a good financial (valuation) model should be. Our brief excursion into corporate finance was intended to suggest some of the agency issues that are part and parcel of a firm's cash-flow determination. That we have elected to focus on pricing issues surrounding those cash-flow streams does not diminish the importance of the many issues surrounding their creation.

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