Portfolio Sorting: Empirical Illustrations

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Object of today

- Today, we will focus on how you can apply portfolio sorting as a method to answer financial research questions
- More specifically, we will examine CAPM through a portfolio sort and, next, we will see how to include macroeconomic information in a portfolio sort

Betting against beta

Motivation

- The underlying assumption of CAPM is that investors can leverage and de-leverage their portfolio
- In real-life, however, we have leverage constraints and instead "leverage" their portfolio by overweight more risky assets
- Indeed, the Capital market line is flat, confirming that high beta stocks earn lower risk-adjusted returns (Jensen et al., 1972)

  $\rightarrow$ Frazzini and Pedersen (2014) (FP) examines several research questions related to the puzzle

A theoretical model

- FP construct an OLG model and introduce margin requirements for the investor
- They show within their model framework that

$$\alpha_i = \psi_t(1 - \beta_i) \tag{1}$$

  where ψ_t measures the lagrange multiplier (i.e., the tightness of the funding constraint)
- Note, that α_i is decreasing in β_i
- Their model delivers 4 additional prediction related to:
  - A β neutral portfolio behaves due to fundings constraint (and shocks to funding constraints)
  - How the difference in β among the cross-section behaves due to funding constraints
  - Lastly which type of investor investing in high β stocks

Data

- While FP examine a wide cross-section of countries and different asset classes, we will focus on the case of US stocks
- more specifically, we will consider common stocks (SHRCD 10 and 11) listed on the NYSE, AMEX, and NASDAQ exchanges (EXCHCD 1, 2, and 3) from January 1986 to December 2019 (equivalent to the dataset from the momentum illustration)

Characteristic

- FP estimate the sorting variable β as the following

$$\hat{\beta}_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m} \tag{2}$$

- $\hat{\sigma}_i$ and $\hat{\sigma}_m$ denote the standard deviation of asset i returns and the market, respectively, calculated using 1-year rolling window
- $\hat{\rho}$ is the estimated correlation between asset i and the market estimated using a five-year rolling window
- Pedersen calculate the correlation using three-day overlapping log-returns by an argument of nonsynchronous trading

Characteristic

- Lastly, they shrink their β estimates towards the cross-sectional mean towards the true cross-sectional average

$$\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i)\beta^{XS}, \tag{3}$$

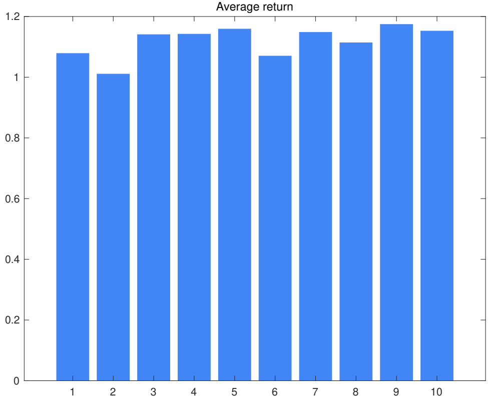
where β^{XS} is set equal to 1 and w_i is set to 0.6

Implemention advice

- The rolling window estimation of β's is a computationally intensive exercise if you consider a standard loop given the size of the daily CRSP dataset
- Using the "movmean", "movsum", and "movstd", you can estimate the β's efficiently!

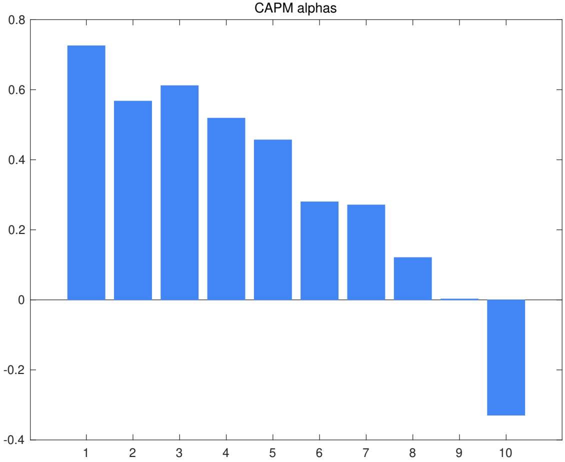
Portfolio analysis

- They consider 10 equal-weighted portfolios with the following average return:



	1	2	3	4	5	6	7	8	9	10
SR	1.11	0.99	1.01	0.90	0.84	0.69	0.68	0.56	0.49	0.36

The CAPM α



$\rightarrow$ The α are (strictly) decreasing in β, in line with the theoretical prediction!

Portfolio analysis

- The question is whether the decline in α is due to CAPM being a misspecified model
- What happens if we adjust the returns for, respectively, the FF3 and FF5 model?

	1	2	3	4	5	6	7	8	9	10
α_{CAPM}	0.73 (3.35)	0.57 (2.88)	0.61 (3.23)	0.52 (2.72)	0.46 (2.55)	0.28 (1.5)	0.27 (1.47)	0.12 (0.55)	0 (0.01)	-0.33 (-0.87)
α_{FF3}	0.63 (3.8)	0.46 (3.33)	0.51 (4.24)	0.4 (3.77)	0.34 (3.43)	0.17 (1.78)	0.17 (1.7)	0.03 (0.24)	-0.05 (-0.33)	-0.38 (-1.39)
α_{FF5}	0.64 (3.61)	0.38 (2.65)	0.43 (3.43)	0.32 (2.82)	0.27 (2.57)	0.16 (1.48)	0.23 (1.84)	0.18 (1.11)	0.28 (1.31)	0.2 (0.68)

- The α's are still generally decreasing in β decile
- All models have issues pricing the low β portfolio!

A market-neutral factor (BAB factor)

- Instead of the standard long-short portfolio (which is not market neutral), FP consider the following rank-based weighting:

$$w_H = k(z - \bar{z})^+ \tag{4}$$

$$w_L = k(z - \bar{z})^- \tag{5}$$

$$k = \frac{2}{\sum_{i=1}^{N_t}(z_i - \bar{z})} \tag{6}$$

where z_i is the rank of asset i

A market-neutral factor (BAB factor)

- The factor is then given as

$$BAB_t = \frac{1}{\beta_t^L}(r_t^L - r_{f,t}) - \frac{1}{\beta_t^H}(r_t^H - r_{f,t}) \tag{7}$$

where $\beta_j = w_j' \beta_t$ and $r_t^j = w_j' r_t$

The BAB factor

- We can now look at the performance of the BAB factor:

Excess return	α_{CAPM}	α_{FF3}	α_{FF5}
0.87	1.25	1.12	0.80
(3.63)	4.79	5.47	3.71

- All α's are highly significant and the excess return is significant

The BAB factor and funding constraints

- To test whether the BAB anomaly is related to funding constraints as proposed by the theoretical model, FP consider the TED spread
- The spread is given as the difference between the interbank rate and the 3-month T-bill rate, and, hence, measures credit risk of the banks in addition to liquidity in the interbank market
- FP find a negative relationship between the TED spread and returns on the BAB factor
- We will not deal further with this, but you can try if you can replicate the finding. We can find the TED spread here: https://fred.stlouisfed.org/series/TEDRATE

Conclusion

- All predictions from the theoretical model (surprisingly) seem to hold empirically
- And, we should simply all betting against beta

Betting against beta and Lottery demand?

A lottery demand explanation?

- We will now examine the paper of Bali, which provides a good example of how to examine whether a variable explains an anomaly by applying a dependent bivariate sort!
- Lottery investors generate demand for stocks with a high probability of short-term upward price movements $\rightarrow$ Typically these stocks have a high β

Data

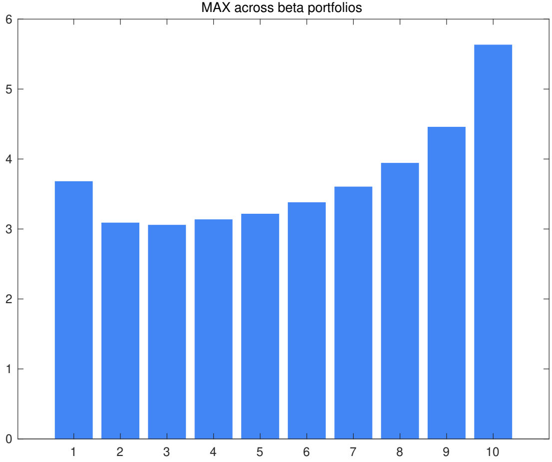
- We will consider the same data as for the BAB factor and, hence, the data is slightly different from Bali
- They exclude all stocks with a price below $ 5

The MAX characteristic

- To identify lottery characteristics, Bali et al. (2017) consider the average of the 5 largest daily returns within the last month
- A univariate sort on MAX gives the following:

1	2	3	4	5	6	7	8	9	10
0.88	1.05	1.02	1.03	0.95	0.92	0.79	0.68	0.44	0.31

MAX across beta portfolios



- Indeed, the average MAX is highest in the high beta portfolio!

Bivariate analysis

- So let us examine whether the β anomaly is still present when conditioning on the lottery demand
- We do that by applying a dependent bivariate sort: first on MAX and secondly on β
- For each of the 10 MAX portfolios, we can then construct L-S portfolios and evaluate the α's

Test for beta anormaly across groups of MAX

- We can now examine whether the beta anomaly exists when we condition on MAX
- Below is FF5 used as the pricing model

	1	2	3	4	5	6	7	8	9	10
					Betas from FP					
α_{FF5}	0.04 (0.15)	0.66 (3.27)	0.29 (1.04)	0.45 (1.60)	0.43 (1.32)	0.50 (1.43)	0.38 (0.88)	0.33 (0.75)	0.40 (0.72)	1.10 (1.69)
					Betas from linear regression					
α_{FF5}	0.37 (1.57)	0.61 (2.91)	0.50 (1.86)	0.54 (1.83)	0.57 (1.75)	0.58 (1.51)	0.77 (1.87)	1.06 (2.28)	1.46 (2.32)	2.00 (3.29)

Test for beta anormaly across groups of MAX

- The S-L only delivers significant α for stocks within the 10th and 20th percentile for the FP betas
- For the one-year rolling window β's which Bali et al. (2017) examine, the results are much different...
- Note that Bali et al. (2017) ends their sample in 2012. All α's are insignificant when only considering data up to 2012...

Betting against betting against beta

Betting against betting against beta

- Novy-Marx and Velikov (2022) comes with a pretty harsh critique of the methodology in the paper of FP
- More specifically, they claim that the results of FP are a result of three non-standard choices made by FP

The first non-standard procedure

- Non-standard procedure #1: rank-weighted portfolio construction.
- if you remember the rank weighting:

$$w_H = k(z - \bar{z})^+ \tag{8}$$

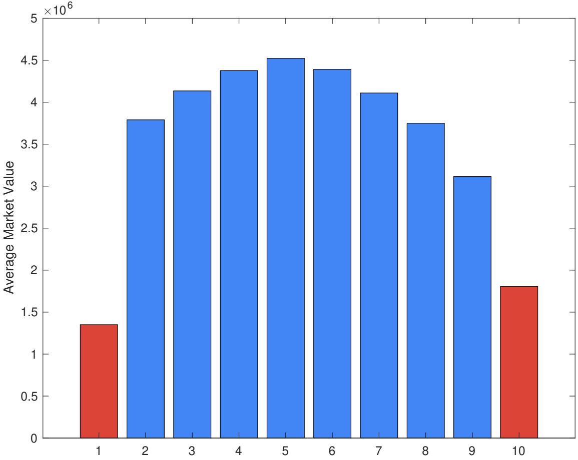
$$w_L = k(z - \bar{z})^- \tag{9}$$

$$k = \frac{2}{\sum_{i=1}^{N_t} (z_i - \bar{z})} \tag{10}$$

- The weighting proposed in FP is essentially an equal weighted portfolio sort: "The procedure creates portfolios that are almost indistinguishable from simple, equal-weighted portfolios"

A result of heigh weights in nano/micro firms

- To examine the claim, we first consider the average market value of a firm in each portfolio:



A new weighting method

- Novy-Marx and Velikov (2022), instead, propose the following weighting (in a footnote):

$$w_{h,i} = \frac{(z - \bar{z})^{+} meq_i}{\sum_j (z - \bar{z})^{+} meq_j} \tag{11}$$

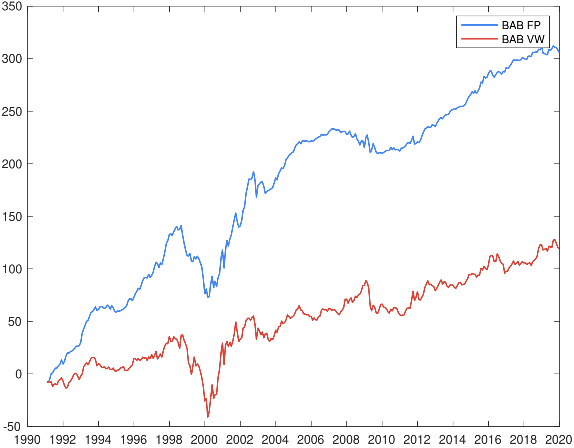
$$w_{l,i} = \frac{(z - \bar{z})^{-} meq_i}{\sum_j (z - \bar{z})^{-} meq_j} \tag{12}$$

- The weights above has the same property that they sum to 1, but the market value meq has an impact

- Holding all things equal: more valuable stocks have higher weight

The value weighted BAB factor

- The proposed weighting gives the following performance over time relative to the original:



$\longrightarrow$ A huge decline in performance!

The value weighted BAB factor

Excess return	α_{CAPM}	α_{FF3}	α_{FF5}
0.34	0.73	0.63	0.15
(1.40)	(3.08)	(3.45)	(0.82)

- The average return is no longer significant
- Conditional on the FF5 model, the value-weighted BAB factor does not provide a significant α!

Sum-up

- The ranking choice in FP has a large impact on the results
- Novy-Marx find that taking transaction costs into account decreases the performance of the BAB factor by 60%
- The point above questions how implementable the strategy is in real-life (and whether the anomaly exists due to inefficiency in financial markets)

Non-standard approach to calculate betas

- The second point is related to the way FP estimates their betas:

$$\hat{\beta}_i^{TS} = \hat{\rho} \frac{\hat{\sigma}_i}{\hat{\sigma}_m} \tag{13}$$

- Novy-Marx show that another way to write this is given as

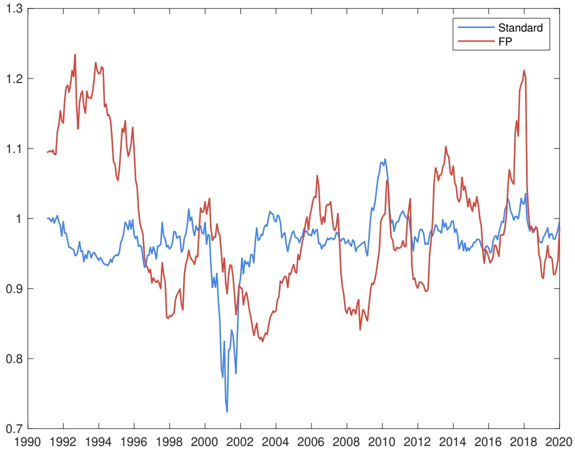
$$\hat{\beta}_i^{TS} = \frac{\rho_5}{\rho_1} \hat{\beta}_i^1 \tag{14}$$

- Novy-Marx claims that this choice makes the beta easier to predict because correlation is increasing in market volatility

  $\rightarrow$ The TED spread can predict the bias and, by construction, the β's (and, hence, the leverage)

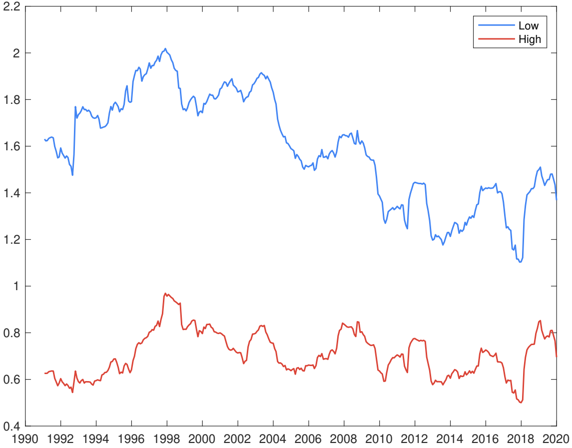
Non-standard way to calculate betas

- To examine the bias introduced by FP, consider the value-weighted average of the FP betas vs. $\hat{\beta}_i^1$
- By construction the true value-weighted beta must be 1 for all periods



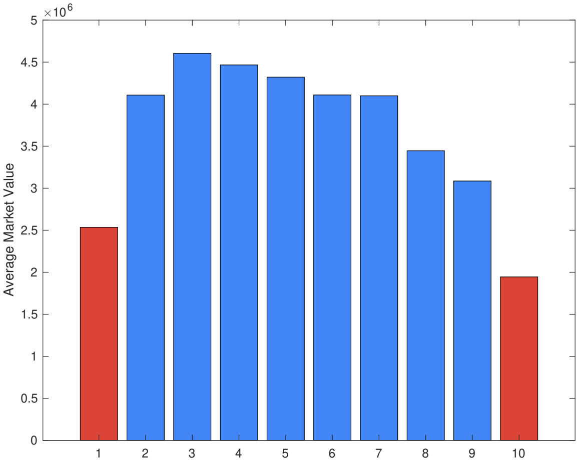
The leverage and short position over time

- FP find that the average long position is 1.4 and the short position is 0.7
- They do not provide any information on how the two evolves over time



Calculate breakpoint on NYSE

- To decrease the spread in size between the portfolios, you can alternatively apply only the NYSE stocks when calculating the breakpoints (Hou et al. (2020) suggest this should be standard)



Additional comments

- Another area of the literature examines whether downward risk is priced differently than upward risk
- For instance, Ang et al. (2006) construct β's for negative and positive market-returns and find negative is much more profitable than positive
- Bollerslev et al. (2021) decompose β into four components conditional on the sign of the market return and asset i. They find that only the negative, negative β compensates investors

Economic uncertainty and the cross-section of stocks

The relevance for you

- The paper we will go now through is a (good) example of how you can construct a factor from macro information
- You can apply the same method and type of discussion with any macro variable of interest
- But always ask yourself; why does it make intuitive sense that the variable is priced in financial markets?

  $\rightarrow$ For now, we will consider the case of the macroeconomic uncertainty index of Jurado et al. (2015)

Motivation

- The ICAPM model of Merton (1973) suggests that investors seek to hedge changes in investment and consumption opportunity sets
- An implication is any variable that correlates with these opportunity sets should be priced in the cross-section
- Prior studies find a link economic uncertainty and the real economy in addition to asset prices (Bloom, 2009, Drechsler, 2013, Augustin and Tédongap, 2021, Ludvigson et al., 2021)

  $\rightarrow$ MU should correlate with the opportunity sets

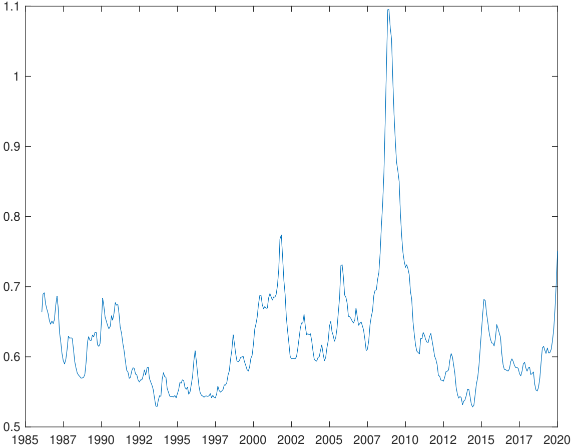
The macroeconomic uncertainty index

- Bali et al. (2017) examine the macroeconomic uncertainty measure of Jurado et al. (2015)
- Jurado et al. (2015) defines uncertainty as volatility on forecast errors, i.e.,

$$\mathcal{U}_{j,t}^{h} = \sqrt{E\left[\left(y_{t+h,j} - E_t(y_{t+h,j})\right)^2 \mid I_t\right]} \tag{15}$$

- The measure is then aggregated (equal-weighted) across the $j \in J$ variables
- Note, that the index is not a vintage dataset meaning that we can discuss whether the index is in investors' information set

The macrouncertainty index



The MU exposure factor

- To examine whether MU is priced in the cross-section of US stocks, Bali et al. (2017) estimates the following pricing model for each asset i

$$\begin{aligned} R^e_{i,t} &= \alpha_i + \beta_{MU,i}MU_t + \beta_{MKT,i}MKT_t + \beta_{SMB,i}SMB_t \\ &+ \beta_{HML,i}HML_t + \beta_{UMD,i}UMD_t + \beta_{LIQ,i}LIQ_t \\ &+ \beta_{IA,i}R_{IA,t} + \beta_{ROE,i}R_{ROE,t} + \varepsilon_{i,t} \end{aligned} \tag{16}$$

- Using a rolling window of 5 year (60 months for 9 parameters)
- The β_{MU} estimates are then saved for the portfolio sort

data

- We consider the FF5 model for MKT, SMB, HML, AI, ROE factors
- The UMB from ? obtained from Kenneth French
- The liquidity factor from Pástor and Stambaugh (2003)
- We consider, again, the CRSP dataset from 1986 to 2019

Univariate portfolio analysis

	1	2	3	4	5	6	7	8	9	10	L-S
						VW					
Excess return	1.12	1.01	0.76	0.77	0.88	0.80	0.58	0.67	0.61	0.75	-0.37
	(2.75)	(3.19)	(2.69)	(3.39)	(4.06)	(3.94)	(2.62)	(2.91)	(2.20)	(1.88)	(-1.20)
α_{FF5}	0.35	0.14	-0.07	-0.08	0.13	0.03	-0.15	-0.11	-0.07	-0.02	-0.37
	(1.51)	(0.66)	(-0.58)	(-1.04)	(1.43)	(0.36)	(-2.60)	(-1.28)	(-0.62)	(-0.13)	(-1.11)
						EW					
Excess return	1.56	1.31	1.25	1.06	1.06	1.09	0.98	0.96	1.02	1.04	-0.52
	(3.33)	(3.86)	(4.24)	(3.92)	(4.07)	(4.39)	(3.75)	(3.56)	(3.16)	(2.50)	(-2.15)
α_{FF5}	0.75	0.39	0.37	0.17	0.19	0.24	0.15	0.12	0.16	0.28	-0.47
	(2.60)	(2.68)	(3.32)	(1.90)	(2.41)	(3.25)	(1.83)	(1.37)	(1.50)	(1.72)	(-1.94)

Discussion of choices

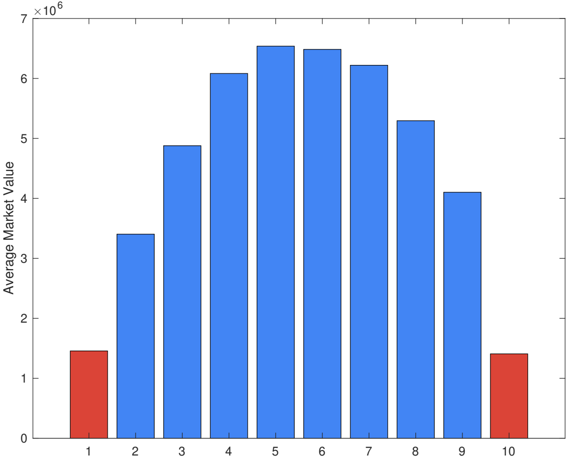
- Bali et al. (2017) make some choices in the construction of the analysis:

  1. The model in eq. (16)
  2. The 5-year rolling window
  3. They calculate breakpoints based on the entire cross-section instead of NYSE(Hou et al., 2020)
  4. They measure economic activity by a three month moving average of CFNAI

Different model specification in eq. (16)

	1	2	3	4	5	6	7	8	9	10	L-S
						EW					
Excess returns	1.51	1.26	1.15	1.04	1.09	1.07	0.98	0.98	1.08	1.16	-0.34
	3.27	3.79	3.92	3.94	4.33	4.31	3.72	3.43	3.38	2.70	-1.33
α_{FF5}	0.67	0.36	0.29	0.19	0.27	0.22	0.13	0.09	0.22	0.39	-0.28
	2.11	2.42	2.40	2.02	3.36	3.06	1.60	1.22	2.37	2.39	-0.97

Market value across portfolios



NYSE based breakpoints

	1	2	3	4	5	6	7	8	9	10	L-S
						VW					
Excess return	0.96	0.95	0.79	0.77	0.90	0.82	0.56	0.59	0.71	0.75	-0.20
	(2.58)	(3.43)	(3.21)	(3.36)	(4.32)	(3.89)	(2.55)	(2.56)	(2.86)	(2.30)	(-0.79)
α_{FF5}	0.14	0.10	-0.09	-0.08	0.20	0.02	-0.16	-0.17	-0.02	0.03	-0.12
	(0.65)	(0.81)	(-0.89)	(-0.87)	(2.10)	(0.27)	(-2.02)	(-1.88)	(-0.24)	(0.21)	(-0.39)

Discussion

- Bekaert et al. (2021) estimates risk aversion in a generalized habit model. They find a 0.82 correlation between economic uncertainty and risk aversion
- From SDF, we know that risk aversion has a one-to-one impact on the SDF and, thus, expected returns
- This naturally motivates that you could consider other variables related to risk aversion such as the variance risk premium of the aggregated market, or Bekaert et al. (2021) has uploaded their risk aversion measure

Brief overview of factors

Stocks

In addition to the papers cited already, you can look into:

- Default risk (Vassalou and Xing, 2004)
- volatility (Ang et al., 2006)
- Liquidity (Pástor and Stambaugh, 2003)
- Financial constraints (Owen et al., 2001)
- Tail risk (Kelly and Jiang, 2014, Bali et al., 2014)
- Profitablity (Fama and French, 2015)
- Bid-ask spreads (Corwin and Schultz, 2012)
- Financial Intermediation (Adrian et al., 2014)
- ETC....

Hou et al. (2020) examine 452 anomalies so you have an endless list of oppotunities!

Corporate bonds

- Bond maturity (Baker et al., 2003)
- MU (Bai et al., 2021)
- Long-run reversals (Bali et al., 2021)
- VaR (Bai et al., 2019)

Commodities

- Carry/basis (Yang, 2013, Koijen et al., 2018)
- Value and momentum (Asness et al., 2013)
- Low beta (Frazzini and Pedersen, 2014)
- Inventories (Gorton et al., 2013)

Factors

- Factor momentum (Ehsani and Linnainmaa, 2019)
- Alternatively, you can construct factor characteristics by the value-weighted difference between L-S portfolios in the anomaly and sort anomalies based on the difference

Currencies

We will discuss how currencies relate to what we have learned so far on Wednesday

References

Adrian, T., E. Etula, and T. Muir (2014). Financial intermediaries and the cross-section of asset returns. *The Journal of Finance 69*(6), 2557–2596.

Ang, A., J. Chen, and Y. Xing (2006). Downside risk. *The review of financial studies 19*(4), 1191–1239.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The journal of finance 61*(1), 259–299.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and momentum everywhere. *The Journal of Finance 68*(3), 929–985.

Augustin, P. and R. Tédongap (2021). Disappointment aversion, term structure, and predictability puzzles in bond markets. *Management Science 67*(10), 6266–6293.

Bai, J., T. G. Bali, and Q. Wen (2019). Common risk factors in the cross-section of corporate bond returns. *Journal of Financial Economics 131*(3), 619–642.

Bai, J., T. G. Bali, and Q. Wen (2021). Is there a risk-return tradeoff in the corporate bond market? time-series and cross-sectional evidence. *Journal of Financial Economics 142*(3), 1017–1037.

Baker, M., R. Greenwood, and J. Wurgler (2003). The maturity of debt issues and predictable variation in bond returns. *Journal of Financial Economics 70*(2), 261–291.

Bali, T. G., S. J. Brown, S. Murray, and Y. Tang (2017). A lottery-demand-based explanation of the beta anomaly. *Journal of Financial and Quantitative Analysis 52*(6), 2369–2397.

Bali, T. G., S. J. Brown, and Y. Tang (2017). Is economic uncertainty priced in the cross-section of stock returns? *Journal of Financial Economics 126*(3), 471–489.

Bali, T. G., N. Cakici, and R. F. Whitelaw (2014). Hybrid tail risk and expected stock returns: when does the tail wag the dog? *The Review of Asset Pricing Studies 4*(2), 206–246.

Bali, T. G., A. Subrahmanyam, and Q. Wen (2021). Long-term reversals in the corporate bond market. *Journal of Financial Economics 139*(2), 656–677.

Bekaert, G., E. C. Engstrom, and N. R. Xu (2021). The time variation in risk appetite and uncertainty. *Management Science*.

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica 77*(3), 623–685.

Bollerslev, T., A. J. Patton, and R. Quaedvlieg (2021). Realized semibetas: Disentangling "good" and "bad" downside risks. *Journal of Financial Economics*.

Corwin, S. A. and P. Schultz (2012). A simple way to estimate bid-ask spreads from daily high and low prices. *The Journal of Finance 67*(2), 719–760.

Drechsler, I. (2013). Uncertainty, time-varying fear, and asset prices. *The Journal of Finance 68*(5), 1843–1889.

Ehsani, S. and J. T. Linnainmaa (2019). Factor momentum and the momentum factor. Technical report, National Bureau of Economic Research.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of financial economics 116*(1), 1–22.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of Financial Economics 111*(1), 1–25.

Gorton, G. B., F. Hayashi, and K. G. Rouwenhorst (2013). The fundamentals of commodity futures returns. *Review of Finance 17*(1), 35–105.

Hou, K., C. Xue, and L. Zhang (2020). Replicating anomalies. *The Review of Financial Studies 33*(5), 2019–2133.

Jensen, M. C., F. Black, and M. S. Scholes (1972). The capital asset pricing model: Some empirical tests.

References

Jurado, K., S. C. Ludvigson, and S. Ng (2015). Measuring uncertainty. *American Economic Review 105*(3), 1177–1216.

Kelly, B. and H. Jiang (2014). Tail risk and asset prices. *The Review of Financial Studies 27*(10), 2841–2871.

Koijen, R. S., T. J. Moskowitz, L. H. Pedersen, and E. B. Vrugt (2018). Carry. *Journal of Financial Economics 127*(2), 197–225.

Ludvigson, S. C., S. Ma, and S. Ng (2021). Uncertainty and business cycles: exogenous impulse or endogenous response? *American Economic Journal: Macroeconomics 13*(4), 369–410.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica: Journal of the Econometric Society*, 867–887.

Novy-Marx, R. and M. Velikov (2022). Betting against betting against beta. *Journal of Financial Economics 143*(1), 80–106.

Owen, L., C. Polk, and J. Saa-Requejo (2001). Financial constraints and stock returns. *Review of Financial Studies 14*, 529–554.

Pástor, L. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns. *Journal of Political economy 111*(3), 642–685.

Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *The journal of finance 59*(2), 831–868.

Yang, F. (2013). Investment shocks and the commodity basis spread. *Journal of Financial Economics 110*(1), 164–184.