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Cross-currency Betting Arbitrage

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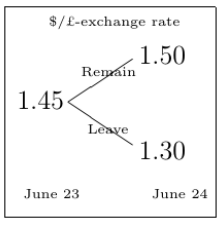
Imagine bookmakers posting these odds on Remain and Leave shortly[1] before the British EU-referendum on June 23, 2016:

EU-referendum outcome	Remain	Leave
Decimal odds	1.25	4.80

If we bet £1/1.25 on Remain and $1/4.80 Leave, we are guaranteed a payback of £1, but at a cost of £1.00833. Nothing particularly untoward here; a – quite modest – *cut* to the bookmakers of 0.83% (in non-annualized terms), no free lunch for us as punters.

However, the situation turns out to be more subtle. First, the punter is allowed to choose which currency the bet is made in; if he bets in £, he is repaid in £ (possibly £0), likewise for $. Second, the choice of currency does not affect the odds that the punters are given. Third, the outcome of the referendum affects the $/£-exchange rate (= the no. $ needed to buy £1). Specifically/heroically, assume that that happens in binomial fashion:[2]

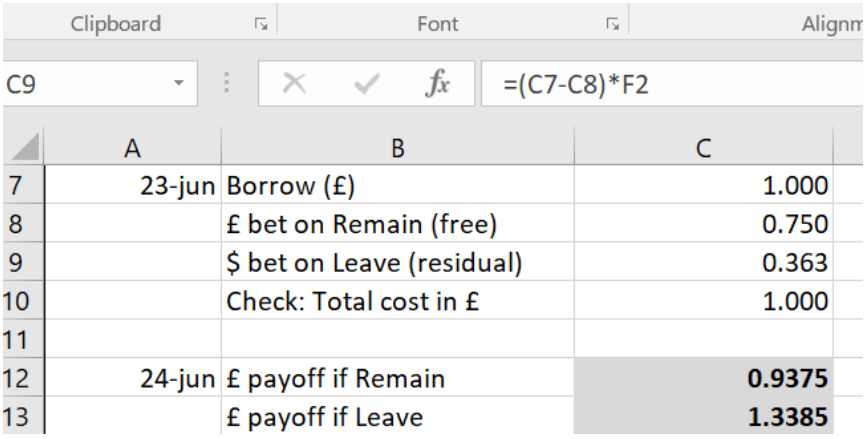
That creates opportunities. Intuitively, we should bet on Remain in £ as the Remain outcome strengthens £ relative to $, and if we lose the bet, we get zero anyway. Similarly, we bet on Leave in $. But, quantitatively: How much do we bet and does it work in the sense of creating a free lunch, an arbitrage? After all, there is a cut to overcome.



[1] So short is the time-horizon that I ignore everything, that has to do with interest rates in this column. Likewise, punters can borrow to their hearts' contents.

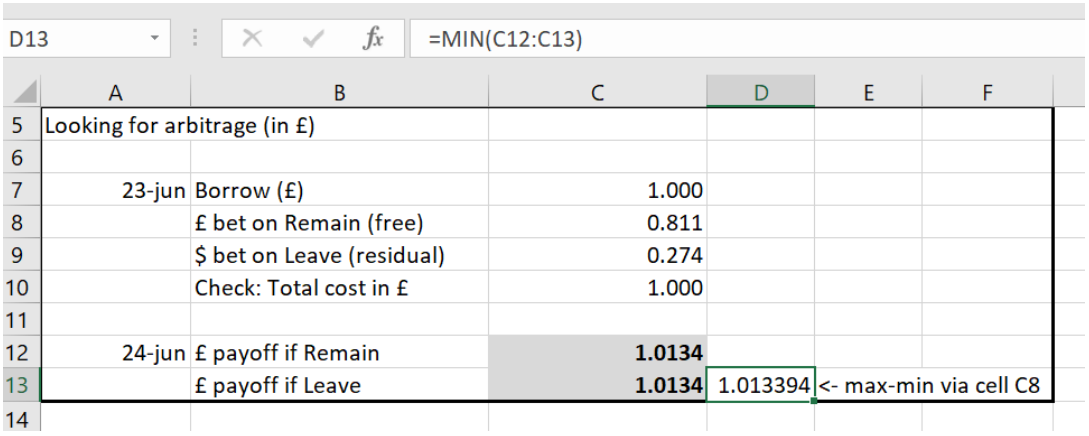
[2] We don't need probabilities *where we are going*. In later used, standard abstract notation, this model specification has multiplicative up- and down-factors u=1.50/1.45 and d = 1.30/1.45.

First, let us set things up in Excel, starting with some bookkeeping:



	A	B	C
7	23-jun	Borrow (£)	1.000
8		£ bet on Remain (free)	0.750
9		$ bet on Leave (residual)	0.363
10		Check: Total cost in £	1.000
11			
12	24-jun	£ payoff if Remain	**0.9375**
13		£ payoff if Leave	**1.3385**

Now put the minimal (over the two outcomes) £ payoff in a cell (say C13) and use Solver to maximize this by changing £ bet amount in cell C8 (put in non-negativity constraints in if you like). Doing that and zooming out gives



	A	B	C	D	E	F
5	Looking for arbitrage (in £)					
6						
7	23-jun	Borrow (£)	1.000			
8		£ bet on Remain (free)	0.811			
9		$ bet on Leave (residual)	0.274			
10		Check: Total cost in £	1.000			
11						
12	24-jun	£ payoff if Remain	**1.0134**			
13		£ payoff if Leave	**1.0134**	1.013394	<- max-min via cell C8	
14						

So for £1 we get more back, always £1.0134 irrespective of the outcome. This is an arbitrage opportunity – and a risk-free one at that.[3]

In Hanke, Poulsen &/Weissensteiner (2019) we go algebraic and consider a *(u,d)*-binomial model for the $/£-exchange rate and define (in obvious notation) the bookmakers' cut *c* via 1/odds_L + 1/odds_R = 1+ *c* . We then show that a betting arbitrage[4] can be constructed if (and only if) one of the two following (non-equivalent) conditions hold:

[3] The reader should try her hand at constructing a risk-free arbitrage by borrowing in $ and betting appropriately. This is also possible, but – perhaps somewhat surprisingly – the return on each $ borrowed is 0.0187, not 0.0134 as in the £-case. Thus, while the analyses in the two different economies are similar, they are not equivalent – a common pitfall and source of what are seemingly paradoxes in exchange rate modelling.

[4] When we say betting arbitrage, it is implicit that we only allow for long positions; we are punters, we cannot be bookmakers.

(A) 1- c*odds_L > d , (B) 1-c*odds_R > 1/u.

Furthermore, an arbitrage that pays off (the same, wlog) strictly positive amount in both future states (like in the numerical example) can be constructed if and only if both condition (A) and (B) hold. The proofs of these statements are straightforward. I will not divulge how long time it took us to get them right.

The source checking reader who is aware of the insights of Neuberger (2016) will note that our paper runs 30+ pages and ask, "What's up with that?" The answer – arguably arrogant and academic – is "Carefully checking/verification of details – both theoretical and practical". That involves:

- Analyzing non-binomial models for the exchange rate – such as for instance the mixing models suggested in Clark & Amen (2018) and Hanke, Poulsen & Weissensteiner (2018). In this case, currency independent odds do not generally give arbitrage, but with reasonable, estimable distributions, it leads to approximate arbitrages or good deals; very high Sharpe ratios (above four).
- Documenting that you can choose which currency to bet in without costs stifling you, and that the currency does not affect odds.
- Showing that by choosing the best odds across bookmakers, you can bring cuts down to the range 0-2%, which is needed for something akin to arbitrage to be realistic. This happens despite individual bookmakers' cuts being much larger, 5-10%. Also interestingly, it very rarely happens that the cut is even slightly negative (which would mean easy arbitrage). Bookmakers are watching each other and know that punters are watching them – as a quick search of "betting arbitrage" or "sure bet" reveals.

By combining the pieces above, we demonstrate that the betting arbitrage is not just a toy example. It happened on many days before the EU-referendum. And it happened in the Trump/Hillary presidential election with the Mexican peso replacing the pound. And it is likely to happen again.

References

Clark, Iain J. & Saeed Amen (2018), "Using FX Volatility Skew to Assess the Implied Probability of Hard Brexit", *Wilmott*, issue 95 (May), pp. 64-69.

Hanke, Michael, Rolf Poulsen & Alex Weissensteiner (2018), "Event-Related Exchange-Rate Forecasts Combining Information from Betting Quotes and Option Prices", *Journal of Financial and Quantitative Analysis*, volume 53, issue 6, pp. 2663-2683.

Hanke, Michael, Rolf Poulsen & Alex Weissensteiner (2019), "Numeraire Dependence in Risk-Neutral Probabilities of Event Outcomes", *Journal of Derivatives*, forthcoming.

Neuberger, Anthony (2016), “How to beat the bookies with a Brexit bet”, *The Conversation*, https://tinyurl.com/ycs27wy9