Asset Pricing

Arbitrage Pricing Theory (APT)

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Outline

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   - Asness, Moskowitz, and Pedersen (2013)
   - Lustig, Roussanov, and Verdelhan (2011)

Arbitrage Pricing Theory (Ross, 1976)

The Arbitrage Pricing Theory (APT) of Ross (1976) is a no-arbitrage model stating that a few (K, say) common factors drive expected returns, i.e.,

$$\widetilde{r}_i = \mu_i + \sum_{k=1}^{K} b_{ik} \widetilde{f}_k + \widetilde{\varepsilon}_i \tag{1}$$

where μ_i is an asset-specific constant, $\widetilde{f}_k$ are factor innovations with $\mathbb{E}\left[\widetilde{f}_k\right] = 0$, b_{ik} are factor loadings, and $\widetilde{\varepsilon}_i$ is an idiosyncratic risk component specific to asset i that can be diversified away under the following assumptions

$$\mathbb{E}\left[\widetilde{\varepsilon}_i\right] = 0 \tag{2}$$

$$\text{Cov}\left[\widetilde{f}_k, \widetilde{\varepsilon}_i\right] = 0 \ \forall i,k \tag{3}$$

$$\text{Cov}\left[\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_h\right] = 0 \ \forall i \neq h \tag{4}$$

$$\text{Var}\left[\widetilde{\varepsilon}_i\right] = \mathbb{E}\left[\widetilde{\varepsilon}_i^2\right] = \sigma^2_{\varepsilon_i} \leq s^2 < \infty \tag{5}$$

A few remarks about the framework

- The substantial part is the assumption that Cov $[\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_h] = 0 \ \forall i \neq h$, which implies that *all* common return characteristics are captured by $\widetilde{f}_k$

- The implications of the APT are only "approximative", but can often be assumed close to exact

- The failure of the CAPM to explain stylized facts suggests that one or more additional factors may be needed to fully characterize the behavior of expected returns

- The market portfolio is but one plausible risk factor that can exist in the economy from the viewpoint of the APT

- In a recent study, Harvey et al. (2016) identify no less than 316 factors that have been used to explain the cross-section of expected returns and Harvey and Liu (2019) update the number to 382 risk factors

Arbitrage

Arbitrage

An arbitrage opportunity is a feasible, self-financing (zero net-investment) trading strategy involving two or more securities with one of the following characteristics

1. It does not cost anything at initiation, and it generates a for sure positive profit by a certain date in the future
2. It generates a positive profit at initiation, and it has a for sure nonnegative payoff by a certain date in the future

The no-arbitrage condition requires that no such arbitrage opportunities exists

* In efficient, competitive asset markets, where arbitrage trades are feasible, arbitrage opportunities should be rare, fleeting, and temporary
* Hence, it is reasonable to assume that market prices reflect an absence of arbitrage opportunities
* This assumption leads to the law of one price, which establishes that securities with identical payoffs should have the same price

No-arbitrage versus equilibrium models

* Equilibrium models make assumptions about supply and demand, market competitiveness, and the rationality and preferences of investors
* This allows us to formulate and solve the investor's (constrained) optimization problem, typically with reference to an expected utility function
* While equilibrium models imply the no-arbitrage condition, they typically come with an extra set of restrictive assumptions
    - An arbitrage opportunity would imply that at least one agent can reach a higher utility without violating any (budget) constraints due to their costless setup
* However, the reverse is not true: No-arbitrage does not imply equilibrium!
* In this set of slides, we therefore want to investigate just how far we can get by assuming no-arbitrage only

Arbitrage pricing example

* An early and classic example of arbitrage pricing originates from the foreign exchange (FX) market literature
* Specifically, we can obtain a no-arbitrage pricing formula for the forward exchange rate from the covered interest rate parity (CIP)
* CIP links spot and forward exchange rates through foreign and domestic interest rates, where the notation is as follows
  - S_t denotes the spot exchange rate expressed as the units of foreign currency per U.S. dollar (USD)
  - F_t denotes the today-available forward exchange rate in units of foreign currency per USD to be purchased one period into the future
  - i_t and $i_t^\star$ denote the US and foreign per-period risk-free rates of return for one-period borrowing and lending

Investing domestically or abroad

- Consider a US investor with current wealth $Y_t = \$1$ that faces the decision between investing at home or abroad

  1. Investing in the US money market (buying domestic bond with interest i_t) provides a return of
  $$Y_{t+1} = (1 + i_t) \tag{6}$$
  2. Selling $\$1$ at the spot price and obtaining S_t units of foreign currency to invest in the foreign money market at $i_t^\star$ and hedge currency risk by selling the proceed $S_t\,(1 + i_t^\star)$ forward
  $$Y_{t+1}^\star = S_t\,(1 + i_t^\star)\,\frac{1}{F_t} \tag{7}$$

- The investments i_t and $i_t^\star$ are identical in terms of maturity, default risk, liquidity risk, etc. The only difference is the currency of denomination

Covered interest parity and the forward rate

- By the law of one price and no-arbitrage, we must have that $Y_{t+1} = Y^{\star}_{t+1}$, implying the following relation known as the covered interest parity

$$(1 + i_t) = \frac{S_t}{F_t}\,(1 + i^{\star}_t) \tag{8}$$

- If this parity is violated, investors have the opportunity to make arbitrage profits of the first type since all quantities are known today
- In the absence of arbitrage, (8) implies a pricing relation for the no-arbitrage forward rate

$$F_t = S_t \frac{(1 + i^{\star}_t)}{(1 + i_t)} \tag{9}$$

- Thus, pricing assets by way of no-arbitrage is attractive in the sense that assumptions regarding investor preferences or beliefs are not required

A formal statement of the model

- The Arbitrage Pricing Theory (APT) assumes a model of financial markets that is both frictionless and competitive in which the returns to each traded asset i are governed by a factor structure

$$\widetilde{r}_i = \mu_i + \sum_{k=1}^{K} b_{ik} \widetilde{f}_k + \widetilde{\varepsilon}_i \tag{10}$$

$$= \mu_i + \mathbf{b}_i^\top \widetilde{\mathbf{f}} + \widetilde{\varepsilon}_i \tag{11}$$

where $\mathbf{b}_i$ is a $K \times 1$ vector of factor loadings, $\widetilde{\mathbf{f}}$ is a $K \times 1$ vector of factor innovations, and $\widetilde{\varepsilon}_i$ is an idiosyncratic error term

- We can similar write the model in matrix form by letting $\widetilde{\mathbf{r}}$ and $\boldsymbol{\mu}$ be $N \times 1$ vectors of (mean) returns, $\mathbf{B}$ an $N \times K$ matrix of factor loadings, $\widetilde{\mathbf{f}}$ a $K \times 1$ vector of factor innovations, and $\widetilde{\boldsymbol{\varepsilon}}$ an $N \times 1$ vector of idiosyncratic errors

$$\widetilde{\mathbf{r}} = \boldsymbol{\mu} + \mathbf{B}\widetilde{\mathbf{f}} + \widetilde{\boldsymbol{\varepsilon}} \tag{12}$$

- We will maintain the assumptions that $\mathbb{E}\left[\widetilde{\mathbf{f}}\right] = \mathbf{0}$, $\mathbb{E}\left[\widetilde{\boldsymbol{\varepsilon}}\right] = \mathbf{0}$, $\mathbb{E}\left[\widetilde{\mathbf{f}}\,\widetilde{\boldsymbol{\varepsilon}}^\top\right] = \mathbf{0}$ and $\mathbb{E}\left[\widetilde{\boldsymbol{\varepsilon}}\,\widetilde{\boldsymbol{\varepsilon}}^\top\right] = \mathbf{V}_{\widetilde{\boldsymbol{\varepsilon}}} = \sigma^2_{\widetilde{\boldsymbol{\varepsilon}}}\mathbf{I}_N$, and potentially $\mathbb{E}\left[\widetilde{\mathbf{f}}\,\widetilde{\mathbf{f}}^\top\right] = \mathbf{V}_{\widetilde{\mathbf{f}}} = \sigma^2_{\widetilde{\mathbf{f}}}\mathbf{I}_K$

The APT restriction

The APT restriction

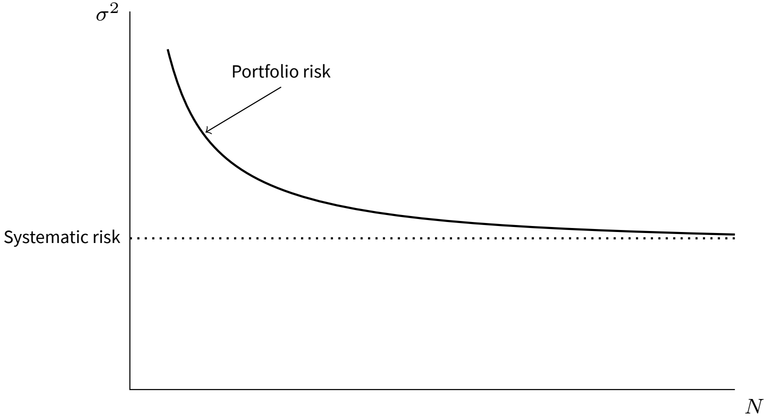
Following Ross (1976), we can impose no-arbitrage on the above model such that there exists a $K \times 1$ vector of factor risk premia $\boldsymbol{\lambda}_K$, which provides us with the following APT restriction

$$\mathbb{E}\left[\tilde{\mathbf{r}}\right] = \boldsymbol{\mu} \approx \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K \tag{13}$$

- The types of no-arbitrage arguments provided by Stephen Ross completely changed the game and has facilitated the discovery of some of the best known asset pricing models
- Ross was able to derive the, at the time, highly successful and hailed CAPM as well as much more general asset pricing models using a very limited number of assumptions
- In the following, we will derive the above statement and show how to derive the CAPM within the APT framework

Diversification eliminates idiosyncratic risk

- One of the main ideas behind the APT is that idiosyncratic risk can be eliminated by constructing a well-diversified portfolio consisting of N assets, where N is assumed to be large



A heuristic derivation of the ATP restriction

- Form an arbitrage (zero-investment) portfolio $\boldsymbol{\omega}_A$ using all assets such that $\boldsymbol{\omega}_A^\top \mathbf{1} = 0$ and whose return is given by

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} = \boldsymbol{\omega}_A^\top \boldsymbol{\mu} + \boldsymbol{\omega}_A^\top \mathbf{B}\widetilde{\mathbf{f}} + \boldsymbol{\omega}_A^\top \widetilde{\boldsymbol{\varepsilon}} \tag{14}$$

- If we assume that each weight in the portfolio is "well-diversified" such that $\omega_i \approx 1/N$, then we have that

$$\boldsymbol{\omega}_A^\top \widetilde{\boldsymbol{\varepsilon}} \approx \frac{1}{N}\sum_{i=1}^{N} \widetilde{\varepsilon}_i \tag{15}$$

which by the law of large numbers and $\mathbb{E}\left[\widetilde{\varepsilon}_i\right] = 0$ implies that

$$\lim_{N\to\infty} \boldsymbol{\omega}_A^\top \widetilde{\boldsymbol{\varepsilon}} \approx \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \widetilde{\varepsilon}_i = 0 \tag{16}$$

which shows that for N large enough we can diversify away idiosyncratic risk

A heuristic derivation of the ATP restriction

- This implies that we can write the approximate factor model for sufficient large N as (approximate because we need $N \to \infty$)

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} \approx \boldsymbol{\omega}_A^\top \boldsymbol{\mu} + \boldsymbol{\omega}_A^\top \mathbf{B}\widetilde{\mathbf{f}} \tag{17}$$

- The next step involves forming the arbitrage portfolio $\boldsymbol{\omega}_A$ such that it has no systematic factor risk, i.e., $\boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, so that the portfolio return becomes

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} \approx \boldsymbol{\omega}_A^\top \boldsymbol{\mu} \tag{18}$$

- In the absence of arbitrage, the expected return on a zero-investment portfolio with no systematic factor risk ($\boldsymbol{\omega}_A^\top \mathbf{1} = \boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$) must be equal to zero

$$\boldsymbol{\omega}_A^\top \boldsymbol{\mu} = 0 \tag{19}$$

No-arbitrage condition

Let $\boldsymbol{\omega}_A$ be a well-diversified portfolio. If $\boldsymbol{\omega}_A^\top \mathbf{1} = 0$ and $\boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, then $\boldsymbol{\omega}_A^\top \boldsymbol{\mu} = 0$

Is the ATP restriction arbitrage free?

* The no-arbitrage conditions imply that $\boldsymbol{\mu}$ is spanned by $\mathbf{1}$ and $\mathbf{B}$, which leads us to suggest the following model for expected returns

$$\boldsymbol{\mu} \approx \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K \tag{20}$$

* We can verify that the restriction is arbitrage free using the no-arbitrage conditions

$$\begin{aligned} \boldsymbol{\omega}_A^\top \boldsymbol{\mu} &\approx \boldsymbol{\omega}_A^\top \mathbf{1}\lambda_0 + \boldsymbol{\omega}_A^\top \mathbf{B}\boldsymbol{\lambda}_K \qquad (21) \\ &= 0\lambda_0 + \mathbf{0}\boldsymbol{\lambda}_K = 0 \qquad (22) \end{aligned}$$

* If a riskless asset exists, then $\lambda_0 = r_f$ such that the APT restriction becomes

$$\mathbb{E}\left[\widetilde{\mathbf{r}}\right] = \boldsymbol{\mu} \approx \mathbf{1}r_f + \mathbf{B}\boldsymbol{\lambda}_K \tag{23}$$

* If a riskless asset does not exist, then λ_0 is the return on a portfolio with no systematic risk, i.e., a zero-beta rate (similar to zero-beta CAPM)

A brief remark on exact factor pricing

- If we are willing to assume that $\widetilde{\varepsilon} = 0$, meaning that the included risk factor(s) account for *all* risk, then we have an exact factor model

$$\widetilde{\mathbf{r}} = \boldsymbol{\mu} + \mathbf{B}\widetilde{\mathbf{f}} \tag{24}$$

- Following the same steps as above, but without having to rely on the law of large numbers, we have that the APT restriction is exact, i.e.,

$$\mathbb{E}\left[\widetilde{\mathbf{r}}\right] = \boldsymbol{\mu} = \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K \tag{25}$$

- If absence of arbitrage is the only assumption, then the model is necessarily only approximative

- This implies that the APT restriction does not directly provide testable restrictions for asset returns, but for N large enough the differences between approximate and exact factor pricing are likely to be very small

APT restriction for the CAPM

- As mentioned, Ross (1976) completely turned things around by being able to obtain the CAPM by only assuming no-arbitrage
- Assume that $K = 1$ and let $\widetilde{f}$ be an unspecified market wide factor with expectation zero and factor loadings $\boldsymbol{\beta}$
- Assume further that a riskless asset exists. In this case, the APT restriction becomes

$$\boldsymbol{\mu} \approx \mathbf{1} r_f + \boldsymbol{\beta} \lambda_M \tag{26}$$

- If we let $\widetilde{f}$ be the market portfolio, then we can form a portfolio $\boldsymbol{\omega}_M$ that satisfies the following CAPM-implied restrictions

$$\boldsymbol{\omega}_M^\top \mathbf{1} = 1 \tag{27}$$

$$\boldsymbol{\omega}_M^\top \boldsymbol{\beta} = 1 \tag{28}$$

Derivation of the CAPM using the APT framework

* Using the APT relation, we can define the return on the market portfolio as

$$\begin{aligned} \mathbb{E}\left[\widetilde{r}_M\right] &= \boldsymbol{\omega}_M^\top \boldsymbol{\mu} && (29) \\ &\approx \boldsymbol{\omega}_M^\top \mathbf{1} r_f + \boldsymbol{\omega}_M^\top \boldsymbol{\beta} \lambda_M && (30) \\ &\approx r_f + \lambda_M && (31) \end{aligned}$$

* Next, solve for λ_M and insert into the APT restriction in (26) to obtain

$$\boldsymbol{\mu} \approx \mathbf{1} r_f + \boldsymbol{\beta} \left(\mathbb{E}\left[\widetilde{r}_M\right] - r_f\right) \tag{32}$$

* Obviously, if we assume exact factor pricing, then we obtain the CAPM exactly
* Thus, we have derived the (approximative) Security Market Line (SML) without the need for any assumptions like mean-variance efficiency, normality of returns, quadratic utility of investors, unlimited borrowing/lending at the same risk-free rate, or general equilibrium. The only assumptions used are absence of arbitrage and that investors are well diversified

Empirical applications of the APT

- In the Arbitrage Pricing Theory (APT), we know that the common variation in returns to each traded asset i are governed by a K–factor structure

$$\widetilde{r}_i = \mu_i + \sum_{k=1}^{K} b_{ik} \widetilde{f}_k + \widetilde{\varepsilon}_i \tag{33}$$

$$= \mu_i + \mathbf{b}_{ik}^{\top} \widetilde{\mathbf{f}} + \widetilde{\varepsilon}_i \tag{34}$$

- The APT itself, however, is agnostic about the sources and identification of the factors themselves and their number
- Moreover, there is nothing in the theory that guarantees that all factor risk premia $\boldsymbol{\lambda}_K$ are strictly positive
- Thus, determining which and how many factors to use in the APT has become a large and important empirical issue in financial economics

Identifying the risk factors

* The aim of this exercise is to identify a small number of factors that determine *all* asset returns

* Generally speaking, one can split the identification into three different categories (see Connor (1995))

    1. **Economic factors:** Macroeconomic variables such as industrial production, GDP, (un)employment, term spreads, and other variables similarly thought to capture business-cycle related risks

    2. **Financial factors:** Portfolio returns, returns on systematically important financial assets (i.e. the short-term interest rate) or returns to factor-mimicking portfolios

    3. **Statistical factors:** For both previous cases, one can use dynamic factors models, principal component analysis, or similar statistical approaches to identify a few common factors from a larger set of data

Macroeconomic factor models

* Macroeconomic factor models originate from the idea that asset prices are determined by exposure to macroeconomic events (e.g., recessions)
* The aim is therefore to uncover a set of macroeconomic variables, or factors, that can reliably proxy underlying systematic state variables
* These state variables should ideally provide good measures of "bad times", that is, times in which marginal utility of consumption is high
* Recall our discussion of the inverse relationship between utility and marginal utility, which prompts investors to prefer assets that payoff in bad times
* In general, we tend to believe that the risk premia offered by different assets should reflect aggregate, macroeconomic risks

The Chen, Roll, and Ross (1986) model

The Chen, Roll, and Ross (1986) model

The Chen et al. (1986) (CRR) model is a first and classic example of a macroeconomic factor model

$$\mu = \lambda_0 + b_{MP}\gamma_{MP} + b_{UI}\gamma_{UI} + b_{DEI}\gamma_{DEI} + b_{UTS}\gamma_{UTS} + b_{UPR}\gamma_{UPR} \tag{35}$$

where we consider the following state variables (in their notation)

1. $MP_t = \ln IP_t - \ln IP_{t-1}$ is the log monthly growth in industrial production
2. $UI_t = I_t - \mathbb{E}_{t-1}[I_t]$ is monthly unanticipated inflation
3. $DEI_t = \mathbb{E}_t[I_{t+1}] - \mathbb{E}_{t-1}[I_t]$ is the change in expected inflation
4. $UTS_t = LGB_t - TB_{t-1}$ is the slope of the term structure
5. $UPR_t = BAA_t - LGB_t$ is the unanticipated default premia

Important insights from the CRR model

* Chen et al. (1986) found that their model could price the cross-section of 20 portfolios sorted on market value (size) quite well
* Neither the market portfolio nor aggregate consumption growth were priced separately in the cross-section (Why is this important?)
* Advantages of their model includes
  1. Factors are chosen from well-founded economic arguments
  2. Provide a direct link between the macroeconomy and asset prices
* Disadvantages of their model includes
  1. Macroeconomic variables are often measured poorly, e.g. problems with publication lags and subsequent revisions
  2. Macroeconomic variables are often published with low frequency (e.g. quarterly)

An incomplete list of alternative macroeconomic models

* Jagannathan and Wang (1996) consider the growth rate of labor income as a proxy for the return on human capital as a state variable (and the default premium (PREM) as a measure of the business cycle)

$$\mu = \lambda_0 + b_M \gamma_M + b_{\text{PREM}} \gamma_{\text{PREM}} + b_{\text{LABOR}} \gamma_{\text{LABOR}} \tag{36}$$

* Lettau and Ludvigson (2001) explore the influence of the consumption-wealth ratio (CAY) on the cross-section of stock returns

$$\mu = \lambda_0 + b_M \gamma_M + b_{\text{CAY}} \gamma_{\text{CAY}} + b_{\text{CAY} \times \text{M}} \gamma_{\text{CAY} \times \text{M}} \tag{37}$$

* Petkova (2006) consider a five-factor macroeconomic model specified as

$$\mu = \lambda_0 + b_M \gamma_M + b_{TERM} \gamma_{TERM} + b_{DEF} \gamma_{DEF} + b_{DP} \gamma_{DP} + b_{TBL} \gamma_{TBL} \tag{38}$$

  where M is the market, and the remaining variables are innovations to the term premium ($TERM$), the default premium (DEF), the log market dividend yield (DP), and the short-term Treasury bill rate (TBL)

Factor-mimicking portfolios

Factor-mimicking portfolios

Factor-mimicking portfolios are portfolios constructed from a set of base asset in such a way that their returns mimick the behavior of a particular risk factor

- Recall that the Arbitrage Pricing Theory (APT) is agnostic about the number, sources, and identification of the factors to be included in the model
- The only prerequisite is that they should be relevant risk factors (or state variables) that captures risk relevant for asset pricing
- Thus, one could argue that building factors that mimick the behavior of asset pricing anomalies may improve upon existing models
- Fama and French (1993) were among the first to take up this agenda by building on their 1992 paper identifying size (ME) and book-to-market equity (BE/ME) ratios as variables with the ability to explain the cross-section of expected returns

The Fama and French (1993) three-factor model

The Fama and French (1993) three-factor model

Fama and French's (1993) return-based three-factor model is specified as follows

$$\mu = \lambda_0 + b_M \gamma_M + b_{SMB} \gamma_{SMB} + b_{HML} \gamma_{HML} \tag{39}$$

where M is the excess return on the market portfolio, SMB is the excess returns to a portfolio mimicking risk related to size, and HML is the excess returns to a portfolio mimicking risk related to value

- The main result in Fama and French (1993) is that these portfolios, created to mimick risk factors related to size and value, capture the cross-sectional variation in returns surprisingly well
- The motivation comes from Fama and French (1992) in which they provide evidence that size and value proxy for common risk factors in returns

Building portfolios of size and value groups

* Fama and French (1993) sort the universe of NYSE, AMEX, and NASDAQ stocks into three value (BE/ME) groups and two size (ME) groups
* The decision to sort firms into three value groups and only two size groups follows the evidence in Fama and French (1992) that value has a stronger role than size
* They then build six portfolios from the intersection of these portfolios, as illustrated below

Table 14.1: Stock sort underlying the Fama and French (1993) factor construction

Biggest (B) ½ of stocks ranked by ME	BL	BM	BH
Smallest (S) ½ of stocks ranked by ME	SL	SM	SH
	Lowest (L) 30% of stocks as ranked by (BE/ME)	Middle (M) 40% of stocks as ranked by (BE/ME)	Highest (H) 30% of stocks as ranked by (BE/ME)

Building the size and value factors

* The "size" factor, referred to as Small-Minus-Big (SMB), is then constructed as

$$SMB_t = \frac{1}{3}\left[SL_t + SM_t + SH_t\right] - \frac{1}{3}\left[BL_t + BM_t + BH_t\right] \tag{40}$$

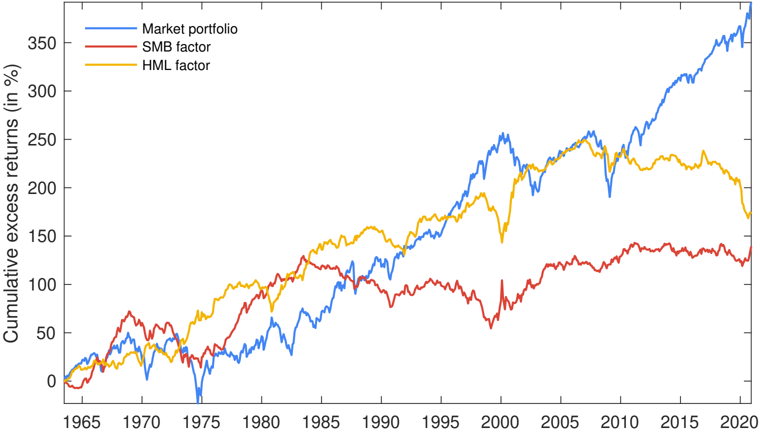
* The "value" factor, denoted High-Minus-Low (HML), is constructed as

$$HML_t = \frac{1}{2}\left[SH_t + BH_t\right] - \frac{1}{2}\left[SL_t + BL_t\right] \tag{41}$$

	MKT	SMB	HML
Panel A: Descriptive statistics (1963:07-2020:12)			
Mean	6.81	2.42	3.01
Std	15.48	10.55	9.93
Sharpe ratio	0.44	0.23	0.30
AR(1)	0.06	0.05	0.18
Panel B: Correlation matrix			
MKT	1.00		
SMB	0.30	1.00	
HML	-0.22	-0.17	1.00

Cumulative profits to size and value trading

* Consider the cumulative returns earned if an investor had followed the strategy over the period 1963:07 to 2020:12



Testing the Fama-French three-factor model

- We already know that the CAPM is unable to explain the cross-section of the returns to the 25 size and value sorted portfolios (see CAPM slides)
- We therefore want to investigate whether the Fama-French three-factor model can overcome this obstacle using the Fama-MacBeth methodology
- As a first step, we run the first-pass regression to obtain the full-sample time series betas

$$
\begin{aligned}
\widetilde{r}_{t,i} - \widetilde{r}_{t,f} = \alpha_i + b_{iM}\left(\widetilde{r}_{t,M} - \widetilde{r}_{t,f}\right) \\
+ b_{i,SMB} SMB_t + b_{i,HML} HML_t + \widetilde{\varepsilon}_{t,i}
\end{aligned}
\tag{42}
$$

- **Note:** The book uses $(SMB_t - r_f)$ and $(HML_t - r_f)$ in their formulation, which, put simply, is **completely wrong** as the factors are already zero-cost portfolios by design

Factor loadings for the market portfolio

* Market loadings are close to one, suggesting that the market factor captures the general risk of being a stock relative to a risk-free asset

	Book-to-market equity (BE/ME)				
	Low	2	3	4	High
	Panel A: Mean annualized excess returns				
Small	4.15	10.09	9.49	11.76	12.62
2	6.78	9.78	10.54	10.56	11.71
3	7.18	9.79	9.22	10.50	11.89
4	8.30	7.87	8.43	9.96	10.12
Large	6.97	6.50	7.03	5.90	7.61
	Panel B: Factor loadings for the market portfolio				
Small	1.11	0.97	0.93	0.89	0.96
2	1.14	1.01	0.97	0.96	1.09
3	1.11	1.03	0.98	0.99	1.08
4	1.07	1.07	1.04	1.03	1.16
Large	0.98	0.98	0.95	1.03	1.13

Factor loadings for the size factor (SMB)

* The size factor captures the size dimension rather well and the loadings are positive (negative) for small (large) stocks

	Book-to-market equity (BE/ME)				
	Low	2	3	4	High
	Panel A: Mean annualized excess returns				
Small	4.15	10.09	9.49	11.76	12.62
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4	8.30	7.87	8.43	9.96	10.12
Large	6.97	6.50	7.03	5.90	7.61
	Panel B: Factor loadings for the size factor				
Small	1.37	1.32	1.09	1.08	1.07
2	1.00	0.90	0.75	0.72	0.88
3	0.74	0.59	0.43	0.43	0.57
4	0.39	0.21	0.16	0.22	0.28
Large	-0.24	-0.20	-0.24	-0.21	-0.16

Factor loadings for the value factor (HML)

* The value factor captures the value dimension rather well and the loadings are positive (negative) for value (growth) stocks

	Book-to-market equity (BE/ME)				
	Low	2	3	4	High
	Panel A: Mean annualized excess returns				
Small	4.15	10.09	9.49	11.76	12.62
2	6.78	9.78	10.54	10.56	11.71
3	7.18	9.79	9.22	10.50	11.89
4	8.30	7.87	8.43	9.96	10.12
Large	6.97	6.50	7.03	5.90	7.61
	Panel B: Factor loadings for the value factor				
Small	-0.29	0.02	0.30	0.47	0.68
2	-0.36	0.12	0.39	0.57	0.78
3	-0.40	0.14	0.41	0.60	0.81
4	-0.40	0.20	0.42	0.58	0.81
Large	-0.36	0.07	0.28	0.66	0.84

Cross-sectional asset pricing test

- Finally, estimate the second-pass regression using the full-sample loadings

$$\widetilde{r}_{t,i} - \widetilde{r}_{t,f} = \gamma_{t,0} + \gamma_{t,M}\widehat{b}_{i,M} + \gamma_{t,SMB}\widehat{b}_{i,SMB} + \gamma_{t,HML}\widehat{b}_{i,HML} + \eta_{t,i} \quad (43)$$

for each month t and form estimates using the standard Fama-MacBeth approach

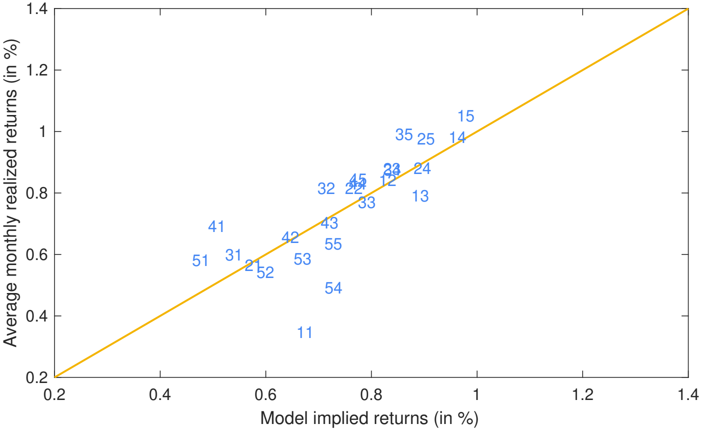
$$\widehat{\gamma}_j = \frac{1}{T}\sum_{t=1}^{T}\widehat{\gamma}_{t,j} \quad \text{and} \quad \sigma^2(\gamma_j) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\widehat{\gamma}_{t,j} - \widehat{\gamma}_j\right)^2 \quad (44)$$

- The results from estimating the Fama and French (1993) three-factor model are presented in the below table

	γ_0	γ_M	γ_{SMB}	γ_{HML}	R^2 (%)
Estimate	1.19	-0.58	0.16	0.27	60.89
s.e.	(0.25)	(0.30)	(0.12)	(0.11)	
tstat	[4.71]	[-1.93]	[1.33]	[2.44]	

Pricing error plot for Fama-French three-factor model

- Consider the pricing error plot for the Fama-French three-factor model, where we note a much improved fit relative to the CAPM



Interpreting the size and value factors

- Although there is no generally accepted story explaining why the SMB and HML factors work so well, they must be capturing fundamental systematic macroeconomic risks (e.g., business cycle risk) or behavioral biases

- On the rational side, potential explanations include

  - **Liew and Vassalou (2000):** SMB and HML are leading indicators of future GDP growth

  - **Vassalou (2003):** A factor mimicking news related to future GDP growth can, together with the market portfolio, explain the 25 size-value portfolios as well as the Fama-French three-factor model

  - **Zhang (2005):** Investment irreversibility and countercyclical risk aversion serves as underlying determinants of the HML factor

- On the behavioral side, potential explanations include

  - **Lakonishok et al. (1994):** Value premium attributable to investor overreaction and over-extrapolation of recent news about firm returns

  - **Barberis and Huang (2001):** Value premium is caused by loss aversion and mental accounting

Jegadeesh and Titman's (1993) momentum factor

- Early research (e.g. De Bondt and Thaler (1985, 1987)) investigates contrarian strategies that exploit return reversals

- Today focus is mostly on momentum strategies of the type studied in Jegadeesh and Titman (1993), where investors bet on return continuations

- Jegadeesh and Titman (1993) uncover continuation in stock prices at intermediate horizons by analyzing 16 relative strength momentum trading strategies based on all NYSE and AMEX stocks over the period 1965:01 to 1989:12, which are intersections of
  - Look-back periods of $J = 3,6,9,12$ months prior cumulative returns
  - Holding periods of $K = 3,6,9,12$ months following portfolio formation

- Stocks are ranked on their momentum signal and sorted into ten portfolios, where the strategy shorts the losers and buys the winners

Overview of Jegadeesh and Titman's main results

* The main results from Jegadeesh and Titman (1993) are represented in the below table, where numbers are to be interpreted as percentages and Panel B introduces a one week lag in portfolio formation

		Panel A				Panel B			
J		$K =$ 3	6	9	12	$K =$ 3	6	9	12
3	Sell	0.0108	0.0091	0.0092	0.0087	0.0083	0.0079	0.0084	0.0083
		(2.16)	(1.87)	(1.92)	(1.87)	(1.67)	(1.64)	(1.77)	(1.79)
3	Buy	0.0140	0.0149	0.0152	.0156	0.0156	0.0158	0.0158	0.0160
		(3.57)	(3.78)	(3.83)	(3.89)	(3.95)	(3.98)	(3.96)	(3.98)
3	Buy-sell	0.0032	0.0058	0.0061	0.0069	0.0073	0.0078	0.0074	0.0077
		(1.10)	(2.29)	(2.69)	(3.53)	(2.61)	(3.16)	(3.36)	(4.00)
6	Sell	0.0087	0.0079	0.0072	0.0080	0.0066	0.0068	0.0067	0.0076
		(1.67)	(1.56)	(1.48)	(1.66)	(1.28)	(1.35)	(1.38)	(1.58)
6	Buy	0.0171	0.0174	0.0174	0.0166	0.0179	0.0178	0.0175	0.0166
		(4.28)	(4.33)	(4.31)	(4.13)	(4.47)	(4.41)	(4.32)	(4.13)
6	Buy-sell	0.0084	0.0095	0.0102	0.0086	0.0114	0.0110	0.0108	0.0090
		(2.44)	(3.07)	(3.76)	(3.36)	(3.37)	(3.61)	(4.01)	(3.54)
9	Sell	0.0077	0.0065	0.0071	0.0082	0.0058	0.0058	0.0066	0.0078
		(1.47)	(1.29)	(1.43)	(1.66)	(1.13)	(1.15)	(1.34)	(1.59)
9	Buy	0.0186	0.0186	0.0176	0.0164	0.0193	0.0188	0.0176	0.0164
		(4.56)	(4.53)	(4.30)	(4.03)	(4.72)	(4.56)	(4.30)	(4.04)
9	Buy-sell	0.0109	0.0121	0.0105	0.0082	0.0135	0.0130	0.0109	0.0085
		(3.03)	(3.78)	(3.47)	(2.89)	(3.85)	(4.09)	(3.67)	(3.04)
12	Sell	0.0060	0.0065	0.0075	0.0087	0.0048	0.0058	0.0070	0.0085
		(1.17)	(1.29)	(1.48)	(1.74)	(0.93)	(1.15)	(1.40)	(1.71)
12	Buy	0.0192	0.0179	0.0168	0.0155	0.0196	0.0179	0.0167	0.0154
		(4.63)	(4.36)	(4.10)	(3.81)	(4.73)	(4.36)	(4.09)	(3.79)
12	Buy-sell	0.0131	0.0114	0.0093	0.0068	0.0149	0.0121	0.0096	0.0069
		(3.74)	(3.40)	(2.95)	(2.25)	(4.28)	(3.65)	(3.09)	(2.31)

Momentum and the Efficient Market Hypothesis

Efficient Market Hypothesis (EMH)

Fama (1970, 1991) are important surveys on market efficiency. In them, he defines market efficiency as: "*A market in which prices always "fully reflect" available information is called efficient*". We can divide the EMH into three subcategories depending on the level of information:

- **Weak form:** Historical price information
- **Semi-strong form:** All publicly available information
- **Strong form:** Insider information

- Momentum is important to the discussion of market efficiency because it attacks it directly at its most basic notion of efficiency: the weak form
- That is, unless we can document that momentum returns are due compensation for some systematic risk (that we are yet to uncover)
- Eugene F. Fama versus Richard H. Thaler: Are Market Efficient?

Are momentum returns compensation for systematic risk?

- **Key question:** Are momentum returns compensation for systematic risk?
- The prevailing answer to this is mostly no, although we are still searching for a sensible risk-based explanation (to support its continued existence)
- Jegadeesh and Titman (1993) undertake a comprehensive analysis of the sources of the profits to their relative strength strategies and find that
  1. Momentum strategies do not pick up high-risk stocks and therefore cannot be viewed as compensation for bearing systematic risk
  2. Momentum profits are not caused by serial correlation in factor-related returns, meaning that this cannot be viewed as a source of the profits either
  3. Lead-lag effects are not an important source of relative strength profits
- Instead, the authors propose that momentum profits may be due to investors underreacting to information about short-term prospects and overreacting to information about long-term prospects

Momentum more generally

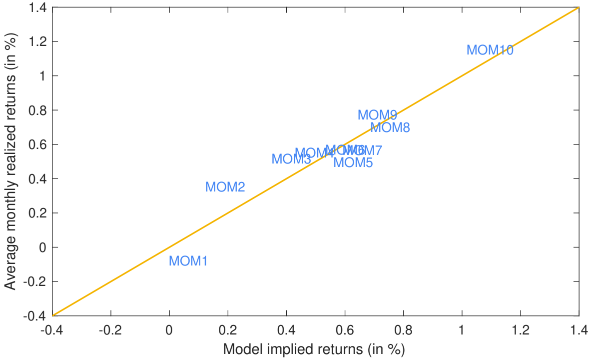
- A widely studied strategy today defines the time $t-1$ momentum signal as the cumulative return from $t-12$ to $t-2$ with a single holding period
- We skip the most recent month to avoid the effect of one-month reversal in stock returns, typically attributed to liquidity and/or microstructure issues (Jegadeesh, 1990, Lo and MacKinlay, 1990)
- Is the Fama-French three-factor model capable of explaining returns to momentum strategies?
- Below we report statistics for ten portfolios sorted on prior returns for the period 1963:07 to 2020:12 along with time series factor loadings for the Fama-French three-factors

	Loser	2	3	4	5	6	7	8	9	Winner
Mean	-0.95	4.22	6.21	6.62	5.93	6.82	6.79	8.41	9.26	13.82
Std	29.08	22.24	18.73	16.92	15.58	15.73	15.12	15.38	16.57	21.25
β_M	1.44	1.23	1.08	1.02	0.97	0.98	0.94	0.94	0.98	1.05
β_{SMB}	0.47	0.15	0.02	-0.05	-0.06	-0.08	-0.12	-0.07	-0.03	0.38
β_{HML}	0.31	0.33	0.32	0.25	0.21	0.16	0.11	0.05	0.01	-0.30

Testing the Fama-French model using momentum

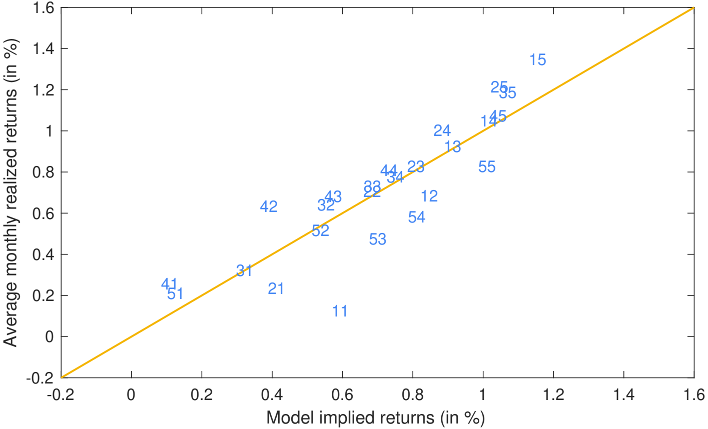
- Running second-pass Fama-MacBeth regressions yields the following results

	γ_0	γ_M	γ_{SMB}	γ_{HML}	R^2 (%)
Estimate	3.04	-2.35	1.07	-0.37	88.91
s.e.	(0.74)	(0.80)	(0.55)	(0.55)	
tstat	[4.10]	[-2.94]	[1.94]	[-0.67]	



Size and momentum sorted portfolios

* Suppose that we instead consider 25 portfolios sorted on size and momentum



Building a momentum factor

- The pervasiveness of momentum returns has prompted the construction of a momentum factor-mimicking portfolio

- Carhart (1997) builds a four-factor model in which he adds a momentum factor to the Fama-French three-factor model, i.e.,

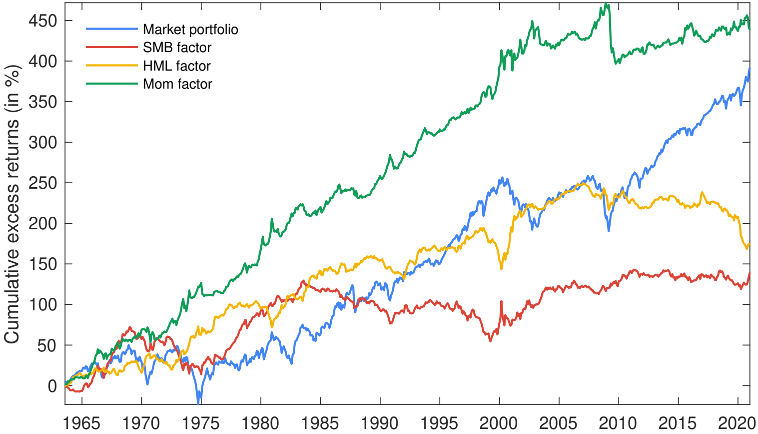
$$\mu = \lambda_0 + b_M \gamma_M + b_{SMB} \gamma_{SMB} + b_{HML} \gamma_{HML} + b_{MOM} \gamma_{MOM} \tag{45}$$

  where the momentum factor can be constructed from the intersections of two size groups and three momentum groups (winners, neutral, losers) akin to the HML factor, i.e.,

$$MOM_t = \frac{1}{2} \left[SW_t + BW_t \right] - \frac{1}{2} \left[SL_t + BL_t \right] \tag{46}$$

How profitable is momentum?

* Consider the cumulative returns to the momentum strategy relative to the other zero-cost investment strategies (1963:07-2020:12)

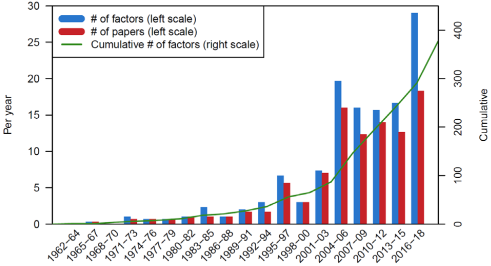


The zoo of factors

Factor zoo

There is by now identified a plethora of factor-mimicking portfolio models and risk factors in the literature. In fact, dealing with the aptly named "factor zoo" is becoming an issue (Cochrane, 2011, Harvey et al., 2016, Feng et al., 2020)

* Harvey and Liu (2019) provide a census of the proverbial factor zoo (Cochrane, 2011, Harvey et al., 2016) Google sheet with factors
* The below figure details the factor production from 1963–2018 and clearly indicates a strong increase (382 factors in top journals alone)



* Journals published through December 2018. Data collection in January 2019.

Prominent factor models in the zoo

* Pástor and Stambaugh (2003) build a four-factor model in which the fourth factor mimicks risk related to illiquidity

$$\mu = \lambda_0 + b_M \gamma_M + b_{SMB} \gamma_{SMB} + b_{HML} \gamma_{HML} + b_{LIQ} \gamma_{LIQ} \tag{47}$$

* Fama and French (2015) build a five-factor model in which the additional factors mimicks risk related to operating profitability and aggressiveness of the firm's investments

$$\begin{aligned} \mu = & \lambda_0 + b_M \gamma_M + b_{SMB} \gamma_{SMB} + b_{HML} \gamma_{HML} \\ & + b_{RMW} \gamma_{RMW} + b_{CMA} \gamma_{CMA} \end{aligned} \tag{48}$$

* Hou et al. (2015) build an empirical q-factor model consisting of four factors related to the market, size, investment, and profitability

$$\mu = \lambda_0 + b_M \gamma_M + b_{ME} \gamma_{ME} + b_{IA} \gamma_{IA} + b_{ROE} \gamma_{ROE} \tag{49}$$

Prominent factor models in the zoo

- Hou et al. (2020) present an augmented q-Factor model that additionally includes an expected investment-to-asset growth factor

$$\mu = \lambda_0 + b_M \gamma_M + b_{ME} \gamma_{ME} + b_{IA} \gamma_{IA} + b_{ROE} \gamma_{ROE} + b_{EG} \gamma_{EG} \tag{50}$$

- Stambaugh and Yuan (2017) build a mispricing model consisting of four factors related to the market, size, firm management, and performance

$$\mu = \lambda_0 + b_M \gamma_M + b_{SMB} \gamma_{SMB} + b_{MGMT} \gamma_{MGMT} + b_{PERF} \gamma_{PERF} \tag{51}$$

- Daniel et al. (2020) propose a theoretically motivated factor model based on investor psychology that augments the market factor with two factors that capture long- and short-horizon mispricing

$$\mu = \lambda_0 + b_M \gamma_M + b_{FIN} \gamma_{FIN} + b_{PEAD} \gamma_{PEAD} \tag{52}$$

A detour to other asset classes

* Our discussion of anomalies and risk factors has so far been focused on U.S. equities in the form of portfolios sorted on different characteristics

* U.S. equities are, of course, not the only asset class that investors care about. They also invest in: international equities and indices, bonds (Government and corporate), foreign exchange (FX) assets, and commodities

* Many of the asset pricing anomalies covered so far are not unique to US equities and their presence across asset classes suggest that they could be driven by some common systematic risk factor

* Moreover, many of the empirical techniques are directly transferable to other classes, e.g. the Fama and MacBeth (1973) method. We will illustrate this fact using an important example from the FX literature

Asness, Moskowitz, and Pedersen (2013)

* Asness et al. (2013) provide a comprehensive investigation of value and momentum across a broad spectrum of asset classes
  - US, UK, Japanese, and European individual equities
  - Global equity indices
  - Currencies
  - Global government bonds
  - Commodity futures
* They find consistent and ubiquitous evidence of value and momentum return premia across all markets under study and that strategies are positively correlated across market

Building value and momentum factors

* The momentum signal is defined as the cumulative return from $t-12$ to $t-2$
* Value is more tricky as no universal definition exists. They use BE/ME for stocks and long-term past return proxies for value for the rest
* Using the signals, we can build value and momentum factors as zero-costs long-short portfolios with weights

$$\omega_{i,t}^{S} = c_t \left(\text{rank}\left(S_{it}\right) - \frac{1}{N_t} \sum_{i=1}^{N_t} \text{rank}\left(S_{it}\right) \right) \tag{53}$$

where $S \in \{\text{Value}, \text{Momentum}\}$ and $c_t = 2/\sum_i \left|\text{rank}\left(S_{it}\right) - \overline{\text{rank}}\left(S_{it}\right)\right|$ is a scaling factor ensuring that the overall portfolio is scaled to one dollar long and one dollar short and where portfolio returns are given by $\widetilde{r}_t^S = \sum_i \omega_{i,t}^S \widetilde{r}_{it}$

* This is similar to a long-short portfolio in the top and bottom deciles, but here using all assets and letting weights be a function of signal strength

Comovements everywhere

- They further find consistent patterns of comovements among the strategies everywhere. In particular
  1. Value strategies in one asset class are correlated with value strategies elsewhere
  2. Momentum strategies in one asset are correlated with momentum strategies elsewhere
  3. Value and momentum strategies are negatively correlated everywhere

	Panel A: Correlation of Average Return Series			
	Stock Value	Nonstock Value	Stock Momentum	Nonstock Momentum
Stock value	0.68*	0.15*	−0.53*	−0.26*
Nonstock value		0.07	−0.16*	−0.13*
Stock momentum			0.65*	0.37*
Nonstock momentum				0.21*

Relation to macroeconomic risk

- Are value and momentum strategies related to macroeconomic risks?
  - Not convincingly. There are modest loadings on business cycle variables, consumption, and default risk
  - However, nothing seems capable of explaining the negative correlation between value and momentum

		U.S. Stocks			Global Stocks		Nonstock Assets		All Asset Classes	
		Value	Momentum		Value	Momentum	Value	Momentum	Value	Momentum
U.S. values	Long-run	0.0004	0.0001	Global values	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
for	consumption growth	(2.06)	(0.33)	for	(0.93)	(0.92)	(0.68)	(0.03)	(1.01)	(0.43)
independent	Recession	−0.0068	−0.0056	independent	0.0037	−0.0044	0.0045	−0.0081	0.0043	−0.0072
variables	dummy	(−1.06)	(−0.73)	variables	(0.66)	(−0.75)	(1.48)	(−2.44)	(1.55)	(−2.26)
	GDP growth	−0.0050	0.0019	(GDP-	−0.0011	0.0023	−0.0005	−0.0034	−0.0006	−0.0020
		(−1.75)	(0.57)	weighted)	(−0.39)	(0.80)	(−0.32)	(−2.08)	(−0.45)	(−1.29)
	Market	−0.3435	0.0219		−0.0615	−0.0709	0.0101	−0.0083	−0.0068	−0.0231
		(−7.46)	(0.40)		(−1.41)	(−1.55)	(0.44)	(−0.32)	(−0.32)	(−0.93)
	TERM	0.2038	−0.0234		0.0523	0.0141	−0.0885	0.0370	−0.0551	0.0316
		(2.64)	(−0.25)		(1.04)	(0.27)	(−3.30)	(1.25)	(−2.23)	(1.11)
	DEF	0.7439	−0.7733		0.2650	−0.3752	−0.0510	−0.0787	0.0240	−0.1490
		(5.25)	(−4.57)		(2.86)	(−3.87)	(−1.03)	(−1.44)	(0.53)	(−2.84)
	R-square	13.1%	5.9%		2.3%	6.4%	3.4%	2.9%	2.9%	4.7%

Relation to liquidity risk

- What about funding and/or market liquidity risk?
  - Funding liquidity risks are consistently negatively correlated with value returns and consistent positively correlated with momentum returns
  - Value performs poorly when funding liquidity rises, whereas momentum thrives
  - The negative value-momentum correlation can be explained by opposite signed exposure to funding liquidity
  - Market liquidity appears to be near unrelated to value and momentum returns

Panel B: Global Liquidity Risk Measures

		Value	Momentum	50/50 Combination	Val – Mom
Funding liquidity risk	TED spread	– 0.0067	0.0094	0.0023	– 0.0161
		(– 1.69)	(2.00)	(0.74)	(– 2.05)
	LIBOR-term repo	– 0.0177	0.0139	– 0.0005	– 0.0316
		(– 2.87)	(1.66)	(– 0.08)	(– 2.36)
	Swap-T-bill	– 0.0076	0.0055	– 0.0012	– 0.0131
		(– 2.15)	(1.31)	(– 0.46)	(– 1.86)
	Funding liquidity PC	– 0.0094	0.0112	0.0013	– 0.0206
		(– 4.74)	(3.58)	(0.58)	(– 4.67)
Market liquidity risk	On-the-run – off-the-run	0.0108	– 0.0001	0.0037	0.0109
		(0.68)	(– 0.01)	(0.32)	(0.34)
	Pástor-Stambaugh	0.0010	– 0.0002	0.0003	0.0011
		(1.06)	(– 0.15)	(0.43)	(0.61)
	Acharya-Pedersen	0.0009	0.0008	0.0020	0.0001
		(0.39)	(0.28)	(1.30)	(0.02)
	Market liquidity PC	– 0.0009	0.0016	0.0012	– 0.0025
		(– 0.74)	(1.21)	(1.00)	(– 1.45)
All liquidity risk	All PC	– 0.0079	0.0093	0.0016	– 0.0172
		(– 3.25)	(4.43)	(0.82)	(– 4.63)

An "everywhere" three-factor model

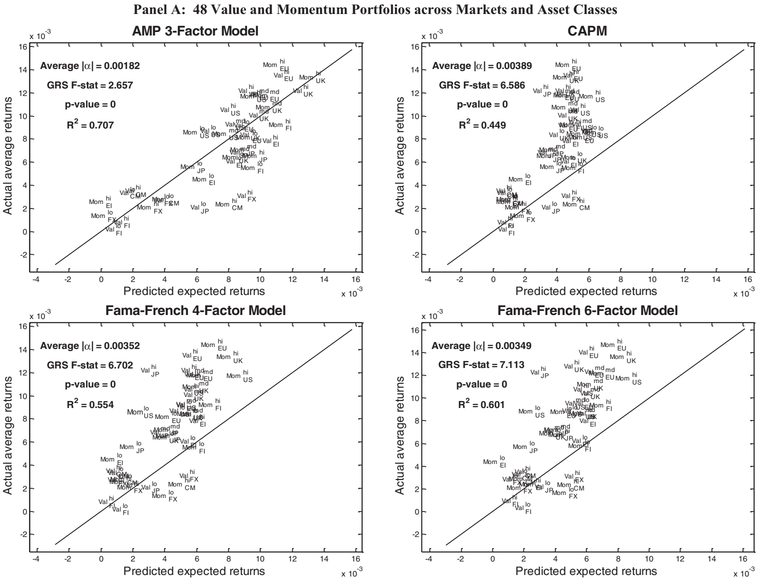
* Asness et al. (2013) further provide a new three-factor model based on their "value and momentum everywhere" factors

$$\begin{aligned} R_{i,t}^P - r_{f,t} = {} & \alpha_i^P + \beta_i^P MKT_t \\ & + v_i^P VAL_t^{\text{everywhere}} + m_i^P MOM_t^{\text{everywhere}} + \varepsilon_{i,t}^P \end{aligned} \tag{54}$$

  where MKT_t is a global market portfolio, $VAL_t^{\text{everywhere}}$ and $MOM_t^{\text{everywhere}}$ are equal-volatility-weighted across-asset-class value and momentum factors

* This model outperforms a host of asset pricing models for their 48 value and momentum portfolios and performs on par with the Fama-French models for the 25 size-value portfolios

Pricing error plot for the global three-factor model



Reviewing the Covered Interest Parity (CIP)

Covered interest parity (CIP)

The covered interest parity (CIP) states that investors are indifferent between domestic and foreign money market investments when currency risk is covered through the forward market

$$(1 + i_t) = \frac{S_t}{F_t}(1 + i_t^\star) \tag{55}$$

- If the CIP condition in (55) holds, then forward discounts (approximately) equal interest rate differentials

$$\frac{F_t - S_t}{S_t} = \frac{i_t^\star - i_t}{(1 + i_t)} \approx i_t^\star - i_t \tag{56}$$

- Akram et al. (2008) and Rime et al. (2019), among others, find that CIP holds up quite well even at high frequencies
- However, Du et al. (2018) document large and persistent deviations during and after the latest financial crisis

Uncovered Interest Parity (UIP)

Uncovered Interest Parity (UIP)

The Uncovered Interest Parity (UIP) states that domestic and "uncovered" foreign money market investments have the same expected return

$$(1+i_t) = \frac{S_t}{\mathbb{E}_t\left[S_{t+1}\right]}\left(1+i_t^{\star}\right) \tag{57}$$

- Note that this strategy involves exchange rate risk as the investor is exposed to fluctuations in the actual spot rate at time $t+1$, i.e. the profit depends on the actual realization of S_{t+1}
- We can re-arrange the UIP condition and subtract 1 from each side to obtain the International Fisher Hypothesis

$$\frac{\mathbb{E}_t\left[S_{t+1}\right]-S_t}{S_t} = \frac{i_t^{\star}-i_t}{(1+i_t)} \approx i_t^{\star}-i_t \tag{58}$$

Unbiasedness hypothesis

Unbiasedness hypothesis

When the forward rate equals the expected future spot rate, the forward rate is said to be an unbiased predictor of the future spot rate. This equality is summarized by the unbiasedness hypothesis

$$F_t = \mathbb{E}_t\left[S_{t+1}\right] \tag{59}$$

- This hypothesis is implied by the covered and the uncovered interest parity, which is easily seen by equating the two conditions

$$\underbrace{\frac{S_t}{\mathbb{E}_t\left[S_{t+1}\right]}\left(1+i_t^\star\right)}_{\text{UIP}} = \left(1+i_t\right) = \underbrace{\frac{S_t}{F_t}\left(1+i_t^\star\right)}_{\text{CIP}} \tag{60}$$

- Bilson (1981) and Fama (1984) find that forward rates are not unbiased predictors of the future spot rate, but that spot rates tend to move either too little or in the wrong direction (relative to the International Fisher Hypothesis)

Fama (1984) regressions

Fama (1984) regressions

Let $s_t = \ln S_t$ and $f_t = \ln F_t$ denote the log spot and one-month forward rate, then Fama (1984) ran the following regressions

$$s_{t+1} - s_t = \alpha + \beta \left(f_t - s_t \right) + \varepsilon_{t+1} \tag{61}$$

$$f_t - s_{t+1} = \delta + \theta \left(f_t - s_t \right) + \nu_{t+1} \tag{62}$$

* Since $s_{t+1} - s_t$ and $f_t - s_{t+1}$ sum to $f_t - s_t$, we have the following relationship

$$\beta + \theta = 1 \tag{63}$$

$$\alpha + \delta = 0 \tag{64}$$

* Under UIP, we should expect to find $\beta = 1$, but one often finds $\beta < 1$ and even $\beta < 0$, which clearly violates the unbiasedness hypothesis

Currency excess returns

Currency excess returns

The realized currency excess return, abstracting from transaction costs, to a long position in foreign currency is

$$RX_{t+1} = \frac{F_t - S_{t+1}}{S_t} \tag{65}$$

which can be interpreted as a simple trading strategy in which investors purchase foreign currency k forward at time t and sell it in the spot market at time $t+1$

* This is equivalent to the forward discount (interest rate differential) minus the spot exchange rate change

$$RX_{t+1} = \frac{F_t - S_t}{S_t} - \frac{S_{t+1} - S_t}{S_t} \tag{66}$$

$$\overset{CIP}{\approx} \frac{(1+i_t^{\star})}{(1+i_t)} - \frac{S_{t+1}}{S_t} \tag{67}$$

The carry trade strategy

Currency carry trade

The currency carry trade strategy consists of borrowing low-interest rate (funding) currencies and lending high-interest rate (investment) currencies

- The excess return to a carry trade for a single currency pair can be defined in terms of the currency excess from (65) as

$$
\begin{aligned}
RX_{t+1}^{\text{Carry}} &= \text{sign}\left(i_t^{\star} - i_t\right) \cdot RX_{t+1} && (68)\\
&= \text{sign}\left(\frac{F_t - S_t}{S_t}\right) \cdot RX_{t+1} && (69)\\
&= \text{sign}\left(F_t - S_t\right) \cdot RX_{t+1} && (70)
\end{aligned}
$$

- Examples of popular currency pairs for single carry trades are AUD/JPY and NZD/JPY due to the high (low) rates in AUD and NZD (JPY)

Currency carry portfolios and risk factors

* Lustig et al. (2011) identify two risk factors in the cross-section of carry trade portfolios: DOL (RX in their terminology) and HML$_{FX}$

* Note that this is equivalent to the Fama-French size and value factors that seek to explain the value and size anomalies found in stock returns

* In particular, Lustig et al. (2011) propose two common factors extracted from currency portfolios sorted on the basis of their forward discounts, i.e. $(F_t - S_t)/S_t$

    1. **DOL**$_t$: The dollar risk factor is the average excess return across the carry trade portfolios

    2. **HML$_{\mathbf{FX},t}$**: The carry risk factor is the return differential between the high interest rate portfolio (largest forward discounts) and the low interest rate portfolio (smallest forward discounts)

Descriptive statistics

Table 1
Currency portfolios—U.S. investor

Portfolio	1	2	3	4	5	6
	Panel I: All Countries					
	Spot change: Δs^j					
Mean	−0.64	−0.92	−0.95	−2.57	−0.60	2.82
Std	8.15	7.37	7.63	7.50	8.49	9.72
	Forward Discount: $f^j - s^j$					
Mean	−2.97	−1.23	−0.09	1.00	2.67	9.01
Std	0.54	0.48	0.47	0.52	0.64	1.89
	Excess Return: rx^j (without b–a)					
Mean	−2.33	−0.31	0.86	3.57	3.27	6.20
Std	8.23	7.44	7.66	7.59	8.56	9.73
SR	−0.28	−0.04	0.11	0.47	0.38	0.64
	Net Excess Return: rx^j_{net} (with b–a)					
Mean	−1.17	−1.27	−0.39	2.26	1.74	3.38
Std	8.24	7.44	7.63	7.55	8.58	9.72
SR	−0.14	−0.17	−0.05	0.30	0.20	0.35
	High-minus-Low: $rx^j - rx^1$ (without b–a)					
Mean		2.02	3.19	5.90	5.60	8.53
Std		5.37	5.30	6.16	6.70	9.02
SR		0.38	0.60	0.96	0.84	0.95
	High-minus-Low: $rx^j_{net} - rx^1_{net}$ (with b–a)					
Mean		−0.10	0.78	3.42	2.91	4.54
		[0.30]	[0.30]	[0.35]	[0.38]	[0.51]
Std		5.40	5.32	6.15	6.75	9.05
SR		−0.02	0.15	0.56	0.43	0.50

Principal components analysis

* Lustig et al. (2011) conduct a principal component analysis to better understand the common factors in the cross-section of carry trade returns

Table 3
Principal components

Panel I: All Countries

Portfolio	1	2	3	4	5	6
1	0.42	0.43	0.18	−0.15	0.74	0.20
2	0.38	0.24	0.15	−0.27	−0.61	0.58
3	0.38	0.29	0.42	0.12	−0.28	−0.71
4	0.38	0.04	−0.35	0.83	−0.03	0.18
5	0.43	−0.08	−0.72	−0.44	−0.03	−0.30
6	0.45	−0.81	0.35	−0.03	0.11	0.06
% Var.	71.95	11.82	5.55	4.00	3.51	3.16

Asset pricing tests

- Having built the currency risk factors, Lustig et al. (2011) conduct standard asset pricing tests to study if HML$_{FX}$ as a priced risk factor

Table 4
Asset pricing—U.S. investor

	All Countries				
	$\lambda_{HML_{FX}}$	λ_{RX}	$b_{HML_{FX}}$	b_{RX}	R^2
GMM_1	5.50	1.34	0.56	0.20	70.11
	[2.25]	[1.85]	[0.23]	[0.32]	
GMM_2	5.51	0.40	0.57	0.04	41.25
	[2.14]	[1.77]	[0.22]	[0.31]	
FMB	5.50	1.34	0.56	0.20	70.11
	[1.79]	[1.35]	[0.19]	[0.24]	
	(1.79)	(1.35)	(0.19)	(0.24)	
$Mean$	**5.08**	**1.33**			

Factor betas and understanding the mechanisms

* Last, to better understand the economic mechanisms, consider the factor loadings from time series regression of portfolio returns upon the factors and a constant

Table 4
Continued

	All Countries				
Portfolio	α_0^j	$\beta_{HML_{FX}}^j$	β_{RX}^j	R^2	$\chi^2(\alpha)$
1	−0.10	−0.39	1.05	91.64	
	[0.50]	[0.02]	[0.03]		
2	−1.55	−0.11	0.94	77.74	
	[0.73]	[0.03]	[0.04]		
3	−0.54	−0.14	0.96	76.72	
	[0.74]	[0.03]	[0.04]		
4	1.51	−0.01	0.95	75.36	
	[0.77]	[0.03]	[0.05]		
5	0.78	0.04	1.06	76.41	
	[0.82]	[0.03]	[0.05]		
6	−0.10	0.61	1.05	93.84	
	[0.50]	[0.02]	[0.03]		
All					6.79

Risk-based explanations for the carry trade

* Although currency-based risk factors do well, they do not provide us with much economic insight. That is, we do not know what fundamental risk-based view the factors represents

* In theory, excess returns must be driven by exposure to some systematic risk (no free lunch argument)

  1. **Volatility risk:** Menkhoff et al. (2012) constructs a measure of global FX volatility that explain carry trade returns about as well as HML$_{FX}$
  2. **Crash risk:** Brunnermeier et al. (2009) and Chernov et al. (2018) argue that carry trade returns are compensation for taking on crash risk
  3. **Peso problems:** Burnside et al. (2011) argue that carry trade payoffs reflect peso problems (similar to crash risk, but the compensation event may be unprecedented)
  4. **Macroeconomic fundamentals:** Della Corte et al. (2016) show that carry trades are exposed to global imbalances in current accounts and Colacito et al. (2020) that currency returns are exposed to business cycle risk as measured by the output gap

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