CHAPTER 7

Risk Aversion and Investment Decisions, Part III: Challenges to Implementation

Chapter Outline

7.1 Introduction

Prior to the advent of modern portfolio theory (MPT), financial researchers (and practitioners) had no comprehensive mental or analytical structure for understanding either how asset "risk" might be measured or how the return patterns of a portfolio's constituent assets interact to determine its overall risk. Although practitioners were certainly aware of the qualitative benefits of portfolio diversification, their understanding often took the form of simply noting that the act of spreading one's wealth across many risky assets (or investment projects) increased the likelihood that some, at least, would pay positive returns.[1] MPT dramatically extended this qualitative understanding by making

[1] The notion of diversification to mitigate investment risk is very old. See Ecclesiastes 11: 1−2; New Revised Standard Version, Harper Collins Study Bible: "Send out your bread upon the waters, for after many days you will get it back. Divide your means seven ways, or even eight, for you do not know what disaster may happen on earth."

precise the measurement of risk overall and the measurement and source of the "gains to diversification."

It is not a large step from descriptive to normative applications of MPT. As a result, MPT has become a fundamental tool for assisting investors in making rational portfolio allocation decisions. The message is clear: there are large gains to diversification and these are achieved by allocating one's wealth between risk-free assets and the available tangency portfolio.

Normative applications of the theory, however, require estimates of the fundamental statistical quantities upon which it is built: the (μ_i, σ_i) for every candidate asset i, and the cross-correlations ρ_{ij} for all asset return pairs. These numbers are necessary to characterize the investor's future risk/return possibilities. How are they to be obtained?

As noted in Box 3.1, it is natural to look to historical return data as the source of these estimates and, more precisely, to use historical average returns, historical standard deviations, and historical correlations as the estimates, respectively, for the required μ_i, σ_i, and ρ_{ij}s. If the returns and return interactions are statistically stationary with a unique ergodic set and if the historical data series is sufficiently long, the aforementioned quantities can be estimated with a high degree of precision.[2] In many applied applications, however, it is customary to base these estimates only on the prior 5 years of monthly return data. This choice follows from the fact that many investors have 1-year (or less) investment horizons and are therefore more concerned with conditional estimates rather than the unconditional ones that long data series are designed to provide. Equity return patterns over the coming year, for example, may be highly dependent on the macroeconomy's present state: is it in recession or expansion?

Furthermore, the present state can change fairly rapidly as we have seen recently.
The financial crisis was preceded by a long period of gradual macroeconomic expansion often referred to as the "Great Moderation." It ended abruptly with the bankruptcies of Lehman Brothers, Bear Stearns, and AIG (all within a period of a few months), and the onset

2 Our notion of stationarity for a discrete time (Markov) process $\{x_t\}$ is as follows; Let s, t be arbitrary time indices and X the state space with $\hat{x}\in X$, and $B\subseteq X, B$ a subset. Define

$$P(s,\hat{x},t,B) = \text{Prob}(x_t \in B; x_s = \hat{x})$$

Then for any integer u, if

$$P(s+u,\hat{x},t+u,B) = \text{Prob}(x_{t+u}\in B; x_{s+u} = \hat{x}) = P(s,\hat{x},t,B)$$

the (Markov) process is said to be stationary.
The same (Markov) process possesses an invariant distribution $\hat{G}(\cdot)$ on X if and only if for any $B\subseteq X$, $\text{Prob}(x_{t+1}\in B) = \int_{x\in X} P(x_{t+1}\in B; x_t = x)\hat{G}(\mathrm{d}x)$. The process is ergodic if the invariant distribution describes the long-run average pattern in the data.

of the "Great Recession." Statistical estimates drawn from "Great Moderation" data were not very informative vis-à-vis stock market behavior in the first year of the financial crisis.

As these thoughts suggest, while investors should be guided by the implications of MPT, they must also be aware of its practical limitations. We elaborate upon these issues in the remainder of the chapter.

7.2 The Consequences of Parameter Uncertainty

In order to construct the efficient frontier of risky assets, an investor or his agent must obtain estimates $\hat{\mu}_i, \hat{\sigma}_i$, and $\hat{\rho}_{ij}$, $\forall i \neq j$ for all risky assets (individual shares, mutual funds, exchange traded funds (ETFs), etc.) under consideration for portfolio inclusion. These estimates, obtained from historical data are imperfect "stand-ins" for the true μ_i, σ_i, and ρ_{ij} $\forall i \neq j$ which are themselves presumed to be stable inclusive of the historical period of estimation and the investor's future investment horizon. As computed in Chapter 3, the $\hat{\mu}_i, \hat{\sigma}_i$, and the $\hat{\rho}_{ij}s$ represent unbiased estimates of their true counterparts, but they may not be precise estimates if the samples from which they are estimated are "too small." Unfortunately, errors in these estimates interact with the technique of mean−variance portfolio optimization to trace out an "estimated efficient frontier" which may be radically distorted relative to its true counterpart.

Most of the distortion has its origin in the misestimation of mean returns (see Box 7.1). To illustrate how mean misestimation can distort an investor's perception of his risk/return tradeoffs, consider the construction of an efficient frontier of six assets with true monthly mean returns of $(\mu_1, \mu_2, \ldots, \mu_6)$ and estimated mean returns of $(\hat{\mu}_1, \hat{\mu}_2, \ldots, \hat{\mu}_6)$. It is the latter quantities that influence actual efficient frontier construction. Roughly speaking, one would expect $\hat{\mu}_j > \mu_j$ for three of the assets and $\hat{\mu}_j < \mu_j$ for the remaining three. When computing the efficient frontier, these errors do not "average out," however, because mean−variance optimization will, by design, overweight (relative to what would be the case using the true values) the former group of assets, and underweight the latter group. Accordingly, the estimated frontier will lie above the true efficient frontier, with the distortion being the least in the region of the minimum variance portfolio where mean return values are downplayed.

BOX 7.1 On the Precision of Mean Estimates

Straightforward estimations typically rely on monthly data going back at most 5 years yielding 60 data points. Suppose that $\hat{\mu}_i = 0.8\%$ and $\hat{\sigma}_i = 6\%$ at monthly frequencies. The standard error of the estimate of the mean is $(\hat{\sigma}_i/\sqrt{J}) = (0.06/\sqrt{60}) = 0.008$, yielding a confidence interval around the true mean of

(*Continued*)

BOX 7.1 On the Precision of Mean Estimates (Continued)

$$\text{Prob}\left(\hat{\mu}_i - 2\left(\frac{\hat{\sigma}_i}{\sqrt{J}}\right) \leq \mu_i \leq \hat{\mu}_i + 2\left(\frac{\hat{\sigma}_i}{\sqrt{J}}\right)\right) = 0.95$$

$$= \text{Prob}(0.008 - 2(0.008) \leq \mu_i \leq 0.008 + 2(0.008)) = 0.95$$

$$= \text{Prob}(-0.008 \leq \mu_i \leq 0.024) = 0.95$$

By implication, we cannot even conclude to a high degree of confidence that μ_i is positive. Variances computed on the basis of 60 data points, however, are more precisely estimated than means. The standard error in this case is $(\hat{\sigma}_i/\sqrt{2J})$.

Neither of these frontiers, however, describes the operative frontier confronting the investor. That frontier must be the one which uses portfolio weights obtained via MPT analysis using the historical estimates in conjunction with the true means, variances, and return correlations. By construction, the operative frontier must lie below the true efficient frontier and in fact need not even be concave. Figure 7.1 illustrates these relationships qualitatively. While the position of the efficient frontier constructed using the true μ_i, σ_i, and $\rho_{ij}s$ is not known, it must lie between its two "half-brother" frontiers, a fact of little use since the position of the operative frontier is similarly unknown. Note that the investor ends up effectively selecting portfolios with the worst risk-return characteristics.

To get some feel for the actual magnitudes involved, we compute the above three frontiers using six assets whose plausible monthly return statistics are described in Table 7.1. The results of this exercise are presented in Figure 7.2.[3]

Note that at a 3% portfolio SD, the difference between the estimated and operative expected returns is roughly.65% per month (0.019−0.0125), or 7.8% on an annualized basis. In this case, the distortions introduced by estimation errors lead to a substantially exaggerated sense of expected portfolio returns.[4] In general mean−variance optimization tends to overweight securities with large estimated returns, small variances, and negative correlations relative to other candidate assets, and vice versa.

[3] To generate the Operative Frontier in Figure 7.2, we first generated a random time series of 24 return entries for each of the securities in Table 7.1 in a manner that respected the stated moments and cross-correlations. From this artificial data, sample means, variances, and cross-correlations were then computed. It is these latter quantities that formed the basis for constructing both Estimated and Operative Frontiers in the following manner: For a given σ, the Estimated Frontier uses the $\hat{\mu}_i, \hat{\sigma}_i$, and $\hat{\rho}_{ij}$ obtained from the generated data to get proportions $(\hat{w}_1, \ldots, \hat{w}_6)$. The Operative Frontier uses these $(\hat{w}_1, \ldots, \hat{w}_6)$ values in conjunction with the true μ_i, σ_i, and ρ_{ij}.

[4] The magnitude of the discrepancy between estimated and operative returns generally declines, however, as the number of available securities increases.

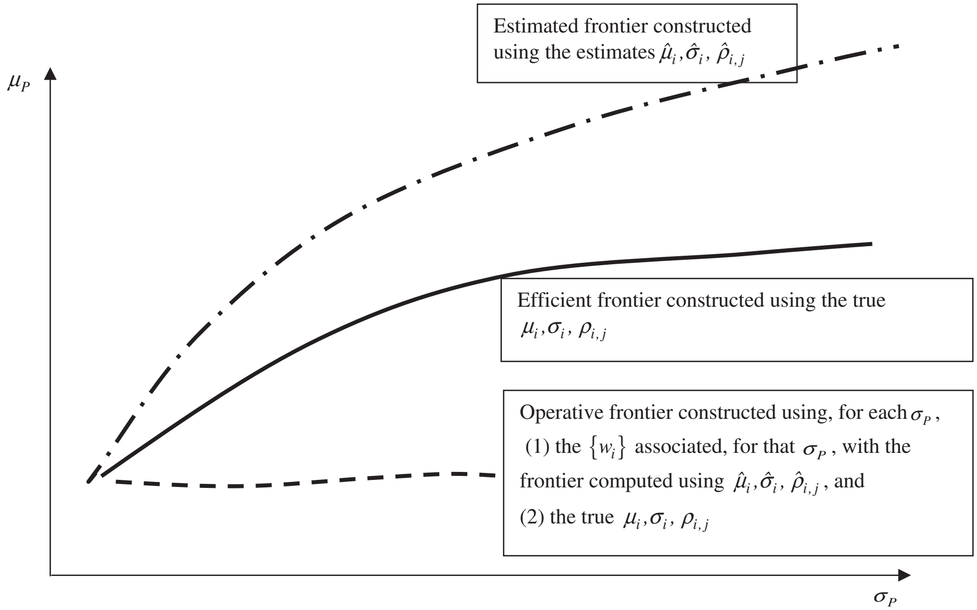

Figure 7.1
True, actual, and operative frontiers: general pattern.

Table 7.1: Underlying moments for the six securities

Security	1	2	3	4	5	6
True mean	0.016	0.014	0.013	0.011	0.01	0.012
True SD	0.06	0.05	0.055	0.04	0.035	0.051
Correlation matrix: for all $i \neq j$, $\rho_{ij} = 0.10$						

The portfolios we have been discussing are collections of risky assets alone. When risk-free assets are included as an investment option in conjunction with eligible portfolios on the estimated frontier, their inclusion may lead to the construction of portfolios with excessive allocations to risky assets, at least relative to what would result using candidate portfolios on the “true” or “operative” frontiers. In a mean−variance investment environment, “overoptimistic” parameter estimates may thus contribute to excessive risk taking. This assertion follows from the fact that the efficient frontier of risk-free and risky assets using the estimated risky asset frontier is steeper than the one based on the operative frontier of risky assets, i.e., its Sharpe ratio is higher. As a result, the investor will tend to allocate a larger fraction (given his $U(\mu_i, \sigma_i)$) of his investible wealth to risky assets (Figure 7.3). If the equity portfolio ends up being very highly leveraged (as in a hedge-fund context) and actual *ex post* returns turn out to be low (as the position of the operative frontier suggests), the result could be cumulatively devastating. Investors, of course, do not know the operative frontiers, but they should be aware that it lies below their estimated one.

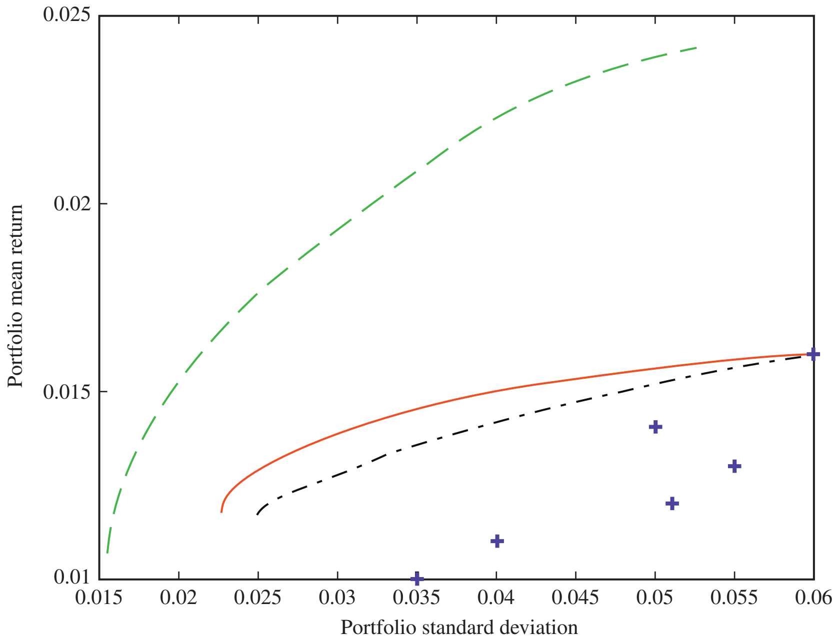

Figure 7.2: True, estimated, and operative frontiers[i]
(i) The three frontiers are pictured in red (true), green (estimated), and black (operative) respectively; the + signs denote the individual securities based on simulated return data. Computations based on 24 monthly return data points.

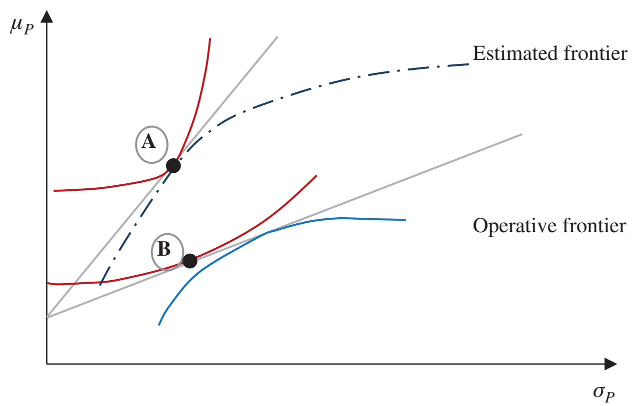

Figure 7.3: Tangency portfolios, estimated and operative frontiers[i]
[i]Consider optimal allocations (A) using estimated and (B) using operative frontiers. The set of mean–variance indifference curves is the same for all frontiers. Note that in this example, the estimated frontier suggests an all stock portfolio while the operative frontier (were it actually to be identified) suggests equal proportions in stocks and risk-free assets.

These observations do not imply that MPT is "wrong" in any way. Due to data limitations, however, it may give rise to misleading conclusions.

There is another aspect of MPT which arises because of its emphasis on mean portfolio returns, but that has nothing to do with estimation issues. This is the tendency of MPT to construct efficient portfolios with large long and short positions when short selling constraints are not imposed. Many investors are prohibited from short selling, or from undertaking short positions as large as unconstrained MPT would recommend. We defer this discussion to Appendix 7.1. The appendix relies on data presented in the next section and thus should be read subsequent to it.

7.3 Trends and Cycles in Stock Market Return Data

When the input data to an efficient frontier calculation is based on historical return series, it is implicitly assumed that the stochastic return relationships on which the data is based will be preserved for the future investment horizon. This is a simple characterization of stationarity. A related issue is the precision of the estimates: precise estimates require lots of data, which means relying on data series into the more distant past. This requirement, however, makes the stationarity assumption more difficult to accept: economic relationships are not static.

In recent years, changing economic relationships have been especially manifest in the international arena. The globalization of industry and finance, in particular, has brought the economies of the world into closer alignment and interconnectedness. There is increasing evidence of a world business cycle, a phenomenon we would expect to translate into increased cross-country stock market return correlations. We also observe increased world financial market integration, with the accompanying increase in cross-border capital flows.[5] Especially in times of financial stress (high return volatility), the ease of capital flows allows the consequences of destructive economic events in one country, most especially, to be transmitted almost instantaneously across all of the world's principal financial markets. The recent financial crisis is a major case in point: while originating in the United States, it quickly spread to Europe and then to Asia.

There are both trend and cyclical aspects to these considerations which we document shortly. In either case, they represent challenges for investors because they suggest that the potential for risk reduction via international diversification may in general be diminishing (trend) and that this phenomenon may be most acutely experienced in cyclical periods of

[5] Global capital flows as a percentage of world GDP rose from 4% in 1994 to 20% in 2007. By 2009, these flows had collapsed to 2.5% of world GDP. See Milesa-Ferretti and Tille (2010).

extreme financial uncertainty ("tail events"). In essence, "when investors need most to be well diversified, the possibilities for diversification disappear." In the remainder of this section, we illustrate these considerations.

7.3.1 Trends in International Stock Market Cross-Correlations

To get some flavor for trends in the inputs to the efficient frontier, let's explore the gains to international diversification across national stock markets. We consider estimates based on historical monthly return data for the period 1.1.1996 through 12.31.2006. These estimates are found in Tables 7.2 and 7.3 for, respectively, the full sample period and the most recent half sample.

First, note the enormous variation in average returns across the two samples. In passing from 10 years of data to the more recent 5 years, average monthly French returns go from

Table 7.2: Major stock markets summary risk/return statistics 1.1.1996–12.31.2006[a]

	FRA	GER	Japan	UK	US	Brazil	Korea
Average monthly return	0.71%	0.65%	−0.21%	0.34%	0.57%	1.57%	0.63%
Standard deviation	5.9%	7.2%	5.6%	4.1%	4.5%	10.4%	11.0%
Correlation table							
France	1						
Germany	0.90	1					
Japan	0.45	0.43	1				
UK	0.79	0.79	0.49	1			
US	0.74	0.77	0.37	0.79	1		
Brazil	0.58	0.60	0.27	0.61	0.65	1	
Korea	0.40	0.39	0.45	0.51	0.48	0.39	1

[a]Authors' calculations.

Table 7.3: Major stock markets summary risk/return statistics 1.1.2002–12.31.2006[a]

	FRA	GER	Japan	UK	US	Brazil	Korea
Average monthly return	−0.40%	−0.38%	0.25%	−0.19%	−0.15%	1.08%	1.36%
Standard deviation	5.9%	7.9%	5.2%	4.1%	4.3%	8.2%	7.4%
Correlation table							
France	1						
Germany	0.95	1					
Japan	0.53	0.49	1				
UK	0.90	0.88	0.51	1			
US	0.89	0.89	0.49	0.87	1		
Brazil	0.67	0.65	0.43	0.67	0.72	1	
Korea	0.74	0.74	0.56	0.71	0.72	0.54	1

[a]Authors' calculations.

0.71% to, more recently, −0.40%, average returns to the Brazilian market index jump by 50% and Korean returns double. At least for shorter sample periods, these numbers illustrate the instability of the mean estimates. Much less variation across the data samples is observed for estimates of σ_j; changes are comparatively small and in two cases, France and the United Kingdom, the estimate is unchanged. Note also that within this sample of countries, stock market returns become uniformly more highly correlated with the passage of time: the correlation matrix in Table 7.3 dominates its counterpart in Table 7.2 entry by entry. While not exhausted, the gains to international diversification across these countries over a purely domestic portfolio appear to be diminishing with time. This fact in itself is not surprising: as national economies become more highly integrated into the world economy and, as such, become more influenced by the world business cycle, we would expect stock returns to become more highly correlated. For countries with long-term historical trading relationships, stock market returns are already highly correlated; see, for example, France and Germany. Note also that there is generally less cross-country variation in standard deviations than in the means for either data set.

Confirming our earlier discussion, the locations of the efficient frontier based on information in, respectively, Tables 7.2 and 7.3 for the G5 subsample (France, Germany, Japan, the UK and the USA) differ substantially. This is captured in Figure 7.4.

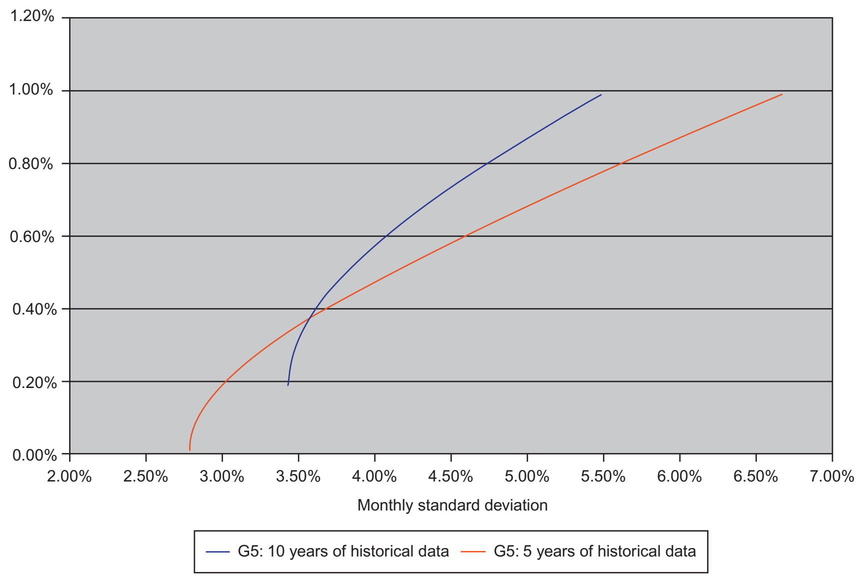

Figure 7.4: Effect of updating the period used to calculate risk/return estimates (most recent 5 years versus 10 years of historical data) for the G5 countries.
Source: Based on data, respectively, in Tables 7.2 and 7.3.

7.3.2 Asset Correlations in Cyclical Periods of High Volatility

Another aspect of parameter instability is directly associated with periods of high stock market return volatility. In particular, cross-country market returns for many of the world's major stock markets tend to become more highly correlated during periods of general market decline, e.g., large losses in the US market tend to coincide with large losses across other major markets. The classic reference in this regard is Longin and Solnik (2001), who measure the extent of these effects across the aggregate stock markets of the United States, Germany, France, the United Kingdom, and Japan using monthly return data for the period 1959−1996. Figure 7.5 captures the essence of their results for US−UK return relationship.

Some interpretation is required. First, the 'black dot' on the vertical axis describes the correlation of US and UK aggregate stock market returns using the full data sample: 0.519. The continuous line represents the conditional correlation of US/UK stock market returns in the subsamples where the returns in either country exceeded the indicated levels (numbers on the horizontal axis). For example, in the periods for which either return was less than −10%, the aggregate US−UK return correlation was 0.676. Generally, similar patterns are observed across all the sample stock markets, i.e., the pattern is similar for the US−Germany, US−France, etc.

The important qualitative implication to note is that the greater the severity of the market decline in either country, the greater the cross-correlation of returns—and the less effective cross-country wealth diversification would be in reducing risk. Note also that this pattern is

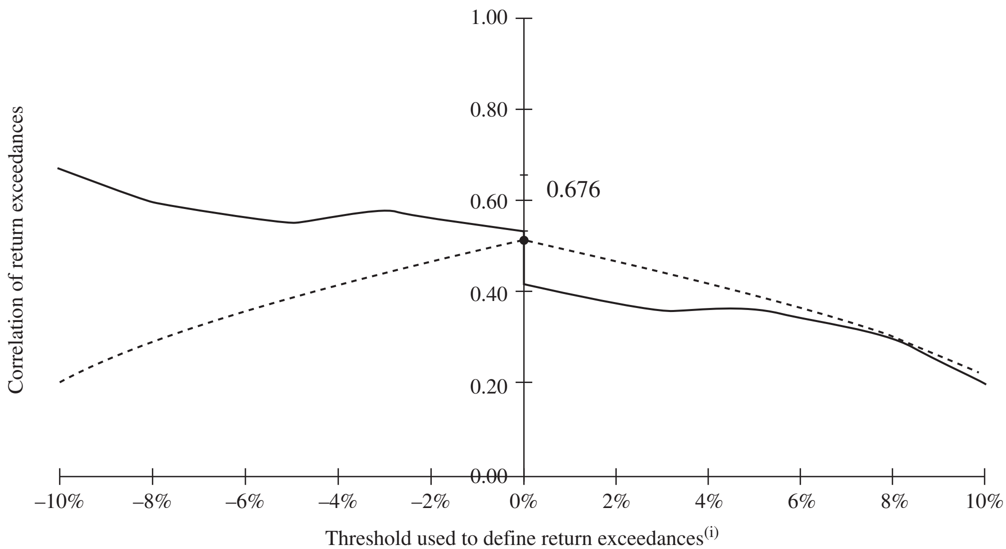

Figure 7.5: Correlation between US and UK aggregate stock market return exceedances
(i) Exceedances signify returns above the corresponding r_f. *Source:* *Longin and Solnik (2001)*.

reversed in the case of positive returns. While Longin and Solnik's data set does not include the period immediately preceding the financial crisis, there is no reason to believe the pattern conveyed in Figure 7.5 has otherwise changed. While these results again do not refute the principle of diversification, they do remind us that the benefits to diversification will likely diminish in periods of severe market stress. The dashed line gives the predictive correlations for the same benchmarks under return normality assumptions where the UK–US return cross-correlation is taken as its full sample value of 0.519.

Similar results were found by Riberio and Veronesi (2002) using a different data set. They report average stock market and industrial production cross correlations across the United Kingdom, Japan, Germany, Canada, France, Switzerland, and the United States exclusively during periods of recession and boom as defined by the National Bureau of Economic Research (NBER). These are reported in Table 7.4.

Focusing on quarterly data, Table 7.4 makes us aware not only of increased stock market return correlations in "bad times" (recessions) but also the underlying increased business cycle correlations that these return correlations reflect. In traditional factor models of security returns (e.g., Chen, Roll and Ross, 1986), industrial production is often selected as an underlying macroeconomic factor. Note also that both correlations are less strong in periods of boom relative to periods of recession which is in line with the Longin and Solnik (2001) results, suggesting that "booms" are essentially more idiosyncratic than recessions.

7.3.3 The Financial Crisis

National stock markets whose returns were already highly correlated generally became even more so during the period of the financial crisis. Table 7.5 provides cross-correlations for the world's major stock markets (by aggregate market value) for the period

Table 7.4: Average correlations for aggregate stock market returns and industrial production in recessions and booms[a]

	Average Correlation Based on Monthly Data	
	Stock Returns	Industrial Production
Recession	0.7847	0.5295
Boom	0.7229	0.5207
	Average Correlation Based on Quarterly Data	
Recession	0.8546	0.6439
Boom	0.7263	0.5558

[a]Based on data from 1970–2000; sample average correlation across all country pairs for United Kindom, United States, Japan, Germany, Canada, France, and Switzerland.
Source: Riberio and Veronesi (2002).

Table 7.5: Summary risk/return statistics for major stock markets during the Financial Crisis, 1.1.2008–12.31.2012[a]

		FRA	GER	Japan	UK	US	Italy	Canada
Average monthly return		−0.09%	0.079%	−0.21%	0.05%	0.30%	−0.72%	0.31%
Monthly standard deviation		8.28%	8.87%	5.26%	6.55%	5.47%	9.40%	7.57%
Correlation table								
	FRA	1						
	GER	0.95	1					
	Japan	0.76	0.78	1				
	UK	0.92	0.89	0.79	1			
	US	0.89	0.90	0.77	0.91	1		
	Italy	0.97	0.92	0.75	0.85	0.85	1	
	Canada	0.82	0.81	0.73	0.78	0.86	0.78	1

[a]We thank G. Bekaert for providing these statistics to us.

January 2008–December 2012 which more than encompasses the worst of the crisis. The Lehman Brothers bankruptcy filing, which is sometimes cited as the start of the "true crisis phase," occurred early in the morning of September 15, 2008.

In comparing Table 7.5 with Table 7.3, much of the message of Section 7.3.2 is confirmed. Where comparable, all the cross-correlations in Table 7.5 exceed or are equal to their counterparts in Table 7.3 and strictly exceed their counterparts in Table 7.2. For countries whose stock market returns were already highly correlated such as Germany and France, there is little change (Table 7.5 versus Table 7.3). Of greater interest is the case of Japan: its stock market returns became much more highly correlated with the rest of the world: on an entry-by-entry basis, the column in Table 7.5 for Japan exceeds its counterpart in Table 7.3, reflecting the worldwide nature of the crisis.

Note also that the return standard deviations are uniformly higher during the crisis than in either earlier period. It is of interest that the highest return standard deviations are observed among the European countries, with Germany's returns the most volatile despite the fact that its macroeconomy was less affected by the crisis than those of France, the United States, and the United Kingdom. This may reflect the lingering sovereign debt crisis in Europe, and Germany's central role in any plan for its alleviation. The European sovereign debt crisis had no counterpart in the United States or the United Kingdom.

Mean monthly returns are, in general, not disastrously low (except in the case of Italy), reflecting the stock market recoveries that prevailed during the latter half of the sample period. Returns were uniformly negative for all stock markets in the year following the Lehman Brothers bankruptcy. As of this writing (May 2014), the macroeconomic consequences of the crisis (e.g., the loss of employment) are still manifest.

7.4 Equally Weighted Portfolios

Our discussion has thus far highlighted some of the difficulties encountered in the actual practice of portfolio formation. We emphasize that these considerations do not represent the failure of MPT, but rather its quantitative limitations due to data availability. In this section, we present one potential manifestation of these data restrictions.

There is substantial evidence that many investors follow a simple, equal-weight allocation rule for the assets they have decided to hold (Benartzi and Thaler, 2001; Huberman and Jiang, 2006). Most mean−variance-based portfolio selection rules, even those with constraints (e.g., no short sales) arrive at portfolio weights, however, that are far from equal. While the origins of this practice may be behavioral, we note that an equally weighted portfolio does not require parameter (means, variances) estimation and thus does not suffer from the consequences of misestimating these quantities as discussed in Section 7.2. Could this fact alone justify the widespread adoption of the equal weighting rule?

Various authors, in particular DeMiguel et al. (2009) and Plyakha et al. (2012), have recently compared the return characteristics of naïve, equally weighted portfolios with monthly rebalancing over and against those arising from various mean−variance motivated sophisticated selection rules. In nearly all cases and using a variety of performance measures (e.g., the classic Sharpe ratio), the equally weighted portfolio strongly outperforms those organized around the mean-variance-efficiency principle.

How is their performance evaluation undertaken? DeMiguel et al. (2009) first select a particular initial historical time interval and assemble the actual return data during this period for all assets under consideration for portfolio inclusion. This data set constitutes the "data sample" and it is used to estimate, for every asset, its mean returns, return standard deviations, and return correlations with other assets. Using these estimates and the portfolio selection technique being evaluated, the resulting portfolio returns are constructed using the actual constituent asset returns in the subsequent time period (so called out of sample portfolio returns) on a monthly updating basis.[6] The various performance measures are then applied to this newly constructed portfolio return data. In particular, DeMiguel et al. (2009)

[6] This "rolling-sample" approach to model evaluation is quite clearly described in the DeMiguel et al. (2009, p. 1927) paper; we paraphrase them as follows: given a T period-long data set, they choose an estimation window of length $M = 60$ or $M = 120$ months, $M < T$. In each month t, starting from $t = M + 1$, they use the data in the previous M months to estimate the parameters (means, variances, etc.) needed to implement a particular strategy: they use these estimated parameters to determine the relative portfolio weights in the portfolio of risky assets. These weights determine the composition of the risky portfolio that is used to determine the actual portfolio return in period $t + 1$ under the given strategy. The process continues by adding the actual return realizations for the various underlying assets next period to the data set while dropping the earliest return until the end of the data set is reached. The result of this procedure is a series of $T-M$ monthly "out of sample" returns generated by the particular portfolio strategy under consideration. It is these returns that are subject to the various measures of performance evaluations.

emphasize three performance measures: the Sharpe ratio, the certainty equivalent (CEQ) return, and a measure of portfolio turnover.[7] The latter measure is suggestive of the transactions costs associated with a particular strategy. The methodology is then applied to the evaluation of 14 strategies using 7 different sets of underlying assets. They find that the 1/*N* strategy delivers a statistically superior Sharpe ratio across all strategies for all but one of the 7 data sets. It is similarly dominant across the board for all strategies and data sets under CEQ evaluation, and is dead last with regard to the turnover measure. These results are due to the parameter misestimation arising from the limited size of the data sets. Comparing the results of standard mean–variance portfolio construction with the (1/*N*) strategy based, in particular, on portfolios of 50 US-based stocks, the authors demonstrate that 6000 monthly data entries would be required for mean–variance allocation to be the dominant strategy. Typical estimations in the finance industry use time series for at most 120 quarters. In general DeMiguel et al. (2009) find that the naïve (1/*N*) strategy dominates all others provided (1) the number of available assets *N* is large (allowing substantial risk reduction even under naïve diversification) and (2) the constituent assets do not have available sufficiently long term data series for precise parameter estimation.

At the start of this section, we mentioned that naïve diversification (the 1/*N* strategy) is often the strategy of choice for investors. In retirement portfolios, it often assumes the form of equal allocations to several distinct mutual funds. Perhaps this phenomenon can be attributed in part to portfolio managers and financial advisors being aware of the sophisticated analysis characteristic of DeMiguel et al. (2009) and the associated literature.[8] It may also be due to investors noticing that this strategy on average "does relatively well" for them. DeMiguel et al. (2009) suggest one possible underlying reason for this relative success.

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7 To be precise, for each strategy k, denote the mean and SD of the sample excess (above r^f) portfolio return series generated as per footnote (8) by, respectively, $\hat{\mu}_k$ and $\hat{\sigma}_k$. The three performance measures are:

i. the Sharpe ratio, $\hat{\mu}_k/\hat{\sigma}_k$;
ii. the CE return is defined as the risk free rate r_k^f that an investor is willing to accept rather than adopting a particular risky portfolio strategy k. In a mean–variance utility context it is defined by $r_k^f \equiv \hat{\mu}_k - (1/2)\sigma_k^2$;
iii. the turnover measure attempts to capture the amount of trading required to implement a particular strategy k. It is defined as the average sum of trades across the N risky assets in the portfolio under a particular strategy implementation: turnover $k = (1/T-M)\sum_{t=1}^{T-M}\sum_{j=1}^{N}\left|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t+}\right|$ where $\hat{w}_{k,j,t+}$ is the portfolio weight under strategy k, just before rebalancing in $t+1$ (after period t price changes) and $\hat{w}_{k,j,t+1}$is the desired portfolio weights for period $t+1$ before prices change in that period.

8 See, for example, Best and Grauer (1991), Bloomfield et al. (1977), Jorion (1991), Michaud (1989), and Plyakha et al. (2012).

7.5 Are Stocks Less Risky for Long Investment Horizons?

Our discussion of the consequences of uncertainty in mean return estimation for the position of the *ex ante* efficient frontier was thus far cast in the familiar one-period setting. As we will see now, these same considerations also have implications for whether stocks are more or less risky for long in contrast to short investment horizons.

7.5.1 Long- and Short-Run Equity Riskiness: Historical Patterns

Are stocks less risky at longer time horizons? How is the question to be addressed? Popular wisdom in this regard is based primarily on Siegel (2008) who argues that, by certain measures, the riskiness of US stocks (we are speaking here of the value-weighted portfolio of all publicly traded US equities) declines for longer investment horizons (30 years), even to the extent of being less risky than US default-free treasury securities on the basis of comparing inflation-adjusted real returns. Siegel's (2008) main assertion is forcefully summarized in Figure 7.6.

Figure 7.6 chooses to measure relative riskiness by focusing on worst- and best-case scenarios. By these measures, stocks have been less risky than bonds at long investment horizons: at 30-year horizons, for example, the worst annualized real return on the US stock market averaged 2.5%, while for long-term bonds and T-bills, the average was, respectively, -2.0% and -1.0%. The dramatic decline, for increasing time horizons, in the range of annualized returns for all three security types suggests strong mean reversion in the return patterns of each.[9] Using standard measures, Campbell and Viceira (2002, 2005) also report conditional variance estimates that decline with the investment horizon.

Given stationarity, ergodicity, mean reversion, and any other statistical property which may enhance the power of historical return data to characterize future return distributions, the evidence presented in Figure 7.6 might be sufficient to sway investors in favor of all stock portfolios, but only for those who can commit to long investment horizons of 20 years or more. Consider a 10-year horizon; the worst stocks did was to lose 4.1% compounded for 10 years which was less than for bonds and T-bills.[10] Suppose this same investor was required, however, to liquidate after only 3 years. Could the investor possibly have lost 38% (the maximum 1-year loss) in each of these periods? In this case, the investor would have been wiped out. The point being made is simply to say that knowing the best and worst average scenarios over long time periods tells us little about the investor's wealth

[9] Mean reversion is the idea that periods of high security returns will be followed by low return periods and vice versa. Formally, a rate of return series (r_t) is said to be mean-reverting if and only if for any integers $0 \leq s < t < v < u$, $\text{cov}(r_t - r_s, r_v - r_u) < 0$. (This is not the only definition: see the Web notes to the present chapter.)

[10] A 4.1% annual loss for 10 years is actually quite bad; an investor's capital would have been depleted by roughly one half under that scenario.

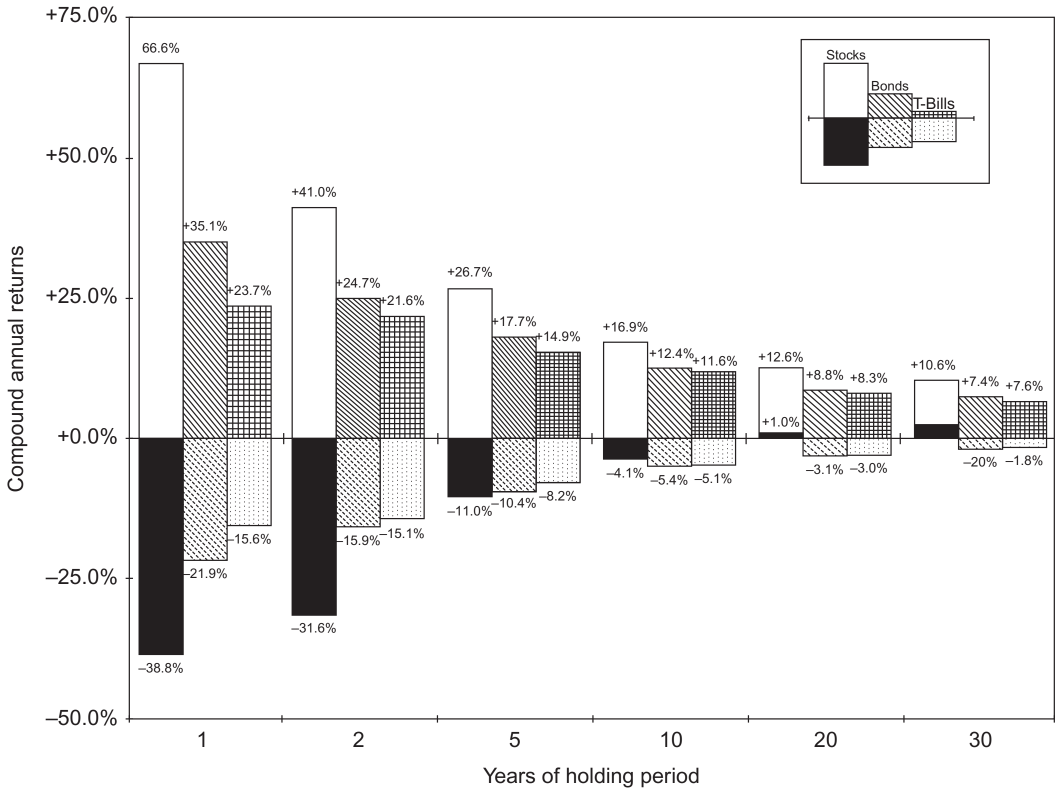

Figure 7.6: Maximum and minimum real holding period returns (1802–1997)[(i)]

[(i)]The maximum and minimum holding period returns, annualized, as reported in the figure, reflect the maximum and minimum returns over all contiguous time horizons of the indicated length during the 1802–1997 period. For example, the minimum (annualized) 30-year compounded equity return during 1802–1997 period was 2.6%. Note that the sample period does not include the Internet Bubble and subsequent collapses of the late 1990s, early 2000s, or the period of the Great Recession. Low negative returns on T-bills signifies periods of high inflation. *Source: Siegel (2008), Figure 2.1.*

volatility within the investment period, something that could be of vital significance in the event of an unexpected health emergency, job loss, etc. In particular, knowledge of the variance of returns would provide information on intermediate wealth volatility, something that is absent from a max–min discussion.[11]

Accordingly, it would be useful to explore "Siegel (2008)-like" results in a model which explicitly accounts for means and variances in an *ex ante* setting. After all, we are

[11] For the United States, the σ of annualized real returns is generally accepted to be around 17%. A 1-year positive return of 66.6% (see Figure 7.6 for a 1-year horizon) is more than three standard deviations from the mean, surely a tail event.

concerned about predictive returns—what future returns will be. Historical returns are useful principally as a guide to estimating moments. We thus need a model of equity return evolution with which to guide our thinking. The random walk model is the natural context in which to first explore these questions, and we introduce this model in Section 7.5.2. In Section 7.5.3, we then modify the random walk model and then revisit the original question: Are stocks more or less risky at longer time horizons?

7.5.2 Intertemporal Stock Return Behavior Through Time: The Random Walk Model

While our analysis so far has been quite detailed as regards to asset return distributions within a period (normality), we still have no working hypothesis as to how asset returns evolve intertemporally. Such a framework is especially important for long-term investors.

The benchmark model of equity return evolution is the random walk: returns are i.i.d. normally distributed across time periods.[12] It may seem surprising to start off with a statement concerning stock returns. We do this because stock returns can reasonably be thought of as stationary stochastic processes. Stock prices are not stationary because there is no upper bound to the values they may achieve and this unboundedness reflects nonstationary behavior. Statistical tests repeatedly confirm, however, that *stock returns* do represent stationary time series, and it is returns that are of interest to investors.[13] Let us agree to measure returns and return statistics (μ, σ) on an annual basis, and let Δt represent an interval of time measured in years. The random walk hypothesis on a stock's rate of return over the time interval $[t, t+\Delta t]$ asserts that

$$\tilde{r}_{t,t+\Delta t} \sim N(\mu\Delta t, \sigma^2\Delta t) \tag{7.1}$$

where μ and σ^2 denote, respectively, the stock's known annualized mean return and the variance of its annual rate of return.[14] It is frequently convenient to express Eq. (7.1) equivalently as

$$\tilde{r}_{t,t+\Delta t} = \mu\Delta t + \sigma\sqrt{\Delta t}\tilde{\varepsilon}_t \tag{7.2}$$

[12] For equity securities, the random walk model is an expression of the notion of "market efficiency." The idea is this: by definition, new information flows to stock market participants in a statistically independent fashion (no predictability). If traders act on this new information to alter their positions immediately, the resulting price and return variation should be statistically independent as well.

[13] The rate of return on wealth essentially captures its growth rate. By analogy to a stock's price, an economy's GDP is potentially without bound and nonstationary. The growth rate of an economy's GDP, however, does represent a stationary time series.

[14] Expression (7.1) means that 1-year stock returns are distributed $N(\mu, \sigma^2)$; for 6-month intervals it is $N(.5\mu, .5\sigma^2)$; for monthly time intervals it is $N(1/12\mu, 1/12\sigma^2)$. The mean and variance are simply scaled up or down by the length of the time intervals.

where $\tilde{\varepsilon}_t \sim N(0,1)$. To see this equivalence, note that it follows from Eq. (7.2) that

$$\begin{aligned} E\tilde{r}_{t,t+\Delta t} &= \mu\Delta t, \text{ and} \\ \sigma^2_{\tilde{r}_{t,t+\Delta t}} &= \sigma^2\Delta t, \text{ as proposed in Eq. (7.1)} \end{aligned}$$

The benchmark random walk model (i.i.d. normal returns cum constant μ and σ) tends to be a very good model of security return evolution when the length of the time periods is short (e.g., a day where $\Delta t = (1/250)$, reflecting the fact of there being roughly 250 trading days per year), and the cumulative time horizon is less than 6 months. At longer horizons, return persistence may creep in: changes in μ and σ must be modeled systematically. In the case of μ, for example, it is reasonable to propose

$$\tilde{\mu}_{t+\Delta t} = (1-\delta)\mu + \delta\mu_t + \tilde{\varepsilon}^{\mu}_{t+\Delta t} \tag{7.3}$$

where μ is the long-run mean (so that μ_t, the period t mean return, fluctuates about μ) and $\text{cov}(\tilde{\varepsilon}_t, \tilde{\varepsilon}^{\mu}_t) = 0$.[15]

While Eq. (7.2) describes the evolution of the period by period rate of return, we have to do a bit more work to understand how the asset's corresponding price evolves.[16] If q^e_t is the price of a stock in period t, then $\tilde{q}^e_{t,t+\Delta t} = q^e_t(1 + \mu\Delta t + \sigma\sqrt{\Delta t}\tilde{\varepsilon}_t)$, assuming no dividend payments. In a world of continuous compounding

$$\begin{aligned} &\ln\left(\frac{\tilde{q}^e_{t+\Delta t}}{q^e_t}\right) \sim N(\hat{\mu}\Delta t, \hat{\sigma}^2\Delta t), \ \textit{or} \\ &\ln \tilde{q}^e_{t+\Delta t} - \ln q^e_t \sim N(\hat{\mu}\Delta t, \hat{\sigma}^2\Delta t) \ \textit{or} \\ &\ln \tilde{q}^e_{t+\Delta t} = \ln q^e_t + \hat{\mu}\Delta t + \hat{\sigma}\sqrt{\Delta t}\tilde{\varepsilon}_t, \ \text{and} \end{aligned} \tag{7.5a}$$

$$\tilde{q}^e_{t+\Delta t} = q^e_t \, e^{\hat{\mu}\Delta t + \hat{\sigma}\sqrt{\Delta t}\tilde{\varepsilon}_t} \tag{7.5b}$$

[15] Heston (1993) has also proposed a model that allows for the stochastic evolution of the return variance:

$$\tilde{\sigma}^2_{t+\Delta t} = \sigma^2_t + \kappa(\sigma^2 - \sigma^2_t)\Delta t + \sigma^2_v\sqrt{\sigma^2_t}\sqrt{\Delta t}\,\tilde{\varepsilon}^{\sigma}_{t+\Delta t} \tag{7.4}$$

where σ^2 is the long run variance, σ^2_t the period t variance, κ a positive constant capturing the speed of convergence of σ^2_t back to σ^2, σ^2_v the volatility of the variance, and $\text{cov}(\tilde{\varepsilon}_t, \tilde{\varepsilon}^{\sigma}_t) = \rho$, which may differ from zero. The presence of the $\sqrt{\sigma^2_t}$ term guarantees that the variance remains positive as $\Delta t \mapsto 0$. For the moment we will set aside generalizations (7.3) and (7.4) and focus on the benchmark formulation (7.2). Note that either generalization (7.3) or (7.4) introduces persistence into the model, and, in the case of Eq. (7.3), mean reversion.

[16] The discussion above applies to a portfolio of assets as well.

with $\tilde{\varepsilon}_t \sim N(0,1)$ and for any time interval Δt. We then say that the price of the asset is lognormally distributed because the ln of its price at time $t + \Delta t$ is normally distributed (recall Section 6.3). It follows that

i. $$E\tilde{q}^e_{t,t+\Delta t} = q^e_t \, e^{\left(\hat{\mu}+\frac{\hat{\sigma}^2}{2}\right)\Delta t} \tag{7.6}$$

ii. $$\mathrm{var}\tilde{q}^e_{t,t+\Delta t} = (q^e_t)^2 \, e^{(2\hat{\mu}+\hat{\sigma})\Delta t}(e^{\hat{\sigma}^2\Delta t} - 1) \tag{7.7}$$

Note that Eqs. (7.6) and (7.7) are the exact counterparts of the expressions in Section 6.6. This is no surprise: the one-period model of Section 6.6 is a special case of what we are considering here (Figure 7.7).

(Expressions (7.5a, b), (7.6), and (7.7) are really very general since we may substitute aggregate investor wealth Y_t invested in a portfolio p for q^e_t, let t = 0, $\hat{\mu} = \hat{\mu}_P, \hat{\sigma} = \hat{\sigma}_P$, and select Δt equal to our investment time horizon.)

When a stock's price evolves according to Eqs. (7.5a, b), the result is a familiar sawtooth pattern. In Figures 7.8 and 7.9, we present two potential price paths (there are an uncountably infinite number of possible paths) for the case where $q^e_t = \$120$, $\hat{\mu} = 0.20$, $\hat{\sigma} = 0.30$ and $\Delta t = (1/250)$. These price paths are generated by feeding into Eq. (7.5b) a sequence of independent draws from $N(0, 1)$ and updating the price accordingly.

If the reader sees a pattern in these figures, we gently remind him that he is mistaken: they are the result of purely i.i.d. draws from $N(0, 1)$.

With this perspective in mind, we return to the question of whether stocks—more precisely, very well diversified stock portfolios such as the S&P$_{500}$ portfolio—are less risky at longer time horizons. If this were to be the case, what additional assumption on the return processes, if any, would have to be true?

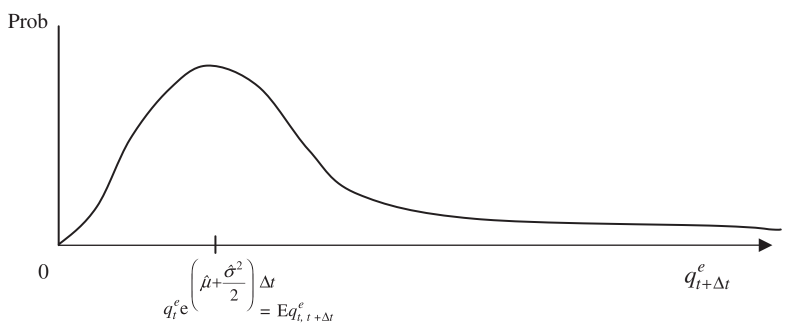

Figure 7.7
Asset price probability density function.

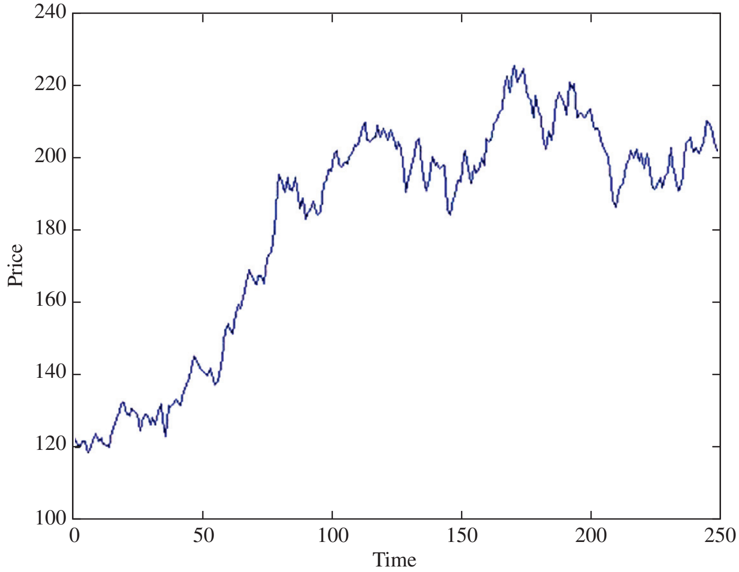

Figure 7.8
Sample price path of 250 periods; $q_t^e = \$120$, $\hat{\mu} = 0.20$, $\hat{\sigma} = 0.30$. *Source: authors' simulations.*

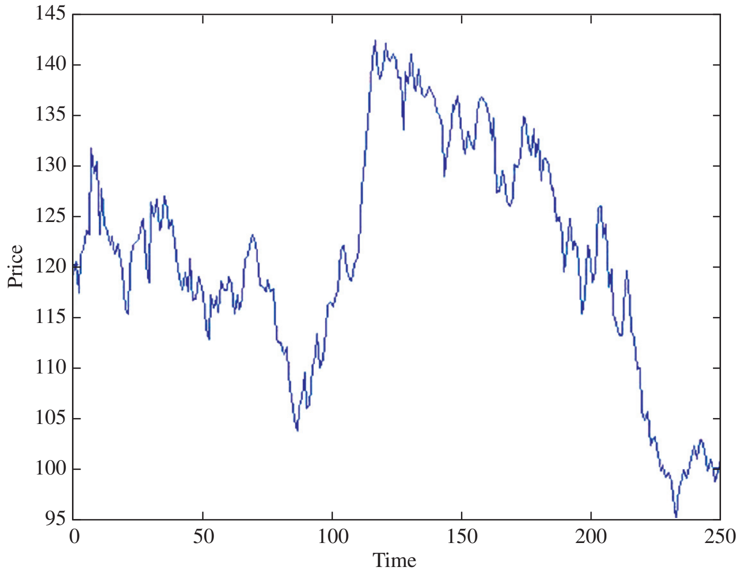

Figure 7.9
Sample price path of 250 periods; $P_0 = \$120$, $\hat{\mu} = 0.20, \hat{\sigma} = 0.30$. *Source: authors' simulations.*

7.5.3 Are Stocks Less Risky in the Long Run? A Predictive Perspective[17]

This is a substantive question from the perspective of an investor looking forward; let us first see what the random walk model has to say. To simplify notation, let $\Delta t = 1$ year. Equation (7.2) then reduces to

$$\tilde{r}_{t+1} = \mu + \sigma^2 \tilde{\varepsilon}_t$$

Investors are interested to know the variance of

$$\tilde{r}_{t,t+J} = \tilde{r}_{t+1} + \tilde{r}_{t+2} + \cdots + \tilde{r}_{t+J}$$

where J denotes their investment horizon. We refer to it as the predictive variance. The question at hand is how the predictive variance changes with J. Assuming μ and σ^2 are precisely estimated, by the i.i.d. assumption underlying a random walk,

$$\text{var}(\tilde{r}_{t,t+J}) = J\sigma^2 \text{ or}$$

$(1/J)\text{var}(\tilde{r}_{t,t+J}) = \sigma^2$: the annualized risk is the same irrespective of the investment horizon. In keeping with the difficulty in estimating means precisely (recall Box 7.1), however, let us revisit this calculation under circumstances where σ^2 is known but μ is not. Let I_t denote all information pertinent to the estimation of μ. In this case

$$\begin{aligned}
\text{var}(\tilde{r}_{t,t+J}) &= J\sigma^2 + \text{var}(J\mu|I_t) \\
&= J\sigma^2 + J^2\,\text{var}(\mu|I_t) \text{ and} \\
\frac{1}{J}\text{var}(\tilde{r}_{t,t+J}) &= \sigma^2 + J\,\text{var}(\mu|I_t) \mapsto \infty \text{ as } J \mapsto \infty
\end{aligned} \tag{7.8}$$

An investor who holds to the random walk model but who is uncertain as to the true mean return should thus generally view stocks as increasingly risky in the long run. This is a very different perspective than Siegel's (2008). If the uncertainty surrounding μ is measured conventionally by the standard error of the estimate of the mean, $(\sigma/\sqrt{T})$ (see Box 7.1), then expression (7.8) reduces to

$$\frac{1}{J}\text{var}(\tilde{r}_{t,t+J}) = \sigma^2\left(1 + \frac{J}{T}\right)$$

Pastor and Stambaugh (2012) compute this quantity for $T = 206$ (the same historical data series as Siegel (2008)) and $J = 50$ (a 50-year time horizon looking forward); in this case:

$$\frac{1}{J}\text{var}(\tilde{r}_{t,t+J}|I_t) = \sigma^2\left(1 + \frac{50}{206}\right) = 1.243\sigma^2$$

[17] The remarks in this section are taken directly from Pastor and Stambaugh (2012).

the predictive return variance grows about 25% at long relative to short (one-year) horizons. Note that if the data set is much shorter, $T = 40$ years, for example, the long-run predictive variance is more than twice σ^2.

Other sources of uncertainty can enter the long-/short-run variance comparison besides one-period return volatility. Suppose a security's return evolution conforms to the following system:

$$\tilde{r}_{t+1} = \mu_t + \tilde{\varepsilon}_{t+1} \tag{7.9a}$$

$$\tilde{\mu}_{t+1} = (1-\beta)E\tilde{r} + \beta\mu_t + \tilde{w}_{t+1} \tag{7.9b}$$

where $\beta < 1$, $\rho_{\varepsilon,w} < 0$ and $\tilde{\varepsilon}_t, \tilde{w}_t$ are mean zero random components. Under specification (7.9a, b), there are five sources of potential uncertainty: (i) uncertainty in future expected returns as per Eq. (7.9b), (ii) i.i.d. uncertainty in $\tilde{\varepsilon}_t$, (iii) uncertainty in the current expected return ($E\tilde{r}$), and (iv) potential parameter uncertainty as regards $(\sigma_\varepsilon, \sigma_w, \rho_{\varepsilon,w}, \beta, E\tilde{r})$ (referred to as estimation risk), and the presence of (v) mean reversion which, in this model, is captured by $\rho_{\varepsilon,w} < 0$.[18] In and of itself, mean reversion leads to a reduction in long-run uncertainty relative to the i.i.d. case. Under this specification, whether stocks are less or more risky in the long-run boils down to whether mean reversion overwhelms or is overwhelmed by the consequences of uncertainty in the other elements.

To gain an understanding of how these five sources of uncertainty interact to determine the predictive variance, Pastor and Stambaugh (2012) estimate the entire system (equations 7.9a,b and associated parameters) using annual USA return data for the period 1802-2007, the same data set as used by Siegel (2008). While we leave the myriad details of this estimation to the interested reader, the relative contributions of the different sources of uncertainty, and how they change as the investor's time horizon increases, are easily seen in Figure 7.10. Note that the vertical axis measures annualized variance and that Panel A represents the vertical sum of the components in Panel B.

As expected, it is only the mean reversion in returns ($\rho_{\tilde{\varepsilon},\tilde{w}} < 0$) that tends to lower the predictive variance as the investor's time horizon increases. While its effect is substantial, it is overwhelmed by the increases in the other sources of uncertainty. In particular, the contribution of future mean uncertainty $\{\tilde{\mu}_{t+j}\}$ increases quite robustly and soon becomes dominant; the contribution from uncertainty as regards the current mean (μ_t) and the model parameters ($\sigma_{\tilde{w}}, \sigma_{\tilde{\varepsilon}}$, etc.) increase with the time horizon as well. By construction, the i.i.d. component's contribution ($\tilde{\varepsilon}_t$) is constant on an annualized basis.

What do we learn from the Pastor and Stambaugh (2012) exercise as regards our original question: "Are stocks more risky in the long run?" The answer appears to be model specific

[18] Pastor and Stambaugh (2012) analyze this model in considerable detail, to which we refer the interested reader.

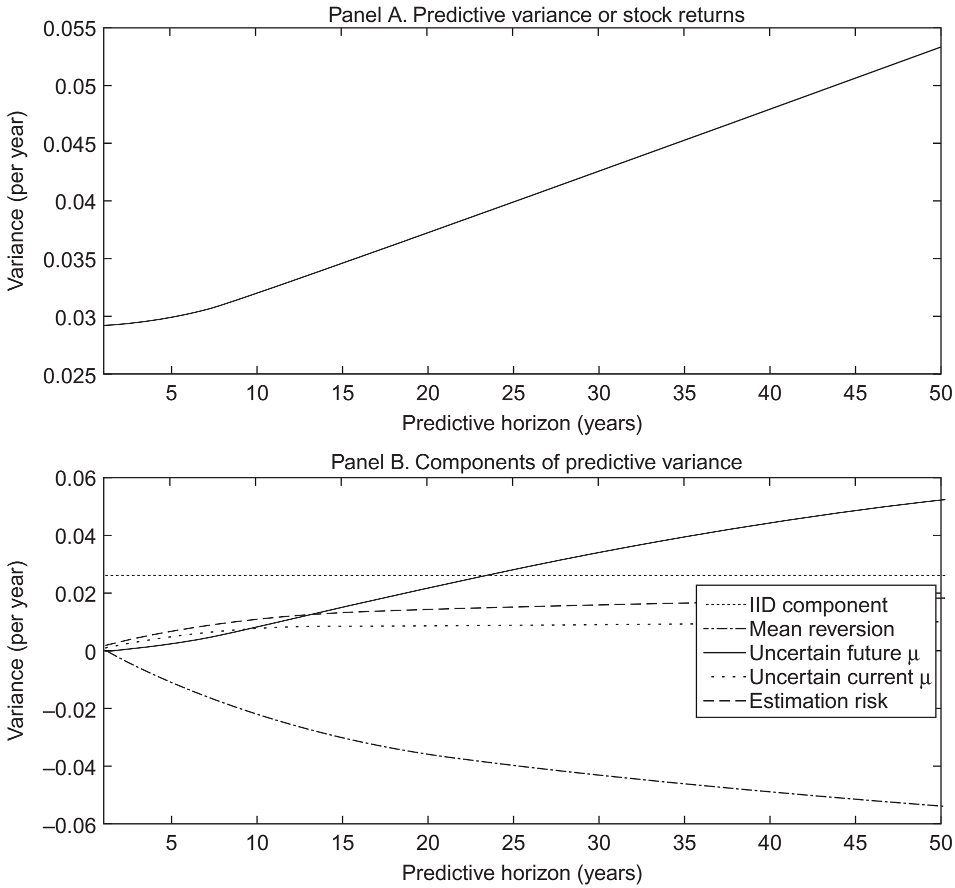

Figure 7.10: Predictive variance of multiperiod return and its components.
Panel A plots the variance of the predictive distribution of long-horizon returns, var($r_{t,t+j}|I_t$). Panel B plots the five components of the predictive variance. All quantities are divided by k, the number of periods in the return horizon. The results are obtained by estimating the predictive system on annual real U.S. stock market returns in 1802 to 2007. *Source: Figure 6 in Pastor and Stambaugh (2012)*.

and largely depends on what the investor "knows – or believes he knows." In particular, uncertainty as regards the initial period's mean return and uncertainty regarding future expected returns can conspire such that long term investors in practice face substantially more return volatility than short-horizon investors.

7.6 Conclusions

First, it is important to keep in mind that everything said so far in this chapter applies regardless if the probability distributions (possibly normal) on returns represent the *subjective* expectations of the particular investor upon whom we are focusing or "objective" market forecasts.

Second, although initially conceived in the context of descriptive economic theories, the success of portfolio theory arose primarily from the possibility of giving it a normative interpretation, i.e., of seeing the theory as providing a guide on how to proceed to identify a potential investor's optimal portfolio. In particular, it points to the information requirements to be fulfilled (ideally). Even if we accept the formal restrictions implied by mean−variance analysis, one cannot identify an optimal portfolio without spelling out expectations on mean returns, standard deviations of returns, and correlations among returns. As this chapter has demonstrated, this is no easy task empirically. Investors may simply fall back on equally weighted portfolios. There is much to commend this alternative. One can view the role of the financial analyst as providing plausible figures for the relevant statistics or offering alternative scenarios for consideration to the would-be investor. This is the first absolutely critical step in the search for an optimal portfolio and one that may have statistical as well as intuitive aspects. The numbers proposed are forecasts and they may not be very precise ones at that. The computation of the (subjective) efficient frontier is the second step, and it essentially involves solving the quadratic programming problem (QP) possibly in conjunction with constraints specific to the investor. The third and final step consists of defining, at a more or less formal level, the investor's risk tolerance and, on that basis, identifying his optimal allocation to risk-free versus risky assets.

Market equilibrium considerations are next.

References

Benartzi, S., Thaler, R., 2001. Naive diversification strategies in defined contribution savings plans. Am. Econ. Rev. 91, 79−98.

Best, M., Grauer, R.R., 1991. On the sensitivity of mean−variance−efficient portfolios to changes in asset means: some analytical and computational results. Rev. Finan. Stud. 4, 315−342.

Bloomfield, T., Leftwich, R., Long, J., 1977. Portfolio strategies and performance. J. Finan. Econ. 5, 201−218.

Campbell, J., Viceira, L., 2002. Strategic asset allocation: Portfolio choice for long term investors. Oxford Univ. Press, Oxford, UK.

Campbell, J., Viceira, L., 2005. The term structure of the risk-return tradeoff. Finan. Analysts. J. 61, 34−44.

Chen, N., Roll, R., Ross, S., 1986. Economic forces and the stock market. J. Bus. 59, 383−403.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naïve diversification: how inefficient is the (1/*N*) portfolio strategy? Rev. Econ. Stud. 22, 1915−1953.

Heston, S., 1993. A closed form solution for options with stochastic volatility. Rev. Finan. Stud. 6, 327−343.

Huberman, G., Jiang, W., 2006. Offering vs. choice in 401(k) plans: equity exposure and the number of funds. J. Finan. 61, 763−801.

Jorion, P., 1991. Bayesian and CAPM estimates of the means: implications for portfolio selection. J. Bank. Finan. 15, 717−727.

Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. J. Finan. 56, 649−676.

Michaud, R., 1989. The Markowitz optimization enigma: is "optimized" optimal? Finan. Analysts J. 45, 31−42.

Milesa-Ferretti, G., Tille, C., 2010. The great retrenchment: international capital flows during the global financial crisis. CEPR, Economic Policy Working Paper.

Pastor, L., Stambaugh, R., 2012. Are stocks really less volatile in the long run? J. Finan. 67, 431−477.

Plyakha, Y., Uppal, R., Vilkov, G., 2012, Why does an equal-weighted portfolio outperform value and price-weighted portfolios? Working Paper, EDHEC Business School.
Riberio, R., Veronesi, P., 2002. The excess co-movement of international stock markets in bad times: a rational expectations equilibrium model, mimeo, University of Chicago, Booth School.
Siegel, J., 2008. Stocks for the Long Run. fifth ed. McGraw Hill, New York, NY.

Appendix 7.1

We can also use the data in Table 7.2 to get an idea of the enormous impact that short selling constraints (see Section 6.4) can have on portfolio composition and the associated portfolio risk and return when the techniques of MPT are applied. Let us suppose that the entries in Table 7.2 represent ex ante estimates in 1.1.1996. By comparing Tables A.7.1 and A.7.2 (both based on Table 7.2), we see that for all cases in which four or more securities (national market portfolios) are eligible for portfolio admission, the lifting of the "no-short-sales" constraint leads to leverage levels exceeding 80% of the initial investment. Mean monthly returns in these cases are roughly 0.15% higher (1.8% annualized). In all cases, the portfolio is configured to have a monthly standard deviation of 5%.

Table A.7.1: Portfolio proportions when $\sigma_p = 5\%$ and short sales are not allowed

Short-Sale Constraints?	No	No	No	No	No
Horizon	120	120	120	120	120
France	122.28%	99.24%	89.59%	79.41%	79.79%
Germany	− 54.85%	− 56.23%	− 67.67%	− 71.09%	− 71.43%
Japan		− 31.24%	− 31.25%	− 21.86%	− 21.22%
UK	32.57%	88.23%	15.69%	20.93%	21.05%
US			93.64%	78.10%	79.06%
Brazil				14.51%	15.29%
Korea					− 2.55%
Mean monthly return	0.62%	0.70%	0.85%	0.89%	0.89%
SD of monthly rate	5.00%	5.00%	5.00%	5.00%	5.00%

Table A.7.2: Portfolio proportions when $\sigma_p = 5\%$ and short sales are permitted.

Short-Sale Constraints?	Yes	Yes	Yes	Yes	Yes
Horizon	120	120	120	120	120
France	62.99%	62.99%	59.09%	17.04%	17.04%
Germany	0.00%	0.00%	0.00%	0.00%	0.00%
Japan	0.00%	0.00%	0.00%	0.00%	0.00%
UK	37.01%	37.01%	0.00%	0.00%	0.00%
US			40.91%	68.82%	68.82%
Brazil				14.14%	14.14%
Korea					0.00%
Mean monthly return	0.57%	0.57%	0.65%	0.74%	0.74%
SD of monthly rate	5.00%	5.00%	5.00%	5.00%	5.00%