Asset Pricing

Introduction and course overview

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Fall 2021

Outline

1. About me
2. Course introduction
3. A first look at Asset Pricing
   - Valuing known cash flows
   - Valuing risky cash flows
   - Equilibrium versus no-arbitrage models
4. Models, stylized facts, and anomalies
   - The equity premium
   - Anomalies
   - The yield curve

About me

Basic information

* Jonas Nygaard Eriksen
* Associate Professor, Department of Economics and Business Economics
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Background

* Ph.D. in Economics and Business Economics (2015)
* MSc. in Finance (2012)

Research interests

* Asset pricing
* Return predictability
* Monetary policy

Lectures

Lectures in weeks 36–49

- Lecture slides and material will be made available in advance of the lectures
- Lecture slides and material are made on a topic-by-topic basis
- I usually write out weekly on Brightspace about the plan

Discussion forums

- I have made discussion forums for each topic on Brightspace
- I encourage you to be active (I will monitor and participate when needed)

Exercises

Exercises

- There will be several exercise sets during the course
- Focus on theory, empirical implementation, and evaluation of models and methods using financial data
- They are exam relevant and I frequently ask about them
- I recommend doing them in groups of 2–4 people
- Solutions will be available in PDF and Matlab

On programming language

- The main language of choice is Matlab
- Used in several other courses across the department
- You are free use your (statistical) software of choice

Exam

Exam

- 20 minutes oral exam without preparation
- No supplementary material allowed
- There will be 8 main topics to draw from at the exam
- Document with more info is available on Brightspace

Structure

- 10 minutes for presenting the drawn topic
- 5 minutes for questions from the co-examiner and myself
- 5 minutes for discussing and giving the grade

Remarks

- You will not have time to cover all in a topic, so prioritize selected material
- We expect you to use the boards to write up important models, equations and illustrations, sketch derivations, and explain how to evaluate theories/models
- Focus will be on intuition and key equations, not only detailed derivations

Overview of main topics and structure

1. Expected utility and risk aversion
2. Mean-variance analysis
3. Capital Asset Pricing Model (CAPM)
4. Arbitrage Pricing Theory (APT)
5. Consumption-based Asset Pricing (CCAPM)
6. Fixed income securities
7. The expectations hypothesis
8. Term structure models

Literature

Main textbooks

* Danthine and Donaldson (2014): Intermediate financial theory (3rd Edition)
* Campbell, Lo, and MacKinlay (1997): The econometrics of financial markets

Articles and lecture notes

* The full list of articles is available in the lecture plan
    1. The articles all highlight key aspects of the development of asset pricing
    2. They are important to the course and exam relevant
    3. Potential inspiration for thesis topics and background
    4. The articles is a mix of classic and newer contributions
* Lecture notes provide additional details and cover material not available elsewhere (also exam relevant)

Overview of learning outcomes

Knowledge and understanding of

- The concepts of utility functions and risk aversion and how they relate to each other and their influence on investor behaviour
- The mean-variance framework for portfolio selection, its assumptions, and the benefits of diversification
- The Capital Asset Pricing Model (CAPM) as an equilibrium theory, its assumptions, its relation to the mean-variance framework, and its empirical failures
- The Arbitrage Pricing Theory (APT) as a no-arbitrage model, its similarities and differences to the CAPM, and how to select and evaluate risk factors
- Asset pricing puzzles such as size, value, and momentum and their implication for standard asset pricing models and market efficiency

Overview of learning outcomes

- The Consumption-based Capital Asset Pricing Model (CCAPM) as an equilibrium theory, its assumptions, and its predictions about the risk-free rate and risk premia on risky assets and how it compares to the CAPM and APT
- Spot rates, forward rates, yields, holding-period returns, and prices and their relation to each other and the yield curve and the concepts of duration and convexity as well as their applicability to interest rate risk management
- Selected techniques and models (e.g., bootstrapping and the Nelson-Siegel model) to estimating the zero-coupon discount curve from market prices and the implications of the chosen method
- The expectations hypothesis (EH), its empirical validity, and its implications for the behaviour of interest rates
- Selected affine term structure models (e.g., Vasicek and Cox-Ingersoll-Ross) from an asset pricing perspective, their underlying assumptions, and how their predictions relate to the EH

Overview of learning outcomes

Skills to

* Discuss and evaluate the use of different utility functions and reflect on their implications for the degree of risk aversion and the behaviour of economic agents
* Solve the mean-variance portfolio optimization problem, evaluate and discuss the outcome, and identify the efficient frontier and the location of the tangency and the minimum variance portfolios
* Estimate and evaluate asset pricing models using standard methods and discuss and reflect on the outcome of the tests and the implications for theories and market efficiency
* Evaluate and assess the predictions of asset pricing models using different values of key parameters and discuss and reflect on the implication for the behaviour of investors

Overview of learning outcomes

- Estimate and evaluate methods for extracting discount and interest rate curves from market prices and compute duration and convexity for fixed income securities
- Test and evaluate the empirical validity of the EH using standard tests and discuss and reflect on the implication for fixed income securities and the behaviour of interest rates
- Estimate and evaluate selected affine term structure models and discuss and reflect on their implications for the behaviour of interest rates

Overview of learning outcomes

Competences to

- Assess and compare relevant utility functions and discuss and reflect on their appropriateness for modelling investor behaviour
- Identify optimal portfolios, the tangency, and the minimum variance portfolios, and discuss and reflect on their properties and relation to asset pricing theory
- Compare asset pricing models and their assumptions and discuss their similarities and differences and reflect on their empirical and theoretical validity
- Evaluate empirical tests of asset pricing models and discuss and reflect on their implications for asset pricing theories

Overview of learning outcomes

- Compare selected methods to estimating the term structure of interest rates and reflect on the implications of the selected method
- Discuss and reflect on the implications of the EH and its empirical and theoretical validity for describing the behaviour of interest rates
- Compare selected affine term structure models and discuss their differences and reflect on their implications for the behaviour of interest rates

Expectations

My expectations

* That you arrive prepared for class
* That you actively participate in the course
* That you work with the material, including readings and exercises

Your expectations

* What do you expect to gain from this course?
* What do you expect from me as a lecturer?

Questions or comments?

QUESTIONS OR COMMENTS TO THE COURSE STRUCTURE?

A first look at Asset Pricing

What is asset pricing?

Asset pricing theory is the attempt to model and understand how individuals value and trade claims to future uncertain payments

- "Uncertain" and "future" are both key adjectives as we have to account for both the *delay* and the *risk* of a payment
- The impact of time is somewhat straightforward and often make up a small fraction of the overall compensation
- Uncertainty, or corrections for risk, is much more important and what makes asset pricing interesting and challenging
- Asset pricing theory can be viewed from a *positive* (how the world does work) and a *normative* (how the world should work) position

Valuing cash flows and pricing asset

Main questions of asset pricing

The main question of modern asset pricing can be formulated in a variety of ways

1. How do we value a risky cash flow?
2. How do we price risky assets?
3. Why do different assets earn different average rates of return?

* They all boil down to the same issue since an asset is nothing more than the right to a stream of future (uncertain) cash flow
* The final statement emphasizes a cross-sectional perspective that will be the main focus of this course (especially for the asset pricing part)
* A low price implies a high expected return, so one can also think of the theory as explaining why some assets pay higher average returns than others

Valuing known cash flows

- If the future cash flow is known and available for sure, then pricing the asset is an almost trivial exercise
- Consider an n-period zero-coupon (discount) bond that pays \$1 dollar at maturity. If the today dollar price of the bond is P_t^n, then the risk-free yield is

$$1 + Y_t^n = \left(\frac{1}{P_t^n} \right)^{\frac{1}{n}} \tag{1}$$

- If we instead observe the risk-free yield Y_t^n, then we can determine the price by discounting the future dollar payment

$$P_t^n = \frac{1}{(1 + Y_t^n)^n} \tag{2}$$

- US Treasury (Government) bills are structured this way and they are important for many pricing exercises in finance

Time value money

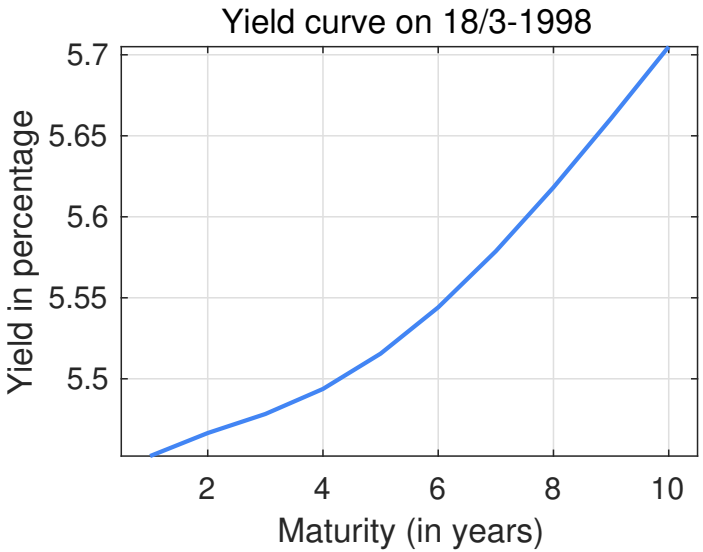
Time value of money

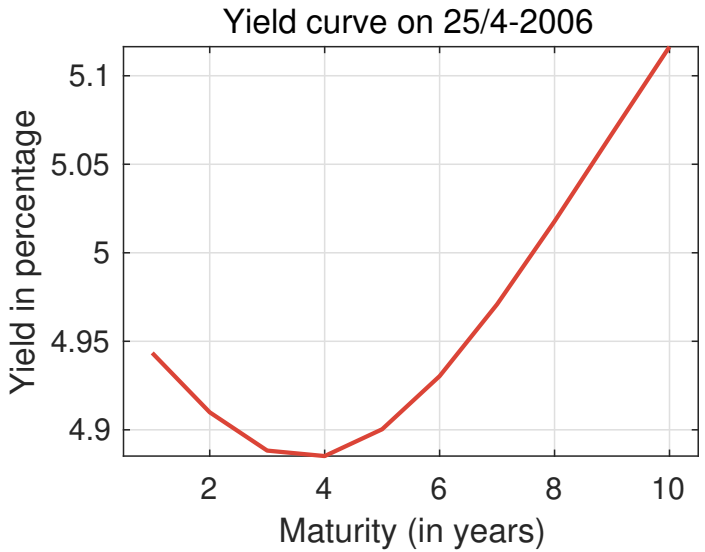
The time value of money states that receiving a dollar today is preferable to receiving a dollar in the future. As such, it is the difference in value between money today and money in the future

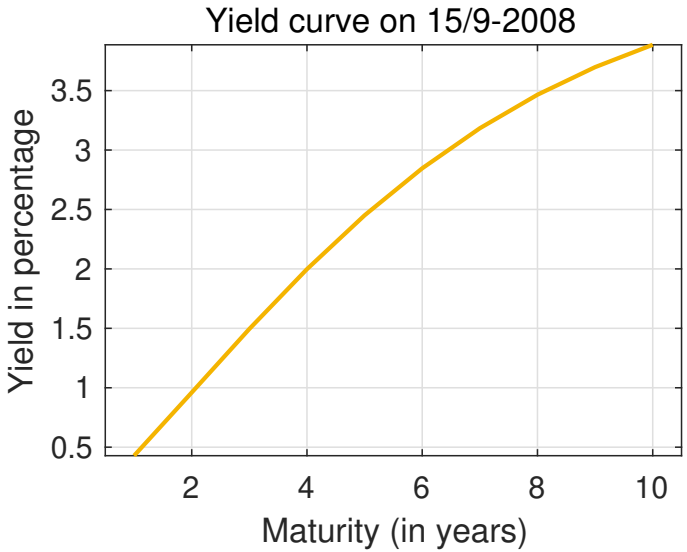
- We can view the price P_t^n as the today price of a known future payment
- When the future payment is completely certain, then this price is a pure reflection of the time value of money
- Observing discount bond prices for different maturities on any given day enables us to determine the full term structure of interest rates
- Although discount bonds are usually short-term, we can derive a full zero-coupon term structure from coupon-paying bonds

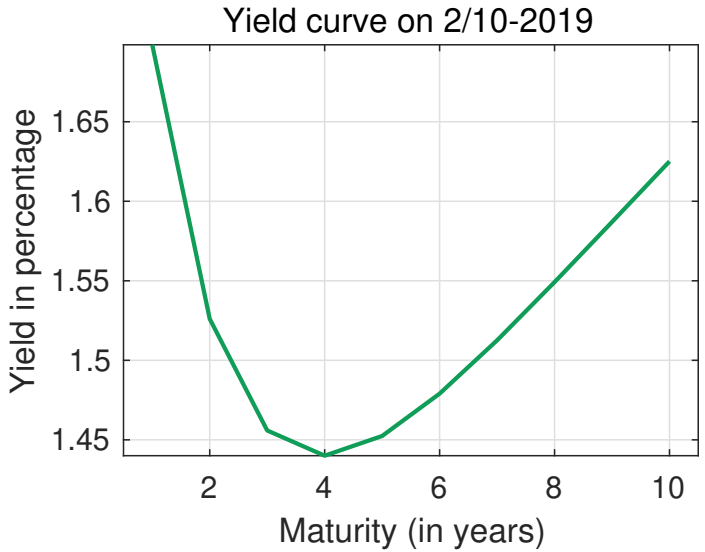
The term structure of interest rates

* The term structure of interest rates can take on many possible shapes and below are real-world examples from randomly selected days









Adding intermediate payments

* Consider an n-period coupon-paying bond that pays a per period coupon rate of C with a face value of \$1
* We can view a bullet bond as a portfolio of zero-coupon bonds and the law of one price and no-arbitrage dictate that we can price such an asset as

$$P_{c,t}^{n} = \sum_{i=1}^{n} \frac{C}{\left(1 + Y_{c,t}^{n}\right)^{i}} + \frac{1}{\left(1 + Y_{c,t}^{n}\right)^{n}} \tag{3}$$

where we note that the risk-free rates used for discounting are those related to discount bonds and the term structure of interest rates

Valuing risky cash flows

- Consider an asset with future risky cash flows $\left\{\widetilde{CF}_t, \widetilde{CF}_{t+1}, \ldots, \widetilde{CF}_T\right\}$ that are uncertain from the viewpoint of today
- Valuing these risky cash flows is essentially what asset pricing is all about
- To value such assets, we typically work with expectations of the future, unknown, random variable

Expected value

Consider a random variable $\widetilde{x}$ that can take on N possible values $\{x_1, x_2, \ldots, x_N\}$ with probabilities $\{\pi_1, \pi_2, \ldots, \pi_N\}$, where $\sum_{i=1}^{N} \pi_i = 1$ and $\pi_i \geq 0$. The expected value of such a random variable is given by

$$\mu = \mathbb{E}\left[\widetilde{x}\right] = \pi_1 x_1 + \pi_2 x_2 + \cdots + \pi_N x_N \tag{4}$$

$$= \sum_{i=1}^{N} \pi_i x_i \tag{5}$$

Valuation strategies for risky cash flows

1. The most common strategy consists of discounting expected values at a rate higher than the risk-free rate

$$P_t = \frac{\mathbb{E}\left[\widetilde{CF}_{t+1}\right]}{\left(1 + r_{f,t+1} + \widetilde{rp}_{t+1}\right)} \tag{6}$$

   where the additional rate is usually referred to as the risk premium. In this approach, estimating $\widetilde{rp}_{t+1}$ is key

2. A similar approach consists of correcting the expected cash flow itself so that discounting can take place at the risk-free rate

$$P_t = \frac{\mathbb{E}\left[\widetilde{CF}_{t+1}\right] - \Pi_{t+1}}{\left(1 + r_{f,t+1}\right)} \tag{7}$$

   where Π_{t+1} is a risk premium, here just in absolute value rather than a rate

Valuation strategies for risky cash flows

3. Another possibility is to "distort" the probabilities so as to down-weight favorable outcomes and over-weight adverse outcomes (so-called risk-neutral probabilities) such that

$$P_t = \frac{\widehat{\mathbb{E}}\left[\widetilde{CF}_{t+1}\right]}{(1 + r_{f,t+1})} \tag{8}$$

where $\widehat{\mathbb{E}}[X] = \widehat{\pi}_1 X_1 + \widehat{\pi}_2 X_2 + \cdots + \widehat{\pi}_N X_N$ is then the expectations operator defined over the "distorted" probabilities

4. Last, one can decompose the future cash flow into its state-by-state elements (so-called Arrow-Debrau pricing)

$$P_t = \sum_{\theta_t \in \Theta_t} q(\theta_t) CF(\theta_t) \tag{9}$$

where θ_t denote a possible state of the world and $q(\theta_t)$ is the today price of $\$1$ if that state realizes in the future (also called the state price)

Equilibrium versus no-arbitrage models

* Although the above strategies are all designed to accomplish the same basic goal, i.e., valuing risky cash flows, we can still group them into two broad categories of asset pricing models: **Equilibrium and no-arbitrage models**

Equilibrium models

The traditional equilibrium approach consists of an analysis of the factors determining the supply and demand for the cash flow (asset) in question. That is, it prices all assets based on the principles of microeconomic theory. Examples include the CAPM and the CCAPM

No-arbitrage models

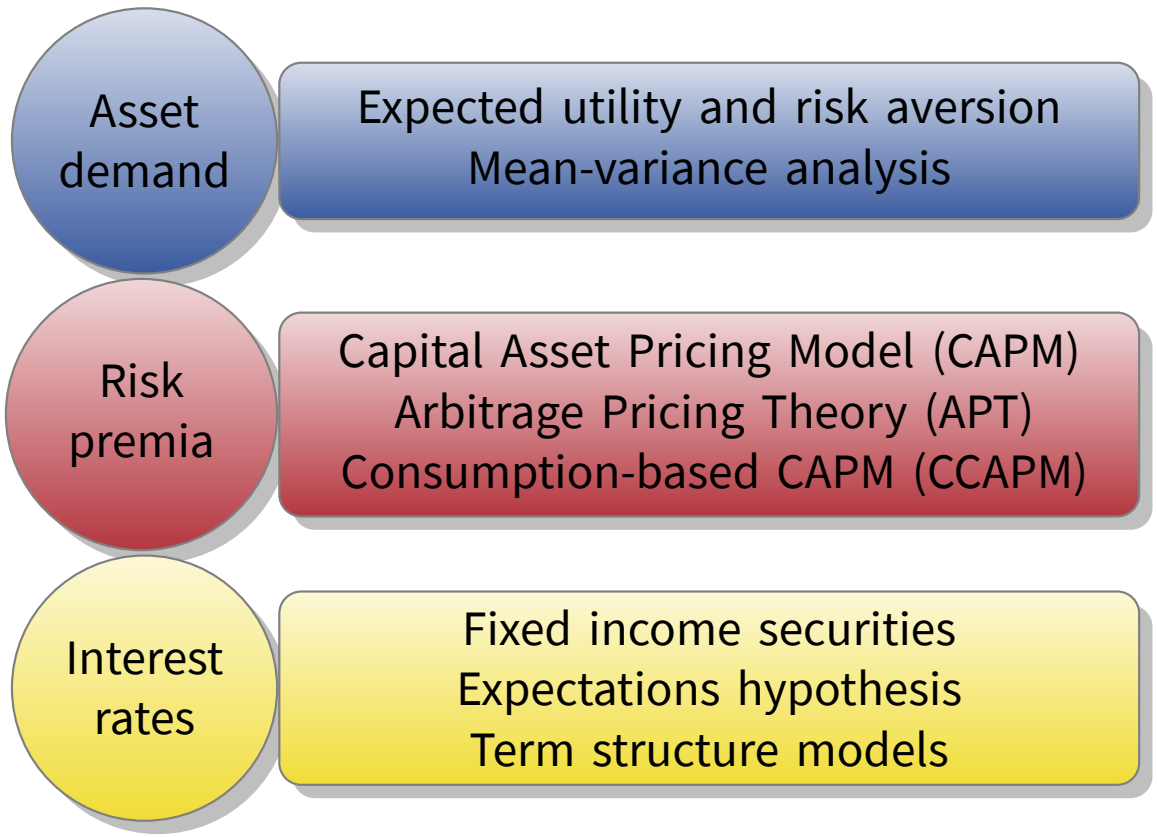
No-arbitrage theories take the prices of some basic assets as given and use those to determine the prices of other assets. That is, the price is the sum of the value of the basic asset. The APT is an example of this approach

Equilibrium versus no-arbitrage models: Remarks

- Equilibrium models requires us to discuss the preferences and attitudes towards risk of investors
- No-arbitrage models are more straightforward than equilibrium models and require fewer assumptions
- The simplicity, however, comes at a cost. No-arbitrage models can never answer questions such as
  - Where do the prices of the basic securities come from (if not through equilibrium)?
  - How do asset prices relate to macroeconomic fundamentals?
- As such, no-arbitrage models are best viewed as a complement to the equilibrium approach, although one should not dismiss the usefulness of the arbitrage approach

The road map going forward

* With the above discussion in mind, we can illustrate the asset pricing road map going forward as



Asset pricing models

* Our goal as financial economists is to understand the behavior of financial markets, meaning building models that can explain asset pricing phenomena

Asset pricing models

Asset pricing models are abstractions of reality and, at best, approximations of the true return generating process. As such, any asset pricing model is a simplification of reality designed to illuminate complex processes

1. A good model must be simple enough to enrich our intuition while at the same time being able to explain the phenomenon under study
2. The abstraction which the model represents must be tailored to the particular questions being asked of it
3. The model should be able to give precise answers to questions we pose concerning the behavior of the real economy

Stylized facts and anomalies

Stylized facts

Stylized facts are well established price, quantity, or return patterns that have been present in financial market data over long periods of time

- To judge whether a model is a good abstraction of reality, it is standard to examine its ability to reproduce a set of stylized facts
- A related concept is asset pricing anomalies by which we mean pervasive return patterns not readily explained by existing models (e.g., momentum within the Capital Asset Pricing Model)
- Again, any good asset pricing should be able to accommodate these anomalies so that the gaps in our understandings are filled

The equity premium

* A broadly diversified portfolio of stocks (e.g., the S&P500) consistently earn average returns substantially in excess of the risk-free rate

Table 2.3: US returns: 1889–2010[a]

	Real Return on a Market Index[b]	Real Return on a Relatively Riskless Security	% Risk Premium
Time period	**Mean**	**Mean**	**Mean**
1889–2010	7.5%	1.1%	6.4%
1889–1978	7.0%	0.8%	6.2%
1926–2010	8.0%	0.8%	7.2%
1946–2010	7.5%	0.8%	6.7%

[a]Data from Mehra (2012); annualized returns.
[b]The $S\&P_{500}$ and its antecedents.

International equity premia

* This observation is not unique to the United States, but indeed extends to international data as well with comparable magnitudes

Table 2.4: The equity premium: the principal capital markets[a]

Country	Time Period	% Risk Premium	Country	Time Period	% Risk Premium
Belgium	1900–2010	5.5%	Sweden	1900–2010	6.6%
Holland	1900–2010	6.5%	UK	1900–2010	6.0%
France	1900–2010	8.7%	Australia	1900–2010	8.3%
Germany	1900–2010	9.8%	Canada	1900–2010	5.6%
Ireland	1900–2010	5.3%	India	1991–2004	11.3%
Italy	1900–2010	9.8%	Japan	1900–2010	9.0%

[a] *Source and details*: Dimson et al. (2010); annualized returns.

* The high premium earned by stocks is termed a puzzle as standard models have trouble replicated these statistics (at least for realistic parameter values)

* The Consumption-based Capital Asset Pricing Model (CCAPM), for example, have trouble doing this for plausible levels of risk aversion

The value premium

- The value premium is a statement about the cross-section of stock returns

Table 2.5: Average annualized excess returns for 10 portfolios sorted on BE/ME[a]

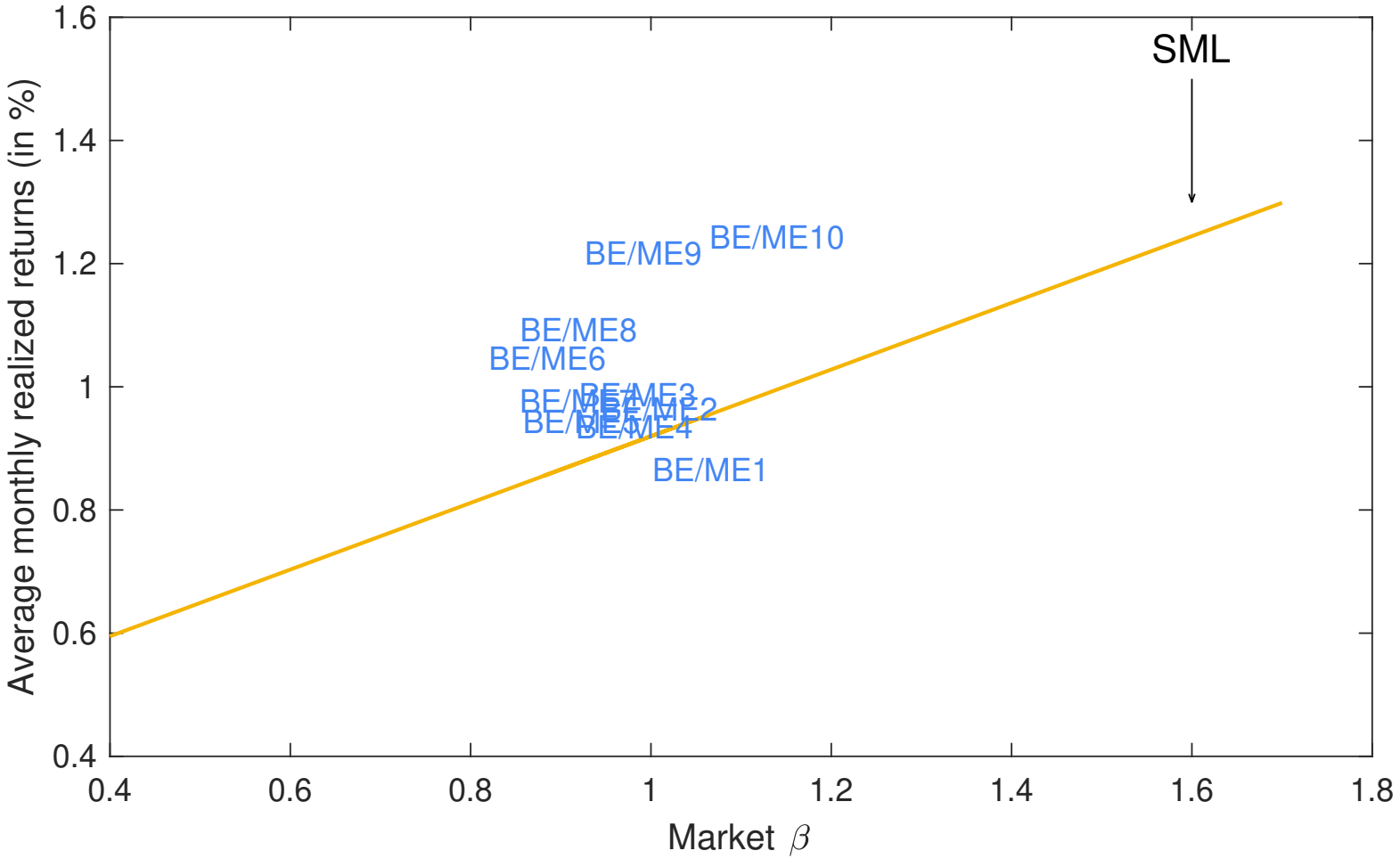
Lowest		→ Increasing (BE/ME) →						Highest	
Port 1	Port 2	Port 3	Port 4	Port 5	Port 6	Port 7	Port 8	Port 9	Port 10
6.76	7.64	7.89	7.65	8.43	8.92	9.02	10.88	11.65	12.75

[a]Based on monthly data for the period 1963.1 through 2011.7. These (value weighted) portfolios are reconstructed (i.e., all the Compustat stocks are reassigned to one of the 10 portfolios) at the end of June of each year based on the end of the previous year's BE and ME values. We thank Tano Santos for making this data available to us.

- It is the observation that stocks with high book-to-market equity ratios (value stocks) on average earn higher excess returns than low book-to-market ratio (growth) stocks
- It is an anomaly (stylized fact) in the sense that standard models such as the CAPM and CCAPM are unable to explain this pervasive return pattern

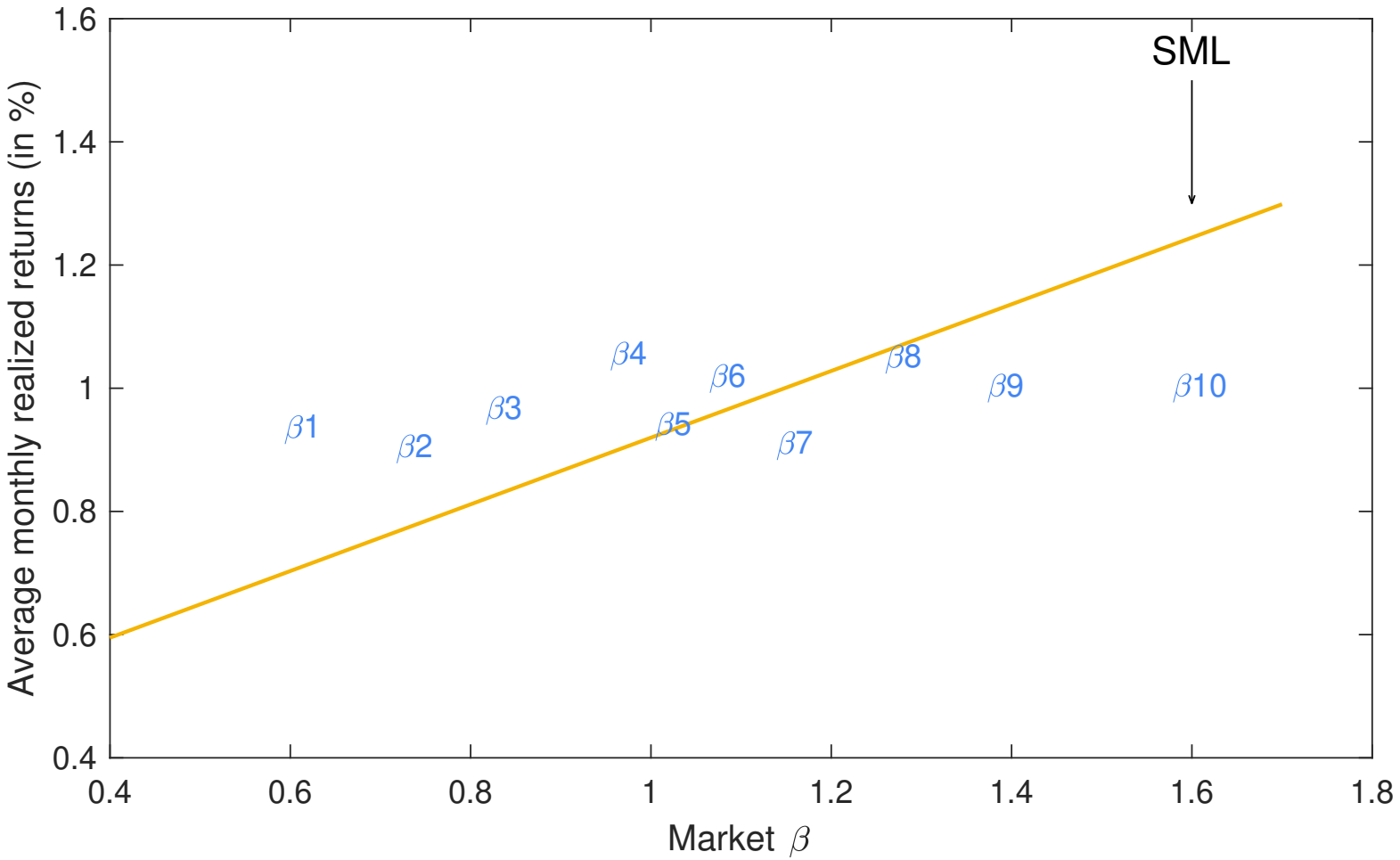
The CAPM cannot explain the value premium

* We can illustrate the failure of the CAPM to explain the value premium by plotting mean excess returns against their *ex post* CAPM-βs and comparing it to the Security Market Line (SML)



And not even beta-sorted portfolios

* Another outstanding puzzle is that the CAPM fails to price the cross-section of beta-sorted portfolios

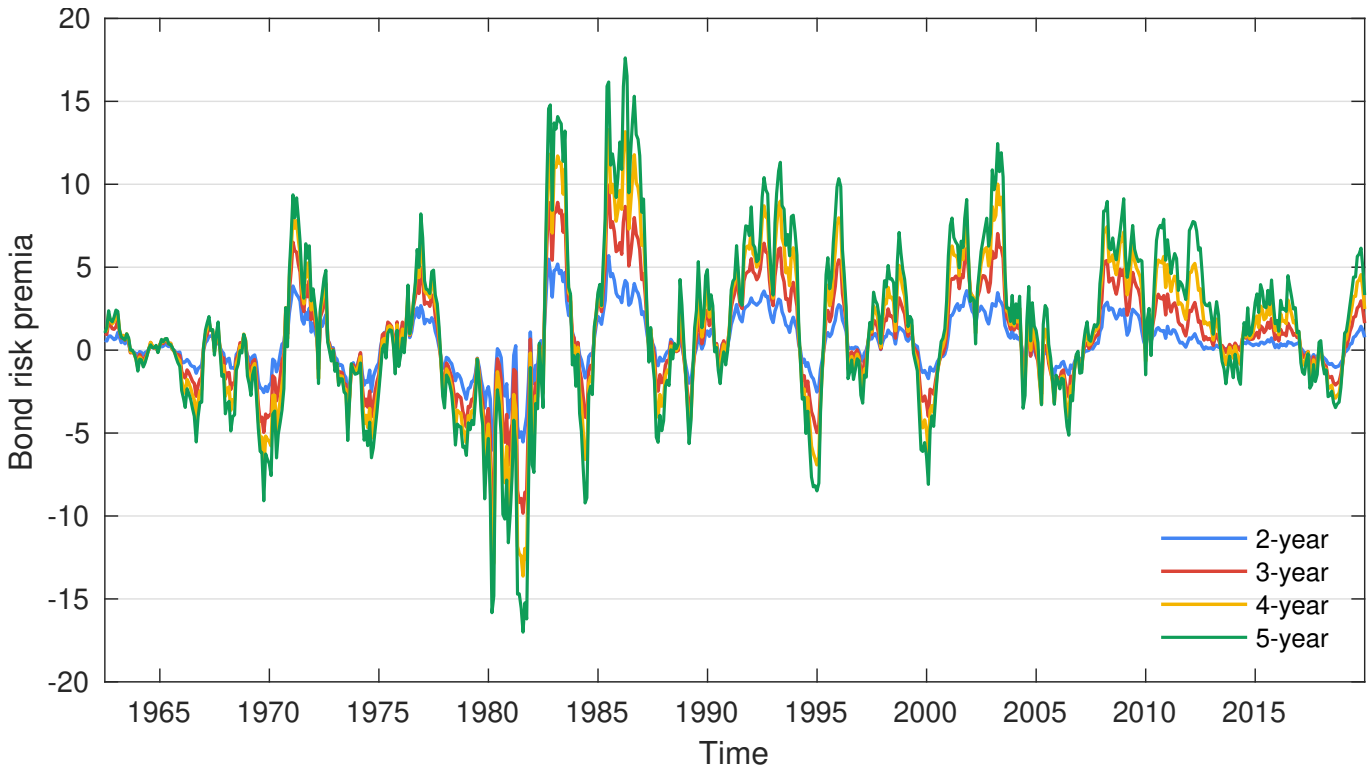


The expectations hypothesis

* The expectations hypothesis states that long-term yields are averages of expected future short-term yields (plus a time-invariant risk premium)

$$y_t^n = \frac{1}{n} \sum_{j=0}^{n-1} \mathbb{E}_t \left[y_{t+j}^1 \right] + \Phi^n \tag{10}$$

* The risk premium Φ^n should be time-invariant according to theory, yet empirical evidence suggests that it is highly time-varying



Term structure models

* Finally, we end the course with affine models that describes how the term structure of interest rates moves over time as a function of a latent state variable x_t and coefficients A_n and B_n

$$y_t^n = n^{-1} \left(A_n + B_n x_t \right) \tag{11}$$