Asset Pricing

The expectations hypothesis

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The expectations hypothesis of the term structure

Implication for bond risk premia

A popular and simple model of the term structure is known as the expectations hypothesis (EH). The EH says that the risk premium on long-term bonds over one-period bonds is constant over time

$$\mathbb{E}_t\left[r_{t+1}^n\right] - y_t^1 = RP^n \tag{1}$$

* The basic idea is that lenders and borrowers decide between long-term and short-term bonds by comparing the n-period return on an n-period bond with the expected return from rolling over one-period bonds for n periods

$$ny_t^n = \sum_{i=0}^{n-1} \mathbb{E}_t\left[r_{t+1+i}^{n-i}\right] \tag{2}$$

* In the absence of frictions, these investment strategies should be identical from the viewpoint of today (up to a constant risk premium)

The EH is a knockout in terms of rejections

* The expectations hypothesis and its implications has been subject to extensive scrutiny in academia, and has found very little empirical support in the data

* There are many potential explanations for this dismissal, but overwhelming evidence suggests that risk premia are not constant

* We will limit our consideration to two versions of the expectations hypothesis

  1. **The pure expectations hypothesis (PEH)**, which says that expected excess returns on long-term bonds over short-term bonds are zero,

  2. **The expectations hypothesis (EH)**, which allows for a constant and maturity-specific risk premia on long-term bonds over short-term bonds

* That risk premia are time-invariant implies that the premium required by investors for bearing risk, such as interest rate risk, inflation risk, reinvestment risk, liquidity risk, and macroeconomic risk, is constant through time

What returns to use?

- Any study of the EH must choose whether to use simple or log holding period returns for the empirical analysis
- With simple returns, we have the inconvenient observation that the PEH in its one-period version is inconsistent with its n-period version
- For example, the PEH equates the one-period expected return on one-period and n-period bonds

$$\left(1+Y_t^1\right)=\mathbb{E}_t\left[1+R_{t+1}^n\right]=\left(1+Y_t^n\right)^n\mathbb{E}_t\left[\frac{1}{\left(1+Y_{t+1}^{n-1}\right)^{n-1}}\right] \tag{3}$$

- Second, it equates the n-period expected return on one-period and n-period bonds

$$\left(1+Y_t^n\right)^n=\left(1+Y_t^1\right)\mathbb{E}_t\left[\left(1+Y_{t+1}^{n-1}\right)^{n-1}\right] \tag{4}$$

What is the problem?

- The issue with simple returns is that both statements in (3) and (4) cannot be true simultaneously due to Jensen's Inequality, i.e.,

$$\mathbb{E}_t\left[\frac{1}{\left(1+Y_{t+1}^{n-1}\right)^{n-1}}\right] \neq \frac{1}{\mathbb{E}_t\left[\left(1+Y_{t+1}^{n-1}\right)^{n-1}\right]} \tag{5}$$

- That is, the pure expectations hypothesis cannot hold in both its one-period form and its n-period form

- As a result, we will focus on the log (pure) expectations hypothesis for the remainder of the slides

- This version is exemplified with the opening prediction in (1) that log holding-period excess returns are constant over time

Bond prices and yields

- The (pure) expectations hypothesis is usually formulated in terms of Treasury discount bonds, allowing us to abstract from intermediate coupon payments

- The time t nominal price of a discount bond with n periods left to maturity, a yield of Y_t^n, and terminal payoff of a nominal dollar is

$$P_t^n = \frac{1}{(1 + Y_t^n)^n} \tag{6}$$

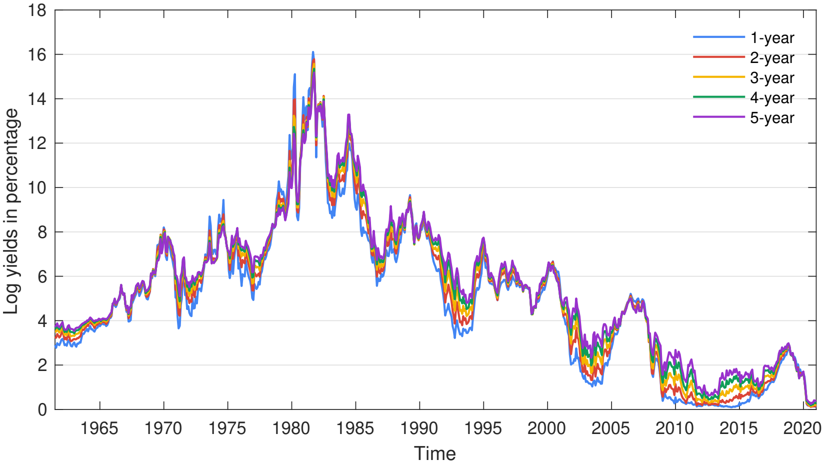
- Taking the natural logarithm to the expression in (6) gives us

$$p_t^n = -n y_t^n \quad \Leftrightarrow \quad y_t^n = -\left(\frac{1}{n}\right) p_t^n \tag{7}$$

  where we use the usual convention of letting lowercase letters denote log values of bond prices and yields

Dynamics of log zero-coupon yields

- Using the Nelson and Siegel (1987)–Svensson (1994) parameters provided by Gürkaynak et al. (2007), we obtain the following time series of annualized continuously compounded yields



Forward rates

Forward rates

We can define the log forward rate at time t for loans between time $t+n-1$ and $t+n$ from (7) as

$$\begin{aligned} f_t^n &= p_t^{n-1} - p_t^n && (8) \\ &= n y_t^n - (n-1)\, y_t^{n-1} && (9) \\ &= y_t^{n-1} + n\left(y_t^n - y_t^{n-1}\right) && (10) \end{aligned}$$

- The expression in (10) reveals that $f_t^n > y_t^{n-1}$ whenever $y_t^n > y_t^{n-1}$, that is, when the term structure is upward sloping — and vice versa
- Recall that the terminal payoff is $\$1$ so that $p_t^0 = 0$, implying the relation that we will take as the risk-free rate

$$f_t^1 = y_t^1 = -p_t^1 \tag{11}$$

Holding period returns

Holding period returns

A holding period return r_{t+1}^{n} in the Treasury bond market from buying and selling discount bonds prior to maturity is defined as follows

$$
\begin{aligned}
r_{t+1}^{n} &= p_{t+1}^{n-1} - p_{t}^{n} && (12)\\
&= n y_{t}^{n} - (n-1)\, y_{t+1}^{n-1} && (13)\\
&= y_{t}^{n} - (n-1)\left(y_{t+1}^{n-1} - y_{t}^{n}\right) && (14)
\end{aligned}
$$

- The holding period return can be viewed as a trading strategy in which we purchase today (at time t) an n–period bond and subsequently sell it one period later as an $n-1$ period bond
- Importantly, the trading activity exposes the investor to various sources of risk, e.g., inflation, reinvestment, and macroeconomic, that may cause bond prices to change unfavorably during the holding period

The expectations hypothesis

- Consider an investor that wants to invest her funds for n periods. She has two options and compares the n-period return on an n-period bond with the expected return from rolling over one-period bonds for n periods

$$y_t^n = \frac{1}{n}\mathbb{E}_t \underbrace{\left(y_t^1 + y_{t+1}^1 + y_{t+2}^1 + \cdots + y_{t+n-1}^1\right)}_{\text{Roll-over strategy}} \tag{15}$$

The expectations hypothesis

The expectations hypothesis postulates that long-term yields equal average expected future short-term yields over the lifetime of the long-term bond plus a term premium that may vary with bond maturity n, but not over time

$$y_t^n = \frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}_t\left[y_{t+j}^1\right] + \Phi^n \tag{16}$$

where Φ^n denote the maturity-dependent, but time-invariant, term premium. All alternative formulations of the EH comes down to different assumptions on Φ^n

Further implications of the (P)EH

* Besides the implication in (16), the (P)EH implies two additional relations with important implications for fixed income markets

  1. First, the (P)EH implies that forward rates should be an unbiased estimator of expected future spot rates

  $$f_t^n = \mathbb{E}_t\left[y_{t+n-1}^1\right] + RP^n \tag{17}$$

  2. Secondly, the (P)EH implies that the expected holding period return should be equal on bonds of all maturities. Formally,

  $$\mathbb{E}_t\left[r_{t+1}^n\right] = y_t^1 + RP^n \tag{18}$$

* The P(EH) implications are that RP^n cannot vary over time, but may depend on the maturity of the discount bonds

* As usual, we will see that the empirical support is limited and that the evidence overwhelmingly points to time-varying risk premia

Equivalence of the statements

- We can note that the EH statements in (16)–(18) are all mathematically equivalent and so contain the same qualitative predictions
- As an example, start from the PEH statement in (17) that $f_t^n = \mathbb{E}_t \left[y_{t+n-1}^1 \right]$ and add these up over n periods to obtain

$$\begin{aligned}
&\mathbb{E}_t \left[y_t^1 + y_{t+1}^1 + y_{t+2}^1 + \cdots + y_{t+n-1}^1 \right] && (19)\\
&= f_t^1 + f_t^2 + f_t^3 + \cdots + f_t^n && (20)\\
&= \left(p_t^0 - p_t^1\right) + \left(p_t^1 - p_t^2\right) + \left(p_t^2 - p_t^3\right) + \cdots + \left(p_t^{n-1} - p_t^n\right) && (21)\\
&= -p_t^n = n y_t^n, && (22)
\end{aligned}$$

  which recovers that long yields equal average expected future short rates
- One can show that all statements are equivalent by similar arguments (See the EH.pdf lecture note for details)

Yield-based tests (Campbell and Shiller, 1991)

Yield-based tests (Campbell and Shiller, 1991)

Campbell and Shiller (1991) ask two important questions about the term structure of interest rates

1. Does the slope of the term structure — the yield spread between longer-term and shorter-term interest rates — predict future changes in interest rates?

2. And if so, is the predictive power of the yield spread in accordance with the expectations theory of the term structure?

* Their main results can be summarized as follows: When the term spread is high, the long rate tends to fall and the short rate tends to rise

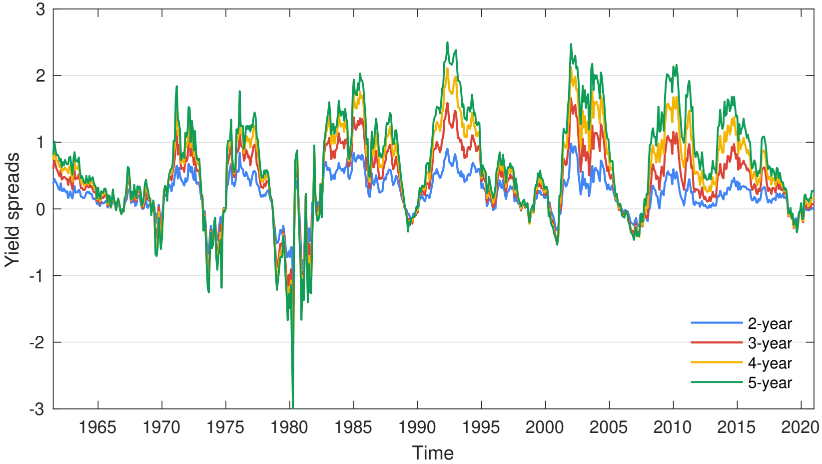
    1. **In contrast to the EH:** When the yield spread is high, the yield on the longer-term bond tends to fall over the life of the short-term bond

    2. **In accordance with the EH:** When the yield spread is high, shorter-term rates tend to rise over the life of the longer-term bond

Yield spreads

* Campbell and Shiller (1991) focus on the behavior through time of a simple measure of the shape of the term structure: the yield spread between an n-period discount bond and a one-period (risk-free) bond

$$s_t^n = y_t^n - y_t^1 \tag{23}$$



Interpreting the yield spread

Yield spread

Recall that $y_t^n = \frac{1}{n}\sum_{i=0}^{n-1} r_{t+i+1}^{n-1}$, which when written in terms of the yield spread $s_t^n = y_t^n - y_t^1$ gives us the following equations

$$s_t^n = \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left[y_{t+i}^1 - y_t^1\right] + \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left[r_{t+1+i}^{n-i} - y_{t+i}^1\right] \tag{24}$$

$$= \frac{1}{n}\sum_{i=1}^{n-1}\mathbb{E}_t\left[(n-i)\,\Delta y_{t+i}^1\right] + \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left[r_{t+1+i}^{n-i} - y_{t+i}^1\right], \tag{25}$$

* These are fundamental to our intuition and tell us that variation in the yield spread arises from one of two sources

  1. Changing expectations about future short-term interest rates and/or
  2. Changing expectations about future term (risk) premia

Getting to know the risk premia components

The term premium

Under the (P)EH, the last term in both equations is a constant equal to the time-invariant term premium

$$\Phi^n = \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left[r_{t+1+i}^{n-i} - y_{t+i}^{1}\right] \tag{26}$$

- The testable prediction that Φ^n is time invariant is shared among all versions of the expectations hypothesis

- We have the following relation between expected returns, yields, and risk premia from (13) and (18)

$$\mathbb{E}_t\left[r_{t+1}^{n}\right] = n y_t^n - (n-1)\,\mathbb{E}_t\left[y_{t+1}^{n-1}\right] = y_t^1 + RP^n \tag{27}$$

such that $\Phi^n = \frac{1}{n}\sum_{i=0}^{n-1} RP^{n-i}$ and $RP^n = n\Phi^n - (n-1)\,\Phi^{n-1}$

The testable yield implications

- Invoking the EH prediction of constant risk premia, we can write the yield spread equations as two testable statements

Testable implications

We have two equations that derive directly from the EH with testable implications

1. First, using the definition of a holding-period return and the constant expected return implication in (27), we get

$$s_t^n = (n-1)\,\mathbb{E}_t\left[y_{t+1}^{n-1} - y_t^n\right] + \underbrace{\mathbb{E}_t\left[r_{t+1}^n - y_t^1\right]}_{RP^n} \tag{28}$$

2. Second, using (25), we can show that

$$s_t^n = \sum_{i=1}^{n-1} \mathbb{E}_t\left[\left(1 - \frac{i}{n}\right)\Delta y_{t+i}^1\right] + \underbrace{\frac{1}{n}\sum_{i=0}^{n-1} \mathbb{E}_t\left[r_{t+1+i}^{n-i} - y_{t+i}^1\right]}_{\Phi^n} \tag{29}$$

Campbell-Shiller long rate regressions

Campbell-Shiller long rate regressions

To test the implications of the expectations hypothesis, Campbell and Shiller (1991) consider the testable implication in (28) that can be re-arranged into

$$\mathbb{E}_t \left[y_{t+1}^{n-1} - y_t^n \right] = -\frac{RP^n}{n-1} + \frac{s_t^n}{n-1}, \tag{30}$$

and conveniently framed as a linear regression model as follows

$$y_{t+1}^{n-1} - y_t^n = \alpha^n + \beta^n \left(\frac{s_t^n}{n-1} \right) + \varepsilon_{t+1}^n \tag{31}$$

- We use that $\alpha^n = -RP^n / (n-1)$ is constant under EH, but may vary with n
- Under the (pure) expectations hypothesis, we should expect to find $\beta^n = 1$ for all n and α^n (zero) negative for all n
- Generally, studies tend to reject the null of $\beta = 1$ and in fact often find a negative beta, which contradicts theory

Empirical results for long rate regressions

- Suppose we test the EH using the Campbell and Shiller (1991) long rate regressions and the Gürkaynak et al. (2007) smoothed yield data
- In this case, we find strong evidence against the implications of the EH

Table 1: Campbell-Shiller long rate regressions

	2-year	3-year	4-year	5-year
α^n	-0.155	-0.076	-0.016	0.031
$\text{se}^{\text{NW}}(\alpha^n)$	(0.238)	(0.221)	(0.207)	(0.196)
β^n	-0.497	-0.767	-1.029	-1.273
$\text{se}^{\text{NW}}(\beta^n)$	(0.564)	(0.629)	(0.670)	(0.698)
$\text{t}^{\text{NW}}(\beta^n = 1)$	-2.656	-2.808	-3.026	-3.259
R^2 (%)	0.94%	1.89%	3.00%	4.12%

Campbell-Shiller short rate regressions

Campbell-Shiller short rate regressions

Another way of testing the implications of the EH, also considered in Campbell and Shiller (1991), is to test whether a high yield spread forecasts long-term increases in short rates. Define the perfect foresight spread as

$$s_t^{n,\star} = \sum_{i=1}^{n-1} \left(1 - \frac{i}{n}\right) \Delta y_{t+i}^1 \tag{32}$$

and test the implication in (29) using the regression model

$$s_t^{n,\star} = \delta^n + \theta^n s_t^n + \nu_{t+1}^n \tag{33}$$

- We use that $\theta^n = -\Phi^n$ is constant under EH, but may vary with n
- Under the expectations hypothesis, we should expect to find $\theta^n = 1$ for all n
- We tend to be unable to reject this hypothesis for longer-term bonds

Empirical results for short rate regressions

- Suppose we test the EH using the Campbell and Shiller (1991) short rate regressions and the Gürkaynak et al. (2007) smoothed yield data
- In this case, we find evidence qualitatively in line with the EH (although not quantitatively so according to the prediction)

Table 2: Campbell-Shiller short rate regressions

	2-year	**3-year**	**4-year**	**5-year**
δ^n	-0.062	-0.219	-0.421	-0.589
$\text{se}^{\text{NW}}\ (\delta^n)$	(0.124)	(0.240)	(0.300)	(0.332)
θ^n	0.207	0.476	0.685	0.769
$\text{se}^{\text{NW}}\ (\theta^n)$	(0.283)	(0.332)	(0.294)	(0.251)
$\text{t}^{\text{NW}}\ (\theta^n = 1)$	-2.796	-1.576	-1.072	-0.919
$\text{R}^2\ (\%)$	0.66%	4.15%	9.83%	13.92%

Predicting bond risk premia

Bond risk premia

We can define an *ex post* measure of bond risk premia – more precisely, log excess holding-period returns – directly form the EH implication in (18)

$$rx^n_{t+1} = r^n_{t+1} - y^1_t \tag{34}$$

where r^n_{t+1} is the log holding period return from (12) and y^1_t is the one-year yield

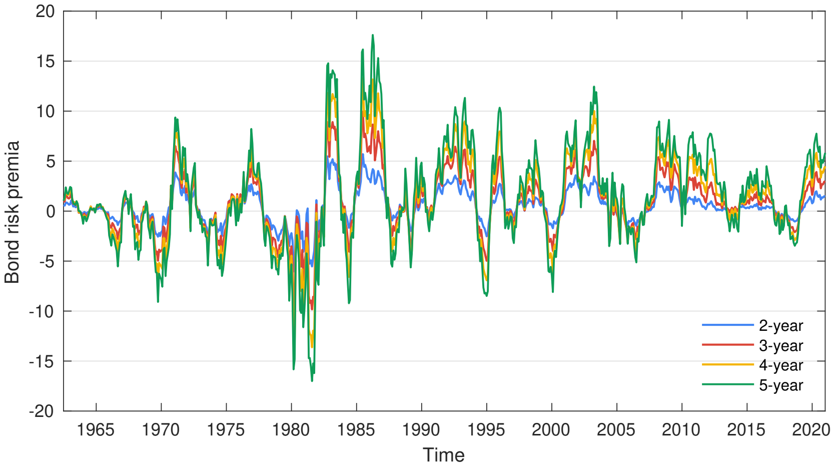
- We can write the expectations hypothesis in terms of bond risk premia as follows

$$y^n_t = \frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}_t\left[y^1_{t+j}\right] + \underbrace{\frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}_t\left[rx^{n-j}_{t+j}\right]}_{\Phi^n} \tag{35}$$

- Hence, if rx^n_{t+1} is constant over time, so is Φ^n. If rx^n_{t+1} varies over time, so will Φ^n and we should really write Φ^n_t instead (A contradiction of the EH)

Time series dynamics of bond risk premia

- Computing and plotting the time series dynamics of risk premia in the Treasury bond market reveals that a time invariance story is unlikely



Predictability of bond risk premia

Predictive regressions

We follow a long tradition in the finance literature and consider a traditional linear predictive regression framework to gauge the predictability of bond risk premia

$$rx_{t+1}^n = \alpha^n + \boldsymbol{x}_t'\boldsymbol{\beta}^n + \varepsilon_{t+1}^n \tag{36}$$

where $\boldsymbol{x}_t$ denotes a vector of time t observable variables that we think may predict and/or explain the observed time-variations in bond risk premia

- We can then interpret a test of $\boldsymbol{\beta} = \mathbf{0}$ as a test of the expectations hypothesis
  - If $\boldsymbol{\beta} = \mathbf{0}$ (we cannot reject the null), then we have no evidence against the EH as the model collapses to $rx_{t+1}^n = \alpha^n + \varepsilon_{t+1}^n$
  - If $\boldsymbol{\beta} \neq \mathbf{0}$ (we reject the null), then we have evidence against the EH as bond risk premia appear to vary in a predictable manner
- The empirical literature has uncovered a broad collection of candidates for $\boldsymbol{x}_t$ that explains part, but not all, of the time-variation in bond risk premia

Fama-Bliss and forward spreads

- Fama and Bliss (1987) ask if forward rates contain information about expected returns on discount bonds, or more specifically, whether forward spreads contain information about expected future holding period excess returns

- To see why the forward spread is a prime candidate predictor, consider the definition of the forward rate in (8), subtract the one-period yield, add and subtract p_{t+1}^{n-1}, and re-arrange to obtain

$$
\begin{aligned}
f_t^n - y_t^1 &= p_t^{n-1} - p_t^n - y_t^1 && (37)\\
&= p_t^{n-1} - p_t^n - y_t^1 + p_{t+1}^{n-1} - p_{t+1}^{n-1} && (38)\\
&= \left(p_{t+1}^{n-1} - p_t^n - y_t^1\right) + \left(-p_{t+1}^{n-1} + p_t^{n-1}\right) && (39)\\
&= \left(r_{t+1}^n - y_t^1\right) + (n-1)\left(y_{t+1}^{n-1} - y_t^{n-1}\right) && (40)
\end{aligned}
$$

where the final expression makes use of the relations in (7) and (12)

Forward spread as a candidate predictor

Finally, take conditional expectations at time t to obtain the expression

$$f_t^n - y_t^1 = \left(\mathbb{E}_t\left[r_{t+1}^n\right] - y_t^1\right) + (n-1)\left(\mathbb{E}_t\left[y_{t+1}^{n-1}\right] - y_t^{n-1}\right) \tag{41}$$

which is an accounting identity that must hold true and thus provides us with some insight into the forecasting mechanisms of the forward spread

* The decomposition reveals that the forward spread consists of two distinct components

  1. The expected holding period return on an n-period bond in excess of the one-period spot rate
  2. The expected change in the yield on an $n-1$ period bond from time t to $t+1$

* This tells us that, if the forward spread varies over time, then it must forecast either bond risk premia, yield changes, or a combination of the two

Fama-Bliss regressions

Fama-Bliss regressions

To test the implications of (41), Fama and Bliss (1987) run maturity-specific regressions for bond risk premia of the form

$$rx_{t+1}^{n} = \alpha^{n} + \beta^{n}\left(f_t^{n} - y_t^{1}\right) + \varepsilon_{t+1}^{n} \tag{42}$$

By the accounting identity, we can also run the mirror-image regression on ex-post yield changes, i.e.,

$$(n-1)\left(y_{t+1}^{n-1} - y_t^{n-1}\right) = \delta^{n} + \theta^{n}\left(f_t^{n} - y_t^{1}\right) + \varepsilon_{t+1}^{n} \tag{43}$$

* We have the following relations between regression coefficients
  1. **Slope coefficients must sum to 1:** $\theta^n = 1 - \beta^n \Rightarrow \theta^n + \beta^n \equiv 1$
  2. **Constant values must sum to 0:** $\delta^n = -\alpha^n \Rightarrow \delta^n + \alpha^n \equiv 0$

Empirical results for FB regressions

- Running the Fama and Bliss (1987) regressions, we find that forward spreads only predicts bond risk premia, which is a contradiction of the EH prediction

Table 3: Fama-Bliss regressions

	2-year	**3-year**	**4-year**	**5-year**
Panel A: Risk premia regressions				
α^n	0.155	0.169	0.085	-0.064
$\text{se}^{\text{NW}}\ (\alpha^n)$	(0.238)	(0.456)	(0.648)	(0.820)
β^n	0.748	0.911	1.078	1.239
$\text{se}^{\text{NW}}\ (\beta^n)$	(0.282)	(0.347)	(0.383)	(0.406)
$\text{t}^{\text{NW}}\ (\beta^n = 0)$	2.656	2.624	2.810	3.053
$\text{R}^2\ (\%)$	7.91%	8.29%	9.25%	10.21%
Panel B: Yield change regressions				
δ^n	-0.155	-0.169	-0.085	0.064
$\text{se}^{\text{NW}}\ (\delta^n)$	(0.238)	(0.456)	(0.648)	(0.820)
θ^n	0.252	0.089	-0.078	-0.239
$\text{se}^{\text{NW}}\ (\theta^n)$	(0.282)	(0.347)	(0.383)	(0.406)
$\text{t}^{\text{NW}}\ (\theta^n = 0)$	0.893	0.256	-0.202	-0.589
$\text{R}^2\ (\%)$	0.96%	0.09%	0.05%	0.42%

Cochrane and Piazzesi (2005)

- Cochrane and Piazzesi (2005) extend the work of Fama and Bliss (1987) by considering the entire term structure of forward rates, which here corresponds to the set $\mathbf{f}_t = \{f_t^1, f_t^2, f_t^3, f_t^4, f_t^5\}$
- While Fama and Bliss (1987) consider maturity-specific regressions, Cochrane and Piazzesi (2005) changed the game by considering a single factor, denoted CP, that predicts bond risk premia for all maturities
- To estimate their factor, they consider the cross-sectional average of bond risk premia across the maturity spectrum

$$\overline{rx}_{t+1} = \frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^n \tag{44}$$

which can be thought of as the annual excess log holding-period return to an equal-weighted portfolio of Treasury bonds with maturities from 2–5 years

The Cochrane-Piazzesi (CP) factor

The Cochrane-Piazzesi (CP) factor

The main contribution of Cochrane and Piazzesi (2005) is to construct a single forecasting factor that predicts bond risk premia for all maturities n. We first run the regression

$$\overline{rx}_{t+1} = \delta + \gamma_1 f_t^{\,1} + \gamma_2 f_t^{\,2} + \gamma_3 f_t^{\,3} + \gamma_4 f_t^{\,4} + \gamma_5 f_t^{\,5} + \varepsilon_{t+1} \tag{45}$$

and build the CP factor as the fitted values from the above regression

$$CP_t = \widehat{\delta} + \widehat{\gamma}_1 f_t^{\,1} + \widehat{\gamma}_2 f_t^{\,2} + \widehat{\gamma}_3 f_t^{\,3} + \widehat{\gamma}_4 f_t^{\,4} + \widehat{\gamma}_5 f_t^{\,5} \tag{46}$$

- They then run predictive regressions of the following kind to test the implications of the EH

$$rx_{t+1}^n = \alpha^n + \beta^n CP_t + \varepsilon_{t+1}^n \tag{47}$$

The tent-shaped plot

- Cochrane and Piazzesi (2005) show that the same function of forward rates forecasts bond risk premia at all maturities. Longer maturities just have greater loadings on this same function

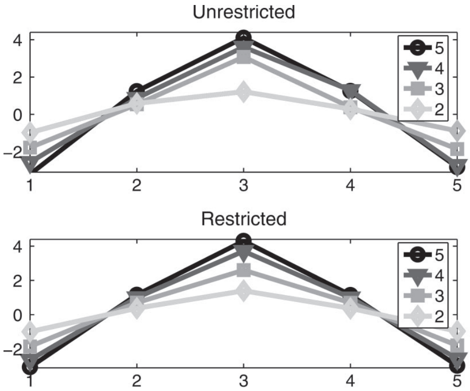


Figure 1. Regression Coefficients of One-Year Excess Returns on Forward Rates

Cochrane-Piazzesi's regression results

- Consider the regression results for the term structure of forward rates and the CP single factor

A. Estimates of the return-forecasting factor, $\overline{rx}_{t+1} = \boldsymbol{\gamma}^\top \mathbf{f}_t + \bar{\varepsilon}_{t+1}$

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	R^2	$\chi^2(5)$
OLS estimates	−3.24	−2.14	0.81	3.00	0.80	−2.08	0.35	
Asymptotic (Large T) distributions								
HH, 12 lags	(1.45)	(0.36)	(0.74)	(0.50)	(0.45)	(0.34)		811.3
NW, 18 lags	(1.31)	(0.34)	(0.69)	(0.55)	(0.46)	(0.41)		105.5
Simplified HH	(1.80)	(0.59)	(1.04)	(0.78)	(0.62)	(0.55)		42.4
No overlap	(1.83)	(0.84)	(1.69)	(1.69)	(1.21)	(1.06)		22.6
Small-sample (Small T) distributions								
12 lag VAR	(1.72)	(0.60)	(1.00)	(0.80)	(0.60)	(0.58)	[0.22, 0.56]	40.2
Cointegrated VAR	(1.88)	(0.63)	(1.05)	(0.80)	(0.60)	(0.58)	[0.18, 0.51]	38.1
Exp. Hypo.							[0.00, 0.17]	

B. Individual-bond regressions

	Restricted, $rx^{(n)}_{t+1} = b_n(\boldsymbol{\gamma}^\top \mathbf{f}_t) + \varepsilon^{(n)}_{t+1}$					Unrestricted, $rx^{(n)}_{t+1} = \boldsymbol{\beta}_n \mathbf{f}_t + \varepsilon^{(n)}_{t+1}$			
n	b_n	Large T	Small T	R^2	Small T	R^2	EH	Level R^2	$\chi^2(5)$
2	0.47	(0.03)	(0.02)	0.31	[0.18, 0.52]	0.32	[0, 0.17]	0.36	121.8
3	0.87	(0.02)	(0.02)	0.34	[0.21, 0.54]	0.34	[0, 0.17]	0.36	113.8
4	1.24	(0.01)	(0.02)	0.37	[0.24, 0.57]	0.37	[0, 0.17]	0.39	115.7
5	1.43	(0.04)	(0.03)	0.34	[0.21, 0.55]	0.35	[0, 0.17]	0.36	88.2

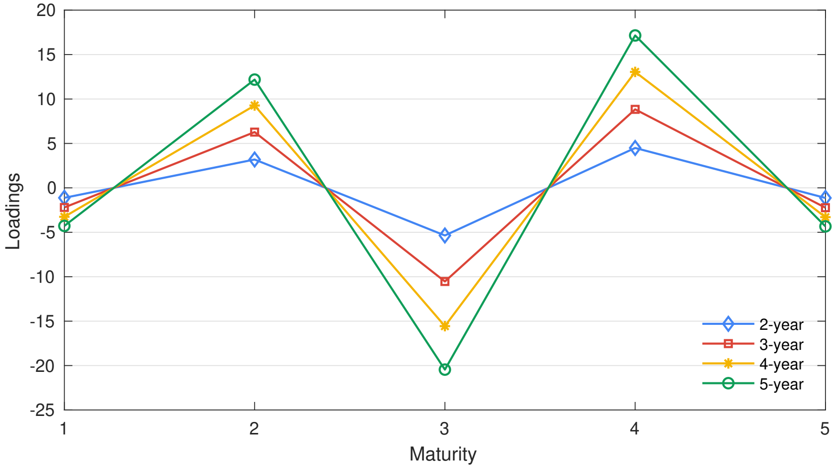
The CP factor also predicts stock returns

- We can view a stock as a long-term bond plus cash-flow risk, so any variable that forecasts bond returns should also forecast stock returns
- The slope of the term structure also forecasts stock returns, as emphasized by Fama and French (1989, 1993), and this fact confirms that the bond return forecast corresponds to a risk premium and not a bond market fad

Right-hand variables		$\boldsymbol{\gamma}^\top \mathbf{f}$	(t-stat)	d/p	(t-stat)	$y^{(5)} - y^{(1)}$	(t-stat)	R^2
1	$\boldsymbol{\gamma}^\top \mathbf{f}$	1.73	(2.20)					0.07
2	D/p			3.30	(1.68)			0.05
3	Term spread					2.84	(1.14)	0.02
4	D/p and term			3.56	(1.80)	3.29	(1.48)	0.08
5	$\boldsymbol{\gamma}^\top \mathbf{f}$ and term	1.87	(2.38)			−0.58	(−0.20)	0.07
6	$\boldsymbol{\gamma}^\top \mathbf{f}$ and d/p	1.49	(2.17)	2.64	(1.39)			0.10
7	All $\mathbf{f}$							0.10
8	Moving average $\boldsymbol{\gamma}^\top \mathbf{f}$	2.11	(3.39)					0.12
9	MA $\boldsymbol{\gamma}^\top \mathbf{f}$, term, d/p	2.23	(3.86)	1.95	(1.02)	−1.41	(−0.63)	0.15

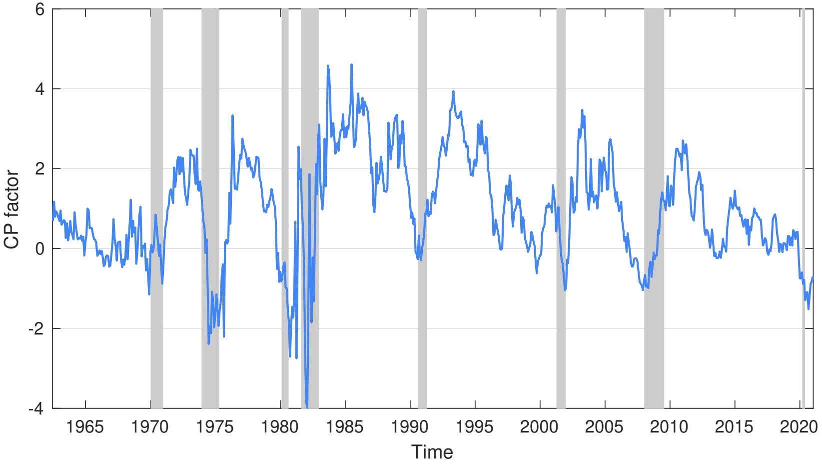
The function of forward rates for our data

* While Cochrane and Piazzesi (2005) find that the same function of forward rates that forecasts bond risk premia at all maturities is tent-shaped, this is somewhat sample-specific and for our data we find the below shape



The CP factor over time

* The evolution of the full-sample CP factor is plotted below, where we note a weak business cycle pattern, i.e., that CP tends to decline before recessions



Empirical results for the CP factor

* Running the Cochrane and Piazzesi (2005) predictive regressions for bond risk premia using the full-sample CP factor provides us with the below results
* Importantly, we note that the same function predicts bond risk premia for all n and that β^n is increasing with maturity

Table 4: Cochrane-Piazzesi regressions

	2-year	**3-year**	**4-year**	**5-year**
α^n	0.080	0.061	-0.014	-0.128
$\text{se}^{\text{NW}}(\alpha^n)$	(0.216)	(0.390)	(0.542)	(0.678)
β^n	0.413	0.812	1.199	1.576
$\text{se}^{\text{NW}}(\beta^n)$	(0.125)	(0.235)	(0.334)	(0.423)
$\text{t}^{\text{NW}}(\beta^n = 0)$	3.307	3.455	3.595	3.725
R^2 (%)	11.01%	12.86%	14.45%	15.79%

Ludvigson-Ng and macro factors

- Our treatment has insofar dealt with information embedded in the yield curve, but do not inform us about macroeconomic drives of bond risk premia

Ludvigson-Ng and macro factors

Ludvigson and Ng (2009) makes significant progress on this question and is an important contribution that addresses two empirical questions

1. Do movements in bond risk premia bear any direct relation to cyclical macroeconomic activity and, if so,
2. Do macroeconomic fundamentals contain information about risk premia that is not already embedded in the zero-coupon term structure?

- A key innovation of Ludvigson and Ng (2009) is to use a large panel of macroeconomic variables than spans many different categories such as employment, prices, and production

The Ludvigson-Ng (LN) factor

The Ludvigson-Ng (LN) factor

Ludvigson and Ng (2009) construct their single macro factor analogous to Cochrane and Piazzesi (2005) from the following regression

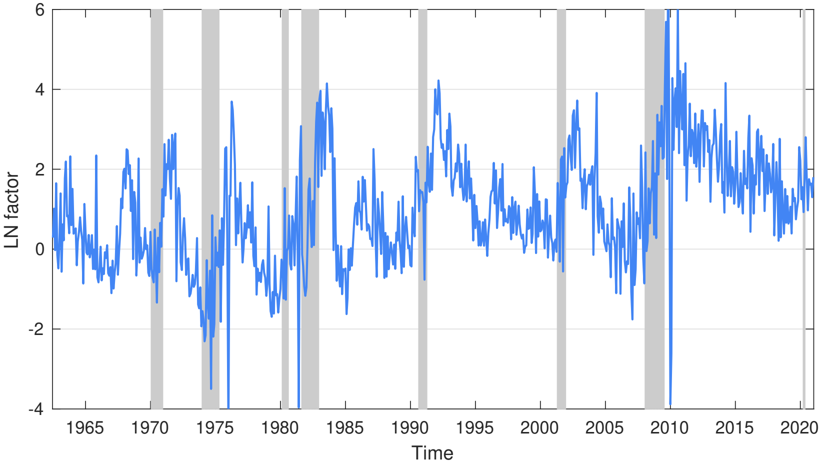
$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{n} = \lambda + \delta_1 \widehat{F}_{1,t} + \delta_2 \widehat{F}_{1,t}^{3} + \delta_3 \widehat{F}_{3,t} + \delta_4 \widehat{F}_{4,t} + \delta_5 \widehat{F}_{8,t} + \nu_{t+1}, \tag{48}$$

where $\widehat{F}_{i,t}$ denotes the ith factor estimated using PCA and the LN factor is constructed as the fitted values from the regression

- They collect more than 130 macroeconomic variables and estimate macro factors using principal component analysis (PCA)
- They argue that eight factors are enough to represent the information in the full panel (explains about 50%) and identify the optimal model for bond risk premia using the BIC criterion

The LN factor over time

* The evolution of the LN factor is markedly cyclical and rises in bad times
* If LN predicts bond risk premia with a positive coefficient, then we have evidence of a cyclical (macroeconomic) component in bond risk premia



Empirical results for the LN factor

- Running the Ludvigson and Ng (2009) predictive regressions for bond risk premia using the full-sample LN factor provides us with the below results

$$rx_{t+1}^{n} = \alpha^{n} + \beta^{n} LN_t + \varepsilon_{t+1}^{n} \quad (49)$$

- β^n is positive for all n, which confirms that bond risk premia contain a countercyclical component

Table 5: Cochrane-Piazzesi regressions

	2-year	**3-year**	**4-year**	**5-year**
α^n	0.030	0.016	-0.007	-0.038
$\text{se}^{\text{NW}}(\alpha^n)$	(0.206)	(0.361)	(0.494)	(0.614)
β^n	0.464	0.859	1.193	1.485
$\text{se}^{\text{NW}}(\beta^n)$	(0.099)	(0.159)	(0.203)	(0.243)
$\text{t}^{\text{NW}}(\beta^n = 0)$	4.676	5.407	5.876	6.105
R^2 (%)	16.45%	16.98%	16.88%	16.54%

Other contributions in this area

- Understanding the dynamics of bond risk premia is a vibrant and active research area. Below are recent contributions
  - Cooper and Priestley (2009) use the output gap and Wright (2011), Ilmanen (1995), Favero et al. (2012), and Joslin et al. (2014) look at macroeconomic variables in general
  - Almeida et al. (2011) show that interest rate options contain information about future bond risk premia and Wright and Zhou (2009) considers jump risk
  - Cieslak and Povala (2015) decompose yields into expected inflation and maturity-specific interest-rate cycles, where the latter forecasts bond risk premia
  - Eriksen (2017) finds that survey-based expectations to macroeconomic fundamentals contain predictive information about future bond risk premia
  - Bauer and Hamilton (2018), on the other hand, question the evidence for predictability and argue for more stringent tests

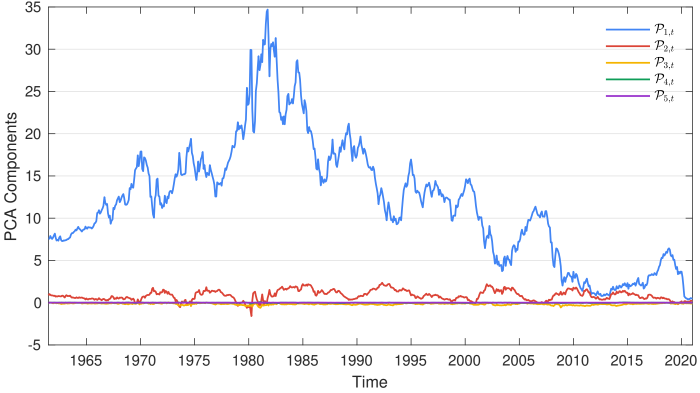
Other contributions in this area

- Ghysels et al. (2018) and Fulop et al. (2020) question the usefulness of macroeconomic data due to revisions, but Eriksen (2017) and Huang et al. (2020) show that carefully identifying the data available at the time of the forecast using vintages of real-time data recovers most of this predictability
- Thornton and Valente (2012) and Sarno et al. (2016) show that out-of-sample predictability does not necessarily leads to utility gains for a mean-variance investor
- Gargano et al. (2019) argue that bond predictability is stronger in recessions, but present in both states. Bianchi et al. (2021) arrive at a similar conclusion
- Andreasen et al. (2021) find in-sample evidence for time-varying parameters in bond prediction models and show that bond risk premia relate positively (negatively) to yield spreads in expansions (recessions)
- Borup et al. (2021) show that the degree of bond return predictability varies over time in a predictable manner related to economic activity and uncertainty

The principal components of yields

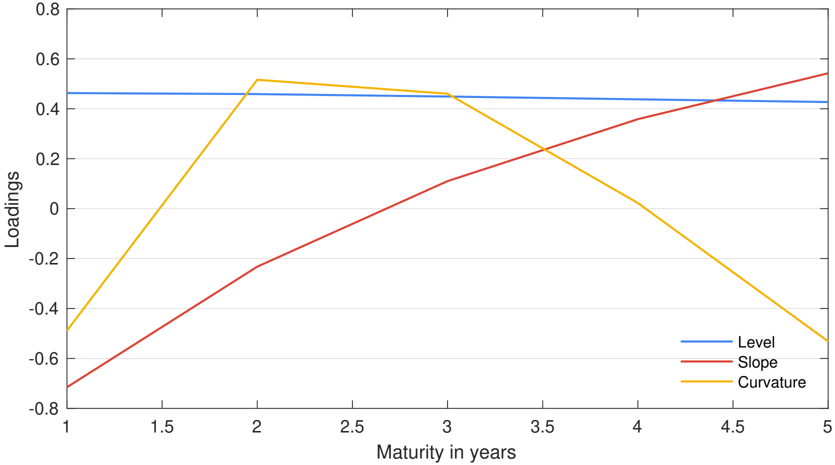
* It turns out that one can quite easily summarize all information in the yield curve by a small set of linear combinations of yields (principal components)

* It is well-established that the first few (first three) principal components of the covariance matrix of yields capture almost all of the cross-sectional variation in the term structure (Litterman and Scheinkman, 1991)

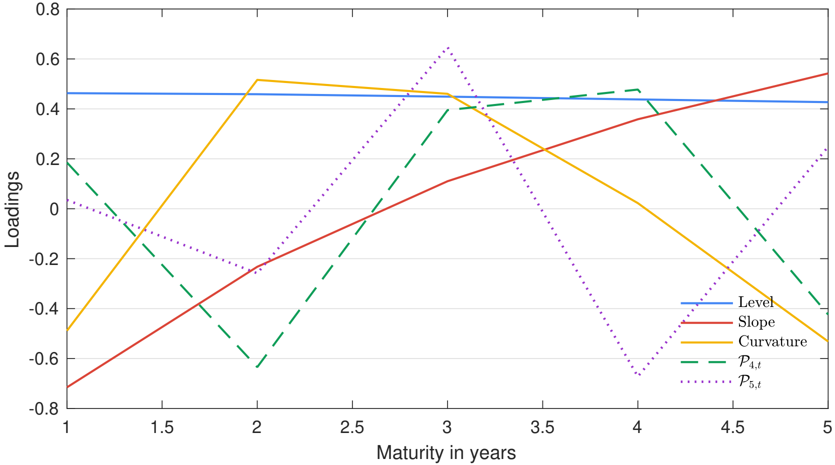


Yield curve factors

* The loadings of the first three components will have a familiar shape as a function of maturity and are commonly referred to as level, slope, and curvature



More yield curve factors



Forecasting the yield curve

- Rather than forecasting yields individually, one can forecast the principal components using, say, a VAR(1) specification

$$\tilde{\mathcal{P}}_{i,t+k} - \tilde{\mathcal{P}}_{i,t} = b_0 + \mathbf{b}_1^\top \tilde{\mathcal{P}}_t \tag{50}$$

  where $\tilde{\mathcal{P}}_t$ denotes a vector of principal components and $\tilde{\mathcal{P}}_{i,t}$ the individual components

- Having obtained forecast of k-period ahead principal components, we can consider a cross-sectional mapping of the form

$$y_{t+k}^{n} = \sum_{i=1}^{F} \widehat{\beta}_{i,n} \tilde{\mathcal{P}}_{i,t+k} + \boldsymbol{\varepsilon}_{t+k} \tag{51}$$

  to obtain a forecast of the entire yield curve without having to forecast each yield individually, but only a small set of principal components

- Variations of this type of approach can be found in Diebold and Li (2006) and Favero et al. (2012). See Duffee (2013) for a recent survey of the literature

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