Deriving the APT restriction*

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*This note provides an overview of the heuristic arguments provided by Ross (1976) in the derivation of the asset pricing restriction embedded in the Arbitrage Pricing Theory (APT). The note draws inspiration from the textbook treatments in Campbell, Lo and MacKinlay (1997), Cochrane (2005), Pennacchi (2008) and Ferson (2019). The note is prepared for use only in the Master's course "Asset Pricing". Please do not cite, circulate, or use for purposes other than this course.

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1. Introduction

A central theoretical implication of the Capital Asset Pricing Model (CAPM) is that the market portfolio of *all* risky assets is the only source of priced risk in the economy. However, this implication is frequently rejected by the data, and the inability of the CAPM to explain stylized facts such as the size premium (Banz, 1981) and the value premium (Fama and French, 1992) suggests that one or more additional factors may be needed to fully characterize the cross-section of expected returns. The Arbitrage Pricing Theory (APT), developed by Ross (1976), can be viewed as an alternative to the CAPM that allows for multiple sources of risk to affect the determination of expected returns in the cross-section. In particular, the APT provides a framework that allows for the inclusion of additional pricing factors using nothing but no arbitrage arguments. In its most basic form, the APT provides an *approximate* relation between expected returns and a set of systematic risk factors that drive common variation in returns. These factors are unspecified as the APT itself is completely agnostic towards the sources and identification of the factors themselves and their number. That the APT is agnostic toward the factors means that the identification of the "true" set of risk factors has become an important issue in asset pricing today, and we will briefly discuss this issue below. Contrasting the APT with the CAPM reveals that the two approaches to modeling asset prices are fundamentally different. The CAPM is an equilibrium model that requires us to make statements about supply and demand and specify investor preferences that ultimately leads us to conclude that the market portfolio is the only factor that matters in equilibrium. Moreover, the CAPM is burdened by a set of restrictive assumptions that are unlikely to ring true in most financial markets today, and the empirical evidence does not favor the prediction that the market portfolio is the only source of priced risk in the economy. The APT, on the other hand, only requires few and relatively mild assumptions: no arbitrage and a market structure to support it, that the returns on all assets are linearly related to a finite number of risk factors, and that the number of assets in the economy is large relative to the number of factors ($N \gg K$). The simplicity does come with a cost: the APT prices an asset relative to other assets, but can never answer the fundamental question of where the prices of the other assets come from. This task has to be delegated to fully specified equilibrium models. With that in mind, it may be worthwhile to point out that equilibrium models imply no arbitrage, but no arbitrage does not imply equilibrium.

Nonetheless, the APT and its ability to investigate multifactor asset pricing models has spurred a rich and ever growing literature that has uncovered a myriad of potential risk factors that can explain the cross-section of expected returns. Commonly applied factor models include the Fama and French (1993) three-factor model, the Fama and French (2015) five-factor model, the Hou, Xue and Zhang (2015) q-factor model, the Hou, Mo, Xue and Zhang (2020a) augmented q-factor model, the Stambaugh and Yuan (2017) mispricing model, or the Daniel, Hirshleifer and Sun (2020) behavioral model. In fact, the proliferation of risk factors in the literature has recently become somewhat of a challenging issue: how to determine which factors are genuine risk factors and how to determine which factors are false and due to spurious findings (Harvey, Liu and Zhu, 2016, Harvey, 2017, Harvey and Liu, 2019)?[1] Part of this problem originates from the APT being agnostic about which factors to include, and so leaves the choice and the argument to the researcher. Clearly, while this agnostic view is a fundamental strength of the model in that it provides a general framework to evaluate risk factor, it also opens up for mining the data until something significant comes out — even if purely by chance. As such, emphasis should be on researchers outlining their reasoning for the inclusion of a given factor (be it economic, financial, or statistical factors). This also includes taking a stand on the expected risk price of the factor by arguing how its movements are expected to covary with the macroeconomy and the marginal utility of investors. Indeed, there is nothing in the framework that guarantees that risk prices are positive. Generally speaking, risk factors that tend to take on high values in good times and low values in bad times (e.g., the market portfolio) usually have positive risk prices, whereas risk factors that tend to take on high values in bad times and low values in good times (e.g., market volatility) usually have negative risk prices. The asset-specific expected return is then determined through the size and sign of the factor loading.

The rest of the note develops as follows. Section 2 details the Arbitrage Pricing Theory and the derivation of the APT restriction. Section 3 outlines how one can obtain an approximate CAPM within the APT framework.

2. The arbitrage pricing theory

This section develops the Arbitrage Pricing Theory (APT) by introducing the factor model of returns, outlining the no-arbitrage conditions, and deriving the asset pricing restriction embedded in the APT by way of the heuristic arguments provided by Ross (1976). The model is build around no arbitrage, which implies that one cannot make a guaranteed profit today or in the future by trading in a feasible, self-financing (zero-cost) investment strategy involving two or more securities. Indeed, in efficient and competitive asset markets, where there are no barriers to trade, arbitrage opportunities should be rare, fleeting, and temporary. If we can accept that notion, then it is reasonable to assume that market prices generally reflects an absence of arbitrage. If markets are free of arbitrage, then the law of one price holds and securities with identical payoff should trade at identical prices. This is a bedrock of standard financial theory.

[1]There is currently a big debate about this issue in the literature, where one side is extremely critical of research practices and claims that most studies fail to replicate (Harvey, 2017, Linnainmaa and Roberts, 2018, Hou, Xue and Zhang, 2020b, Chordia, Goyal and Saretto, 2020, Tian, 2020) and the other side defends the literature and argues that results are likely true on average (McLean and Pontiff, 2016, Chen and Zimmermann, 2020, Jacobs and Müller, 2020, Chen and Zimmermann, 2021, Jensen, Kelly and Pedersen, 2021). Both sides have merits, and the truth – in my view – is properly somewhere in the middle: while some factors are definitely false discoveries, some are true risk factors. The big question is just how to correctly identify them, and the literature has recently started to make significant progress in that direction (see, e.g., Feng, Giglio and Xiu (2020) and Giglio and Xiu (2021)).

2.1. The factor model

The Arbitrage Pricing Theory (APT) of Ross (1976) is a no-arbitrage model stating that a few (K, say) common factors drive expected returns, i.e.,

$$\widetilde{r}_i = \mu_i + \sum_{k=1}^{K} b_{ik} \widetilde{f}_k + \widetilde{\varepsilon}_i \tag{1}$$

where $\mathbb{E}\left[\widetilde{r}_i\right] = \mu_i$ is an asset-specific constant equal to the expected return on asset i, $\widetilde{f}_k$ are risk factor innovations with $\mathbb{E}\left[\widetilde{f}_k\right] = 0$ and defined as $\widetilde{f}_k \equiv \widetilde{F}_k - \mathbb{E}\left[\widetilde{F}_k\right]$, b_{ik} denotes the factor loadings, and $\widetilde{\varepsilon}_i$ is an idiosyncratic risk component specific to asset i. The model claims that idiosyncratic risk can be diversified away under a set of somewhat mild assumptions. In particular, the model assumes that

$$\mathbb{E}\left[\widetilde{\varepsilon}_i\right] = 0 \tag{2}$$

$$\mathrm{Cov}\left[\widetilde{f}_k, \widetilde{\varepsilon}_i\right] = 0 \ \forall i,k \tag{3}$$

$$\mathrm{Cov}\left[\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_h\right] = 0 \ \forall i \neq h \tag{4}$$

$$\mathrm{Var}\left[\widetilde{\varepsilon}_i\right] = \mathbb{E}\left[\widetilde{\varepsilon}_i^2\right] = \sigma_{\varepsilon_i}^2 \leq s^2 < \infty. \tag{5}$$

The assumptions put structure on the factor model, but are not generally restrictive. We can interpret the assumptions as follows. The first assumption implies that the model is correct on average, the second assumption that nothing is omitted and that no relevant factor is left out of the model and hidden in the error term, the third assumption implies that *all* common return variation is captured by the factors in the model so that no cross-sectional correlation remains in the idiosyncratic error terms, and finally that the idiosyncratic variance is bounded from above. The third assumption in (4) is critical to the approach, and provides the economic bite to the model because this restriction implies that the model can be used as a pricing equation. Since the model is a no arbitrage pricing model that relies on the assumption that idiosyncratic risk can be diversified away for a large enough number of assets, pricing implications are usually only approximate. In principle, this implies that we cannot reject the theory (unless we can provide evidence of systematic and exploitable arbitrage opportunities) and a formal test of the proposed asset pricing model cannot be performed unless we have exact factor pricing (Shanken, 1982). We will keep this in mind, but otherwise proceed under the assumption that models can be evaluated.

2.2. Diversification in a single-factor model

To illustrate how diversification can eliminate idiosyncratic risk in the APT model, consider a simple one-factor model

$$\widetilde{r}_i = \alpha_i + b_i \widetilde{f} + \widetilde{\varepsilon}_i, \tag{6}$$

where the above assumptions apply. Suppose that investors invest in well-diversified portfolios with portfolio weights $\omega_i \approx \frac{1}{N}$ so that the portfolio return can be characterized by the single-factor model as

$$\begin{aligned}
\widetilde{r}_p &= \sum_{i=1}^{N} \omega_i \widetilde{r}_i && (7)\\
&= \sum_{i=1}^{N} \omega_i \left(\alpha_i + b_i \widetilde{f} + \widetilde{\varepsilon}_i\right) && (8)\\
&= \sum_{i=1}^{N} \omega_i \alpha_i + \sum_{i=1}^{N} \omega_i b_i \widetilde{f} + \sum_{i=1}^{N} \omega_i \widetilde{\varepsilon}_i && (9)\\
&= \alpha_p + b_p \widetilde{f} + \sum_{i=1}^{N} \omega_i \widetilde{\varepsilon}_i. && (10)
\end{aligned}$$

We are interested in the last term that represents the idiosyncratic risk of the portfolio. Applying the usual OLS variance decomposition to the factor model for the portfolio return yields

$$\begin{aligned}
\sigma_p^2 &= b_p^2 \sigma_f^2 + \mathrm{Var}\left[\sum_{i=1}^{N} \omega_i \widetilde{\varepsilon}_i\right] && (11)\\
&= b_p^2 \sigma_f^2 + \sum_{i=1}^{N} \omega_i^2 \mathbb{E}\left[\widetilde{\varepsilon}_i^2\right], && (12)
\end{aligned}$$

where we make use of the assumptions that $\mathbb{E}\left[\widetilde{\varepsilon}_i\right] = 0$ and $\mathrm{Cov}\left[\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_h\right] = 0 \ \forall i \neq h$ to arrive at the final expression. Next, using that the portfolio is well diversified ($\omega_i \approx \frac{1}{N}$) and that $\mathbb{E}\left[\widetilde{\varepsilon}_i^2\right] = \sigma_{\varepsilon_i}^2 \leq s^2 < \infty$ we get the desired result

$$\begin{aligned}
\sum_{i=1}^{N} \omega_i^2 \mathbb{E}\left[\widetilde{\varepsilon}_i^2\right] &\approx \frac{1}{N^2} \sum_{i=1}^{N} \sigma_{\varepsilon_i}^2 && (13)\\
&= \frac{1}{N^2} \sum_{i=1}^{N} s^2 && (14)\\
&= \frac{s^2}{N} \to 0 \quad \text{as} \quad N \to \infty. && (15)
\end{aligned}$$

That is, idiosyncratic risk can be eliminated trough diversification if N $(> k)$ is sufficiently large. This notion plays a central role in the APT and allows us to focus on systematic factor risk. That is, like the CAPM, the APT focuses on the compensation required by investors for being exposed to systematic risk. The main difference is that the APT allows that risk to originate from multiple sources.

2.3. The APT restriction

The APT assumes a model of financial markets that is both frictionless and competitive in which arbitrage opportunities are swiftly traded away by market participants. The return to each traded asset i is governed by a factor model

$$\widetilde{r}_i = \mu_i + \sum_{k=1}^{K} b_{ik} \widetilde{f}_k + \widetilde{\varepsilon}_i \tag{16}$$

$$= \mu_i + \mathbf{b}_i^\top \widetilde{\mathbf{f}} + \widetilde{\varepsilon}_i, \tag{17}$$

where $\mathbf{b}_i$ is a $K \times 1$ vector of factor loadings, $\widetilde{\mathbf{f}}$ is a $K \times 1$ vector of factor innovations, and $\widetilde{\varepsilon}_i$ is an idiosyncratic error term. To facilitate the derivation of the APT restriction, we will write the model in matrix form by defining $\widetilde{\mathbf{r}}$ and $\boldsymbol{\mu}$ as $N \times 1$ vectors of (mean) returns, $\mathbf{B}$ as an $N \times K$ matrix of factor loadings, $\widetilde{\mathbf{f}}$ a $K \times 1$ vector of factor innovations, and $\widetilde{\boldsymbol{\varepsilon}}$ as an $N \times 1$ vector of idiosyncratic errors. The model becomes

$$\widetilde{\mathbf{r}} = \boldsymbol{\mu} + \mathbf{B}\widetilde{\mathbf{f}} + \widetilde{\boldsymbol{\varepsilon}}, \tag{18}$$

where we maintain the same assumption as before. These can be expressed in matrix form as $\mathbb{E}\left[\widetilde{\mathbf{f}}\right] = \mathbf{0}$, $\mathbb{E}\left[\widetilde{\boldsymbol{\varepsilon}}\right] = \mathbf{0}$, $\mathbb{E}\left[\widetilde{\mathbf{f}}\,\widetilde{\boldsymbol{\varepsilon}}^\top\right] = \mathbf{0}$ and $\mathbb{E}\left[\widetilde{\boldsymbol{\varepsilon}}\,\widetilde{\boldsymbol{\varepsilon}}^\top\right] = \mathbf{V}_{\widetilde{\boldsymbol{\varepsilon}}} = \sigma^2_{\widetilde{\boldsymbol{\varepsilon}}}\mathbf{I}_N$. The assumptions may occasionally be complemented with an assumption that factors are othogonal, i.e., $\mathbb{E}\left[\widetilde{\mathbf{f}}\,\widetilde{\mathbf{f}}^\top\right] = \mathbf{V}_{\widetilde{\mathbf{f}}} = \sigma^2_{\widetilde{\mathbf{f}}}\mathbf{I}_K$, but this is not strictly needed for the development presented here.

The asset pricing restriction Following Ross (1976), we can impose no-arbitrage on the above model and claim that there exists a $K \times 1$ vector of factor risk premia $\boldsymbol{\lambda}_K$, which provides us with the following APT restriction

$$\mathbb{E}\left[\widetilde{\mathbf{r}}\right] = \boldsymbol{\mu} \approx \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K \tag{19}$$

Heuristic proof of the APT restriction The heuristic proof of the APT restriction provided by Ross (1976) proceeds as follows. First, form an arbitrage portfolio $\boldsymbol{\omega}_A$ using all N assets that requires zero net investment such that $\boldsymbol{\omega}_A^\top \mathbf{1} = 0$. The return on this arbitrage portfolio can be described by the factor model

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} = \boldsymbol{\omega}_A^\top \boldsymbol{\mu} + \boldsymbol{\omega}_A^\top \mathbf{B}\widetilde{\mathbf{f}} + \boldsymbol{\omega}_A^\top \widetilde{\boldsymbol{\varepsilon}}. \tag{20}$$

Suppose that investors are well-diversified so that each element of $\omega_i \approx \frac{1}{N}$ in absolute magnitude. That is, no single weight is "too large" in the portfolio. Then the second step involves invoking the law of large numbers to diversify idiosyncratic risk

$$\lim_{N\to\infty} \boldsymbol{\omega}_A^\top \widetilde{\boldsymbol{\varepsilon}} \approx \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \widetilde{\varepsilon}_i = 0 \tag{21}$$

so that the return on the arbitrage portfolio for sufficiently large N becomes (approximately)

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} \approx \boldsymbol{\omega}_A^\top \boldsymbol{\mu} + \boldsymbol{\omega}_A^\top \mathbf{B} \widetilde{\mathbf{f}}, \tag{22}$$

where we emphasize that the pricing relation is now approximate because we need $N \to \infty$ for idiosyncratic risk to be completely diversified away. In words, this step implies that idiosyncratic risk becomes negligible in a well-diversified portfolio. The third step involves forming the arbitrage portfolio $\boldsymbol{\omega}_A$ in such a way that it has no systematic factor risk, i.e., $\boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, so that the portfolio return becomes

$$\boldsymbol{\omega}_A^\top \widetilde{\mathbf{r}} \approx \boldsymbol{\omega}_A^\top \boldsymbol{\mu}. \tag{23}$$

The fourth and final step is to make the argument that a zero-investment portfolio with no systematic factor risk must have an expected return of zero if there are no arbitrage opportunities in the market. That is, for a portfolio with $\boldsymbol{\omega}_A^\top \mathbf{1} = \boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, we must have that

$$\boldsymbol{\omega}_A^\top \boldsymbol{\mu} = 0. \tag{24}$$

This is essentially the proof, and it relies on the following set of no arbitrage conditions that underlie the APT framework.

No arbitrage conditions The heuristic arguments lead us to a set of no arbitrage conditions. In particular, let $\boldsymbol{\omega}_A$ be a well-diversified portfolio. If $\boldsymbol{\omega}_A^\top \mathbf{1} = 0$ and $\boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, then $\boldsymbol{\omega}_A^\top \boldsymbol{\mu} = 0$.

We are then going to claim that there exist constants λ_0 and $\boldsymbol{\lambda}_K$ so that $\boldsymbol{\mu} \approx \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K$, where λ_0 is interpreted as a zero-beta rate (the risk-free rate if such as asset exist) and $\boldsymbol{\lambda}_K$ is a vector of factor risk premia associated with the K factors in the model. The no arbitrage restrictions require that $\boldsymbol{\omega}_A^\top \boldsymbol{\mu} = 0$ for $\boldsymbol{\omega}_A^\top \mathbf{1} = 0$ and $\boldsymbol{\omega}_A^\top \mathbf{B} = \mathbf{0}$, which implies that $\boldsymbol{\mu}$ is spanned by $\mathbf{1}$ and $\mathbf{B}$. That is,

$$\boldsymbol{\mu} \approx \mathbf{1}\lambda_0 + \mathbf{B}\boldsymbol{\lambda}_K. \tag{25}$$

If a risk-free asset exists in the economy, then $\lambda_0 = r_f$ must be its return. Even if there is no risk-free rate in the economy, then λ_0 can be interpreted as the return on a zero-beta portfolio, i.e., all portfolios defined by $\boldsymbol{\omega}^\top \mathbf{1} = 1$ and $\boldsymbol{\omega}^\top \mathbf{B} = \mathbf{0}$. Since $\mathbf{B}$ measures the exposure to factor risk, $\boldsymbol{\lambda}_K$ is readily seen to represent the risk premium associated with the kth factor. Importantly, if the factor is traded, then the risk premia equals its expected excess return. The APT restriction in (25) can be seen as an asset pricing restriction in that any proposed factor model should satisfy the equation, otherwise the model is an incomplete description of expected returns.

As a final note on the APT restriction, we demonstrate that the APT restriction is arbitrage free using the no arbitrage conditions, i.e.,

$$\boldsymbol{\omega}_A^\top \boldsymbol{\mu} \approx \boldsymbol{\omega}_A^\top \mathbf{1}\lambda_0 + \boldsymbol{\omega}_A^\top \mathbf{B}\boldsymbol{\lambda}_K \tag{26}$$

$$= 0\lambda_0 + \mathbf{0}\boldsymbol{\lambda}_K = 0. \tag{27}$$

We note that the above does not represent a rigorous proof of the APT because it assumes that we can ignore the residuals. A separate literature deals with this issue at length.

3. An approximate CAPM

Ross (1976) completely revolutionized asset pricing with these simple arguments, and he was even able to derive the – at the time – highly successful and hailed CAPM using these simpler assumptions. We provide a simple illustration of this derivation here. Assume that $K = 1$ and let $\widetilde{f}$ be an unspecified market wide factor with expectation zero and factor loadings $\boldsymbol{\beta}$. Suppose further that a riskless asset exists in the economy so that the APT restriction becomes

$$\boldsymbol{\mu} \approx \mathbf{1}r_f + \boldsymbol{\beta}\lambda_M \tag{28}$$

If we wish to specify $\widetilde{f}$ as the market portfolio, then we need to form a portfolio $\boldsymbol{\omega}_M$ that satisfies the following CAPM-implied restrictions

$$\boldsymbol{\omega}_M^\top \mathbf{1} = 1 \tag{29}$$

$$\boldsymbol{\omega}_M^\top \boldsymbol{\beta} = 1 \tag{30}$$

Using the APT restriction in (28), we can determine an approximate expression for the expected return om the market portfolio as

$$\mathbb{E}\left[\widetilde{r}_M\right] = \boldsymbol{\omega}_M^\top \boldsymbol{\mu} \tag{31}$$

$$\approx \boldsymbol{\omega}_M^\top \mathbf{1} r_f + \boldsymbol{\omega}_M^\top \boldsymbol{\beta}\lambda_M \tag{32}$$

$$\approx r_f + \lambda_M, \tag{33}$$

which immediately tells us that $\lambda_M = \mathbb{E}\left[\widetilde{r}_M\right] - r_f$ such that when inserted into the APT restriction in (28) yields an approximate CAPM, i.e.,

$$\boldsymbol{\mu} \approx \mathbf{1}r_f + \boldsymbol{\beta}\left(\mathbb{E}\left[\widetilde{r}_M\right] - r_f\right). \tag{34}$$

Thus, we have derived the (approximative) Security Market Line (SML) without the need for any assumptions like mean-variance efficiency, normality of returns, quadratic utility of investors, unlimited borrowing/lending at the same risk-free rate, or general equilibrium.

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