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Pricing Currency Risks

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ABSTRACT

The currency market features a small cross-section, and conditional expected returns can be characterized by few signals: interest differential, trend, and mean reversion. We exploit these properties to construct the ex ante mean-variance efficient portfolio of individual currencies. The portfolio is updated in real time and prices all prominent currency trading strategies, conditionally and unconditionally. The fraction of risk in these assets that does not affect their risk premiums is at least 85%. Extant explanations of carry strategies based on intermediary capital or global volatility are related to these unpriced components, while consumption growth is related to the priced component of returns.

IN THIS PAPER, WE ARGUE that research on cross-sectional (CS) currency pricing can depart fruitfully from the factor mining path established by the equity literature. That is, because a direct solution of the mean-variance optimization problem in the presence of conditional information is feasible in the exchange-rate setting. The value of this approach, if it is empirically successful, is obvious: one obtains a measure of the so-called unconditional mean-variance efficient (UMVE) portfolio of currencies that prices *all*

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admissible dynamic trading strategies in these currencies unconditionally. As a result, as Hansen and Richard (1987) show, the UMVE portfolio correctly prices the full cross-section of currencies and trading strategies associated with them conditionally as well. Measurement of the UMVE portfolio can therefore help direct research that seeks to understand currency pricing.

UMVE portfolio construction requires two critical ingredients: estimates of the conditional mean and the conditional covariance matrix of currency excess returns. This is where exchange rates have an advantage over equities. First, the size of the cross-section of exchange rates is small, not exceeding 40, as compared to thousands of stocks. This feature dramatically affects the precision of the covariance matrix regardless of the specifics of the estimation method. Second, exchange rates have a particular economic structure that helps one hypothesize the functional form of the conditional means. Research that goes back decades suggests the importance of three key ingredients of conditional expectations. Specifically, Covered Interest Parity (CIP, Keynes (1923)), Uncovered Interest Parity (UIP, Porter (1971)), and Random Walk Hypothesis (RWH, Meese and Rogoff (1983)) lead us to the interest rate differential. Purchasing Power Parity (PPP, Cassel (1918)) suggests a measure of mean reversion. And research on weak-form market efficiency in nominal exchange rates (Cornell and Dietrich (1978)) indicates a measure of trend. These three factors remain the pillars of the modern currency trading strategies under the names of carry, value, and momentum, respectively.

We work with a sample of G10 currencies, the most commonly used data in the literature, with monthly returns based on currency forward rates from January 1985 to May 2020. We construct monthly conditional expected excess returns using the three aforementioned signals. The loading on the signal related to the interest rate differential is fixed to be consistent with the RWH, while the loadings on the other signals are estimated on an expanding basis via panel regressions. We construct a conditional covariance matrix of currency excess returns using daily data within each month. Importantly, the approach uses data that are available to investors in real time.

If the conditioning information that drives these estimates is sufficiently rich, the resulting UMVE portfolio will in theory price not only the excess returns on individual currencies but also any dynamic strategy, including those considered in the literature. Yet, because these estimates are out-of-sample (OOS) forecasts and because we have to assume specific forecasting models, it is not given that the resulting UMVE portfolio would perform well (e.g., have a high Sharpe ratio [SR] or price the test assets). Therefore, to validate our UMVE construction, we test the implied one-factor model using a broad set of test assets and show that it is not rejected, while competing alternatives are rejected.

We consider nine leading trading strategies in addition to individual currencies as test assets. The dollar strategy represents the currency version of "the market." The dollar carry strategy goes long or short the dollar strategy depending on the average interest rate differential, or, more precisely, the average currency forward discount. Next, we evaluate the CS and time-series

(TS) carry strategies. These strategies are followed by two versions of CS and TS momentum depending on whether one uses the past month or year as the measure of past performance. We also study CS value, which sorts currencies based on the real exchange rate (RER). As emphasized by Asness, Moskowitz, and Pedersen (2013) and Koijen et al. (2018), besides being the most common strategies in the foreign exchange market, they are more broadly considered in other asset classes. Lastly, because the Hansen and Richard (1987) result requires correct unconditional pricing of literally all trading strategies, we consider additional 81 strategies that are constructed by changing the base currency from U.S. dollar (USD) to one of the nine other currencies. For instance, the Japanese yen (JPY) strategy would involve averaging returns with respect to the JPY (and then expressing these returns in USD terms). We also consider the effect of changing the numeraire (e.g., converting the dollar strategy returns to JPY).

The UMVE is tradeable in real time, has an SR in excess of one, and, conveniently, can be compared directly to existing trading strategies using standard regression-based tests. We verify that it prices both excess returns on nine individual currencies versus the USD and the nine above-mentioned trading strategies via the unconditional and conditional pricing tests of Gibbons, Ross, and Shanken (1989) and Chernov, Lochstoer, and Lundeby (2022), respectively. We also use the above-mentioned additional strategies with a non-USD base or measurement currency and again fail to reject the estimated UMVE.

As an alternative, we consider a variance-based approach to construct conditional expectations of currency returns. This approach is motivated by the work of Lustig, Roussanov, and Verdelhan (2011) and Verdelhan (2018), which implies that the dollar and carry strategies explain (co)variances of currency returns in both the cross-section and the timeseries. This observation implies that the dollar and carry betas should predict currency returns. We demonstrate that the resulting OOS forecast is dramatically different from our advocated conditional mean. The resulting UMVE portfolio has a low SR of 0.3 and is rejected by our tests.

Having validated the UMVE portfolio, we turn to evaluating its properties. First, we find that it explains only a small fraction of the time-series variation in strategy returns. For instance, less than 1% of the variation in the dollar strategy returns (the currency "market portfolio") are priced, while the same for carry strategies is at most 15%.[1] Thus, there are large unpriced

[1] To interpret the evidence, it is convenient to think of the SDF as consisting of two components. The first is the UMVE expressed in terms of a numeraire that is not currency-specific (e.g., a world currency index). The second reflects the change in numeraire to a given currency. We show that because the SR of the UMVE is slightly greater than one while the volatility of depreciation rates is about 0.1 (per annum), it must be the case that most of the valuation effect stems from the UMVE component of the stochastic discount factor (SDF). Furthermore, strategy returns are exposed to common sources of variation in currencies that are not related to the UMVE. These common sources are important for explaining the variation in strategy returns (e.g., Verdelhan (2018)), but not their risk premiums as per the above. As a result, regressions of strategy returns on the UMVE result in low R^2s. See the Internet Appendix for details. The Internet Appendix is available in the online version of the article on *The Journal of Finance* website.

components in strategy returns and we show that hedging out these unpriced components in an OOS manner dramatically increases the SR of the hedged strategy returns. As an OOS model check, we verify that the average returns to the unpriced components are indeed not significantly different from zero.

Related, we document that the portfolios that are the most important for time-series covariation among currencies and the currency trading strategies have little relation to the UMVE portfolio. In other words, the main factors that drive the covariance matrix of returns are not an important source of risk for the marginal currency investor. For instance, the first two principal components (PCs) of individual currency returns capture 75% of the return variation and are strongly associated with the dollar and CS carry strategies, consistent with Verdelhan (2018).[2] However, these PCs explain only around 5% of the variation in UMVE returns. In fact, all nine of these PCs only explain around 10% of the UMVE returns, which indicates that conditional currency risk premiums vary strongly and underscores the importance of correct conditioning.

The currency trading strategies we consider involve substantial timing of individual currencies and therefore potentially account for these conditional dynamics. We investigate whether the strategies' PCs are capable of explaining variation in the UMVE. The answer is no, although there is some improvement. The first three PCs of the currency trading strategies explain around 20% of the variation in the UMVE returns. Using all of the PCs, this increases to 50%. This evidence is consistent with the uncovered large unpriced component of strategy returns. It also echoes the rejection of the UMVE constructed using variance-based conditional expectations. Taken together, these results suggest a large role of optimal timing implicit in the construction of the UMVE above and beyond the timing already implicit in existing trading strategies.

General equilibrium models have struggled to explain exchange rate moments. The current generation of models based on habit (Verdelhan (2010)), long-run risks (Bansal and Shaliastovich (2013), Colacito et al. (2018)), and rare disasters (Farhi and Gabaix (2016)) tend to focus on explaining the forward premium puzzle (and to less extent the carry trade). However, as our analysis shows, the most popular carry strategy explains only about 15% of the variation in our pricing factor. Further, the UMVE portfolio, which is perfectly negatively correlated with the SDF, has approximately zero skewness of -0.05 and modest excess kurtosis of 3.07. This suggests a relatively modest effect of disaster risk pricing in our sample. Thus, there is much left to explain and our empirical model can help guide the specification and tests of future models on currency risk and return.

We provide initial evidence along these lines by first regressing the UMVE returns on various candidate explanatory factors. We find that quarterly and three-year consumption growth, a proxy for long-run consumption risk, are both significantly positively related to the UMVE returns, consistent with Lustig and Verdelhan (2007) and Zviadadze (2017). We next find that the

[2] Aloosh and Bekaert (2022) and Greenaway-McGrevy et al. (2018) offer alternative approaches to the analysis of the time-series behavior of exchange rates.

factors in the Fama and French (2015) five-factor model are only weakly related to the currency UMVE portfolio, with only the value (HML) and investment (CMA) factors having a significant relationship. However, here the R^2s are only about 1%, so economically, there appears to be no relation between priced risks in the equity and currency markets. These results extend those in Burnside (2012), who finds a similar weak relation between carry trade returns and equity factors, to the UMVE portfolio. We also find that intermediary capital factors and shocks to equity and currency variance are unrelated to the UMVE returns. This may appear puzzling, given that previous literature highlights these factors as potential explanations for the carry trade, but we show that this correlation comes from the large unpriced component in carry strategy returns.

Finally, we show that the conditional maximum SR (MSR) has a downward trend over the sample, largely caused by a trend decline in interest differentials across countries. The cyclical variation in the MSR is *negatively* related to measures of the conditional variance of consumption and equity returns and to currency depreciation. In sum, our paper provides a rich set of facts that can help guide future equilibrium models of currency market risk and return.

A number of papers consider conditional mean-variance efficient portfolios (CMVE) using the forward discount alone as a signal (Baz et al. (2001), Ackermann, Pohl, and Schmedders (2017), Daniel, Hodrick, and Lu (2017), Maurer, To, and Tran (2020), Maurer, To, and Tran (2022)).[3] With the exception of Maurer, To, and Tran (2022), these papers do not examine whether CMVE explains other versions of carry or individual currency returns. We reject this version of our model with respect to our full model, which includes mean reversion and trend signals, using the Barillas and Shanken (2017) test. We also reject it using the conditional pricing test, even when only using carry-based strategies as test assets. Della Corte, Sarno, and Tsiakas (2009) depart from the lone forward discount signal by considering three monetary fundamentals variables. They perform an OOS analysis of exchange rate predictability in a mean-variance framework where the allocation choice is between the risk-free asset and one of the three currencies they consider.

In contemporaneous and independent work, Nucera, Sarno, and Zinna (2022) use the risk-premium PC analysis of Lettau and Pelger (2020) to extract latent factors of a currency SDF. They find that the implied SDF includes a strong dollar factor and two weak carry and momentum factors, whereas evidence of a value factor is scant. This is in contrast to our results: we find that the dollar strategy contains a small fraction of priced risk, the momentum strategy contains a larger fraction, and the carry and value strategies contain the largest fractions.

A large literature implements model-free SDF projections in the context of unconditional pricing of assets. Many authors use Hansen and

[3] As detailed later in this paper, there is an infinite number of CMVE portfolios, with UMVE being one of them. UMVE is the only CMVE portfolio that satisfies the unconditional linear beta pricing relation, which is a staple of cross-sectional asset pricing tests.

Jagannathan (1991) bounds as a diagnostic in their models.[4] A variation on the unconditional version of the Hansen and Jagannathan (1991) approach is that of entropy minimization as advocated by Stutzer (1996) in the context of derivatives pricing. Ghosh, Julliard, and Taylor (2019) develop and apply this framework to cross-sectional asset pricing. Korsaye, Trojani, and Vedolin (2020), Orlowski, Sokolovski, and Sverdrup (2021), and Sandulescu, Trojani, and Vedolin (2021) are examples of international applications of this framework. We depart from this work by constructing real-time conditional SDF projections. Our approach allows for an evaluation of any trading strategies in the set of assets involved in the projection.

Our strategies that are used as test assets have origins in the large literature on international asset pricing; such work includes Asness, Moskowitz, and Pedersen (2013), Burnside, Eichenbaum, and Rebelo (2011), Burnside et al. (2011), Daniel, Hodrick, and Lu (2017), Koijen et al. (2018), Lustig, Roussanov, and Verdelhan (2011) (2014), Menkhoff et al. (2012b), and Moskowitz, Ooi, and Pedersen (2012).

This paper is organized as follows. Section I discusses implementation and testing of general linear factor models and the specific application to exchange rates. Section II describes all the empirical results. Section III concludes.

I. Linear Factor Models and Exchange Rates

This section describes the theoretical underpinnings of the UMVE portfolio, how we estimate its weights in the context of exchange rates, and how the estimated UMVE portfolio can be tested.

A. The UMVE Portfolio

We seek to correctly price currency risks both conditionally and unconditionally. As pointed out by Hansen and Richard (1987) and Jagannathan (1996), one can achieve this by constructing the UMVE portfolio.

Suppose that we have N basis assets with an $N \times 1$ vector of excess returns R^e_{t+1}. The conditional mean of this vector is $\mu_t = E_t(R^e_{t+1})$ and its conditional covariance matrix is $\Omega_t = V_t(R^e_{t+1})$. An admissible trading strategy p in the basis assets has an $N \times 1$ vector of weights w_{pt} that are determined only by information available up until time t. The resulting excess portfolio return is then $R_{p,t+1} = w_{pt}^\top R^e_{t+1}$.

[4] The SDF frontier associated with the unconditional mean-variance frontier could be used as informationally efficient SDF bounds. An important literature, starting with Gallant, Hansen, and Tauchen (1990), explores such frontiers *in-sample*. For instance, Ferson and Siegel (2003, 2009) explore efficient bounds based on the unconditional frontier. Bekaert and Liu (2004) argue that using the optimal weights reported is robust to misspecification of the conditional moments of returns since the solution still describes a valid portfolio strategy that expands the boundary in-sample. In contrast, we do not explore the connection of the UMVE portfolio, a single point on the UMV frontier, to these bounds, and our analysis is conducted *OOS*.

The UMVE portfolio is a dynamic trading strategy in these assets and obtains the MSR, both conditionally and unconditionally. Ferson and Siegel (2001) and Jagannathan (1996) show that the UMVE portfolio weights are

$$w_t^* = \frac{1}{1 + \mu_t^\top \Omega_t^{-1} \mu_t} \Omega_t^{-1} \mu_t. \tag{1}$$

Denote the excess return on this portfolio by $R_{t+1}^* = w_t^{*\top} R_{t+1}^e$.

The UMVE portfolio accounts for all risks conditionally in the sense that the following conditional linear beta pricing relationship holds for any admissible strategy p:

$$E_t(R_{p,t+1}) = \beta_{pt} E_t(R_{t+1}^*), \tag{2}$$

where $\beta_{pt} = \mathrm{Cov}_t(R_{p,t+1}, R_{t+1}^*)/V_t(R_{t+1}^*)$. The UMVE portfolio also implies the unconditional linear beta pricing relation

$$E(R_{p,t+1}) = \beta_p E(R_{t+1}^*), \tag{3}$$

where $\beta_p = Cov(R_{p,t+1}, R_{t+1}^*)/V(R_{t+1}^*)$, for any p.

The latter result may seem surprising in light of Jagannathan and Wang (1996), who point out that taking unconditional expectations of equation (2) yields $E(R_{p,t+1}) = E(\beta_{pt})E(R_{t+1}^*) + \mathrm{Cov}(\beta_{pt}, E_t(R_{t+1}^*))$. That is, in general, a conditional linear beta-pricing model does not imply an unconditional linear beta-pricing model. However, the specific case of the UMVE portfolio is an exception.

Indeed, portfolios with weights that are proportional to $\Omega_t^{-1}\mu_t$ are all CMVE and thus satisfy equation (2). There is an infinite number of such portfolios because leverage does not affect the conditional SR. It is the particular time variation in leverage (market timing) implied by the scalar term $(1 + \mu_t^\top \Omega_t^{-1} \mu_t)^{-1}$ in equation (1) that delivers the UMVE portfolio and the validity of the unconditional linear beta pricing in equation (3). Thus, opposite from what the language might seem to imply, the UMVE is also CMVE, while the converse need not be true.

There is a tight link between the UMVE and the SDF. Specifically,

$$M_{t+1}^* = 1 - (R_{t+1}^* - E(R_{t+1}^*)) \tag{4}$$

has the property that

$$E_t(M_{t+1}^* R_{p,t+1}) = 0 \tag{5}$$

for all admissible trading strategies p in the set of basis assets. This SDF can be viewed as a conditional projection of the true SDF, M_{t+1}, onto the payoff space of the basis assets. Alternatively, it can be viewed as an unconditional linear projection of the true SDF onto the payoff space of all admissible trading strategies in the basis assets. Since our focus is on excess returns only, we normalize the unconditional mean of this SDF to one. See Chernov, Lochstoer, and Lundeby (2022) for a recent distillation of these results.

As is well known, even though the true SDF is strictly positive under no-arbitrage, the projected SDF can take negative values. However, this does not matter for the pricing of the assets the projection is made onto. In fact, $M_{t+1} = M^*_{t+1} + \eta_{t+1}$, where η has the property that $E_t(\eta_{t+1} R_{p,t+1}) = 0$ for all admissible trading strategies in the set of basis assets. See Cochrane (2005) for a treatment of these relations.

B. Exchange Rates

In practice, estimating μ_t and Ω_t could be difficult or infeasible. For instance, for equities, it is difficult to estimate Ω_t because N is large, and there is not much consensus on estimating μ_t. As a result, the equity literature has proposed various characteristic-sorted return factors that are presumed to span the UMVE portfolio (see, e.g., Fama and French (1993)). Real-time implementability of the trading strategies implied by these characteristics is paramount because a look-ahead in factor construction biases tests of the null hypothesis that a given factor model prices the cross-section of assets (see, e.g., Black, Jensen, and Scholes (1972), Lo and MacKinlay (1990)).

Starting with Lustig, Roussanov, and Verdelhan (2011), the literature on currency risk and returns has followed the approach in equities by creating factors based on characteristics such as the interest rate differential (carry), or an equal-weighted portfolio of currencies (dollar), assuming that these factors span the UMVE portfolio. This literature has also emphasized the real-time implementability of the factors.

We note that exchange rates differ from equities in two very important respects. First, the cross-section of "assets" is limited. Indeed, the largest data panel considered in the literature does not exceed 40 currencies at any given point in time (see, e.g., Hassan and Mano (2019), Lustig, Roussanov, and Verdelhan (2011), Menkhoff et al. (2012a)), while most studies limit their attention to 10 currencies representing developed economies. Second, a voluminous literature on the forecasting of exchange rates suggests that few variables outperform the RWH OOS.

These observations prompt us to directly construct the returns R^*_{t+1} using the theoretical definition of the UMVE portfolio as given in equation (1). That is, we estimate the conditional expected return and covariance matrix of the currency returns and use these inputs to construct R^*_{t+1}. We do so in an OOS fashion, where we only use data up to time t to estimate μ_t and Ω_t, so our UMVE portfolio is implementable in real time.

C. Implementing the UMVE for Exchange Rates

Let the USD be the measurement (numeraire) currency, that is, all exchange rates are expressed in USD per unit of foreign currency. Let S^i_t and F^i_t denote the spot exchange rate and the one-month forward exchange rate of country i, respectively.

The payoff of a forward contract (when buying one unit of the foreign currency) is $S^i_{t+1} - F^i_t$. One common way to scale this payoff to define excess returns is to divide by F^i_t :

$$R^{ei}_{t+1} = (S^i_{t+1} - F^i_t)/F^i_t. \tag{6}$$

This definition implies that the amount of foreign currency bought is one "forward" USD. Thus, this is an excess return to a trading strategy regardless of whether CIP holds or not.

Next, we estimate Ω_t. First, for each month t, we compute the realized covariance matrix based on daily depreciation rates. Specifically, defining $X_{j,t}$ as the vector of percentage changes in spot exchange rates over day j in month t, we calculate the realized covariance matrix as

$$\hat{\Sigma}_t = \sum_{j=1}^{D_t} X_{j,t} X_{j,t}^\top,$$

where D_t is the number of trading days in the month. Given a potentially large cross-section and short time series, we apply the shrinkage method of Ledoit and Wolf (2020) to $\hat{\Sigma}_t$ to improve the mean-squared error of this sample covariance estimate. Denote the resulting shrunk matrix by $\tilde{\Sigma}_t$. We then apply a simple exponentially weighted average to $\tilde{\Sigma}_t$ to arrive at the estimate of the conditional covariance matrix for month $t+1$ currency percentage price changes:

$$\Omega_t = (1 - \lambda)\tilde{\Sigma}_t + \lambda \Omega_{t-1},$$

where we set $\lambda = 0.94$ following, for example, the RiskMetrics model. Finally, to get to the conditional covariance matrix of currency excess returns, we premultiply and postmultiply this matrix by a diagonal matrix with the $(i, i)^{th}$ element set to S^i_t/F^i_t.

Our starting point for the estimation of μ_t is the RWH for spot exchange rates. The reason is simple: the hypothesis leads to an exceptionally robust forecaster of exchange rates (Meese and Rogoff (1983)). This is also a natural baseline given research in finance, starting with Baz et al. (2001), which uses RWH to develop trading strategies.

The RWH implies that expected excess currency returns are given by

$$\mu_{it} \equiv E_t(R^{ei}_{t+1}) = \gamma \cdot \left(S^i_t/F^i_t - 1\right),$$

with $\gamma = 1$. This is a particular violation of UIP, which posits $\gamma = 0$. We refer to $S^i_t/F^i_t - 1$ as the (normalized) forward discount.

Next, we add mean reversion and trend signals for exchange rate forecasting. See Bilson (1984) and Sweeney (1986), respectively, for early contributions in the context of trading strategies. Thus, the use of the RWH, as well as mean reversion and trend signals, are all concepts that were in the public domain at the start of our sample in 1985.

Our mean-reversion signal is motivated by the literature on the role of RER in forecasting and capturing risk premiums. The RER is defined as

$$Q_t^i = S_t^i \cdot P_t^i / P_t, \tag{7}$$

where P_t and P_t^i are the U.S. and foreign consumer price index (CPI), respectively. The weak form of PPP implies mean reversion in the RER. Thus, when the RER is far from its long-run mean, it should forecast the currency depreciation.

Combining the RER-based signal with the RWH goes as far back as Bilson (1984). More recently, Chernov and Creal (2021) and Dahlquist and Pénasse (2022) show that PPP implies that the RER forecasts nominal depreciation rates and that the currency risk premium depends on the RER. Eichenbaum, Johannsen, and Rebelo (2021) demonstrate that the RER outperforms the random walk in the forecasting of exchange rates at horizons beyond one year.

As Jorda and Taylor (2012) emphasize, the RER's long-run mean is not a clearly defined object empirically. We divide each RER by its five-year smoothed lag (specifically, the average RER from 4.5 to 5.5 years ago) as a way to remove the dependence on the long-run mean while still preserving the long-run nature of mean-reversion signals:

$$\widetilde{Q}_t^i \equiv Q_t^i \cdot \left(\frac{1}{13} \sum_{j=-6}^{6} Q_{t-60+j}^i \right)^{-1}.$$

In a last step, we cross-sectionally demean the signal at each time t to create a cross-sectional ranking of "cheap" and "expensive" currencies. Specifically, our signal is

$$z_{Qt}^i \equiv \widetilde{Q}_t^i - \frac{1}{N} \sum_{i=1}^{N} \widetilde{Q}_t^i. \tag{8}$$

This definition has the virtue of removing any time and currency fixed effects. The specific form of the reference point is motivated by the cross-sectional model of Asness, Moskowitz, and Pedersen (2013), who select the specifics to be comparable with the equity and commodity literatures.

Our trend signal is motivated by the academic and practitioner literature on the use of moving averages in trading and forecasting exchange rates. In contrast to the macro literature, the finance literature suggests that using past performance could be fruitful in improving trading performance (see, e.g., Sweeney (1986), Kho (1996), Okunev and White (2003), and references therein). Following this work, we add a simple trend signal, namely, a one-year depreciation rate.

In summary, we forecast excess returns OOS via

$$\mu_{it} = \gamma_t^i \cdot \left(S_t^i / F_t^i - 1 \right) + \delta_t^i \cdot z_{Qt}^i + \phi_t^i \cdot \left(S_t^i / S_{t-12}^i - 1 \right). \tag{9}$$

We set $\gamma_t^i = 1$ to match the RWH baseline. The coefficients δ_t^i and ϕ_t^i are reestimated each month t using historical exchange rates up to time t, as detailed below. Estimating γ_t^i in a similar fashion does not materially change the results.

An alternative approach to estimating conditional expectations of currency returns would be to assume that currency risk premiums are driven by exposures to the main drivers of currency return variance. Lustig, Roussanov, and Verdelhan (2011) argue that the carry and dollar factors explain the cross-section of expected currency returns. Verdelhan (2018) demonstrates that these two risk factors capture most of the time-series variation in currency returns and are closely related to the first two PCs of currency returns. Thus, the conditional pricing relation $E_t(R^e_{it+1}) = \beta_{it}\lambda_t$ suggests that one could forecast currency returns via

$$\mu_{it} = \lambda_t^{carry} \cdot \beta_{it}^{carry} + \lambda_t^{dollar} \cdot \beta_{it}^{dollar}, \tag{10}$$

where a factor (carry or dollar) is a portfolio of the currencies, $R^F_{p,t+1} = (w^F_t)^\top R^e_{t+1}$. The Internet Appendix describes computation of the betas. We estimate λ_t^{carry} and λ_t^{dollar} using data only up to time t. We refer to this specification of the forecast as the variance-based expected return forecast, and consider this in our analysis as well.

D. Testing Linear Factor Models

D.1. Tests

Equation (2) holds mechanically if the conditional expectations on both the left- and right-hand sides are computed using our estimates of μ_t and Ω_t, regardless of whether these estimates are close to the true objects. Thus, because we do not observe the true conditional means and covariances, we cannot test our model using equation (2) directly.

The analysis of Hansen and Richard (1987) helps resolve this problem. They show that one can evaluate all of the conditional implications of equation (2) by testing equation (3) using all admissible trading strategies as test assets. Because our model's factor represents a return on a traded asset (the UMVE), the model implies that $\alpha_p = 0$ in the TS regression

$$R_{p,t+1} = \alpha_p + \beta_p R^*_{t+1} + \varepsilon_{p,t+1} \tag{11}$$

for each test asset p (e.g., Cochrane (2005), Section 12.1).

Averages of returns and estimated β_p's do not equal their expected values in a finite sample. This implies that point estimates of α_p are nonzero, even if our estimates of μ_t and Ω_t equal their true values.

Further, it is not clear a priori that our estimates of μ_t and Ω_t, which are constructed on an OOS basis, are correct, that is, there is no guarantee that the resulting UMVE portfolio would have the ex ante MSR. This is another source of nonzero α_p. We therefore validate our estimates of μ_t, Ω_t, and the resulting UMVE weights by performing standard Gibbons, Ross, and Shanken (1989) (GRS) joint tests of $\alpha_p = 0$ across a large set of trading strategies p proposed in the literature.

We further follow Chernov, Lochstoer, and Lundeby (2022) and use multi-horizon returns (MHR) on the selected assets to generate additional test assets that are endogenous to the model of the SDF. Specifically, using the projected SDF associated with the UMVE in equation (4), we test whether

$$E(M^*_{t,t+h} R^i_{t,t+h}) = 1$$

for a range of horizons h. Here, the multihorizon SDF is $M^*_{t,t+h} = \prod_{j=1}^{h} M^*_{t+j}$. The multihorizon gross return is $R^i_{t,t+h} = \prod_{j=1}^{h} R^i_{t+j}$, where $R^i_{t+1} = R^{ei}_{t+1} + R^f_t$, with the latter denoting the one-month U.S. gross funding rate.

The set of test assets includes the same currency strategies as well as the UMVE portfolio. Chernov, Lochstoer, and Lundeby (2022) show that unconditional MHR pricing allows for testing most, if not all, aspects of conditional model misspecification.

D.2. Trading Strategies

Next, we describe the nine leading trading strategies proposed in the literature. We use the USD as the base and measurement currency. In the Internet Appendix, we discuss implications of using the strategies with another base and/or measurement currency.

The portfolio excess return of a trading strategy is given by

$$R_{p,t+1} = \sum_{i=1}^{N} w^i_{pt} R^{ei}_{t+1}, \tag{12}$$

where w^i_{pt} is the portfolio's weight in currency i at time t and N is the number of currencies. The weight of a given portfolio p can be based on a signal, z^i_{pt}, and chosen such that the portfolio has an exposure to the USD or not. We use so-called rank and sign weights based on these signals. See Asness, Moskowitz, and Pedersen (2013), Koijen et al. (2018), and Moskowitz, Ooi, and Pedersen (2012) for further discussions of such weights.

We use rank weights for CS strategies. The rank of a currency is based on the signal, and the weight is based on the rank according to

$$w^i_{pt} = \kappa \left(\text{rank}(z^i_{pt}) - N^{-1} \sum_{i=1}^{N} \text{rank}(z^i_{pt}) \right), \tag{13}$$

where the scaling constant κ makes the portfolio one USD long and one USD short (and hence USD neutral). With nine currencies versus the USD, the possible weight values are $+0.4$, $+0.3$, $+0.2$, $+0.1$, 0.0, -0.1, -0.2, -0.3, and -0.4. Note that the weights depend on the currency ranks, the long and short positions sum to $+1$ and -1, respectively, and the net exposure to the USD is zero.

We use sign weights for TS strategies. The weights are then $+1$ or -1, depending on the sign of the signal, and the net exposure to the USD can be positive or negative. We further scale these sign weights by N to get a portfolio volatility similar to those of the CS strategies. However, this scaling does not affect inference about a strategy's risk-adjusted performance.

Note that the dollar strategy differs from both CS and TS approaches as it is an equal-weighted average of the individual currency returns (Lustig, Roussanov, and Verdelhan (2011)).[5] The dollar strategy can be seen as an equal-weighted market portfolio of currencies. It simply goes long all currencies versus the USD.

We consider three carry strategies. The dollar carry strategy uses the average forward discount (across all currencies) as a signal. Specifically, it goes long (short) all currencies versus the USD when the average forward discount is positive (negative) (Lustig, Roussanov, and Verdelhan (2014)). Hence, the dollar carry strategy is a conditional version of the dollar strategy above: when the average forward discount is positive, it goes long the dollar strategy; when the forward discount is negative, it goes short the dollar strategy.

The CS-carry strategy uses an individual currency's forward discount as a signal and the ranking weights as described above. Currencies with relatively high forward discounts have positive weights, while currencies with relatively low forward discounts have negative weights (similar to Lustig, Roussanov, and Verdelhan (2011), who construct a high-minus-low carry portfolio rather than using the rank weights). Recall that the CS strategies are USD neutral.

The TS-carry uses the sign of the individual currency's forward discount as a signal. It goes long (short) currencies with a positive (negative) discount (Burnside et al. (2011), Daniel, Hodrick, and Lu (2017)). At each point in time, a varying number of currencies may have a positive or negative forward discount, so there is a time-varying exposure to the USD.[6]

We consider two CS momentum strategies, which use the currency's performance as a signal. The CS-mom 1 strategy uses the performance in the most recent month as a signal (Burnside et al. (2011), Menkhoff et al. (2012b)), and the CS-mom 12 strategy uses the performance in the most recent year skipping

[5] Verdelhan (2018) uses an equal-weighted average of currency depreciation rates, as opposed to currency excess returns, when studying variation in exchange rates and refers to it as the dollar factor. This means that the dollar factor does not include the effect of the forward discount (i.e., the interest rate differential under his assumption that the CIP holds), but he controls for this in his analysis. He also considers excess returns on portfolios based on the exposure to his dollar factor and refers to the difference between portfolios of high- and low-dollar exposures as the global component of the dollar factor (or simply "dollar global"). He reports that the global component is highly correlated with the dollar factor itself (0.85).

[6] Hassan and Mano (2019) decompose violations of UIP into three portfolio-based components: a cross-currency, a between-time-and-currency, and a cross-time component. Empirically, the cross-currency component drives the correspondence to CS-carry, whereas the cross-time component drives the correspondence to dollar carry. The between-time-and-currency component, theoretically shared by both CS-carry and dollar carry, is empirically less important. The decomposition allows Hassan and Mano (2019) to link dollar carry to the regression-based forward premium puzzle (see, e.g., Fama (1984), Bansal and Dahlquist (2000), Backus, Foresi, and Telmer (2001)).

the most recent month as a signal (Asness, Moskowitz, and Pedersen (2013)). Specifically, weights are rank-based as described above.

We also consider two TS momentum strategies, which use the sign of the currency's recent performance as a signal. The TS-mom 1 strategy uses the currency's prior-month performance (Burnside, Eichenbaum, and Rebelo (2011)) and the TS-mom 12 strategy uses the currency's performance over the prior 12 months as a signal (Moskowitz, Ooi, and Pedersen (2012)). They both go long (short) currencies with positive (negative) performance. Similar to Moskowitz, Ooi, and Pedersen (2012), the latter TS strategy also scales the weights inversely with the conditional return volatilities (same estimates as those used in the construction of the UMVE portfolio).

The CS-value strategy uses the RER signal in equation (8), such that a relatively low (high) RER today indicates that the foreign currency is cheap (expensive) (Asness, Moskowitz, and Pedersen (2013)). Specifically, weights are again based on the rank weights as described above.

This set of trading strategies that are based on the USD perspective comprises our main set of test results. However, this set may be incomplete. The first concern that arises relates to the choice of measurement currency, or numeraire. For example, an investor funding herself in JPY, and receiving a payoff in JPY, faces a different excess return than a USD-funded investor in the same currency i. The second concern relates to the choice of base currency. All of the CS strategies that we discuss are not exposed to the USD by construction. With respect to the remaining strategies, one could contemplate a different base, for example, exposure to the euro (EUR), even if the measurement currency continues to be USD. Lastly, the Hansen and Richard (1987) result requires unconditional testing of a model using *all* admissible strategies, and we do not have a formal metric to assess how close our choice is to the full set. We address these concerns in the Internet Appendix.

II. Evidence

A. Data

We construct a data set of daily spot and one-month forward exchange rates expressed as USD per unit of the foreign currency for the G10 currencies (AUD, CAD, EUR spliced with DEM, JPY, NZD, NOK, SEK, CHF, GBP, USD) from January 1, 1976 to May 31, 2020. Daniel, Hodrick, and Lu (2017) offer compelling arguments for exclusion of emerging currencies and the European currencies other than the EUR. In particular, emerging currencies reflect credit risk of the respective sovereigns, and thus, the economic composition of the associated currency risk premiums is different (see, e.g., Na et al. (2018), Chernov, Creal, and Hördahl (2020)).

We use data from several providers through Datastream. Our monthly data set includes the last day of every month from the daily data set. Forward exchange rates for the Australian dollar (AUD) and the New Zealand dollar

(NZD) are available from December 1984, and thus January 1985 is the common starting month for currency excess returns.

We also collect monthly CPIs from the Organisation for Economic Co-operation and Development (OECD) for the period January 1976 to May 2020. Only quarterly CPIs are available for the AUD and NZD, so we forward-fill the quarterly values to get monthly observations. Given that the CPI is published with a lag, and we want to ensure that all variables are observable at time t, we construct the RER in equation (7) using CPIs lagged by three months.

The returns on the trading strategies are constructed using data going as far back as possible. All strategies use all nine currencies versus the USD from January 1985. Note that since both the value and the momentum signals rely on spot exchange rates, for which we have data going back to 1976, we have nine years of data to estimate the first conditional means and covariance matrix for January 1985, when the currency excess return sample starts. We next expand the sample each month by one month to update the estimates in an OOS fashion. This strategy gets us to the target equation (9) in two steps. First, we forecast percentage changes in spot rates via

$$S^i_{t+1}/S^i_t - 1 = \bar{\delta}_t \cdot z^i_{Qt} + \bar{\phi}_t \cdot \left(S^i_t/S^i_{t-12} - 1\right) + \varepsilon^i_{t+1}.$$

The coefficients $\bar{\delta}_t$ and $\bar{\phi}_t$ are reestimated via a panel regression using historical data up to month t. Then, using the definition of the return on a forward position given in equation (6), the time t expected excess currency return is obtained via

$$\begin{aligned}\mu_{it} &= (S^i_t/F^i_t) \cdot E_t(S^i_{t+1}/S^i_t) - 1 \\ &= (S^i_t/F^i_t - 1) + (S^i_t/F^i_t)\bar{\delta}_t \cdot z^i_{Qt} + (S^i_t/F^i_t)\bar{\phi}_t \cdot \left(S^i_t/S^i_{t-12} - 1\right),\end{aligned}$$

with $(S^i_t/F^i_t)\bar{\delta}_t$ and $(S^i_t/F^i_t)\bar{\phi}_t$ corresponding to δ^i_t and ϕ^i_t in equation (9), respectively.

In the case of the alternative conditional forecasts of excess returns, we run a panel regression of next month's realized currency returns on the conditional betas as implied by the expected return specification in equation (10). We use data only up to time t to estimate λ^{carry}_t and λ^{dollar}_t, with the caveat that since equation (10) calls for forecasting excess returns directly, we cannot use the depreciation rates from 1976 to 1984 as a "burn-in" sample. Instead, we use the sample from 1985 to 1994 to estimate constant loadings on the betas for this period. We then update the loadings in real time, in exactly the same fashion as in our main model. The factor risk premiums are therefore allowed to change over time as more data become available.

B. Preliminary Evidence

Before we proceed with testing the model, we check whether the main objects that we use for constructing the UMVE, $E_t(R^{ei}_{t+1})$ and $V_t(R^{ei}_{t+1})$, are plausible. Specifically, we check whether they predict R^{ei}_{t+1} and $(R^{ei}_{t+1} - E_t(R^{ei}_{t+1}))^2$, respectively.

Panel A of Table I gives the results from regressing R^{ei}_{t+1} on $E_t(R^{ei}_{t+1})$ and $(R^{ei}_{t+1} - E_t(R^{ei}_{t+1}))^2$ on $V_t(R^{ei}_{t+1})$ for individual currencies in a panel setting. Under the null hypothesis that we have identified the correct conditional means and variances, the expected slope coefficients equal one. Indeed, slopes in both regressions are found to be significantly different from zero and insignificantly different from one. Even pointwise these coefficients are pretty close to one, at 0.84 and 0.93, respectively.

In contrast, when we use the variance-based estimate of the conditional expectation, labeled $E_t(R^{ei}_{t+1})$ (var), the point estimate is much lower at 0.573, and insignificantly different from both zero and one. We also regress $E_t(R^{ei}_{t+1})$ from our advocated model on the alternative specification in equation (10). The slope is 0.425 with an adjusted R^2 of 0.066. Thus, the two approaches produce substantively different versions of conditional means.

Panel B of Table I performs the same analysis for strategies. The conditional expected return and variance of each strategy are computed using the expected return vector and covariance matrix for the underlying individual currencies, along with the conditional portfolio weights of each strategy. The conclusion is the same as for individual returns. Because the strategies involve multiple returns, the results suggest that the conditional covariances of returns are estimated well.

To better understand these predictability results, we display the coefficients δ^i_t and ϕ^i_t from our model of expected excess returns in equation (9). The currency-specific values of these two coefficients are so similar that it is hard to distinguish them visually. Thus, Figure 1 plots their cross-sectional averages. For ease of interpretation, we standardize the coefficients in the figure, so they correspond to the annualized expected return response to a one-standard-deviation increase in the RER or trend signal. As we use an expanding window to estimate them, it is expected that they stabilize toward the end of the sample, which they do around −2.1% and 1.5%, respectively. The signs are stable and as expected over the sample—a high relative RER implies low future returns, consistent with mean reversion, while a high trend signal implies high future returns, consistent with momentum. The figure also gives the implied coefficient on the forward discount, 3.4%, standardized in the same manner as the other coefficients. The magnitudes of the coefficients imply economically large variation in conditional expected returns arising from all three signals, with the forward discount (carry) having the largest impact followed by the RER and trend signals.

Panel C of Table I documents the magnitude of time variation in $E_t(R^{ei}_{t+1})$ and $V^{1/2}_t(R^{ei}_{t+1})$ for both individual currencies and strategies. In both cases, the conditional expectations are as variable as the conditional standard deviation, indicating substantial time variation in the UMVE portfolio weights and value from timing currency strategy investments.

Table I

Predictive Ability of Conditional Expectations and Variance

Panel A reports panel regressions of currency excess returns R^{ei}_{t+1} on the conditional expected excess returns as estimated from our advocated model, $E_t(R^{ei}_{t+1})$, and from a variance-based alternative estimate of the mean, $E_t(R^{ei}_{t+1})$ (var). We also regress $(R^{ei}_{t+1} - E_t(R^{ei}_{t+1}))^2$ (using the advocated estimate of the conditional expectation) on the conditional variance as estimated by our model, $V_t(R^{ei}_{t+1})$. Panel B presents the same for the strategy returns. Standard errors are clustered by month. Panel C reports summary statistics for estimated conditional means and standard deviations. $E[E_t(R^{ei}_{t+1})]$ denotes the grand (across time and currencies) average risk premium, $E[V_t^{1/2}(R^{ei}_{t+1})]$ is the grand average conditional standard deviation, $V^{1/2}[E_t(R^{ei}_{t+1})]$ is the standard deviation of conditional expected returns, and $V^{1/2}[V_t^{1/2}(R^{ei}_{t+1})]$ is the standard deviation of conditional standard deviations. As in Panels A and B, "var" refers to the variance-based estimate of the conditional mean. Panel D reports the SR of the UMVE portfolio as a function of the signals used in its construction, along with the "alpha" of the full-model UMVE portfolio regressed on alternative UMVE portfolios. "Fwd disc" refers to forward discount. The numbers in Panels C and D are annualized (except for t-statistics and R^2_{adj}s), and all UMVE portfolios are normalized to have the same volatility as that of the dollar strategy.

Panel A.		Currency Returns			
R^{ei}_{t+1}		R^{ei}_{t+1}		$(R^{ei}_{t+1} - E_t(R^{ei}_{t+1}))^2$	
$E_t(R^{ei}_{t+1})$	0.838	$E_t(R^{ei}_{t+1})$ (var)	0.573	$V_t(R^{ei}_{t+1})$	0.931
s.e.	0.253	s.e.	0.256	s.e.	0.068
R^2_{adj}	0.009	R^2_{adj}	0.004	R^2_{adj}	0.053
Panel B.		**Strategy Returns**			
R^{ei}_{t+1}		R^{ei}_{t+1}		$(R^{ei}_{t+1} - E_t(R^{ei}_{t+1}))^2$	
$E_t(R^{ei}_{t+1})$	0.791	$E_t(R^{ei}_{t+1})$ (var)	0.748	$V_t(R^{ei}_{t+1})$	1.055
s.e.	0.237	s.e.	0.279	s.e.	0.087
R^2_{adj}	0.003	R^2_{adj}	−0.003	R^2_{adj}	0.072

(*Continued*)

Table 1—*Continued*

Panel C.	Summary Statistics of Conditional Means and Standard Deviations					
	$E[E_t(R^{ei}_{t+1})]$	$E[E_t(R^{ei}_{t+1})]$ (var)	$E[V_t^{1/2}(R^{ei}_{t+1})]$	$V^{1/2}[E_t(R^{ei}_{t+1})]$	$V^{1/2}[E_t(R^{ei}_{t+1})]$ (var)	$V^{1/2}[V_t^{1/2}(R^{ei}_{t+1})]$
Currencies	1.09	4.51	10.71	4.36	2.37	4.77
Strategies	2.55	1.57	7.90	3.39	2.81	3.62

Panel D.	UMVE Portfolios			
Signals used	SR	alpha	t-Stat	R^2_{adj}
All signals	1.075	n/a	n/a	n/a
Fwd disc	0.871	3.59	2.54	0.515
Fwd disc/Trend	0.871	3.26	2.63	0.586
Fwd disc/RER	0.998	1.47	2.13	0.796
Var-based	0.441	7.51	5.53	0.094

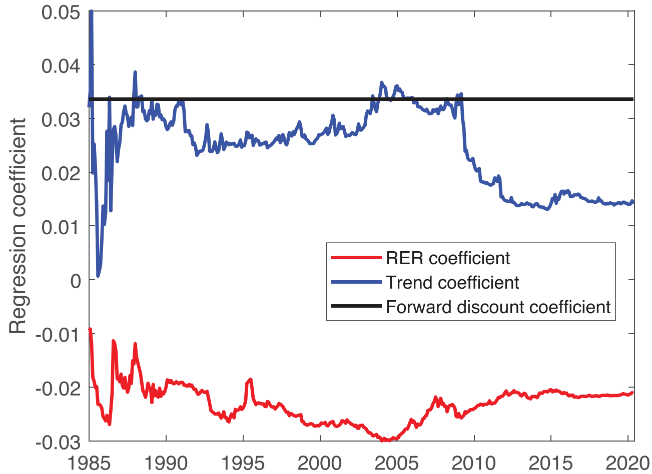

Figure 1. Loadings on signals in expected excess currency returns. The loadings on the real exchange rate (RER), δ_t^i, and on the trend, ϕ_t^i, are estimated in real time using an expanding window. The figure plots standardized values of cross-sectional averages of δ_t^i and ϕ_t^i, as well as the standardized (unit) coefficient on the forward discount, $S_t^i/F_t^i - 1$. The standardization makes the coefficient values correspond to the annualized expected return response to a one-standard-deviation increase in the respective signals. (Color figure can be viewed at wileyonlinelibrary.com)

Panel D explores the role of the three signals that we use to construct conditional expectations in terms of their effect on the UMVE portfolio. In the first row, we report the unconditional SR for the full model, which equals 1.075. We emphasize that UMVE portfolio returns correspond to a real-time implementable trading strategy. In the subsequent rows, we consider UMVEs formed using the forward discount signal alone, the forward discount and trend, the forward discount and RER, and the variance-based estimate of the conditional expectation. All of these combinations result in lower SRs. The variance-based SR is particularly low at 0.4 and lower than the carry SRs reported in the literature. This evidence suggests that conditional expected returns do not line up with conditional exposures to the PCs of currency returns. We expand on this finding in the subsequent analysis.

To assess the economic significance of using all of the signals, we implement the Barillas and Shanken (2017) test and check whether the UMVE formed using all three signals has an alpha with respect to any of these four alternative UMVEs. All of the alphas are significant. The corresponding adjusted R^2s vary between 50% and 80% for signal-based strategies and the R^2 is only 9.4% for the variance-based strategy. We find no significant alphas when we reverse the direction of the regressions. Thus, the UMVE portfolio from the first row explains UMVE returns from other versions of our model, but not the other way around.

Table II

Testing the UMVE

Panel A shows the annualized Sharpe ratio (SR), average excess return, and t-statistic of the average excess return to each currency, along with its "alpha," "beta," and R^2 with respect to the UMVE portfolio. The t-statistics are heteroskedasticity-adjusted. Panel B reports the same for the strategy returns. The sample is monthly from January 1985 to May 2020.

Panel A. Currency Returns

Currency	SR	$E[R^e]$	t-Stat	α	t-Stat	β	t-Stat	R^2_{adj}
AUD	0.241	2.83	1.44	0.73	0.34	0.24	2.46	0.025
CAD	0.099	0.74	0.59	−0.03	−0.02	0.09	1.47	0.007
CHF	0.149	1.68	0.89	3.22	1.56	−0.04	−0.43	0.014
EUR	0.145	1.52	0.86	1.83	0.95	−0.21	−2.53	−0.002
GBP	0.217	2.17	1.29	1.08	0.65	0.18	1.95	0.008
JPY	0.054	0.60	0.32	2.45	1.20	0.12	1.38	0.022
NOK	0.205	2.26	1.22	1.27	0.63	0.14	1.68	0.005
NZD	0.427	5.29	2.54	3.72	1.67	−0.18	−2.06	0.012
SEK	0.134	1.48	0.80	0.28	0.14	0.13	1.57	0.008

Panel B. Strategy Returns

Strategy	SR	$E[R^e]$	t-Stat	α	t-Stat	β	t-Stat	R^2_{adj}
Dollar	0.258	2.06	1.54	1.61	1.14	0.05	0.84	0.000
Dollar Carry	0.576	4.56	3.43	2.40	1.75	0.25	4.28	0.062
CS-Carry	0.469	4.14	2.79	0.43	0.28	0.43	5.85	0.151
TS-Carry	0.685	3.44	4.08	1.25	1.47	0.26	8.47	0.163
CS-Mom 1	0.179	1.46	1.06	0.61	0.36	0.10	1.19	0.007
CS-Mom 12	0.175	1.47	1.04	−1.22	−0.82	0.31	4.88	0.087
TS-Mom 1	0.440	2.73	2.62	1.45	1.30	0.41	4.79	0.034
TS-Mom 12	0.473	5.19	2.82	1.65	0.86	0.15	2.65	0.088
CS-Value	0.529	4.02	3.15	0.98	0.69	0.35	5.71	0.136

C. Testing the Model's Implications

Table II presents our initial testing results. The Internet Appendix summarizes the split-sample analysis. Panel A reports summary statistics for individual currencies (average excess returns and sample SRs). The SR for the UMVE reported in Table I, Panel D, exceeds the largest currency SR, NZD, by 2.5 times. Panel A also shows alphas and betas from individual regressions (11). The largest t-statistic for an alpha is 1.67 for NZD. The largest adjusted R^2 from regressing a return on the UMVE is 2.5% for AUD.

Panel B shows the same information for strategies. The UMVE SR exceeds the largest strategy SR, TS-carry, by a factor of 1.6. None of the strategies has a significant alpha relative to the UMVE, with the highest individual t-statistic of 1.75 for the dollar carry strategy. On average, the alphas are about 70% smaller than the mean excess return on the strategies. The adjusted R^2s in these regressions are low, which implies that only a small component of the variation in these portfolios' returns is being priced. The carry strategies and

Table III
Asset Pricing Tests

We contrast our main model of the UMVE, labeled "optimal," with alternative methods of UMVE construction. The row labeled "fwd disc" refers to the case in which the conditional expectations of excess returns are constructed using the forward discount only. The row labeled "var-based" corresponds to case in which the conditional expectation is estimated using the variance-based approach. The GRS p-value is for the standard joint test of all "alpha" equal to zero. The MHR p-value is for the conditional test of the model as implied by MHRs. The horizons are 1, 3, 6, 12, 24, and 48 months. See the text for details. The "OOS" p-value is for the joint out-of-sample test, which evaluates whether all average dynamically hedged strategy returns are equal to zero. To implement the OOS test, we compute the conditional beta on the UMVE portfolio OOS and thus hedge out the model-implied priced component in real time. This test is also an unconditional test of each model's conditional implications. The columns under "All strategies" correspond to tests using all nine currency trading strategies as tests assets, whereas the columns under "Dollar + all carry" only uses the dollar, dollar carry, CS-carry, and TS-carry. The sample is monthly from January 1985 to May 2020.

	All Strategies			Dollar + All Carry		
Model	GRS	MHR	OOS	GRS	MHR	OOS
Optimal	0.470	0.277	0.399	0.467	0.568	0.384
Fwd disc	0.010	0.001	0.006	0.309	0.030	0.133
Var-based	0.000	0.040	0.025	0.010	0.287	0.199

CS-value exhibit the largest exposures to the UMVE with an R^2 around 15%, followed by momentum around 9%, and the rest with an R^2 ranging between 0% and 6%. Notably, the dollar strategy returns do not reflect any priced risk, consistent with the argument and evidence in Boudoukh et al. (2018).

Table III presents asset pricing tests of the alternative UMVE portfolios. The first row displays the GRS and MHR tests applied to the nine strategies for our advocated UMVE (labeled "optimal") under the columns "all strategies." The MHR test uses strategy returns at the 1-, 3-, 6-, 12-, 24-, and 48-month horizons, and also adds the UMVE portfolio itself as one of the test assets. The p-values are 0.47 and 0.28, respectively. These results imply that the candidate UMVE prices single-horizon returns to the strategies both unconditionally and conditionally.

This failure to reject is meaningful. Consider a simpler model in which conditional expectations are computed on the basis of the forward discount alone and test it using the first four strategy portfolios, which do not rely on momentum or value signals (see the second row of Table III, under the columns "dollar + all carry"). GRS fails to reject with a p-value of 0.31, but MHR does reject with a p-value of 0.03. In other words, the UMVE based only on the conditional mean returns implied by the forward discount does not price long-run returns to the dollar and carry-based strategies. We further reject the version of the UMVE based on the variance-based specification of the conditional mean with both tests when the full set of test assets is used. When we consider the limited "dollar + all carry" test set, GRS rejects also, while MHR fails to reject.

Since our advocated UMVE portfolio construction is the only method that is not rejected in any of the tests, we conclude that it yields a good proxy for the true UMVE portfolio. One might worry that the failure to reject is because of low power. However, we reject a number of alternative models, which suggests that this is not an issue. Moreover, in the Internet Appendix, we provide a formal evaluation of the power of the GRS test in our specific case and conclude that the test has good power properties.

D. Time-Series Implications

We analyze the factor structure implied by our model by performing PC analysis on the covariance matrix of currency returns. In this subsection, we consider the unconditional covariance matrix of individual currencies. In the next subsection, we consider conditional strategies.

Panel A of Figure 2 displays the contributions of each of the nine PCs to the overall variation. The first PC explains 58%, the second 18%, and the third 9% of the variation.

Panel B shows the loadings of the first three PCs on the individual currency returns. We see that the first PC loads similarly on each currency, suggesting that it is related to the dollar strategy. Indeed, the correlation between this first PC and the dollar factor is 0.998. The second PC goes long the high-interest currencies and short the low-interest currency and is thus related to carry—their correlation is 0.718. The third PC is harder to associate with an extant strategy, though a possible interpretation is geographical with largely positive loadings for European countries and largely negative loadings for the other countries. These results are consistent with Verdelhan (2018), who argues that dollar and carry are the main factors driving currency returns.

Panel C examines how much of the UMVE portfolio's variation can be explained by the PCs. The Internet Appendix presents split-sample analysis. Rather than reporting the individual contribution of each PC as in Panel A, we now report their cumulative contribution. Starting with the first PC and gradually adding one after another, we also compute how much of the variance of UMVE returns can be explained by a given number of PCs. The answer is not much. It is close to zero if we use the first PC alone. Recall that this PC explains almost 60% of the variation in individual currency returns. If we use all nine PCs, we progress to about 10% of the variation. This is consistent with low R^2 reported in Panel B of Table II. The flipside of this result is that UMVE has a significant alpha with respect to the PCs as can be seen in Panel D of Figure 2.

We conclude that time-series variation in currency returns can be summarized by three factors, two of which are close to the dollar and CS-carry strategies. However, these factors are weakly related to the UMVE, which prices the cross-section of both currencies and strategies. This suggests that timing based on the conditional dynamics of currency returns is an important ingredient of the pricing success.

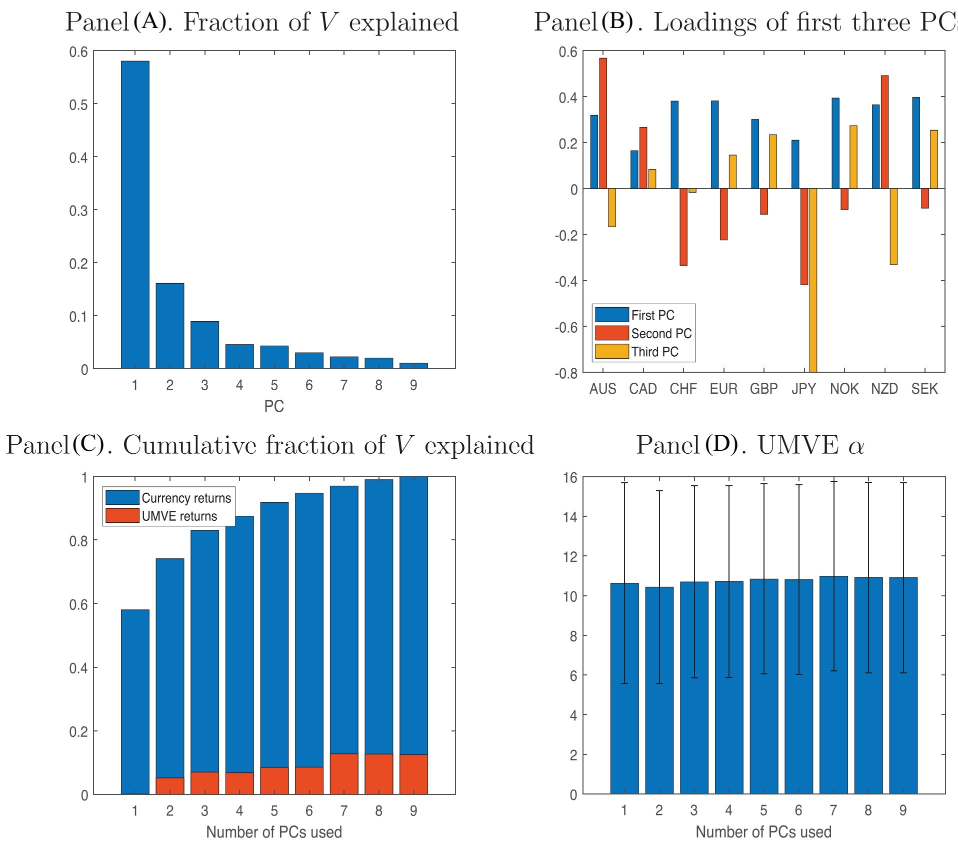

Figure 2. Principal component analysis of currencies. Panel A shows the fraction of variance across the nine individual currencies that each principal component (PC) explains. The PCs are obtained from the unconditional covariance matrix of individual currency returns. Panel B shows the loadings of the three first PCs on each currency. Panel C shows in blue the cumulative amount of currency variance explained as one goes from using one to all nine PCs. In red is the R^2 of a regression of the UMVE returns on an increasing number of PCs. Panel D shows the annualized "alpha" of a regression of the UMVE returns on an increasing number of PCs. The error bars correspond to the 95% confidence interval computed using heteroskedasticity-adjusted standard errors. The sample is monthly from January 1985 to May 2020. (Color figure can be viewed at wileyonlinelibrary.com)

E. Factor Timing

The currency strategies involve a substantive amount of currency timing. Thus, a natural starting point is to investigate how well these trading strategies can explain UMVE portfolio returns. Figure 3 addresses this issue via a PC analysis of strategies, which parallels that of individual currencies in Figure 2.

Panel A shows individual contributions of PCs to the overall variation in strategy returns. Here, the first three PCs explain 65% of the variation. Panel

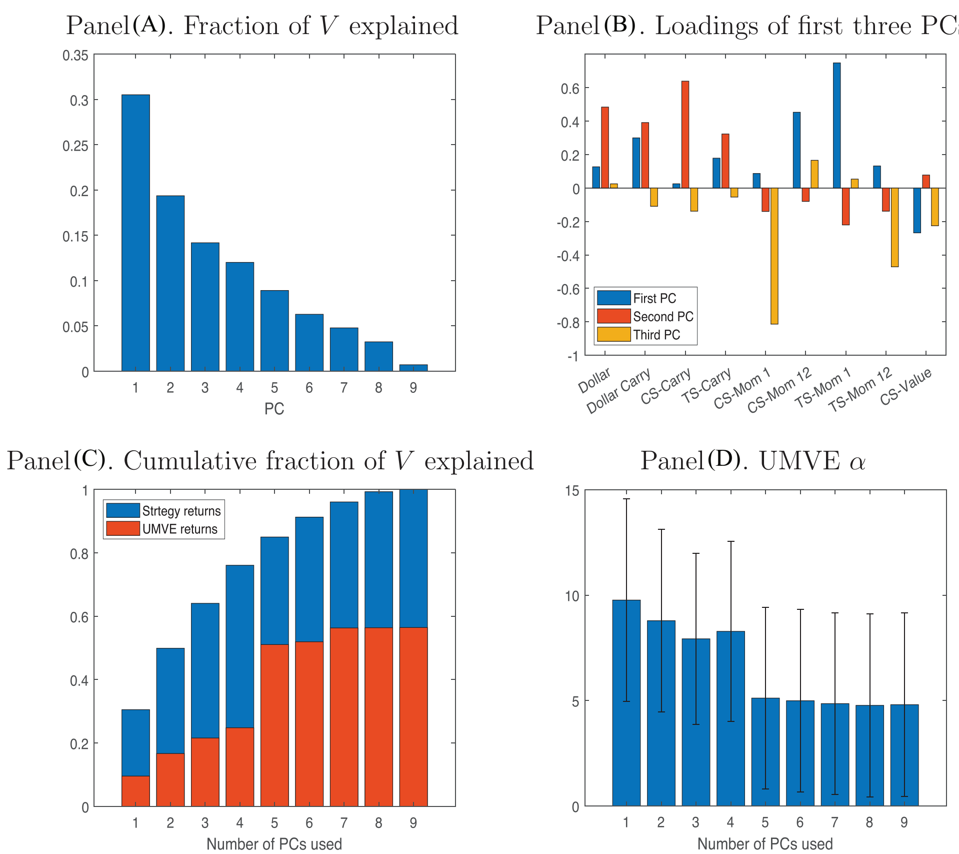

Figure 3. Principal component analysis of strategies. Panel A shows the fraction of variance across the nine currency trading strategies that each principal component (PC) explains. The PCs are obtained from the unconditional covariance matrix of these currency strategies. Panel B shows the loadings of the three first PCs on each strategy. Panel C shows in blue the cumulative amount of strategy variance explained as one goes from using one to all nine PCs. In red is the R^2 of a regression of the UMVE returns on an increasing number of these PCs. Panel D shows the annualized "alpha" of a regression of the UMVE returns on an increasing number of PCs. The error bars correspond to the 95% confidence interval computed using heteroskedasticity-adjusted standard errors. The sample is monthly from January 1985 to May 2020. (Color figure can be viewed at wileyonlinelibrary.com)

B displays the loadings of these three PCs on each strategy, but no clear interpretation emerges regarding what these "factors of strategies" represent. Panel C shows how much of the variance of UMVE returns can be explained by strategy PCs. The first three PCs explain only about 20% of the variation in UMVE returns. Thus, again, the main drivers of the time-series variation in asset returns are not the main drivers of priced risk, as summarized by the UMVE. The first five PCs can explain about 50% of the variation, and this number stays the same after adding the remaining PCs.

The improvement from 10% to 50% is a testament to the importance of currency timing implicit in the strategies. Yet, a lot left is on the table. UMVE alphas with respect to these PCs continue to be significant as documented in Panel D.

We next consider the timing of the nine currency strategies that is implicit in the UMVE portfolio. If the nine strategies considered in this paper were conditionally noncollinear, then, because strategy weights are known, one could reexpress the UMVE as a portfolio of strategies. The weights in such a portfolio would change every period, so one would still have to establish optimal portfolio weights in terms of currency returns, or alternatively, directly model conditional expected returns and the covariance matrix of the strategies themselves.

In practice, some of the strategy returns are closely related to each other. For instance, the dollar carry is conditionally perfectly correlated with the dollar. The different carry and momentum strategies are likewise related to each other. Thus, we cannot construct a portfolio of strategies that tracks the UMVE perfectly.

Instead, to obtain further intuition about the nature of the UMVE portfolio, we use four strategies and strive to get as close to the UMVE as possible. The four strategies are: the dollar, the sum of the CS- and TS-carry, the sum of the four momentum strategies, and CS-value. We then apply equation (1) to strategy returns. We can implement this equation because our model produces real-time conditional expectations and variance for each currency and the portfolio weights for each strategy are known.

Regressing the UMVE-tracking portfolio on our UMVE portfolio gives an R^2 of 84%. Its SR is 0.93 versus 1.07 for the UMVE portfolio. The Barillas and Shanken (2017) test rejects the null that the UMVE-tracking portfolio can explain the UMVE with a *p*-value of 0.2%. Thus, we cannot replicate the optimal portfolio with extant trading strategies. We report deviations of these portfolio weights from those of the UMVE in the Internet Appendix.

Despite its failure to fully explain the UMVE, it is still informative to study the properties of the resulting strategies-based proxy. Figure 4 displays the 12-month moving averages of the weights for the four strategies in this tracking portfolio. As a reference point, Asness, Moskowitz, and Pedersen (2013) famously suggest to consider 50% in momentum and 50% in value. Consistent with the spirit of that suggestion, the sample averages of the strategy weights are positive. In contrast to the Asness, Moskowitz, and Pedersen (2013) recommendation, however, the optimal strategy weights have strong time variation. Indeed, the ratio of the sample standard deviation of the weights divided by their sample mean is 1.21. We note that time variation in portfolio weights does not imply higher transaction costs than a constant-weight strategy as all of the trading strategies are based on one-month forward contracts. Thus, all strategies, simple and complicated, require reestablishment of a new portfolio every period.

Panel A shows that the portfolio weight on the Dollar strategy is not related to the business cycle and in general decreasing over the sample. The portfolio

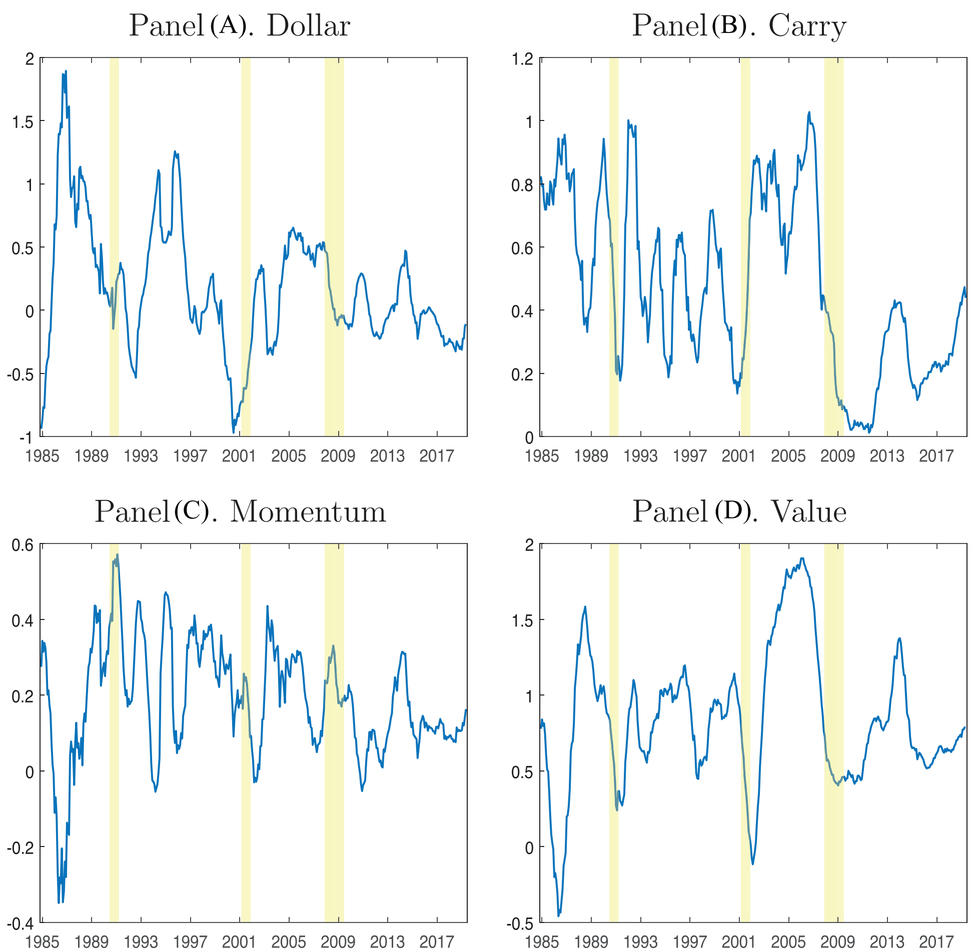

Figure 4. The UMVE tracking with strategies. We deduce portfolio positions in the strategies that allow us to track the UMVE portfolio. We consider the dollar strategy (we discard the dollar carry because it is conditionally perfectly correlated with the dollar), the sum of CS- and TS-carry (labeled as carry), the sum of CS- and TS-momentum (both 1 and 12 months; labeled as momentum), and CS-value. We report the 12-month moving average of the UMVE tracking portfolio weights, which are expressed in decimal form. They have been scaled so that the unconditional standard deviation of UMVE returns matches the unconditional standard deviation of the dollar factor. The sample is monthly from January 1985 to May 2020. (Color figure can be viewed at wileyonlinelibrary.com)

weight on the Carry strategies (Panel B) tends to be high in the period prior to the financial crisis and low thereafter. This is due to a strong decrease in the cross-sectional standard deviation of forward discounts (interest differentials) over the sample. The conditional variance of Carry strongly increases during the financial crisis and then drops throughout the post-2009 period, which is why the Carry portfolio weight increases after being close to zero at the end of the financial crisis. The portfolio weight on momentum (Panel C) is overall decreasing after its peak in the early 1990s, with increasing weight in recessions. The latter is due to the cross-sectional standard deviation of the momentum signal increasing more than the conditional variance of returns in such periods. The value strategy (Panel D) tends to have low weight in recessions and high weight in expansions, though there is substantial variation also not associated with the business cycle. There is no trend in the weight on value over the sample.

Figure 5 evaluates the economic significance of the documented time-varying weights by comparing the UMVE-tracking portfolio to alternative portfolios of the same four strategies. In addition to the 50% in momentum and 50% in value portfolio, we consider equal weighting (25% in each of the four strategies) and a volatility timing strategy that uses the inverse of the conditional variance of each strategy (obtained from our conditional covariance matrix estimate) as the portfolio weights. We display TS of the difference between the conditional SR of the UMVE-tracking portfolio and one of these three alternatives. The differences in annualized SR are large, ranging between 0.32 for equal-weighted and 0.42 for momentum-value strategies, on average.

According to the Barillas and Shanken (2017) test reported in Panel D, the fixed-weight strategies cannot explain the UMVE-tracking portfolio (i.e., the latter has significant "alpha" with respect to the former). The latter can explain the former despite being a suboptimal proxy for the UMVE. In sum, there is substantial time variation in the conditional mean and variance to the strategy returns that leads to significant gains for a mean-variance investor from timing these portfolios.

F. Unpriced Components of Returns

The low R^2s of strategy returns in Table II suggest large unpriced components in these returns. Daniel et al. (2020), in the context of equity strategies, make a strong case for hedging out unpriced components to enhance strategy performance. It is easy to construct a conditional hedge in our setting as we have an explicit UMVE portfolio.

In equation (2), we use the conditional covariance matrix of the underlying currencies to construct, in real time, the conditional beta of each strategy on the UMVE,

$$\beta_{pt} = \frac{w_{pt}^{\top}\Omega_t w_t^*}{w_t^{*\top}\Omega_t w_t^*},$$

where w_{pt} is a vector of strategy weights as in equation (12). A portfolio with systematic exposure, that is, the hedged portfolio, is simply $\beta_{pt}R_{t+1}^*$. We refer to the residual as the hedging portfolio return,

$$R_{h,t+1} = R_{p,t+1} - \beta_{pt}R_{t+1}^*,$$

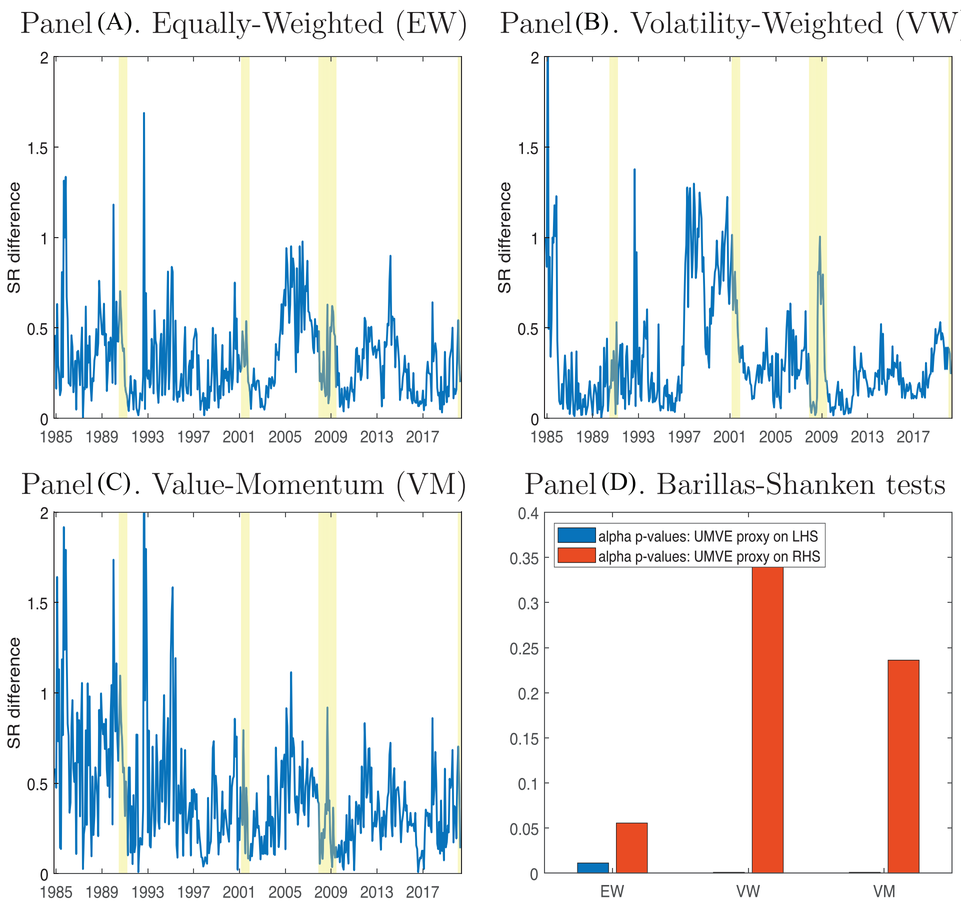

Figure 5. Portfolios of strategies versus the UMVE-tracking portfolio. Panels A, B, and C display the difference between the conditional Sharpe ratios (SRs) of the UMVE tracking portfolio from Figure 4 and those of the following alternatives constructed from the same four strategies: 25% in each; weights inversely related to the conditional variance of each strategy (obtained from our conditional covariance matrix for individual depreciation rates); and 50% in CS-mom-12 and 50% in CS-value. Panel D displays p-values for the Barillas-Shanken (2107) test of whether the UMVE tracking portfolio can be explained by an alternative portfolio (labeled UMVE proxy on LHS) or whether a fixed-weight portfolio can be explained by the UMVE tracking portfolio (labeled UMVE proxy on RHS). The sample is monthly from January 1985 to May 2020. (Color figure can be viewed at wileyonlinelibrary.com)

where the model implies that $E(R_{h,t+1}) = 0$. Since our time t estimates of both the UMVE portfolio weights and the conditional covariance matrix only use data available at time t, this hedging strategy is implementable in real time. Thus, testing whether the sample average of $R_{h,t+1}$ equals zero amounts to an OOS test of our model.

Figure 6 compares the SRs of the original strategies to those of the hedging and hedged components. The SRs for the hedging components are close to zero

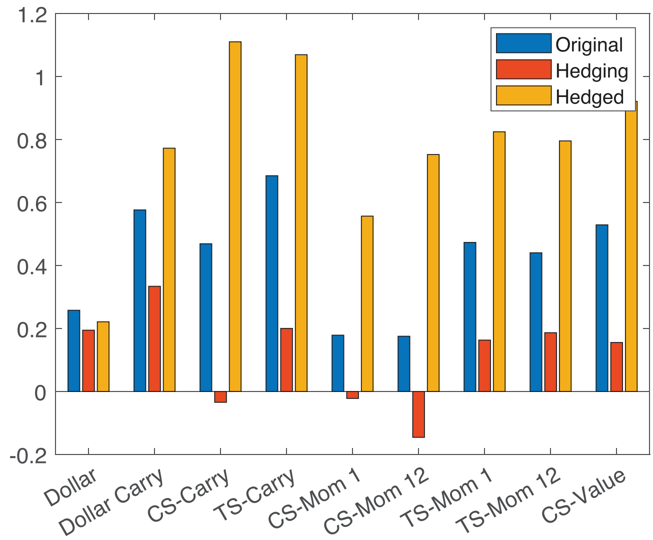

Figure 6. Sharpe ratios of original, hedging, and hedged strategies. The blue bars show the sample annualized Sharpe ratios (SRs) of each strategy. The red bars show the SR for each strategy's hedging portfolio, defined as the part of the strategy returns that is unpriced according to the model. The yellow bars show the SR of the factor returns when unpriced risks are hedged out. Hedging uses real-time conditional betas, so all portfolios are tradeable in real time. The sample is monthly from January 1985 to May 2020. (Color figure can be viewed at wileyonlinelibrary.com)

as should be the case for the unpriced component if our model is well specified. The SRs for the hedged components are much larger than those for the original strategies as anticipated. For instance, for CS-carry, the SR goes from about 0.5 to slightly higher than one. The one exception is the dollar strategy, where hedging does not seem to make much of a difference.

Table III reports GRS tests applied to the real-time hedging return of the strategies in columns labeled "OOS" for out-of-sample. Specifically, we jointly test whether the average hedged returns to the strategies are zero, as implied by the model. The use of the conditional betas means that this is an unconditional test of the models' conditional implications. The approach is similar in spirit to that undertaken by Lewellen and Nagel (2006). Overall, this OOS test yields results similar to the standard GRS test in the same table: only our full model is not rejected when tested on all strategy returns.

The Internet Appendix discusses an affine model that interprets the evidence in the tradition of Lustig, Roussanov, and Verdelhan (2011), among many others.

G. Properties of the Projected SDF

The estimated UMVE corresponds to the linear projection of the SDF on the set of currency excess returns. Figure 7 plots TS of both the implied condi-

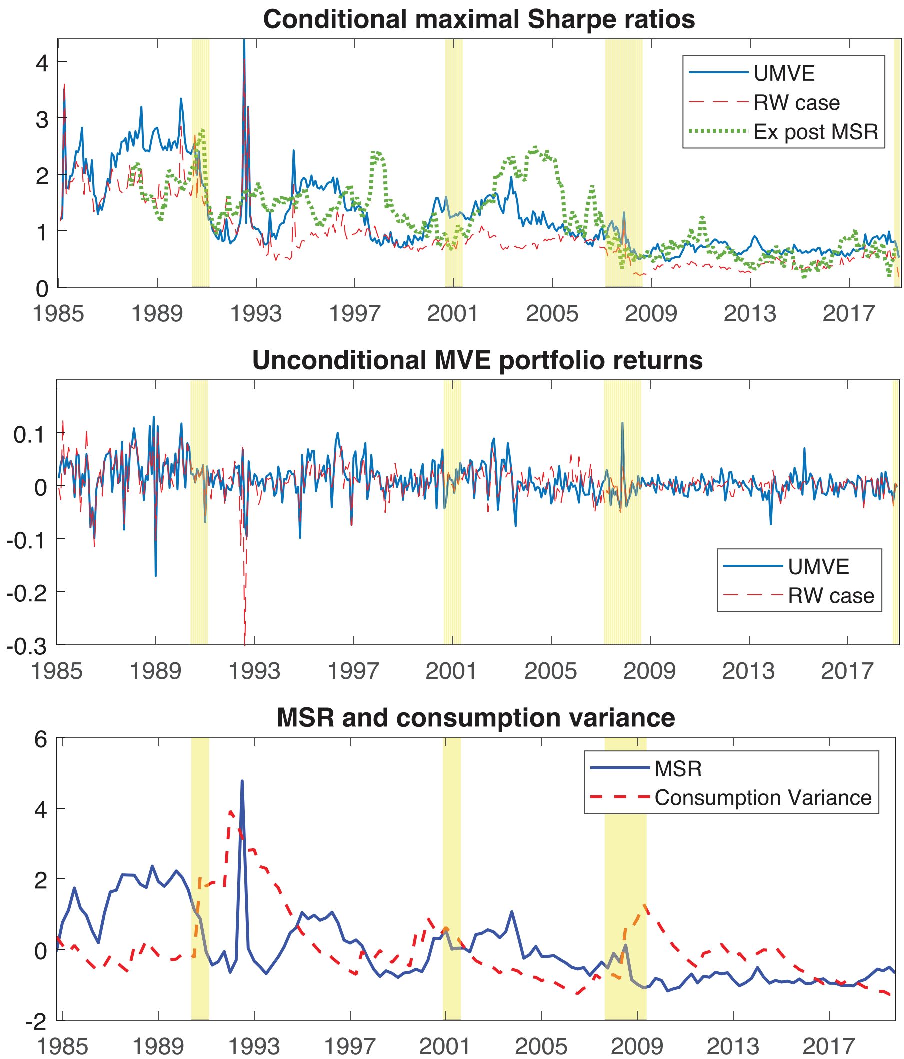

Figure 7. Properties of the UMVE. The top panel shows the annualized conditional maximum Sharpe ratio (MSR) as implied by the UMVE estimated in the paper. The blue solid line corresponds to the full model, whereas the dashed red line shows the case in which only the forward discount is used for estimating currency risk premiums. In addition, the green line depicts an ex post MSR, computed over the past 36 months, that serves as a quantitative benchmark. The middle panel shows the realized returns to the UMVE portfolio implied from this SDF. The color code is the same as for the top panel. Returns have been scaled to have the same standard deviation as that of the dollar strategy returns. The blue solid line in the bottom panel is the same conditional UMVE-based MSR as in the top panel. The dashed red line shows the conditional variance of quarterly real per capita nondurable+services log consumption growth, as estimated by a GARCH(1,1). Both series are standardized to mean zero and unit variance to facilitate comparison. (Color figure can be viewed at wileyonlinelibrary.com)

tional MSR and returns to the UMVE portfolio, which are perfectly negatively correlated with this SDF. The MSR varies strongly over the sample, from a high of about 4 to a low of about 0.5.

We see that the MSR experiences its largest value in a dramatic move during the 1992 Exchange Rate Mechanism crisis. Apart from this event, the main take away is a substantial decline over the sample, mainly caused by a trend decrease in interest rate differentials across countries. In particular, the same decline is shared by the MSR of the model that only uses the RWH when estimating currency expected returns (red dashed line). Notably, recessions, indicated by yellow bars in the figure, are associated with a *decline* in the MSR.

To get a sense of whether these magnitudes are reasonable, we also display an ex post realized MSR. Specifically, at each date t, we use the past 36 months of returns to compute the sample SR for each strategy. We then select and plot the largest SR. These SRs range between 0.15 and 2.8 and feature a similar declining pattern throughout the sample. These rough computations suggest plausibility of our estimated price of risk.

The middle panel of Figure 7 shows the time series of returns to the UMVE portfolio (the dashed red line displays the case of using the forward discount only). As can be seen, the returns are relatively smooth, with little evidence of a disaster event. For instance, the Global Financial Crisis is associated with small negative returns and one big *positive* spike. The sample skewness and excess kurtosis are −0.05 and 3.07, respectively, which suggest modest pricing of disaster risks in currency returns.[7] Moreover, we can see a slow decrease in the volatility of UMVE returns over the sample, consistent with the pattern in the MSR.

As a next step, we attempt to relate these two objects to observable variables that have the potential to capture currency risk premiums. Following the literature, we focus on macro and financial variables. Among macro variables, we have a particular interest in consumption, as it has a rich history and connection to equilibrium models. Among financial variables, we consider the Fama and French (2015) five-factor model as the de-facto SDF projection on the space of excess equity returns. Finally, we consider measures of intermediary leverage. The full list of candidate covariates and the motivations for considering them are offered in the Internet Appendix.

Table IV reports the results. Panel A displays univariate regressions for the significant covariates. We see that quarterly consumption growth and three-year consumption growth, a proxy for the long-run consumption component, are significant. This result is consistent with Lustig and Verdelhan (2007) and Zviadadze (2017), who argue that these variables are important for explaining the CS-carry premium. Our results generalize theirs because we document a

[7] We emphasize that this conclusion is not a mechanical implication of how we construct the UMVE, that is, portfolio weights are constructed using only the mean and variance of excess returns. Our portfolio is represented by a linear combination of excess returns. If these returns exhibit systematic exposure to crash risk, that will show up in the UMVE. Ex ante, or by construction, nothing prevents this from happening. If crash risk is important for risk pricing, assets that load on crash risk will have high SR and high weights in the UMVE.

Table IV

Covariates

Panel A reports results from regressing the UMVE portfolio returns on various factors in univariate regressions. Log real per-capita nondurable+services consumption growth is denoted by Δc. HML is the high-minus-low book-to-market factor from the Fama-French five-factor model, whereas CMA is the Conservative-minus-Aggressive investment factor from the same. Panel B shows regressions of the CS-carry strategy, the priced component of CS-carry and the unpriced component of CS-carry returns on the intermediary capital factor of He, Kelly, and Manela (2017), HKM, as well as shocks to realized volatility in the equity and FX markets. The priced and unpriced components are obtained in an OOS fashion, as described in the text. Panel C displays a regression of the three-year change in the MSR on the three-year change in conditional consumption variance (obtained from a GARCH(1,1)), equity variance, and FX variance. We use three-year changes instead of levels due to a downward drift in the price of risk over the sample period. The t-statistics are heteroskedasticity-adjusted. The sample is monthly from January 1985 to May 2020.

Panel A. UMVE Returns vs. Observable Factors

	Quarterly Δc	3-Year Δc	HML	CMA
Beta	32.452	6.740	0.011	0.021
t-stat	2.516	3.167	2.039	2.724
R^2_{adj}	0.031	0.073	0.006	0.013
N	141	131	425	425

Panel B. UMVE, CS-Carry, Its Unpriced Component and Covariates

	CS-Carry			CS-Carry Priced			CS-Carry Unpriced		
	HKM	Equity RV	FX RV	HKM	Equity RV	FX RV	HKM	Equity RV	FX RV
Beta	0.111	−2.066	−6.023	0.012	−0.252	−1.350	0.099	−1.814	−7.122
(t-stat)	3.482	−6.743	−6.628	1.236	−1.470	−1.924	3.572	−4.954	−4.980
R^2_{adj}	0.084	0.117	0.151	0.003	0.006	0.015	0.086	0.116	0.124
N	425	425	425	425	425	425	425	425	425

Panel C. MSR versus aggregate variance

	Δ Cons. Variance	Δ Equity RV	Δ FX RV
Beta	−22508.189	−1.478	−12.372
t-stat	−3.928	−0.717	−2.806
R^2_{adj}	0.140	0.000	0.032
N	129	399	399

significant relation to the UMVE, and not just carry returns. This generalization also explains why we find strong statistical significance. Focusing on the UMVE portfolio removes the unpriced components of returns and thus helps recover a more precise relationship with consumption growth as compared to a regression of strategy returns, like CS-carry, on consumption. The R^2s of the regressions are only 3% and 7%, indicating that factors other than consumption risk may be important for understanding the currency market conditional risk-return trade-off.

Panel A also reports the results for the only Fama-French factors that have a significant relation to the UMVE portfolio, namely, value (HML) and investment (CMA). Even though they are significant, the relation to the UMVE returns is weak as the betas are small. Further, the R^2s are low, at 0.6% and 1.3%, respectively.

Panel B focuses on the properties of the CS-carry, a strategy that has received a lot of attention both in the empirical and the equilibrium literatures. The left columns of the panel, labeled "CS-carry," relate returns to this strategy to intermediary leverage (He, Kelly, and Manela (2017)), realized variance of equity (Lustig, Roussanov, and Verdelhan (2011)), and exchange rates (Menkhoff et al. (2012a)). We construct the shocks to variance in the same manner as the authors of the two latter papers while we get the intermediary factor from the former authors' webpage. Consistent with the literature, the returns have a strongly statistically significant correlation with these three variables.

However, it is the *unpriced* component of CS-carry returns that is related to these variables, not the priced component. The middle columns of Panel B give the results from regressing the real-time hedged component of the CS-carry return on these variables. Recall from Section 3.6 that this is the OOS-priced return component. The slope coefficients are now insignificant. In the right columns, we show instead the corresponding results using the hedging component (the OOS-unpriced component) of CS-carry. In this case, the slope coefficients are strongly significant and similar to those using the raw CS-carry returns. Recall from Figure 6 that the SR of the unpriced component is effectively zero, while the SR of the priced component is greater than one. Thus, these variables cannot explain the average return to the CS-carry strategy. This is not entirely surprising given the results in Table III, which shows that only about 15% of the variation in CS-carry returns is priced. The different conclusion relative to prior literature can also be understood as a case of omitted factors (Giglio and Xiu (2021)), or more generally, misspecified factors (Jagannathan and Wang (1998)).

Panel C relates the MSR to various measures of aggregate uncertainty: conditional consumption variance, realized equity variance, and realized foreign exchange (FX) variance. The conditional variance of consumption growth is estimated using a GARCH(1,1) model. Due to the downward trend in the price of risk in the sample, we run these regressions in three-year differences. The conditional MSR, that is, the conditional volatility of the SDF, is *negatively* related to consumption variance and FX variance. Thus, periods of high uncertainty are associated with a lower SR in the currency market. The sign is also negative for equity variance, although it is insignificant in this case. The bottom panel of Figure 7 plots the MSR versus the conditional variance of consumption growth. The common trend and the negatively correlated cyclical relation are both clearly visible.

We investigate further the drivers behind the time variation in the component of the MSR that is correlated negatively with consumption variance. We go through a series of exercises where we relate the projection of the MSR onto consumption variance to one of the three variables: (i) cross-sectional dispersion in conditional expectations, (ii) the level (average) of conditional expectations, and (iii) the conditional currency return variance, which is calculated as the sum of the eigenvalues of the conditional currency return covariance matrix. Item (i) implies higher conditional expected returns to judiciously chosen long-short strategies if the spread between expected returns widens during times of low consumption variance. In its turn, that would translate into a higher MSR. Item (ii) leads to a higher MSR if the cross-sectional average level of returns matters. Lastly, if currency returns become less volatile when consumption variance is low as per item (iii), then, all else equal, the MSR increases. We find that only the first variable is significantly related to the consumption variance in three-year changes (we difference to avoid spurious regression bias due to highly persistent series), with a t-statistic of 3.05.

III. Conclusion

We construct an estimate of the UMVE portfolio of currencies. This is feasible as most of the literature focuses on the G10 currencies and the corresponding nine exchange rates versus the USD. A small cross-section makes estimation of the conditional covariance matrix relatively precise. We then use standard expanding panel regressions and a limited set of longstanding currency drivers to obtain conditional expected returns. These inputs are all that is needed to construct the real-time portfolio weights.

The benefit of this approach is that it is a theoretically motivated empirical characterization of the risk-return trade-off. The use of the conditional covariance matrix means we also characterize the factor structure in realized returns, which allow us to discuss priced versus unpriced sources of common variation in currency returns. Our analysis leads to a number of new insights.

First, the UMVE portfolio has a sample SR in excess of one and is not explained by any existing factors. At the same time, the UMVE portfolio accounts for known currency factors risk premiums and is not rejected as the pricing factor in standard model tests. Thus, our proposed estimation methodology is validated in the data and uncovers heretofore undiscovered sources of risk.

Second, we show that most of the common variation in currency returns is due to unpriced risks, that is, factors that do not command a risk premium. We show that existing currency strategies do indeed have large uncompensated components that can be hedged away to obtain much higher SRs.

Third, we document large gains to timing many of the existing factors, owing to the fact that the conditional dynamics of these factors are strongly time-varying. In fact, the standard deviation of conditional risk premiums is as large as the level of unconditional risk premiums.

Fourth, the priced currency risks are effectively unrelated to priced risks in the equity market, shocks to equity and FX variances, and intermediary capital. Consumption growth, however, is related to priced risks, though economically the relation is not very strong. That said, our results support a role for consumption risk in understanding currency risk and returns.

Finally, the conditional MSR of currency returns is strongly downward trending over the sample, with a cyclical component that is *negatively* related with measures of the conditional variance of macroeconomic aggregates and market returns.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.