Unit 3: Inference for Categorical and Numerical Data

2. Inference for the difference of two proportions (3.2)

3/10/2021

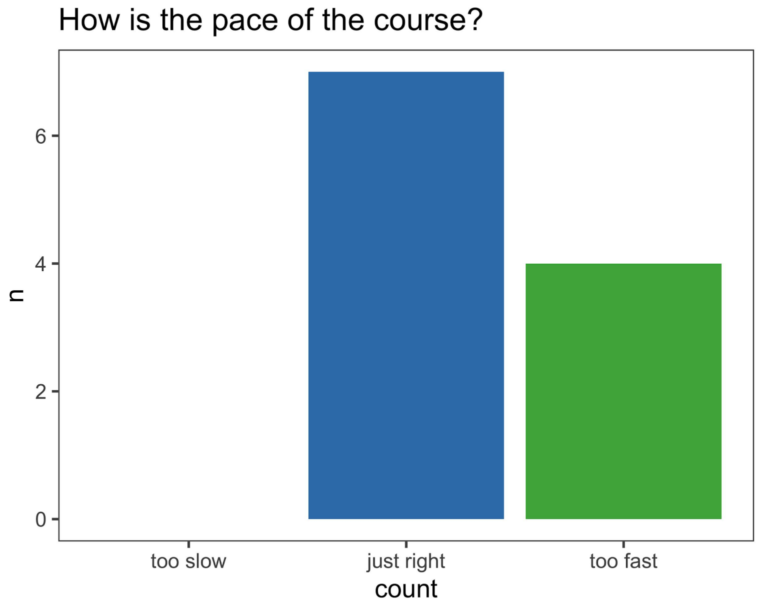
Quiz 5 - Confidence Intervals

Results of the Eberly Early Course Survey

11 total responses (26 total students)

My impression:

We’re clearly moving too fast
(this seems to come more from labs)



The Good

Lecture

- Organization, availability of slides and recordings
- ability to ask questions

Labs

- Interesting material
- Piazza is helpful for answering questions

The Bad

Quizzes are too hard relative to in-class material
(and maybe too long)

Labs (and R in general) moving too fast

- Some discomfort with R and RStudio
- Desire for a practice lab which helps nail down R concepts

Too much stuff in general

- Quizzes, labs, homeworks, etc etc

Recap from last time

1. We can use the CLT to make inferences about proportions

2. Confidence intervals can be used to make inferences about a population proportion

3. Confidence intervals can be used to do hypothesis tests

Key ideas

1. You can use the Normal approximation for the difference of two proportions

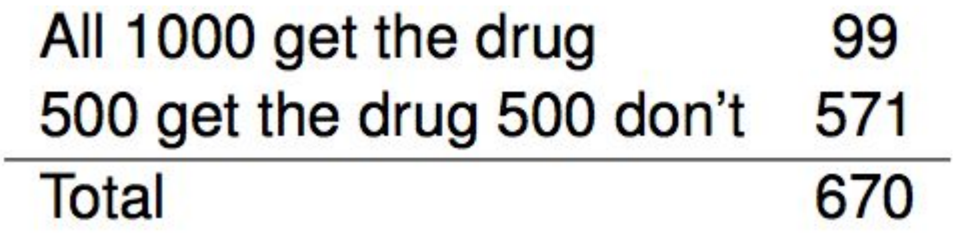
2. The margin of error is not just the sum of the margin of errors for each proportion

3. If you think two proportions come from the same population, you can use a pooled estimate

Results from the NSF SEEI2012

National Science Board
SCIENCE & ENGINEERING INDICATORS 2016

The National Science Foundation asked this question as part of a survey on general scientific literacy in 2010. Here are the results:



All 1000 get the drug	99
500 get the drug 500 don't	571
Total	670

Estimating the population parameter

We would like to estimate the proportion of all Americans who have good intuition about experimental design, i.e. would answer “500 get the drug 500 don't?” What are the parameter of interest and the point estimate?

Parameter of interest: proportion of all Americans who have good intuition about experimental design.

p: a population proportion

Point estimate: proportion of sampled Americans who have good intuition about experimental design.

$\hat{p}$: a sample proportion

Sample proportions are also nearly Normally Distributed

The Central Limit theorem for proportions says that the sample proportion will be nearly normal with mean equal to the population mean *p* and standard error $\sqrt{\frac{p(1-p)}{n}}$

$$\hat{p} \sim Normal\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

Only holds under the assumptions of the Central Limit Theorem:

- Independent samples
- N large enough (~10 success, ~10 failures)

Melting ice caps

Scientists predict that global warming may have big effects on the polar regions within the next 100 years. One of the possible effects is that the northern ice cap may completely melt.

Would it bother you if this actually happened?

(a) Yes

(b) No

Results from the NSF SEEI2012

National Science Board
SCIENCE & ENGINEERING INDICATORS 2016

The National Science Foundation asked this question as part of a survey on general scientific literacy in 2016. Here are the results:

	SEEI2016	**85309-20**	**85309-21**
Yes	578	17	
No	104	0	
Total	680	17	

Estimating the population difference

Parameter of interest: Difference between the proportions of all students and all Americans who would be bothered a great deal by the northern ice cap completely melting.

$p_{class} - p_{US}$

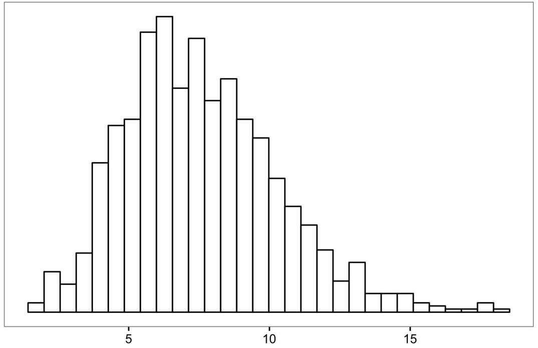
Point estimate: Difference between the proportions of sampled students and sampled Americans who would be bothered a great deal by the northern ice cap completely melting.

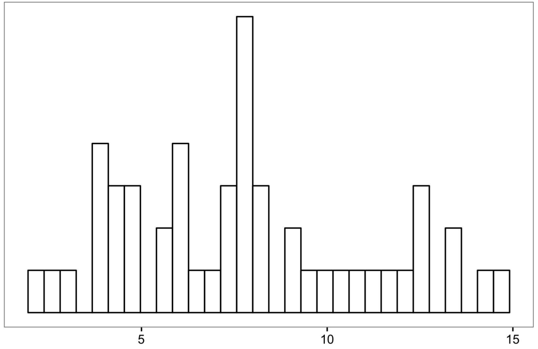
$\hat{p}_{class} - \hat{p}_{US}$: a sample proportion

A reminder about the Central Limit Theorem

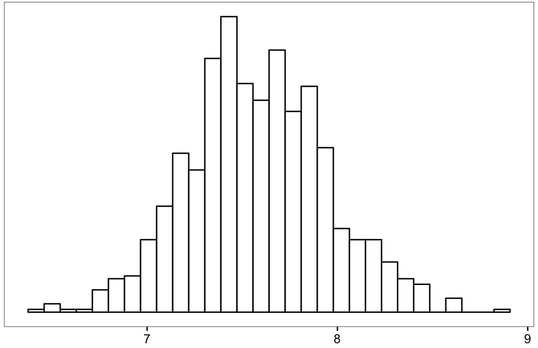
When I draw **independent samples** from the population, as sample size **approaches infinity,** the distribution of means approaches normality

Many statistical methods we use leverage this relationship
(t-test, linear regression, ANOVA, etc)





Take the mean,
Repeat many times...



Inference for comparing proportions

Details almost the same as before…

CI: point estimate ± margin of error

HT: Use $Z = \dfrac{point\ estimate - null\ value}{Standard\ Error}$

We just need the appropriate standard error for the point estimate (SE$_{\textit{class-US}}$)

$$SE_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Practice Question 1: Why the new SE estimate?

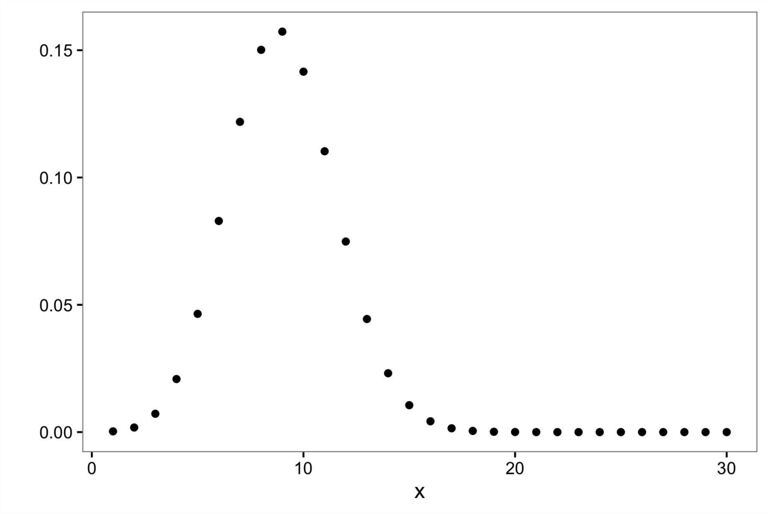
Naïve intuition: Find the SE for the class data, find the SE for the US data.
Add them up

$$SE_{\hat{p}_1} = \sqrt{\frac{p_1(1-p_1)}{n_1}}$$

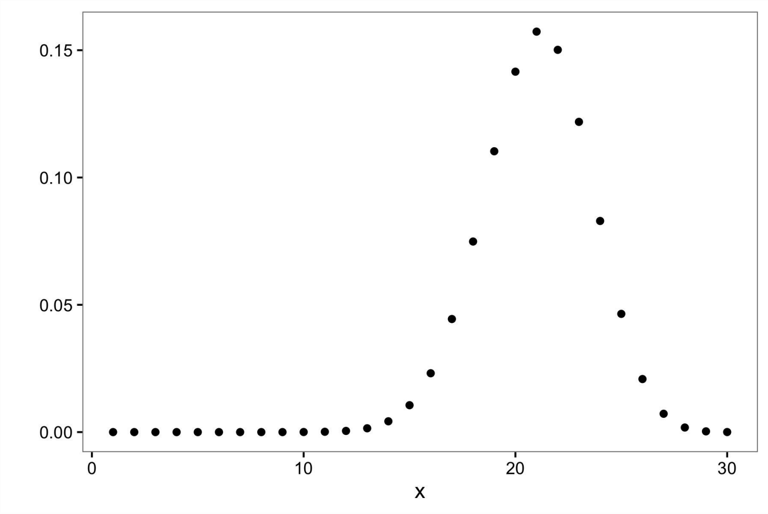
$$SE_{\hat{p}_2} = \sqrt{\frac{p_2(1-p_2)}{n_2}}$$

$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$$

Why is the correct SE estimate smaller?



p=.3



p=.7

Conditions for CI for difference of proportions (Normal approx)

Independence within groups

The people in the US group are sampled independently of each-other.

The people in the class group are sampled independently of each-other.

Independence between groups

The sampled students and US residents are independent of each-other

Success-failure

At least 10 observed successes and 10 observed failures in each group.

Difference of proportions are also nearly-normally distributed

Construct a 95% confidence interval for the difference between the proportions of students and Americans who would be bothered by the melting of the northern ice cap (p_{class} - p_{US}).

$$\hat{p}_1 - \hat{p}_2 \pm Z^* \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$.977 - .85 \pm 1.96 \times \sqrt{\frac{.977 \times .023}{43} + \frac{.85 \times .15}{680}}$$

$$.127 \pm .0522$$

$$(.0748, .179)$$

Practice Question 2

Which of the following is the correct set of hypotheses for testing if the proportion of students who would be bothered a great deal by the melting of the northern ice cap differs from the proportion of all Americans who do?

(a) $H_0: p_{class} = p_{US}$
$H_A: p_{class} \neq p_{US}$

(b) $H_0: \hat{p}_{class} - \hat{p}_{US} = 0$
$H_A: \hat{p}_{class} - \hat{p}_{US} \neq 0$

(c) $H_0: p_{class} - p_{US} = 0$
$H_A: p_{class} - p_{US} \neq 0$

(d) $H_0: p_{class} = p_{US}$
$H_A: p_{class} < p_{US}$

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$H_A: \hat{p}_{\text{class}} - \hat{p}_{\text{US}} \neq 0$

(c) $H_0: p_{\text{class}} - p_{\text{US}} = 0$
$H_A: p_{\text{class}} - p_{\text{US}} \neq 0$

(d) $H_0: p_{\text{class}} = p_{\text{US}}$
$H_A: p_{\text{class}} < p_{\text{US}}$

A pooled estimate of the population proportion

If you think that two samples come from the same population (*p*). Or you want to test whether they do, you used a *pooled estimate* of *p̂*.

$$\hat{p} = \frac{\# \; of \;\; successes_1 + \# \; of \;\; successes_2}{n_1 + n_2}$$

$$\hat{p}_1 - \hat{p}_2 \sim N\left(0, \sqrt{\frac{\hat{p}_{pool}(1 - \hat{p}_{pool})}{n_1} + \frac{\hat{p}_{pool}(1 - \hat{p}_{pool})}{n_2}}\right)$$

Testing H_0: No difference between the samples

$$\hat{p}_{pool} = \frac{578 + 42}{680 + 43} = .858$$

$$SE_{pool} = \sqrt{\frac{.858 \times .142}{680} + \frac{.858 \times .142}{43}}$$

$$= .0549$$

$$.977 - .85 \sim N(0, .0549)$$

$$.127 \sim N(0, .0549)$$

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pnorm(.127, mean=0, sd=.0549)=.99
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Key ideas

1. You can use the Normal approximation for the difference of two proportions

2. The margin of error is not just the sum of the margin of errors for each proportion

3. If you think two proportions come from the same population, you can use a pooled estimate