EECS 490 – Lecture 8

Functional Programming Examples

Announcements

- Homework 2 due on Friday
- Project 2 due Fri 10/6

Review: Environment of Use

- A function passed as a parameter has three environments that can be associated with it
  - The environment where it was defined
  - The environment where it was referenced
  - The environment where it was called
- Scope policy determines which names are visible in the function
  - Static/lexical scope: names visible at the definition point
  - Dynamic scope: names visible at the point of use
- In dynamic scope, point of use can be where a function is referenced or where it is called

Review: Binding Policy

- *Shallow binding*: non-local environment is environment from where a function is called
- *Deep binding*: non-local environment is environment from where a function is referenced

```
int foo(int (*bar)()) {
  int x = 3;
  return bar();
}

int baz() {
  return x;
}

int main() {
  int x = 4;
  print(foo(baz));
}
```

Non-local environment in shallow binding

Non-local environment in deep binding

C-like code with dynamic scope.

Agenda

- Nested Functions

- Example: Iterative Improvement

Nested Functions and Closures

- The ability to create a function from within another function is a key feature of functional programming
- Static scope requires that the newly created function have access to its definition environment
- A *closure* is the combination of a function and its enclosing environment
- Variables from the enclosing environment that are used in the function are *captured* by the closure

Nested Functions and State

- A closure encompasses state that can be accessed by the newly created function

```
def make_greater_than(threshold):
    def greater_than(x):
        return x > threshold
    return greater_than
```

threshold captured from non-local environment

```
>>> gt3 = make_greater_than(3)
>>> gt30 = make_greater_than(30)
>>> gt3(2)
False
>>> gt3(20)
True
>>> gt30(20), gt30(200)
(False, True)
```

Modifying Non-Local State

- Languages may allow non-local variables to be modified

```
def make_account(balance):
    def deposit(amount):
        nonlocal balance
        balance += amount
        return balance
    def withdraw(amount):
        nonlocal balance
        if 0 <= amount <= balance:
            balance -= amount
            return amount
        else:
            return 0
    return deposit, withdraw
```

```
>>> deposit, withdraw = \
        make_account(100)
>>> withdraw(10)
10
>>> deposit(0)
90
>>> withdraw(20)
20
>>> deposit(0)
70
>>> deposit(10)
80
>>> withdraw(100)
0
>>> deposit(0)
80
```

We will come back to data abstraction using functions later.

Decorators

- A common pattern in Python is to transform a function or class by applying a higher-order function to it, called a *decorator*

- Standard syntax for decorating functions:

```
@<decorator>
def <name>(<parameters>):
    <body>
```

- Mostly equivalent to:

```
def <name>(<parameters>):
    <body>

<name> = <decorator>(<name>)
```

Trace Example

- Example: decorator that traces function calls

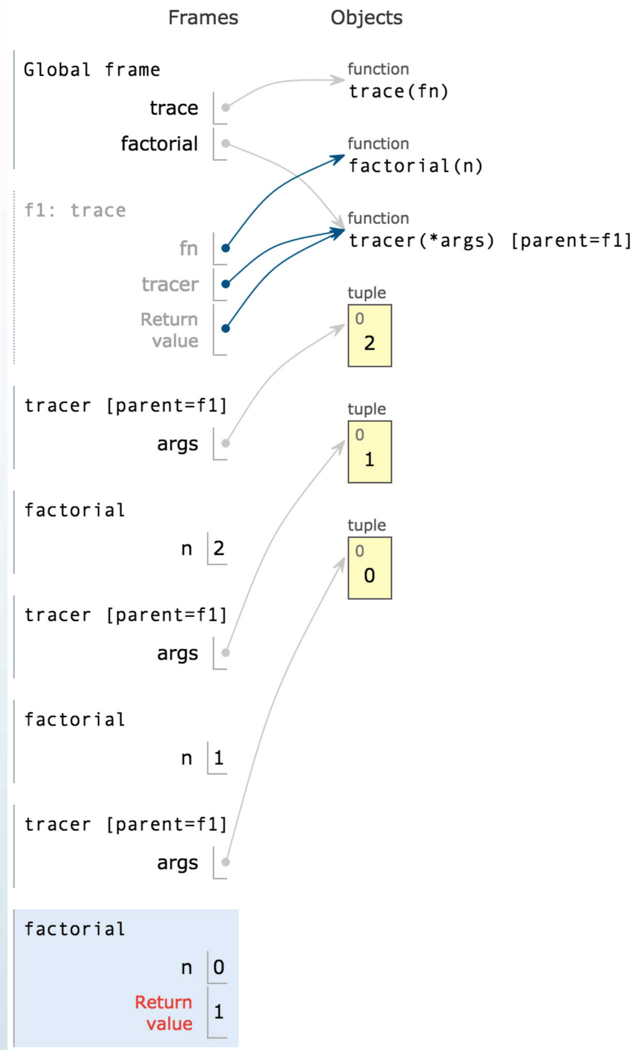
```
def trace(fn):
    def tracer(*args):
        strs = (str(arg) for arg in args)
        print('{}({})'.format(fn.__name__,
                              ', '.join(strs)))
        return fn(*args)
    return tracer

@trace
def factorial(n):
    if n == 0:
        return 1
    return n * factorial(n - 1)
```

```
>>> factorial(5)
factorial(5)
factorial(4)
factorial(3)
factorial(2)
factorial(1)
factorial(0)
120
```

Mutual Recursion

- A decorated recursive function results in *mutual recursion* where multiple functions make recursive calls indirectly through each other

```
>>> factorial(2)
factorial(2)
factorial(1)
factorial(0)
2
```



Partial Application

- Specify some arguments to a function, then specify remaining arguments later
- If function takes *n* arguments and *k* are supplied, results in function that takes *n-k* arguments

```
def partial(func, *args):
    def newfunc(*nargs):
        return func(*args, *nargs)
    return newfunc

>>> power_of_two = partial(pow, 2)
>>> power_of_two(3)
8
>>> power_of_two(7)
128
```

C++ has `bind()` template to do this in `<functional>`.

Currying

- Transforms a function that takes *n* arguments into a series of *n* functions that each in one argument

- In some languages, all functions are curried

```
def curry2(func):
    def curriedA(a):
        def curriedB(b):
            return func(a, b)
        return curriedB
    return curriedA

>>> curried_pow = curry2(pow)
>>> curried_pow(2)(3)
8
```

Uncurrying

- We can also do the reverse transformation

```
def uncurry2(func):
    def uncurried(a, b):
        return func(a)(b)
    return uncurried

>>> uncurried_pow = uncurry2(curried_pow)
>>> uncurried_pow(2, 3)
8
```

Break Time!

- We'll start again in five minutes.

Iterative Improvemnt

- A pattern of computation that begins with an initial guess and repeatedly updates the guess until it is close enough to the intended value

- We can implement this using higher-order functions:

```
def improve(update, is_close, guess):
    while not is_close(guess):
        guess = update(guess)
    return guess
```

Golden Ratio

- Can be computed by repeatedly adding the inverse of a number to 1

```
def golden_update(guess):
    return 1/guess + 1
```

- The ratio φ satisfies the equation $\varphi^2 = \varphi + 1$

```
def is_golden(guess):
    return approx_eq(guess * guess, guess + 1)

def approx_eq(x, y):
    return abs(x - y) < 1e-10
```

```
>>> improve(golden_update, is_golden, 1)
1.618033988702426
```

Scheme Implementation

- Improvement function uses tail recursion

```
(define (improve update close? guess)
   (if (close? guess)
       guess
       (improve update close? (update guess))))

(define (golden-update guess)
   (+ (/ 1 guess) 1))

(define (golden? guess)
   (approx-eq? (* guess guess) (+ guess 1)))

(define (approx-eq? x y)
   (< (abs (- x y)) 1e-10))
```

Square Root

- We can use iterative improvement, but we don't want to write separate `update` and `is_close` functions for each input

- Instead, we define these functions dynamically as local functions:

```
(define (isqrt a)
  (define (update x)
    (average x (/ a x)))
  (define (close? x)
    (approx-eq? (* x x) a))
  (improve update close? 1.0))

(define (average x y)
  (/ (+ x y) 2))
```

Newton's Method

- Generalized method to find the roots of a mathematical function using iterative improvement:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- We can implement this using a higher-order function:

```
(define (find-root f df)
  (define (update x)
    (- x (/ (f x) (df x))))
  (define (close? x)
    (approx-eq? (f x) 0))
  (improve update close? 1.0))
```

Nth Roots

- We can now use Newton's method to compute the nth root of a number:

```
(define (nth-root n a)
  (define (f x)
    (- (expt x n) a))
  (define (df x)
    (* n (expt x (- n 1))))
  (find-root f df))
```

```
> (nth-root 2 4)
2.000000000000002
> (nth-root 4 4)
1.4142135623730951
> (nth-root 8 16)
1.414213562373095
```