

Lambda Calculus and Operational Semantics

1. As in lecture, let us define the following:

```
true =  $\lambda t. \lambda f. t$   
false =  $\lambda t. \lambda f. f$   
or =  $\lambda a. \lambda b. a a b$   
not =  $\lambda b. b \text{ false true}$ 
```

Using the above encodings, evaluate the following expression using normal order evaluation:

`or (false) (not false)`

2. Using the simple imperative language shown in lecture, write out the derivation trees for the following expressions and statements using operational semantics

$P \rightarrow S$ $S \rightarrow \text{skip} \mid S; S \mid V = A$ if B then S else S end while B do S end $A \rightarrow N \mid V \mid (A + A) \mid (A - A) \mid (A * A)$ $B \rightarrow \text{true} \mid \text{false} \mid (A \leq A) \mid (B \text{ and } B) \mid \text{not } B$ $V \rightarrow \text{Identifier}$ $N \rightarrow \text{IntegerLiteral}$	$x = -1$ 1. $((x * 3) - 2)$ 2. $((x \leq 3) \text{ and } (0 \leq x))$
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------

3. Suppose we wanted to add the ternary conditional $(B ? A : A)$ to our language, where $(b ? a_1 : a_2)$ evaluates to a_1 if b evaluates to true and a_2 otherwise. Write rules for the evaluating a ternary conditional using big-step operational semantics.