

VE477

Introduction to Algorithms

Assignment 3

Manuel — UM-JI (Fall 2017)

Reminders

- Write in a neat and legible handwriting or use \LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb, and object)
- Be critical on your results

Ex. 1 — *Hamiltonian path*

1. Explain and present Depth-First Search (DFS).
2. Explain and present topological sorting.
3. Write the pseudo-code of a polynomial time algorithm which decides if a directed acyclic graph contains a Hamiltonian path.
4. Prove its complexity.
5. To what complexity class does the Hamiltonian path problem belong?

Ex. 2 — *Critical thinking*

1. Is the function $\lceil \log n \rceil!$ bounded by a polynomial?
2. Is the function $\log^* \log n$ asymptotically larger than $\log \log^* n$?
3. Given eight balls of similar size but where one is lighter, detect which one it is, while minimizing the number of weighings. Provide the pseudocode.

Ex. 3 — *Rubik's Cube*

In about half a page explain the game and at least two algorithms to solve it. Provide references.

Ex. 4 — *The \mathcal{NP} classe*

Prove that the following problems are in \mathcal{NP} .

1. Does a given graph have a simple path?
2. Is a given integer composite?
3. Does a given graph have a vertex cover of size k , for some integer k ?

Ex. 5 — *PRIMES is in \mathcal{P}*

The PRIMES problem consists in deciding if a given integer n is prime. A simple algorithm to solve PRIMES is trial division which runs in time $\mathcal{O}(n)$. Is it sufficient to prove that PRIMES is in \mathcal{P} ? Explain.

Hint: use the Prime Number Theorem.