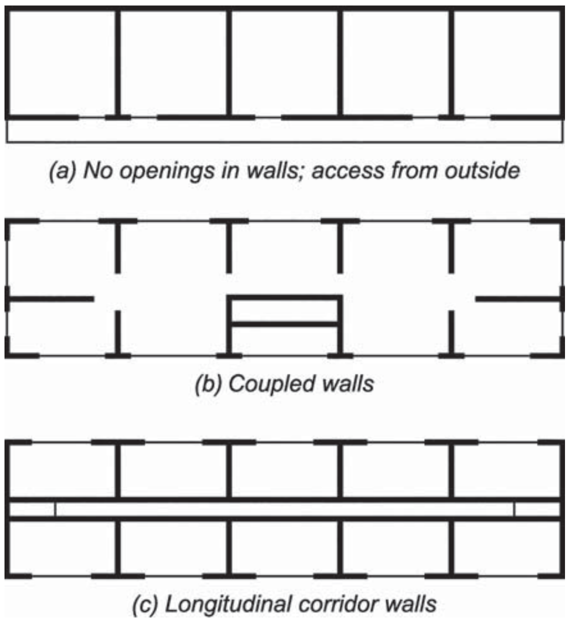

Fig. 2.4.2.2c—Example of shear wall layouts.

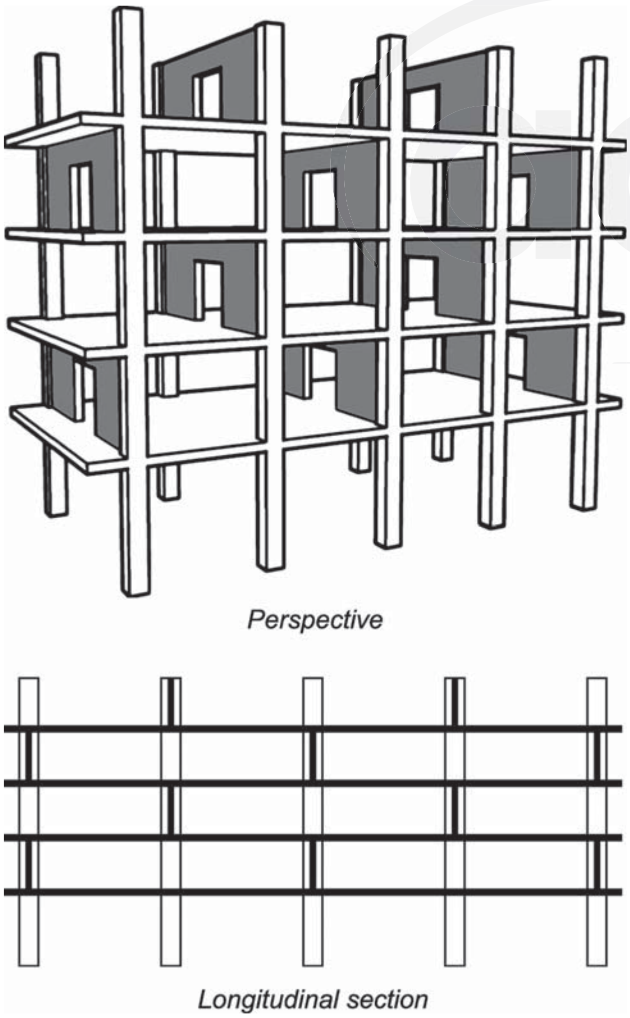

Fig. 2.4.2.3—Staggered wall-beam system.

moments in the columns and girders, and reduces the effects of shear lag.

2.4.3 *Dual systems*—Dual systems consist of combining two of the structural systems discussed in the previous section. They are used to achieve specific response characteristics, particularly with respect to seismic behavior. Some of the more common dual systems are discussed in 2.4.3.1 through 2.4.3.6.

2.4.3.1 *Wall-frame systems*—Rigid-jointed frames and isolated or coupled structural walls can be combined to produce an efficient lateral-force-resisting system. Because of the different shear and flexural lateral deflection characteristics of moment frames and structural walls, careful attention to the interaction between the two systems can improve the structure's lateral response to loads by reducing lateral deflections (Fig. 2.4.3.1).

The wall's overturning moment is greatly reduced by interaction with the frame. Because drift compatibility is forced on both the frame and the wall, and the frame-alone and wall-alone drift modes are different, the building's overall lateral stiffness is increased. Design of the frame columns for gravity loads is also simplified in such cases, as the frame columns are assumed to be braced against sidesway by the walls.

The wall-frame dual system permits the structure to be designed for a desired yielding sequence under strong ground motion. Beams can be designed to experience significant yielding before inelastic action occurs at the bases of the walls. By creating a hinge sequence, and considering the relative economy with which yielded beams can be repaired, wall-frame structures are appropriate for use in higher seismic zones. However, note that the variation of shears and overturning moments over the height of the wall and frame is very different under inelastic versus elastic response conditions.

2.4.3.2 *Outrigger systems*—An outrigger system uses orthogonal walls, girders, or trusses, one or two stories in height, to connect the perimeter columns to central core walls, thus enhancing the structure lateral stiffness (Fig. 2.4.3.2)

In addition to the outrigger girders that extend out from the core, girders or trusses are placed around the perimeter of the structure at the outrigger levels to help distribute lateral forces between the perimeter columns and the core walls. These perimeter girders or trusses are called "hat" or "top-hat" bracing if located at the top, and "belt" bracing if located at intermediate levels. Some further reductions in total drift and core bending moments can be achieved by increasing the cross section of the columns and, therefore, the axial stiffness, and by adding outriggers at more levels. Outriggers are effective in increasing overall building stiffness and, thus, resist wind loads with less drift. Design of outrigger-type systems for SDC D through F must consider the effect of the high local stiffness of the outriggers on the inelastic response of the entire system. Members framing into the outriggers should be detailed for ductile response.

2.4.3.3 *Tube-in-tube*—For tall buildings with a reasonably large service core, it is generally advantageous to use

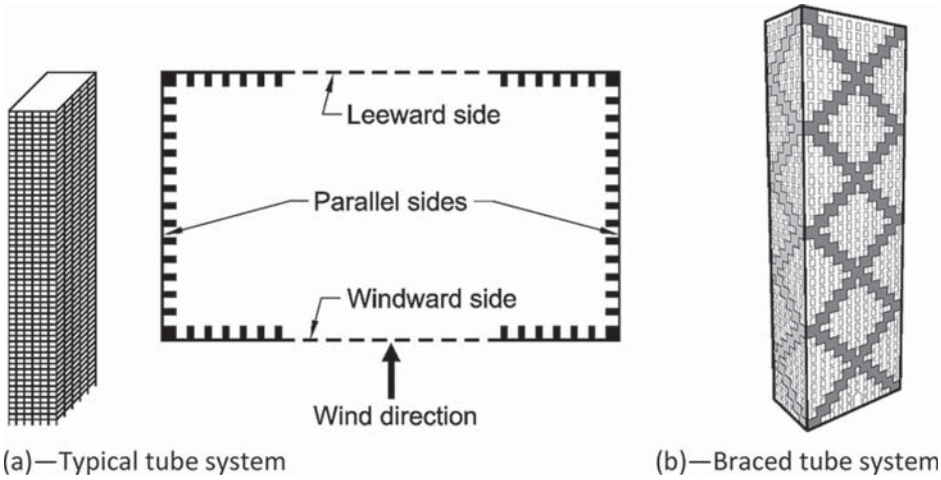

Fig. 2.4.2.4—Tube systems.

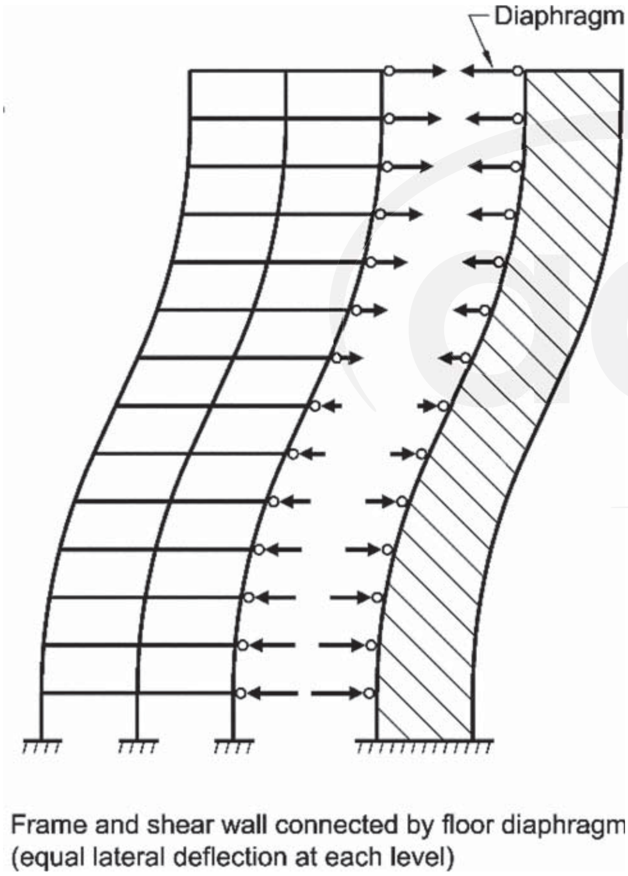

Fig. 2.4.3.1—Shear wall and moment frame system.

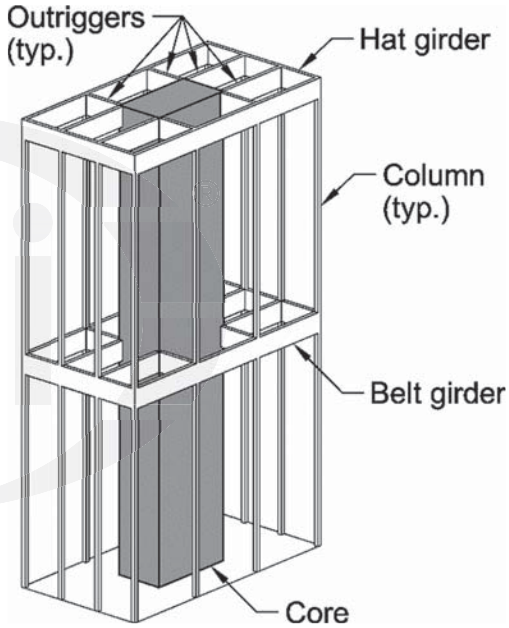

Fig. 2.4.3.2—Outrigger system.

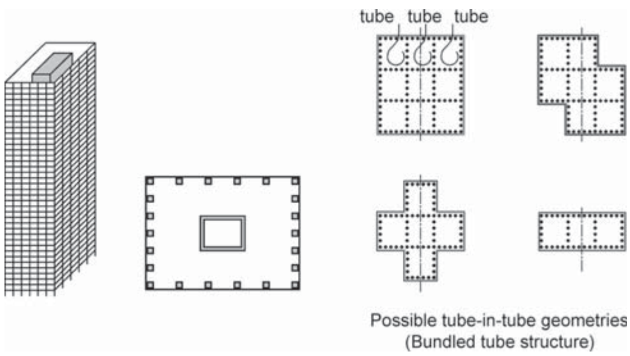

Fig. 2.4.3.3—Tube-in-tube and bundled tube systems.

shear walls enclosing the entire service core (inner tube) as part of the lateral-force-resisting system. The outer tube is formed by the closely-spaced column-spandrel beam frame. A bundled tube system consists of several framed tubes bundled into one larger structure that behaves as a multicell perforated box (Fig. 2.4.3.3).

The tube-in-tube system combines the advantages of both the perimeter framed tube and the inner shear walls. The inner shear walls enhance the structural characteristics of the perimeter framed tube by reducing the shear deformation of the columns in the framed tube. The tube-in-tube system can be considered a refined version of the shear wall-frame interaction type structure.

2.4.3.4 *Bundled tubes*—A bundled tube system consists of several framed tubes bundled into one larger structure that behaves as a multicell perforated box. Individual tubes can be terminated at different heights. The bundled tube system offers considerable flexibility in layout and possesses large torsional and flexural stiffness.

2.4.3.5 *Mixed concrete-steel structures*—Mixed concrete-steel systems consist of interacting concrete and steel assemblies. The resulting composite structure displays most or all of the advantages of steel structures (large spans and lightweight construction) as well as the favorable characteristics of concrete structures (high lateral stiffness of shear walls and cores, and high damping). Engineers must address the differential vertical creep and shrinkage between steel and concrete to prevent uneven displacement. Because the erection of steel and concrete structures involves different building trades and equipment, engineers who design mixed construction should consider scheduling issues.

2.4.3.6 *Precast structures*—Precast concrete members are widely used as components in frame, wall, and wall-frame systems. Mixed construction, consisting of precast concrete assemblies connected to a cast-in-place concrete core, is also used. The efficiency of such systems depends on the extent of standardization, the ease of manufacture, the simplicity of assembly, and the speed of erection.

Precast floor systems include large standardized reinforced (and usually prestressed) concrete slabs, with or without interior cylindrical voids (hollow core), as well as prefabricated rib slabs. Rigid-jointed frames are usually assembled from H- or T-units, and shear walls and cores are assembled from prefabricated single-story panels. Planning and designing appropriate connection details for panels, frame members, and floor assemblies is the single most important operation related to prefabricated structures.

Three main types of connections are described as follows:

1. Steel reinforcement bars protruding from adjacent precast members are made continuous by mechanical connectors, welding or lap splices, and the joint between the members is filled with cast-in-place concrete. If welding is used, the engineer should specify appropriate welding procedures to avoid brittle connections.
2. Steel inserts (plates and angles) provided in the precast members are bolted or welded together and the gaps are grouted.
3. The individual precast units are post-tensioned together across the joint, with or without a mortar bed.

The behavior of a precast system subjected to seismic loading depends to a considerable degree on the characteristics of the connections. Connection details can be developed that ensure satisfactory performance under seismic loadings, provided that the engineer pays particular attention to steel ductility and positive confinement of concrete in the joint area.

2.5—Floor subassemblies

Selection of the floor system significantly affects a structure's cost as well as the performance of its lateral-force-resisting system. The primary function of a floor system is to resist gravity load. Additional important functions in most buildings are:

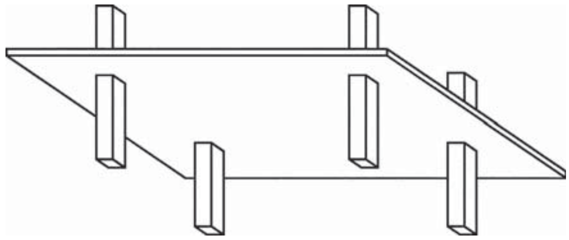

Fig. 2.5.1—Two-way flat plate system.

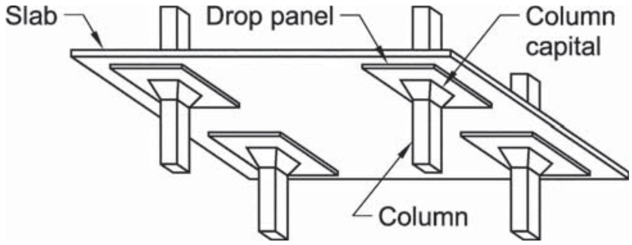

Fig. 2.5.2—Flat slab with drop panels and capitals.

(a) Diaphragm action: The slab's in-plane stiffness maintains the plan shape of the structure, and distributes horizontal forces to the lateral-force-resisting system.

(b) Moment resistance: The flexural stiffness of the floors may be an integral and necessary part of the lateral-force-resisting system.

Concrete structures are commonly analyzed for lateral loads assuming the floor system acts as a diaphragm, infinitely stiff in its plane. This assumption is not valid for all configurations and geometries of floor systems. Factors affecting diaphragm stiffness are: span-to-depth ratio of the slab's plan dimensions relative to the location of the lateral-load-resisting members, slab thickness, locations of slab openings and discontinuities, and type of floor system used.

The floor system flexural stiffness can add to the lateral stiffness of the structure. If the slab is assumed to act as part of a frame to resist lateral moments, engineers usually limit the effective slab width (acting as a beam within the frame) to between 25 and 50 percent of the bay width.

2.5.1 *Flat plates*—A flat plate is a two-way slab supported by columns, without column capitals or drop panels (Fig. 2.5.1).

The flat plate system is a very cost-effective floor for commercial and residential buildings. Simple formwork and reinforcing patterns, as well as lower overall building height, are advantages of this system. In designing and detailing plate-column connections, particular attention must be paid to the transfer of shear and unbalanced moment between the slab and the columns (ACI 318, Chapters 8 and 15). This is achieved by using a sufficient slab thickness or shear reinforcement (stirrups or headed shear studs) at the slab-column joint, and by concentrating slab flexural reinforcement over the column area.

2.5.2 *Flat slabs with drop panels, column capitals, or both*—The shear strength of flat slabs can be improved by thickening the slab around columns with drop panels, column capitals (either constant thickness or tapered), shear caps, or a combination (Fig. 2.5.2). Like flat plates, flat slab

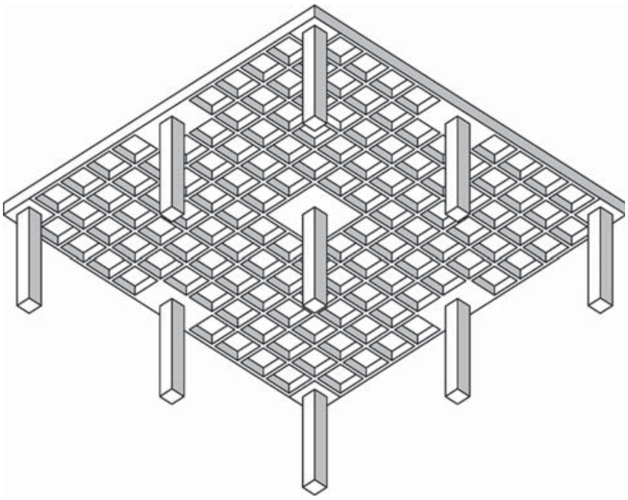

Fig. 2.5.3—Two-way grid (waffle) slab.

systems normally act as diaphragms transmitting lateral forces to columns and walls.

Drop panels increase a slab's flexural and shear strength at the column, and thus improve the ability of the flat slab to participate in the LFRS. Shear caps and column capitals improve the slab shear strength by increasing the slab thickness around the column. To improve the slab shear strength without increasing the slab thickness, engineers can provide closely spaced stirrups or shear studs radiating out from the column.

Lateral-force-resisting systems consisting only of flat slab or flat plate frames, without ductile frames, structural walls, or other bracing members, are unsuitable in high seismic areas (SDC D through F).

2.5.3 *Two-way grid (waffle) slabs*—For longer spans, a slab system consisting of a grid of ribs intersecting at a constant spacing can be used to achieve an appropriate slab depth for the longer span with much less dead load than a solid slab (Fig. 2.5.3).

The ribs are formed by standardized dome or pan forms that are closely spaced. The slab thickness between the ribs is thin and normally governed by fire rating requirements. Some pans adjacent to the columns are omitted to form a solid concrete drop panel, to satisfy requirements for transfer of shear and unbalanced moment between the slab and columns.

A waffle slab provides an adequate shear diaphragm. The solid slab adjacent to the column provides significant two-way shear strength. Slab flexural and punching shear strength can be increased by the addition of closely spaced stirrups radiating out from the column face in two directions. Stirrups may also be used in the ribs. Because a waffle slab behaves similarly to a flat slab, LRFSs consisting only of waffle slab frames are unsuitable in high seismic design areas (SDC D through F).

2.5.4 *One-way slabs on beams and girders*—One-way slabs on beams and girders consist of girders that span between columns and beams that span between the girders. One-way slabs span between the beams. This system provides a satisfactory diaphragm, and uses the girder-

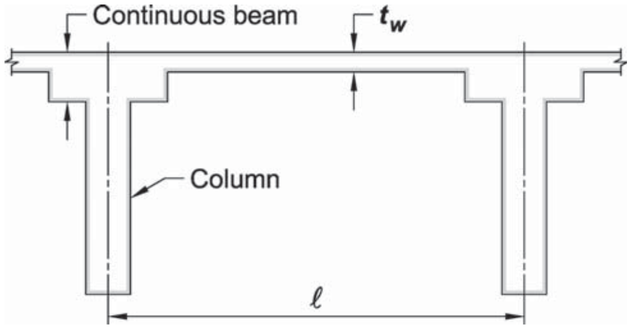

Fig. 2.5.6—One-way banded one-way slab.

column frames and beam-column frames to resist lateral loads. Adequate flexural ductility can be obtained by proper detailing of the beam and girder reinforcement.

The beams and slabs can be placed in a composite fashion (with precast elements). If composite, shear connectors are placed at the beam-slab interface to ensure composite action. This system can provide good lateral force resistance, provided that the shear connectors are detailed with sufficient strength and ductility. Some examples of this type of slab system include:

(a) Precast concrete joists with steel shear connectors between the top of the beam and a cast-in-place concrete slab. The concrete joists are usually fabricated to readily support the formwork for the cast-in-place slab. In this system, the joists are supported on walls or cast-in-place concrete beams framing directly into columns.

(b) Steel joists with the top chord embedded in a cast-in-place concrete slab. The slab formwork is supported from the joists, which supports the fresh slab concrete.

(c) Steel beams supporting a noncomposite steel deck with a cast-in-place concrete slab. Note that ACI 318 does not govern the structural design of concrete slabs for composite steel decks.

2.5.5 *One-way ribbed slabs (joists)*—One-way ribbed slab (joist) systems consist of concrete ribs in one direction, spanning between beams, which span between columns. The size of pan forms available usually determines rib depth and spacing. As with a two-way ribbed system, the thickness of the thin slab between ribs is often determined by the building's fire rating requirements.

This system provides an adequate shear diaphragm and is used in a structure whose lateral resistance comes from a moment-resisting frame or shear walls. One row of pans can be eliminated at column lines, giving a wide, flat beam that may be used as part of the LFRS. Even if the slab system does not form part of the designated LRFS, the engineer should investigate the actions induced in the ribs by building drift.

2.5.6 *One-way banded slabs*—A one-way banded slab is a continuous drop panel (shallow beam) spanning between columns, usually in the long-span direction, and a one-way slab spanning in the perpendicular direction (Fig. 2.5.6).The shallow beam can be reinforced with closely spaced stirrups near the support to increase the slab's shear strength. This

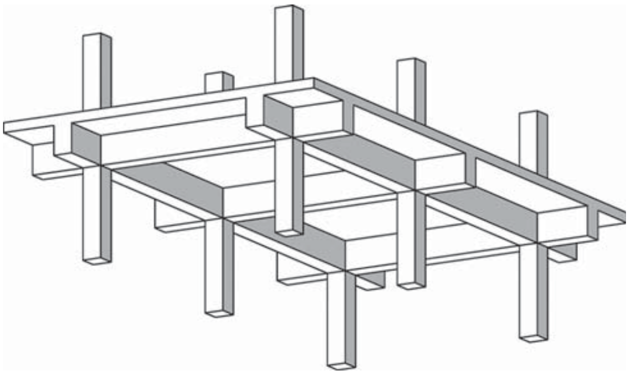

Fig. 2.5.7—Two-way slab with edge beams around perimeter.

system is also sometimes referred to as wide-shallow beams with one-way slabs.

A structure using this type of floor system is less stiff laterally than a structure using a ductile moment frame with beams of normal depth. Lateral-force-resisting systems consisting only of flat slab or flat plate frames, without ductile frames, structural walls, or other bracing members, are not suitable in SDC D through F.

2.5.7 *Two-way slabs with edge beams*—As shown in Fig. 2.5.7, the slab is supported by beams in two directions on the perimeter column lines. This system is useful where a beam-column frame is required as part of the LFRS. The slab provides high diaphragm stiffness, and the perimeter beams can provide sufficient lateral stiffness and strength though frame action for use in SDC D through F.

For longer spans, a two-way grid (Section 2.5.3) slab may be used instead of a flat plate.

2.5.8 *Precast slabs*—Precast, one-way slabs are usually supported by bearing walls, precast beams, or cast-in-place beams. Precast slabs may be solid, hollow-core slabs, or single- or double-T-sections. They are sometimes topped by a thin cast-in-place concrete layer, referred to as a "topping slab."

Welded connections are normally used to transfer in-plane shear forces between precast slabs and their supports. Because precast slabs are individual units interconnected mechanically, the ability of the assembled floor system to act as a shear diaphragm must be examined by the engineer. Boundary reinforcement may be required, particularly where the lateral-force-resisting members are far apart. In areas of high seismicity, the connections between the precast slab system and the lateral-load-resisting system must be carefully detailed. A concrete topping bonded to the precast slab improves the ability of the slab system to act as a shear diaphragm, and can be used in SDC D through F.

2.6—Foundation design considerations for lateral forces

A foundation design must consider the weight of the building, live loads, and the transmission of lateral forces to the ground. A distinction should be drawn between external forces, such as wind, and inertia forces that result from the building's response to ground motions during an earthquake.

External lateral forces can include static pressures due to water, earth or fill, and equivalent static forces representing the effects of wind pressures, where a gust factor or impact factor is included to account for their dynamic nature.

The soil type and strata usually dictate whether the foundation is deep or shallow. A soils report from a licensed geotechnical engineer provides the detailed information and foundation recommendations that the licensed design professional (LDP) needs to design the foundation. For shallow footings, the geotechnical engineer provides an allowable soil bearing pressure for the soil at the foundation elevation. That pressure limit targets a certain amount of soil deflection, and includes consideration of the anticipated use of the building. If allowable soil pressure is less than 2500 lb/ft^2, the soil is very soft and deep foundation options are usually considered. Other soil situations, such as expansive clay or nonstructural fill, may preclude the use of shallow foundations. If the building is below grade, concrete walls can be part of the foundation system.

The two types of deep foundations are caissons (also known as piers) and piles. If hard rock is not far below existing grade, caissons can transfer a column load directly to the bedrock. Bearing values for solid rock can be more than 10 kip/ft^2. Caissons are large in diameter, usually starting at approximately 30 in. Piles are generally smaller in diameter, starting at around 12 in., and can be cast-in-place in augered holes or precast piles that are driven into place. Piles are usually designed for lighter loads than caissons are. Groups of piles may be used where bedrock is too deep for a caisson. Tops of piles or caissons are bridged by pile caps and grade beams to distribute column loads as needed.

Shallow foundations are referred to as footings. Types of footings are isolated, combined, and mat. Isolated rectangular or square footings are the most common types. Combined footings are often needed if columns are too close together for two isolated footings, if an exterior column is too close to the boundary line, or if columns are transmitting moments to the footing, such as if the column is part of a lateral-force-resisting system. If the column loads are uniformly large, such as in multistory buildings, or if column spacing is small, mat foundations are considered.

2.6.1 *Resistance to lateral loads*—The vertical foundation pressures resulting from lateral loads are usually of short duration and constitute a small percentage of the total vertical load effects that govern long-term soil settlements. Allowing a temporary peak in vertical bearing pressures under the influence of short-term lateral loads is usually preferred to making the footing areas larger.

The geotechnical engineer should report the likelihood of liquefaction of sands or granular soils in areas with a high groundwater table, or the possibility of sudden consolidation of loose soils when subjected to jarring. The capacity of friction piles founded in soils susceptible to liquefaction or consolidation should be checked.

2.6.2 *Resistance to overturning*—The engineer should investigate the safety factor of the foundation against overturning and ensure it is within the limits of the local building code. Overturning calculations should be made with removable soil fill or live load completely removed and should be based on a safe (low) estimate of the building's actual dead load.

2.7—Structural analysis

The analysis of concrete structures "shall satisfy compatibility of deformations and equilibrium of forces," as stated in Section 4.5.1 of ACI 318. The LDP may choose any method of analysis as long as these conditions are met. This discussion is intended to be a brief overview of the analysis process as it relates to structural concrete design. For more detailed information on structural analysis, refer to Chapter 3 of this Handbook.

2.8—Durability

Reinforced concrete structures are expected to be durable. The design of the concrete mixture proportions should consider exposure to temperature extremes, snow and ice, and ice-removing chemicals. Chapter 19 of ACI 318-14 provides mixture requirements to protect concrete and reinforcement against various exposures and deterioration. Chapter 20 of ACI 318-14 provides concrete cover requirements to protect reinforcement against steel corrosion. For more information, refer to Chapter 4 of this Handbook.

2.9—Sustainability

Reinforced concrete structures are expected to be as sustainable as practical. ACI 318 allows sustainability requirements to be incorporated in the design, but they must not override strength and serviceability requirements.

2.10—Structural integrity

The ACI Code concept of structural integrity is to "improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability" (ACI Committee 442 1971). The Code addresses this concept by providing system continuity through design and detailing rules within the beam and two-way slab chapters.

2.11—Fire resistance

Minimum cover specified in Chapter 20 of ACI 318-14 is intended to protect reinforcement against fire; however, the Code does not provide a method to determine the fire rating of a member. The International Building Code (IBC) 2015 Section 722 permits calculations that determine fire ratings to be performed in accordance with ACI 216.1 for concrete, concrete masonry, and clay masonry members.

2.12—Post-tensioned/prestressed construction

The introduction of post-tensioning/prestressing to concrete floor, beams, and wall elements imparts an active, permanent force within the structural system. Because cast-in-place structural systems are monolithic, this force affects the behavior of the entire system. The engineer should consider how elastic and plastic deformations, deflections, changes in length, and rotations due to post-tensioning/prestressing affect the entire system. Special attention must be given to the connection of post-tensioned/prestressed members to other members to ensure the proper transfer of forces between, and maintain a continuous load path. Because the post-tensioning/prestressing force is permanent, the system creep and shrinkage effects require attention.

2.13—Quality assurance, construction, and inspection

The International Standardization Organization (ISO) defines "quality" as the degree to which a set of inherent characteristics fulfills a set of requirements. The goal of quality assurance is to establish confidence that projects are built in compliance with project construction documents. Chapter 26 of ACI 318-14 contains requirements to facilitate the implementation of competent construction documents, material, construction, and inspection.

REFERENCES

American Concrete Institute

ACI 216.1-14—Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies

ACI 352R-02—Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures

ACI 442R-71—Response of Buildings to Lateral Forces

ACI 442R-88—Response of Concrete Buildings to Lateral Forces

American Society of Civil Engineers

ASCE 7-10—Minimum Design Loads for Buildings and other Structures

International Code Council

IBC 2015 International Building Code

Authored documents

Ali, M. M., and Moon, K. S., 2007, "Structural Development in Tall Buildings: Current Trends and Future Prospects," *Architectural Science Review*, V. 50, No. 3, pp. 205-223.

CHAPTER 3—STRUCTURAL ANALYSIS

3.1—Introduction

Structural engineers mathematically model reinforced concrete structures, in part or in whole, to calculate member moments, forces, and displacements under the design loads that are specified by a standard such as ASCE 7-10. In all conditions, equilibrium of forces and compatibility of deformations must be maintained. The stiffnesses values of individual members for input into the model, under both service loads and factored loads, are discussed in detail in ACI 318-14, Chapter 6. The factored moments and forces resulting from the analysis are used to determine the required strengths for individual members. The calculated displacements and drift are also checked against commonly accepted serviceability limits.

3.2—Overview of structural analysis

3.2.1 *General*—The analysis of concrete structures "shall satisfy compatibility of deformations and equilibrium of forces," as stated in Section 4.5.1 of ACI 318-14. The licensed design professional (LDP) may choose any method of analysis as long as these conditions are met. ACI 318-14, Chapter 6, is divided into three levels of analysis: 1) elastic first-order; 2) elastic second-order; and 3) inelastic second-order. In addition, ACI 318 permits the use of strut-and-tie modeling for the analysis of discontinuous regions.

Except as noted in Chapter 18, ACI 318 provisions state that the designer may assume that reinforced concrete members behave elastically under design loads. It is also generally acceptable to model concrete members with constant sectional properties along the member length. These assumptions simplify analysis models but they may differ from the actual behavior of the concrete member.

3.2.2 *Elastic analysis*—Most concrete structures are modeled using an elastic analysis. The stability of columns and walls must be considered by both first-order and second-order analysis. For first-order analysis, end moments of columns and walls are conservatively amplified to account for second-order, or P-Δ, effects. For second-order analysis, the second-order effects are calculated directly considering the loads applied on the laterally deformed structure. A series of analyses are made where the secondary moment from each analysis is added to the subsequent analysis until equilibrium is achieved.

3.2.3 *Inelastic analysis*—Inelastic second-order analysis determines the ultimate capacity of the deformed structure. The analysis may take into account material nonlinearity, member curvature and lateral deformation (second-order effects), duration of loads, shrinkage and creep, and interaction with the supporting foundation. The resulting strength must be compatible with results of published tests. This analysis is used for seismic retrofit of existing buildings; design of materials and systems not covered by the code; and evaluation of building performance above code minimum requirements (Deierlein et al. 2010). This handbook does not include discussion of the inelastic analysis approach.

Table 3.2.5—Common analysis types and tools

<table>
<tr><th>Analysis type</th><th>Applicable member or assembly</th><th>Analysis tool</th></tr>
<tr><td rowspan="12">First-order
Linear elastic
Static load
Hand calculations</td><td>One-way slab</td><td>Analysis tables*</td></tr>
<tr><td>Continuous one-way slab</td><td>Simplified method in Section 6.5 of ACI 318-14</td></tr>
<tr><td rowspan="2">Two-way slab</td><td>Direct design method in Section 8.10 of ACI 318-14</td></tr>
<tr><td>Equivalent frame method in Section 8.11 of ACI 318-14</td></tr>
<tr><td>Beam</td><td>Analysis tables*</td></tr>
<tr><td>Continuous beam</td><td>Simplified method in Section 6.5 of ACI 318-14</td></tr>
<tr><td>Column</td><td>Interaction diagrams*</td></tr>
<tr><td rowspan="2">Wall</td><td>Interaction diagrams*</td></tr>
<tr><td>Alternative method for out-of-plane slender wall analysis in Section 11.8 of ACI 318-14</td></tr>
<tr><td>Two-dimensional frame</td><td>Portal method in Section 3.3.3 of this chapter</td></tr>
<tr style="display:none"><td></td><td></td></tr>
<tr style="display:none"><td></td><td></td></tr>
<tr><td rowspan="3">First-order
Linear elastic
Static load
Computer programs</td><td rowspan="2">Gravity-only systems</td><td>Spreadsheet program based on the analysis tools for hand calculations above</td></tr>
<tr><td>Program based on matrix methods but only analyze floor assemblies for gravity loads</td></tr>
<tr><td>Two-dimensional frames and walls</td><td>Program based on matrix methods without iterative capability</td></tr>
<tr><td rowspan="2">Second-order
Linear elastic
Static or dynamic load
Computer programs</td><td>Two-dimensional frames and walls</td><td>Programs based on matrix methods with iterative capability</td></tr>
<tr><td>Three-dimensional structure</td><td>Programs based on finite element methods with iterative capability</td></tr>
<tr><td>Second-order
Inelastic</td><td>Three-dimensional structure</td><td>Beyond the scope of this Handbook</td></tr>
</table>

*Information can downloaded from the ACI website; refer to Table of Contents.

3.2.4 *Strut-and-tie*—The strut-and-tie method in Chapter 23, ACI 318-14, is another analysis method that is permitted by ACI 318. This method does not assume that plane sections of unloaded members remain plane under loading. Because this method also provides design provisions, it is considered both an analysis and design method. This method is applicable where the sectional strength assumptions in ACI 318, Chapter 22, do not apply for a discontinuity region of a member or a local area.

3.2.5 *Analysis types and tools*—ACI 318 identifies three general types of analysis see Section 3.2.1: 1) first-order

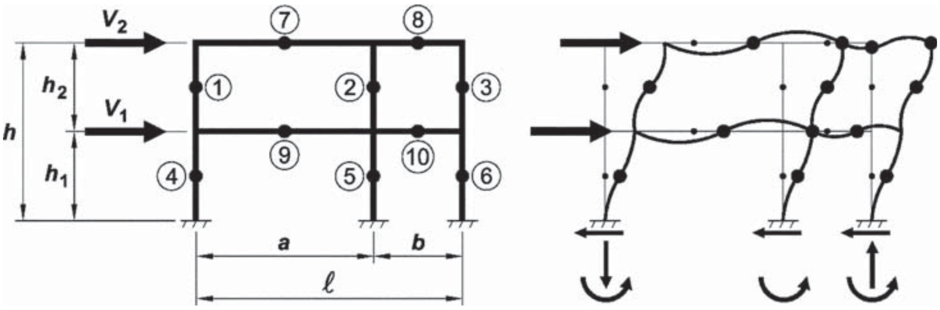

Fig. 3.3.3—Frame analyzed by portal method.

linear elastic; 2) second-order linear elastic; and 3) second-order inelastic. Table 3.2.5 shows some common analysis tools used for different analysis methods, loads, and systems.

3.3—Hand calculations

3.3.1 *General*—Before computers became widely available, designers used simplified code equations to calculate gravity design moments and shears (Section 6.5, 8.10, and 8.11 in ACI 318) along with a simplified frame analysis, such as the portal method, to calculate frame moments and shears due to lateral forces. In very limited applications, the design of an entire building using hand calculations is still possible with today's set of building codes. For the large majority of building designs, however, a hand calculation design approach is not practical due to the large number and complexity of design load combinations necessary to fully meet ASCE 7-10 requirements.

3.3.2 *Code design equations for moment and shear*—The simplified code equations are useful for purposes of preliminary estimating or member sizing, for designing isolated members or subassemblies, and to complete rough checks of computer program output. Because these equations and expressions are easy to incorporate into electronic spreadsheets and equation solvers, they continue to be helpful. In the member chapters of this handbook, examples of hand calculations are provided.

3.3.3 *Portal method*—The portal method was commonly used before computers were readily available to calculate a frame's moments, shears, and axial forces due to lateral forces (Hibbeler 2015). This method has been virtually abandoned as a design tool with the widespread use of commercial design software programs. The portal method has limitations as stated in the assumptions and considerations that follow, but is still a useful tool for the designer. With complex, three-dimensional modeling becoming commonplace, there is always a chance of modeling error. The portal method allows the designer to independently and quickly find approximate moments and shears in a frame. This can be very useful for spot-checking the program results (Fig. 3.3.3).

The basics assumptions to the portal method are:

(a) Apply only the lateral load to the frame
(b) Exterior columns resist the overturning from lateral loads
(c) Shear at each column is based on plan tributary area
(d) Inflection points are assumed to be located at midheight of column and midspan of beams
(e) Shear in the beam is the difference between column axial forces at a joint
(f) Beam axial force is to be zero

These assumptions reduce a statically indeterminate problem to a statically determinate one.

The following should be considered when using this method:

(a) Discontinuity in geometry or stiffness, such as setbacks, changes in story height, and large changes in member sizes, can cause member moments to differ significantly from those calculated by a computer analysis.
(b) The lateral deformation will be larger than the lateral deformation calculated by a computer analysis.
(c) Axial column deformation is ignored.

3.4—Computer programs

3.4.1 *General*—Computing power and structural software have advanced significantly from the time computers were introduced to the designer. Numerous complex computer programs and specialized analysis tools have been developed, taking advantage of increasing computer speeds. Currently, designers commonly use finite element analysis to design structures. A multistory building only takes minutes of computing time on a personal computer compared to the past, when it would take several hours on a large mainframe computer. Computer software has also greatly improved: user interfaces have become more intuitive; members can be automatically meshed; and input and output data can be reviewed graphically and tabular in a variety of preprogrammed or user-defined menus.

Although three-dimensional models are becoming commonplace, many engineers still analyze the building as a series of two-dimensional frames. Matrix methods mentioned in Table 3.2.5 are programs based on the direct stiffness method. Simpler programs model the structure as discrete members connected at joints. The members can be divided into multiple elements to account for changes in member properties along its length. The two-dimensional stiffness method is relatively easy to program and evaluate. A more sophisticated use of the direct stiffness method is the finite element method. The structure is modeled as discrete elements connected at nodes. Each member of the structure consists of multiple rectangular elements, which more accurately determine the behavior of the member. The act of modeling members as an assembly of these discrete elements is called "meshing" and can be a time-consuming task. A large amount of data is generated from this type of analysis, which may be tedious for the designer to review and process. For straightforward designs, a designer may more efficiently analyze the structure by dividing it into parts and using less complicated programs to analyze each part.

3.4.2 *Two-dimensional frame modeling*—A building can be divided into parts that are analyzed separately. For example, structures are often symmetrical with regularly spaced columns in both directions. There may be a few isolated areas of the structure where columns are irregularly spaced. These columns can be designed separately for gravity load and checked for deformation compatibility when subjected to the expected overall lateral deflection of the structure.

Buildings designed as moment-resisting frames can often be effectively modeled as a series of parallel planar frames. The complete structure is modeled using orthogonal sets of crossing frames. Compatibility of vertical deflections at crossing points is not required. The geometry of beams can vary depending on the floor system. For slab-column moment frames, it may be possible to model according to the equivalent frame method in Section 8.11 in ACI 318. For beam-column moment frames, it is permitted to model T-beams, with the limits on geometry given in Section 6.3.2 of ACI 318; however, it is often simpler to ignore the slab and model the beams as rectangles. For beams in intermediate or special moment frames, the assumption of a rectangular section may not be conservative; refer to Sections 18.4.2.3, 18.6.5.1, and 18.7.3.2 in ACI 318.

The stiffness of the beam-column joint is underestimated if the beam spans are assumed to extend between column centerlines and the beam is modeled as prismatic along the entire span. Many computer programs thus allow for the beam to be modeled as spanning between faces of columns. To do this, a program may add a rigid zone that extends from the face of the column to the column centerline. If the program does not provide this option, the designer can increase the beam stiffness in the column region 10 to 20 percent to account for this change of rigidity (ACI 442-71).

Walls with aspect ratios of total height to width greater than 2 can sometimes be modeled as column elements. A thin wall may be too slender for a conventional column analysis, and a more detailed evaluation of the boundary elements and panels may be necessary. Where a beam frames into a wall that is modeled as a column element, a rigid link should be provided between the edge of the wall and centerline of the wall. All walls can be modeled using finite element analysis. Where a finite beam element frames into a finite wall element, rotational compatibility should be assured.

Walls with openings can be more difficult to analyze. A wall with openings can be modeled as a frame, but the rigidity of the joints needs to be carefully considered. Similar to the beam-column joint modeling discussed previously, a rigid link should be modeled from the centerline of the wall to the edge of the opening (Fig. 3.4.2a). Finite element analysis and the strut-and-tie method, however, are more commonly used to analyze walls with openings.

For lateral load analysis, all of the parallel plane frames in a building are linked into one plane frame to enforce lateral deformation compatibility. Alternately, two identical frames can be modeled as one frame with doubled stiffness, obtained by doubling the modulus of elasticity. Structural walls, if present, should be linked to the frames at each floor level (Fig. 3.4.2b). Note that torsional effects need to be considered after the lateral deformation compatibility analysis is run. For seismic loads, rigid diaphragms are required to account for accidental torsion according to Section 12.8.4.2 in ASCE 7-10. For wind loads, a torsional moment should be applied according to Fig. 27.4-8 in ASCE 7-10.

3.4.3 *Three-dimensional modeling*—A three-dimensional model allows the designer to observe structural behavior that a two-dimensional model would not reveal. The effects of structural irregularities and torsional response can be directly analyzed. Current computer software that provides three-dimensional modeling often use finite element analysis with automatic meshing. These high-end programs are capable of running a modal response spectrum analysis, seismic response history procedures, and can perform a host of other time-consuming mathematical tasks.

To reduce computation time, concrete floors are sometimes modeled as rigid diaphragms, reducing the number of dynamic degrees of freedom to only three per floor (two horizontal translations and one rotation about a vertical axis). ASCE 7-10 allows for diaphragms to be modeled as rigid if the following conditions are met:

(a) For seismic loading, no structural irregularities and the span-to-depth ratios are 3 or less (Section 12.3.1.2 in ASCE 7-10)
(b) For wind loading, the span-to-depth ratios are 2 or less (Section 27.5.4 in ASCE 7-10)

If a rigid diaphragm is assumed, the stresses in the diaphragm are not calculated and need to be derived from the reactions in the walls above and below the floor. A semi-rigid diaphragm requires more computation power but provides a distribution of lateral forces and calculates slab stresses. A semi-rigid diaphragm can also be helpful in analyzing torsion effects. For more information on torsion, refer to the code and commentary in Section 12.8 of ASCE 7.

3.5—Structural analysis in ACI 318

3.5.1 *Arrangement of live loads*—Section 4.3.3 in ASCE 7 states that the "full intensity of the appropriately reduced live load applied only to a portion of a structure or member shall be accounted for if it produces a more unfavorable load effect than the same intensity applied over the full structure or member." This is a general requirement that acknowledges greater moments and shears may occur with a pattern load than with a uniform load. There have been a variety of methods used to meet this requirement. Cast-in-place concrete is inherently continuous, and Section 6.4 in ACI 318 provides acceptable arrangements of pattern live load for continuous one-way and two-way floor systems.

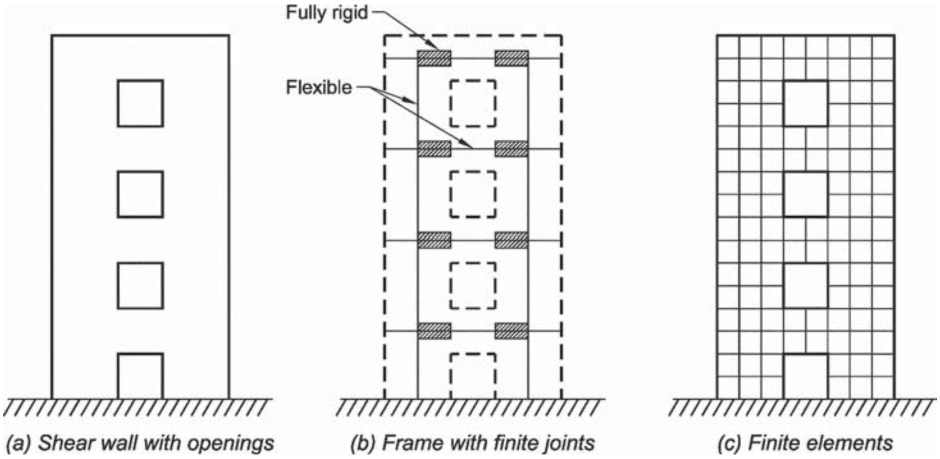

Fig. 3.4.2a—Element and frame analogies.

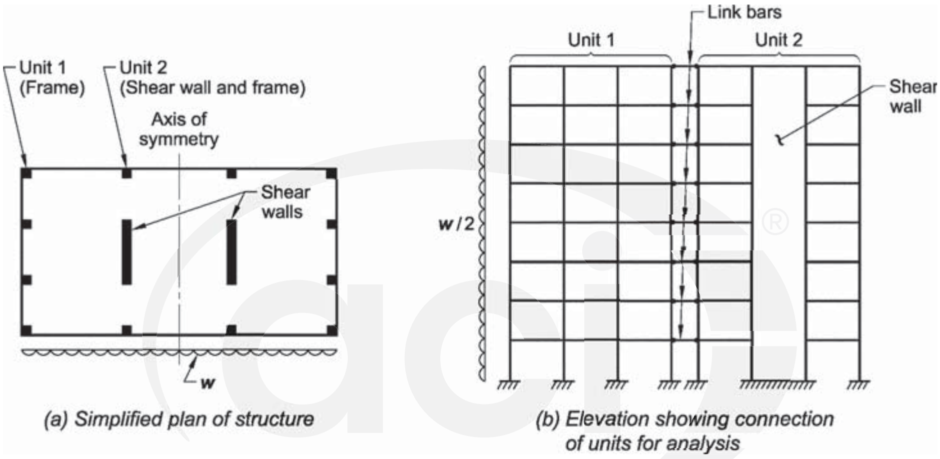

Fig. 3.4.2b—Idealization for plane frame analysis.

3.5.2 *Simplified method of analysis for nonprestressed continuous beams and one-way slabs*—Section 6.5 in ACI 318 provides approximate equations for conservative design moments and shears, which greatly simplifies the design of continuous floor members. This method is probably used more often to estimate initial member sizes for computer input, or for initial cost estimates, than for final design.

3.5.3 *First-order analysis*—Code requirements for first-order analysis are provided in Section 6.6 in ACI 318.

3.5.3.1 *Section properties*—Section properties for elastic analysis are given in Table 6.6.3.1.1(a) of ACI 318. The moment of inertia values have a stiffness reduction ϕ_k of 0.875 already applied. These properties are acceptable for the analysis of the structure for strength design. For service level load analysis, the moment of inertia values in Table 6.6.3.1.1(a) can be multiplied by 1.4. Table 6.6.3.1.1(b) offers a more accurate estimation of stiffness by including the effects of axial load, eccentricity, reinforcement ratio, and concrete compressive strength. These equations can also be used to calculate member stiffness at factored load levels by using the factored axial load and moment, as presented, but the equations can be used to calculate member stiffness for any given axial load and moment. These moment-of-inertia equations also have the 0.875 stiffness reduction ϕ_k already applied. Section 6.6.3.1.2 of ACI 318 also allows a simplification of using $0.5I_g$ for all members in a lateral load analysis. This is helpful for hand-calculation methods such as the portal method.

It is important to note that the stiffness reduction factor used for moment of inertia discussed previously is for global building behavior. The moment of inertia for second-order effects related to an individual column or wall should have a stiffness reduction ϕ_k of 0.75, as discussed in R6.6.4.5.2 of ACI 318.

3.5.3.2 *Slenderness effects*—A first-order analysis in ACI 318 assumes that only primary stresses are calculated. Secondary stresses caused by the lateral deflection caused by the design loads are not calculated. First-order analysis is typical when hand-calculation methods are used or basic matrix analysis computer programs are used that are not programmed for iterative analysis.

This method ignores the P-Δ effects, which are the second-order moments caused by vertical loads acting on the building's laterally deformed configuration (Fig. 3.5.3.2). To approximately account for these secondary effects, a moment magnifier is applied to first-order column design moments.

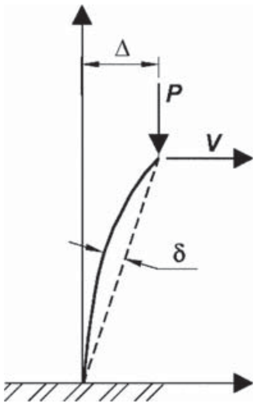

Fig. 3.5.3.2—P-Δ effects.

The designer must account for slenderness in a first-order analysis. Figure R6.2.6 in ACI 318 provides a flow chart that illustrates the options to account for slenderness. In summary, slenderness can be neglected if the column or wall meets the requirement of Section 6.2.5 in ACI 318. If slenderness cannot be neglected, the next step is to determine if the building story being analyzed is sway or nonsway. If the story is nonsway, the column or wall end moments are only magnified for *P*-δ effects along the member. If the story is sway, the column or wall end moments are magnified for *P*-Δ effects along the member (*P*-δ) and at the ends due to story drift (*P*-Δ).

3.5.3.3 *Superposition*—Linear analysis allows for superposition to be used when combining loads. This is very helpful when performing hand calculations. The designer calculates the member moment, shear, and axial load for each load. The reactions are then superimposed according to the applicable load combination. Many hand-calculation tools, such as the moment magnification method, assume that the designer is performing a linear analysis with superposition of multiple load effects.

3.5.3.4 *Redistribution of moments*—ACI 318 allows for the designer to adjust design slab and beam moments and shears by taking advantage of the ductility provided through the code detailing requirements. Ductile detailing is required for continuous fibers at supports and midspan. Moment redistribution can be very helpful in creating economical designs. For example, in a final design, moment redistribution may permit the designer to specify uniform beam sizes over multiple beam spans. If column spacing is not uniform, some beam design moments may be slightly lower than the beam required moments. Once steel yielding has developed at factored loads, however, the moments will redistribute to regions that have not yet yielded, and the beam design moments will satisfy the beam required moments throughout the multiple beam spans.

3.5.4 *Second-order analysis*—In Section 6.7 of ACI 318, a second-order analysis assumes that the effect of loads on the laterally deformed structure is included in the computer analysis. The initial *P*-Δ effects on the member due to story drift are computed. A computer algorithm then automatically carries out a series of iterative analyses using the new deflection values until the solution converges to the final secondary moments. Note that linear material properties are used with this method, but the results of a second-order analysis is a nonlinear solution. This is referred to as "geometric nonlinearity." This means that the load cases cannot be computed separately and then combined for the calculation of the secondary moments.

Software should be checked to determine how it accounts for *P*-Δ effects. Software can easily calculate the additional moment due to building lateral deformation but some software does not calculate the secondary moments along the member length. The designer may have to model the column as at least two segments to capture this effect. Even though a column member is modeled by two elements, the designer must account for the smaller stiffness reduction factor for the moment of inertia (refer to Section 3.5.3.1), because deflection along the member is a local effect. Because of the difficulty of appropriately capturing the secondary moment along the column length, many programs calculate the secondary moments due to lateral deformation and use 6.6.4.5 of ACI 318 in a post-processing program to account for the secondary moment along the length of the column.

3.5.5 *Inelastic second-order analysis*—The consideration of the nonlinear behavior of structures arising from nonlinearity of the stress-strain curve for concrete and steel reinforcement, particularly under large deformations, can become important in seismic analysis. Nonlinearities in structural response, whether arising from material properties as for concrete or steel, loading conditions (for example, axial load effects on bending stiffness) or geometry (for example, second-order moments) are best handled by numerical iterative or step-by-step procedures. For inelastic second-order analysis, the principle of superposition should not be used. Nonlinear analysis is beyond the scope of this Handbook. Several references that provide further information on nonlinear analysis are ASCE 41-13, FEMA report, and Deierlein et al. (2010).

3.5.6 *Finite element analysis*—The finite element analysis of concrete structures is permitted by ACI 318 and can be used to satisfy each the first-order, elastic second-order, and inelastic second-order analyses as long as the element types are compatible with the response required.

Section 6.9 in ACI 318 was added to acknowledge that finite element analysis is a widely used and acceptable tool for analysis. Many programs are based on finite element analysis and have sophisticated auto-mesh capabilities. Finite element analysis is a tool that may be used for either linear or nonlinear analyses, but care should be exercised in selecting element types, numerical solver methods, and nonlinear element properties.

3.6—Seismic analysis

For seismic loads, the structure may go through multiple cycles of significant inelastic deformations. ASCE 7 provides the equivalent lateral force (ELF) analysis procedure (Section 12.8 of ASCE 7) to allow a linear elastic analysis even though the structure will actually behave inelastically. The ELF procedure is a commonly used design method and is adequate for most structures. The ELF analysis assumes

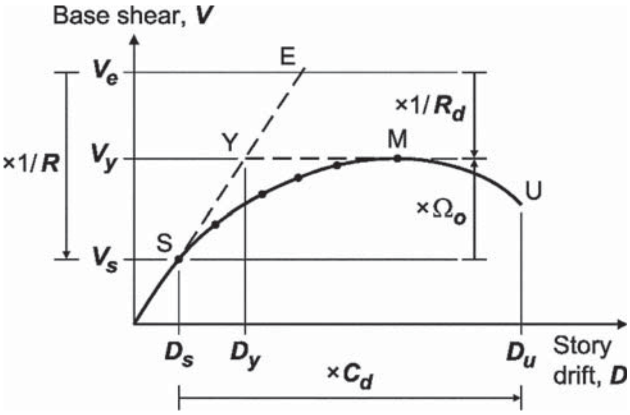

Fig. 3.6—Inelastic force-deformation curve (SEAOC Seismology Committee 2008).

an approximately uniform distribution of mass and stiffness along the building height with minor torsional effects. Structures analyzed using the ELF procedure for seismic loads must comply with several limitations. Depending on the Seismic Design Category (SDC), the building height and type, and the type of structural irregularities, a Modal Response Spectrum analysis (Section 12.9 of ASCE 7) or Seismic Response History procedure, either linear or nonlinear (Ch. 16 of ASCE 7), may be required. Regardless of irregularities, the ELF procedure is acceptable for all buildings in SDC A and B up to 160 ft in height.

Section 12.3 of ASCE 7 provides limitations related to types of structural irregularities. ASCE 7 describes five horizontal irregularities: 1) torsion; 2) reentrant corner; 3) diaphragm discontinuity; 4) out-of-plane offset; and 5) nonparallel systems. ASCE 7 also describes five vertical irregularities: 1) stiffness-soft story; 2) weight; 3) vertical geometry; 4) in-plane discontinuity in lateral force-resisting systems (LFRSs); and 5) discontinuity in lateral strength.

Because the actual structure will undergo greater deflections and stresses than predicted by an ELF analysis, ASCE 7 and ACI 318 have additional requirements to account for the anticipated behavior. Lateral-force-resisting systems for concrete structures are defined in ASCE 7 and Chapter 18 of ACI 318. Each LFRS has a response modification coefficient R, overstrength factor Ω, and deflection amplification factor C_d (Fig 3.6) used in analysis. These factors account for the difference between the actual expected design forces and displacements and the estimated behavior. Detailed explanations of these factors and their application can be found in ASCE 7 and FEMA P-750.

For reinforced concrete structures, ACI 318 provides structures with the ability to deform inelastically by enforcing special seismic detailing requirements. The seismic detailing requirements in Chapter 18 of ACI 318 are additive to the detailing requirements in the member chapters, or the seismic detailing requirements supersede the member chapter requirements. The detailing requirements for a particular LFRS need to be applied even if seismic loads do not govern the required strength of the structure.

REFERENCES

American Concrete Institute

ACI Committee 442-77—Response of Buildings to Lateral Forces

American Society of Civil Engineers

ASCE 7-10—Minimum Design Loads for Buildings and Other Structures

ASCE 41-13—Seismic Evaluation and Retrofit Rehabilitation of Existing Buildings

Federal Emergency Management Agency

FEMA 440-05—Improvement of Nonlinear Static Seismic Analysis Procedures

FEMA P-750—NEHRP Recommended Seismic Provisions for New Buildings and Other Structures

Authored references

Deierlein, G. G.; Reinhorn, A. M.; and Willford, M. R., 2010, "Nonlinear Structural Analysis for Seismic Design," *NEHRP Seismic Design Technical Brief No. 4 (NIST GCR 10-917-5)*, National Institute of Standards and Technology, Gaithersburg, MD, 36 pp.

Hibbeler, R., 2015, *Structural Analysis*, ninth edition, Prentice Hall, New York, 720 pp.

SEAOC Seismology Committee, 2008, "A Brief Guide to Seismic Design Factors," *Structure Magazine*, Sept., pp. 30-32. http://www.structurearchives.org/article.aspx?articleID=756

CHAPTER 4—DURABILITY

4.1—Introduction

Durability of structural concrete is its ability, while in service, to resist possible deterioration due to the surrounding environment, and to maintain its engineering properties. This can be accomplished by proper proportioning and selection of materials for the concrete mixture design. Other aspects influencing durability include reinforcing bar selection, detailing, and construction practices. The ACI 318 Code provides minimum requirements to protect the structure against early serviceability deterioration. Depending on exposure conditions, structural concrete may be required to resist chemical or physical attack, or both. The attack mechanisms the Code covers include exposure to freezing and thawing, soil and water sulfates, wetting and drying, and reinforcement corrosion due to chlorides. All these failure mechanisms depend on transport of water through concrete. For this reason, it is essential to understand the mechanisms themselves and how different concrete-making materials, including admixtures and their proportions, influence concrete's resistance to these mechanisms.

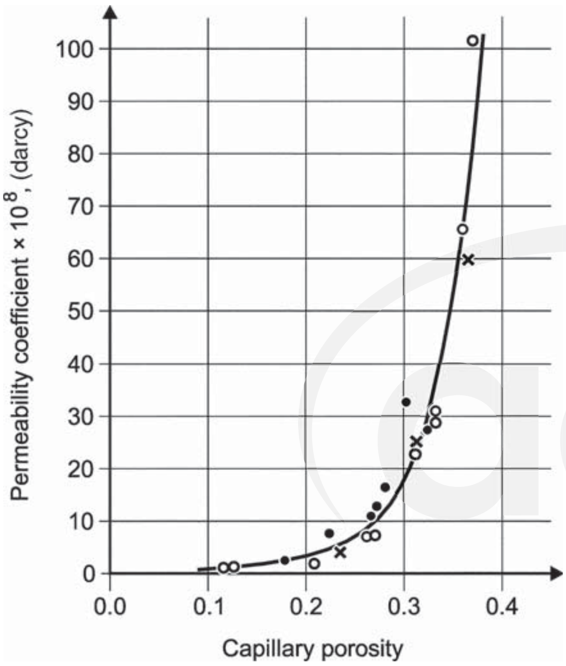

Fig. 4.1.1a—Permeability versus capillary porosity for cement paste. Different symbols designate different cement pastes (Powers 1958).

4.1.1 *Permeability*—Permeability can be defined as "the ease with which a fluid can flow through a solid" or as "the ability of concrete to resist penetration by water or other substances (liquid, gas, or ions)" (ACI 365.1R; Kosmatka and Wilson 2011). Low-permeability concretes are more resistant to resaturation, freezing and thawing, sulfate and chloride ion penetration, and other forms of chemical attack (Kosmatka and Wilson 2011). Concrete permeability is related to porosity (volume of voids/pores in concrete) and connectivity of these pores. Out of the pores present in concrete, capillary pores in cement paste are relevant to concrete durability, as they are responsible for the transport properties of concrete (ACI 201.2R; Kosmatka and Wilson 2011). The influence of capillary porosity in cement paste on permeability was reported by Powers (Fig. 4.1.1a) (Powers 1958).

Concrete permeability, diffusivity, and electrical conductivity can be reduced with lower water-cement ratios (*w*/*c*), the use of SCMs, and extended moist curing (Kosmatka and Wilson 2011). Effects of the *w*/*c* and duration of the moist curing on permeability is presented in Fig. 4.1.1b.

4.1.2 *Freezing and thawing*—When water freezes in concrete, it causes cement paste to dilate destructively by generating hydraulic and osmotic pressure. While hydraulic pressure forces water away from the freezing water-filled capillary cavities, osmotic pressure is produced by water

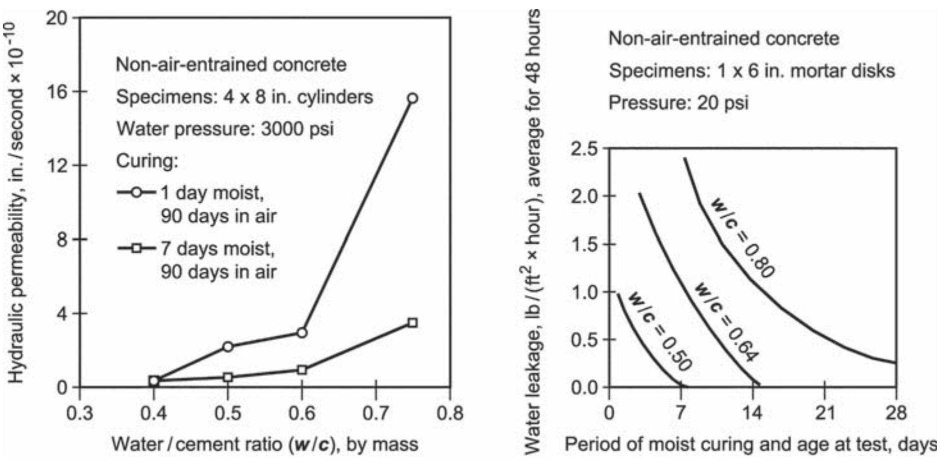

Fig. 4.1.1b—(left) Effect of w/c *and initial curing on hydraulic (water) permeability; and (right) effect of* w/c *and curing duration on permeability (leakage) of mortar (Kosmatka and Wilson 2011).*

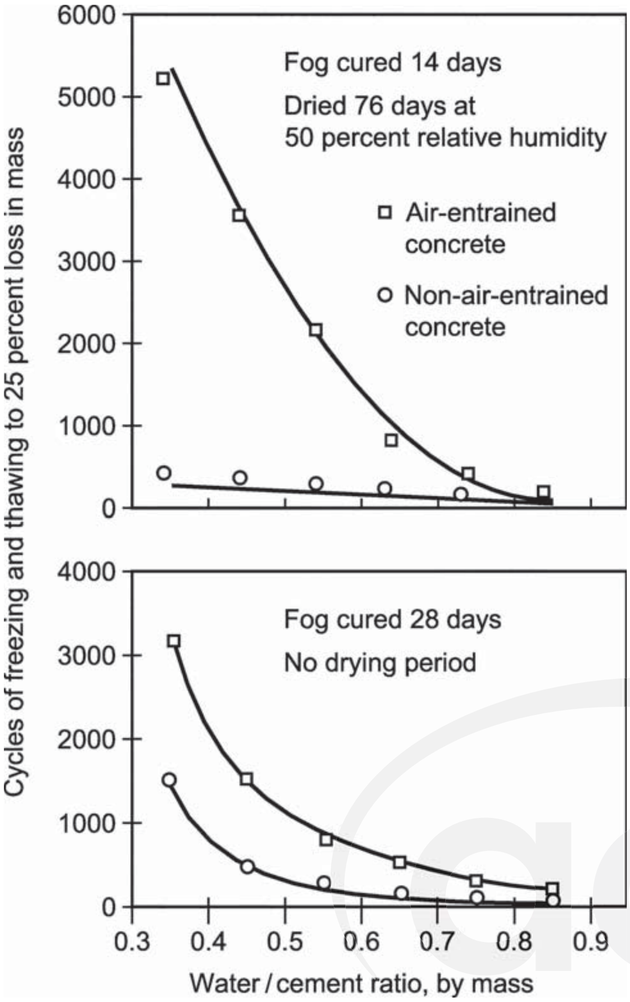

Fig. 4.1.2—Effect of w/c*, air-entrainment, and curing/drying on resistance to freezing and thawing of concrete (Kosmatka et al. 2008).*

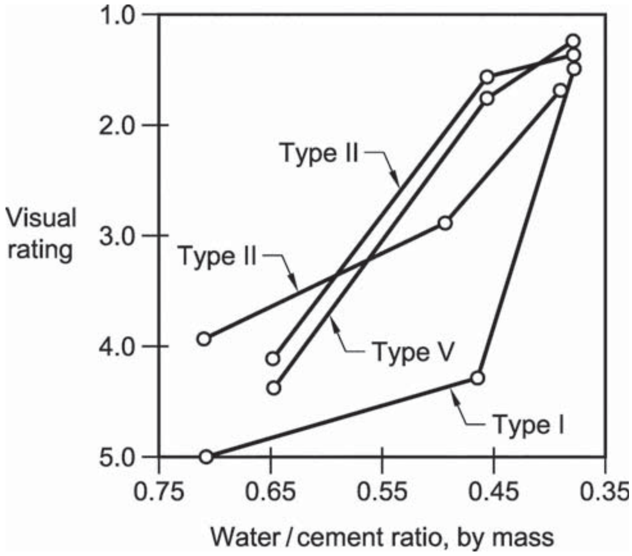

Fig. 4.1.3—Effect of w/c ratio on sulfate resistance for different ASTM C150/C150M types of cement, (lower visual rating indicates better resistance) (Stark 1989).

entering partly frozen capillary cavities (Powers 1958). Hydraulic pressures in cement paste are generated by the 9 percent expansion of water when it freezes and changes to ice. For the freezing to take place, a capillary has to reach its critical saturation: 91.7 percent filled with water (Kosmatka and Wilson 2011). Osmotic pressures develop due to various concentrations of alkali solutions in the paste.

When pressure in concrete due to freezing exceeds the tensile strength of concrete, some damage occurs, especially if concrete is saturated with water and exposed to repeated cycles of freezing and thawing. The resulting damage is cumulative as it increases with additional repetitions of freezing and thawing cycles. Deterioration due to freezing and thawing can appear in the form of cracking, scaling, disintegration, or all three of these (Kosmatka and Wilson 2011).

Low permeability and low absorption are main characteristics needed for concrete to be frost resistant, while air-entraining admixtures are used to control the pressure generated in concrete paste during freezing-and-thawing cycles. In other words, high resistance to freezing and thawing is associated with entrained air, low *w/c*, and a drying period prior to freezing-and-thawing exposure, which is demonstrated by data presented in Fig. 4.1.2.

4.1.3 *Sulfates*—Sulfates present in soil and water can react with hydrated compounds in the hardened cement paste and induce sufficient pressure to disintegrate the concrete. However, formation of new crystalline substances due to those reactions is partly responsible for the expansion. If water can freely diffuse out of capillaries in the cement paste, the volume of growing crystals cannot exceed the space available to them. At the same time, however, the swelling pressure can also arise from the diffusion of the sulfate salts into the gel pores, which disturbs the equilibrium between the gel and its surrounding liquid phase, resulting in the movement of more water from the outside into the gel pores (Hewlett 1998).

Although ordinary portland cements are most susceptible to sulfate attack, the use of sulfate-resistant cements will not stop the sulfate attack, either (Fig. 4.1.3). Resistance to sulfate attack can be greatly increased by decreasing the permeability of concrete through reduction of the water-cementitious material ratio (*w/cm*) (Stark 1989).

4.1.4 *Corrosion*—Alkaline nature of concrete (pH greater than 13) will induce formation of a passive, noncorroding layer on reinforcing steel. If, however, chloride ions are present in concrete, they can reach and disrupt that layer and lead to corrosion of steel in the presence of water and oxygen. Once corrosion initiates, corrosion products form and may cause cracking, spalling, or delamination of concrete. This allows for easier access of aggressive agents to the steel surface and increases the rate of corrosion. Cross-sectional area of the corroding steel will decrease and the load-carrying capacity of the member will be reduced (Neville 2003).

Chlorides can be introduced to concrete with materials used to produce the mixture (contaminated aggregate or water, or some admixtures), with deicing chemicals, or through marine exposure (seawater or brackish water). To reduce likelihood of corrosion initiation, total chloride-ion content should not exceed a certain concentration value, referred to as the chloride threshold. A literature review of reported chloride threshold values revealed "that there is no single threshold value, but a range based on the conditions and materials in use" and were found to vary from 0.1 to 1 percent by mass of cement (Taylor et al. 1999). ACI 318-14 limits water-soluble chlorides to 0.15 percent by mass of cement for concrete exposed to external chlorides or seawater. Value of 0.40 percent total chloride by mass of cement is given in British and European Standards (Neville 2003; Whiting et al. 2002).

Corrosion of the reinforcing steel in concrete can be reduced or prevented by minimizing the *w/cm* ratio (permeability), ensuring maximum cover depth of concrete over steel (Stark 2001), and use of corrosion inhibitors or corrosion resistant steel.

4.2—Background

To produce durable structural concrete, concrete materials and mixture proportions are selected based on design strength requirements, anticipated exposure conditions, and required service life of the structure. The selection of materials and mixture proportions has to be accompanied by appropriate field practices, such as quality control, testing, inspection; and proper placement, finishing, and curing practices.

As stated in Section 1.3.1 of ACI 318-14, "*The purpose of this Code is to provide for public health and safety by establishing minimum requirements for strength, stability, serviceability, and integrity of concrete structures.*" Section 4.8 of ACI 318-14 addresses global durability requirements related to material selection for concrete mixtures and corrosion protection of reinforcement. ACI 318-14, Chapters 19 and 20, provide detailed durability requirements for concrete and reinforcing steel, respectively. ACI 318-14, Chapter 26, discusses what durability requirements must be specified in a project's construction documents.

The Code's durability focus is mainly on concrete resistance to fluid penetration, which is primarily affected by the *w/cm* and the composition of those materials. The use of SCMs, such as Type F and Type C fly ashes, slag cement, silica fume, calcined shale, calcined clay or metakaolin, or their combinations, can result in a significant improvement in concrete durability. The SCMs affect concrete properties in many ways, depending on their type, dosage, and other mixture proportions and composition. In general, SCMs have the following impacts on hardened concrete properties (Kosmatka and Wilson 2011):

- Increase long-term strength
- Have varied effect on early age strength gain (Type F fly ash, calcined shales, and clays lower early strength; silica fume and metakaolin increase early strength gain)
- Reduce permeability and absorption
- Improve resistance to corrosion
- Increase sulfate resistance (with the exception of Type C fly ash, which may have either a positive or negative effect)
- Have no significant impact on abrasion resistance, drying creep and shrinkage, and freezing and thawing
- May reduce resistance to deicer scaling

The Code does not cover all topics related to concrete durability. It does not include recommendations for extreme exposure conditions (that is, acids, high temperature, or exposure to fire), alkali-aggregate reaction, or abrasion. The Code commentary (R4.8) identifies the importance of preventive maintenance; however, the topic is not explicitly addressed in the Code. Additionally, the Code does not cover waterproofing, routine inspections, condition assessment, or service life prediction. Information related to these topics are found in other ACI documents, including:

- ACI 201.2R—Guide to Durable Concrete
- ACI/TMS 216.1—Code Requirements for Determining Fire Resistance of Concrete and Masonry Construction Assemblies
- ACI 221.1R—Report on Alkali-Aggregate Reactivity
- ACI 362.1R—Guide for the Design and Construction of Durable Concrete Parking Structures
- ACI 222R—Protection of Metals in Concrete Against Corrosion
- ACI 222.2R—Report on Corrosion of Prestressing Steels
- ACI 222.3R—Guide to Design and Construction Practices to Mitigate Corrosion of Reinforcement in Concrete Structures
- ACI 224.1R—Causes, Evaluation, and Repair of Cracks in Concrete Structures
- ACI 311.4R—Guide for Concrete Inspection
- ACI 365.1R—Service-Life Prediction
- ACI 515.2R—Guide to Selecting Protective Treatments for Concrete
- ACI 562—Code Requirements for Evaluation, Repair, and Rehabilitation of Concrete Buildings and Commentary

4.3—Requirements for concrete in various exposure categories

The Code addresses durability by requiring that four exposure categories be assigned to each concrete member. The four exposure categories are:

1. F: concrete exposed to moisture and cycles of freezing and thawing (with or without deicing chemicals);
2. S: concrete in contact with soil or water containing deleterious amounts of water-soluble sulfate ions;
3. W: concrete in contact with water but not exposed to freezing and thawing, chlorides, or sulfates;
4. C: concrete exposed to conditions that require additional protection against corrosion of reinforcement.

Each exposure category is divided into exposure classes that define severity of the exposure, starting with 0 for a negligible exposure. Once all structural members are assigned exposure classes and the concrete mixtures for these members satisfy those various requirements, the Code's minimum durability requirements are met.

4.3.1 *Freezing and thawing (F)*—The volume of ice is 9 percent larger than water. As water freezes in saturated concrete, cement phase and aggregates are subject to internal pressure, which then causes concrete tensile stresses. If those stresses are greater than the tensile strength of concrete, cracking will occur. The cumulative expansion after many cycles of freezing and thawing may lead to significant concrete damage. One method to protect concrete from freezing-and-thawing damage is to reduce moisture penetration so it does not become critically saturated; however, this is not always possible. The other method is to generate small air bubbles in fresh concrete by addition of an air-entraining admixture, which creates voids for the freezing water to expand into without creating internal stress.

The Code requires concrete in structural members exposed to cycles of freezing and thawing to be protected by using air-entrained concrete. Air entrainment significantly improves resistance of saturated concrete to freezing and thawing. ACI 212.3R provides an in-depth discussion on these materials, their applications, dosage rates, effects on fresh and hardened concrete, and other factors they influence.

The specified amount of air entrainment depends primarily on frequency of exposure to water (exposure class), but also on nominal maximum aggregate size and concrete compressive strength. To achieve similar freezing-and-thawing protection, higher air content is generally required for concrete mixtures with smaller nominal maximum aggregate size. For example, concrete with 3/8 in. aggregate requires 50 percent higher air content than concrete with 2 in. aggregate (ACI 318-14, Table 19.3.3.1). The Code requires that the licensed design professional (LDP) specify the nominal maximum aggregate size for each concrete mixture in the construction documents. Nominal maximum aggregate size depends on locally available aggregates, as well as construction issues such as size of formwork, member depth, and clear bar spacing. The criteria for maximum size selection are given in Section 26.4.2.1 of ACI 318-14. Table 19.3.3.1 lists target air content for Classes F1, F2 and F3, depending on the nominal maximum aggregate size.

Another factor affecting selection of target air content is compressive strength. An air content reduction of 1 percent is allowed for concrete with specified compressive strength exceeding 5000 psi (ACI 318-14, Section 19.3.3.3). The reason for air content reduction is that concretes with higher strengths are characterized by lower *w*/*cm* and reduced porosity, which improve resistance to freezing-and-thawing cycles.

For example, a structural member in Exposure Class F2 with 1/2 in. nominal maximum aggregate size requires concrete with a target air content of 7 percent (or 6 percent for concrete with compressive strength exceeding 5000 psi). Because exact air content is difficult to achieve, the Code allows tolerance for air content in as-delivered concrete of ±1.5 percentage points. This is consistent with the tolerances in ASTM C94/C94M and ASTM C685/C685M (Section R26.4.2.1(a)(5)). The required air content range, therefore, is from 5.5 to 8.5 percent (or 4.5 to 7.5 percent for concrete with compressive strength exceeding 5000 psi).

Additional requirements or limitations, such as minimum compressive strength, minimum *w*/*cm*, or limits on cementitious materials, depend on the exposure class assigned to a particular member. Interior members, foundations below the frost line, or structures in climates where freezing temperatures are not anticipated are assigned Exposure Class F0. These conditions, therefore, do not require air entrainment and there is no limit on maximum *w*/*cm* or on the use of cementitious materials. The minimum compressive strength for concrete in Exposure Class F0 is the Code minimum: 2500 psi.

Freezing-and-thawing cycles have little effect on concrete that is not critically saturated. Structural members exposed to freezing and thawing cycles, but with low likelihood of being saturated, are assigned exposure class F1. Concrete for this exposure must be air entrained (Table 19.3.3.1 of ACI 318) in case there is occasional saturation during freezing. In addition, the concrete should have maximum *w*/*cm* of 0.55 and at least 3500 psi compressive strength.

Exposure Classes F2 and F3 are assigned to concrete in structural members with a high likelihood of water saturation during freezing. The distinction between the two classes is that Class F2 anticipates no exposure to deicing chemicals or seawater, while Class F3 does. Concrete in F2 and F3 exposure classes must be air entrained (Table 19.3.3.1 of ACI 318) and have a maximum *w*/*cm* of 0.45 and 0.40, respectively. The minimum concrete compressive strengths for F2 and F3 classes are 4500 and 5000 psi, respectively. The most severe class of exposure, F3, also has a limit on cementitious materials in concrete mixtures, given in ACI 318, Table 26.4.2.2(b).

The summary of requirements for concrete in Exposure Category F is listed in Table 19.3.2.1 of ACI 318.

4.3.2 *Sulfate (S)*—All soluble forms of sulfate, sodium, calcium, potassium, or magnesium have a detrimental effect on concrete. Depending on the sulfate form, they react with hydrated cement phases and result in formation of ettringite or gypsum. Depending on the reaction product, the concrete either expands and cracks (ettringite), or softens and loses strength (gypsum). The most effective measure to reduce the effects of sulfate reactions, apart from reducing moisture ingress, is to use cements with a low content of tricalcium aluminate (C_3A). A more detailed discussion on sulfate's effect on concrete can be found in ACI 201.2R.

Exposure Category S applies to structural members that will likely be affected by external source of sulfates, which predominantly come from exposure to soil, groundwater, or seawater. The exposure classification (class) is selected based on the concentration of sulfate ions (SO_4^{2-}), which should be determined in accordance with ASTM C1580 for soil samples and with ASTM D516 or ASTM D4130 for water samples. The Code requires the LDP to specify the exposure class by comparing field test results with concentration ranges in Table 19.3.1.1 of ACI 318. Note that seawater exposure is classified as S1 even though the sulfate concentration (in seawater) is usually higher than 1500 ppm. The reason for lower class for seawater is the presence of chloride ions, which inhibit expansive reaction due to sulfate attack.

Class S0 is assigned to concrete in members not exposed to sulfates and there is no restriction on *w*/*cm*, or type or limit on cementitious materials. The only requirement for concrete classified as S0 is the minimum compressive strength be at least 2500 psi. Greater minimum compressive strength and maximum *w/cm* limits are imposed on concrete in Exposure Classes S1 through S3. For these exposure classes, the type of cement is the major requirement.

A summary of all requirements for concrete in Exposure Category S is listed in Table 19.3.2.1 of ACI 318-14.

4.3.3 *In contact with water (W)*—The durability of structural members in direct contact with water, such as foundation walls below the groundwater table, may be affected by water penetration into or through concrete. Apart from external systems, such as drainage systems or waterproofing membranes for foundations, the most effective way to reduce concrete permeability is to keep the *w/cm* low.

Concrete for members assigned to Exposure Class W0 has no unique requirements except that it has a minimum compressive strength of 2500 psi. Concrete in structural members assigned to Exposure Class W1 requires low permeability. Table 19.3.2.1 of ACI 318-14 requires *w/cm* not to exceed 0.50 and compressive strength to be at least 4000 psi. Note that additional requirements are imposed if the member's durability is to be affected by reinforcement corrosion, sulfate exposure, or exposure to cycles of freezing and thawing. Recommendations for the design and construction of water tanks and reservoirs are provided in ACI 350.4R, ACI 334.1R, and ACI 372R.

Requirements for concrete in Exposure Class W are listed in Table 19.3.2.1 of ACI 318-14.

4.3.4 *Corrosion (C)*—Corrosion of reinforcement may significantly affect durability and structural capacity of a member. Reinforcement corrosion usually occurs as a result of the presence of chlorides or steel depassivation due to carbonation. Corrosion products (rust) are larger in volume than the original steel and therefore exert internal pressure on the surrounding concrete, causing it to crack or delaminate. A significant loss of reinforcing bar cross section leads to increased steel stresses under service load and reduced member nominal strength. Because moisture and oxygen must be present at the steel surface for corrosion to occur, the quality of concrete and the reinforcing bar cover are of great importance. Corrosion can be mitigated by proper mixture design and construction practices; application of sealers, coatings, or membranes that protect concrete from moisture and chloride penetration; use of corrosion resistant reinforcement; or inclusion of corrosion inhibitors in the mixture to elevate the corrosion threshold concentration. Refer to ACI 222R, ACI 222.3R, and ACI 212.3R for additional information.

Each exposure class within the corrosion exposure category has a limit on water-soluble chloride-ion content in concrete. The chloride-ion content is measured in accordance with ASTM C1218/C1218M, which requires the sample be representative of all concrete-making ingredients—that is, cementitious materials, fine and coarse aggregate, water, and admixtures. Because chloride limits are imposed even on concrete in Exposure Class C0, all structural concrete must comply with the Code's maximum chloride ion limits. Chloride limits for nonprestressed concrete, expressed as percent of cement weight, are 1 percent for Class C0, 0.30 percent for Class C1, and 0.15 percent for Class C2. Chloride limit for prestressed concrete is 0.06 percent by cement weight, regardless of exposure class. Apart from chloride limits, Exposure Classes C0 and C1 have no additional requirements, as there is no limit on *w/cm* and the minimum compressive strength is 2500 psi.

Class C2 requires concrete strength of at least 5000 psi, a maximum *w/cm* of 0.40, and reinforcing steel specified cover to satisfy the Code's minimum concrete cover provisions. The minimum concrete cover depends on exposure to weather, contact with ground, type of member, type of reinforcement, diameter and arrangement (bundling) of reinforcement, method of construction (cast-in-place or precast), and if the member is prestressed. Tables 20.6.1.3.1, 20.6.1.3.2, and 20.6.1.3.3 of ACI 318-14 provide cover provisions for cast-in-place nonpressessed, cast-in-place pressessed, and precast nonpressessed or pressessed concrete members, respectively. If the design requires bundled bars, check Section 20.6.1.3.4 of ACI 318-14 for specific requirements. Concrete cover requirements in corrosive environments or other severe exposure conditions are more stringent and are provided in Section 20.6.1.4 of ACI 318-14.

Requirements for concrete in Exposure Class C are listed in Table 19.3.2.1 of ACI 318-14.

4.4—Concrete evaluation, acceptance, and inspection

Durability requirements are met once concrete proportions and properties satisfy the minimums set by the Code. To assure that the delivered concrete achieves the desired durability, the LDP should specify concrete evaluation and acceptance criteria consistent with ACI 318-14, Section 26.12 and field inspection consistent with ACI 318-14, Section 26.13.

4.5—Examples

The following examples illustrate one approach of implementing minimum durability requirements of the Code. In some cases, durability requirements for material properties may exceed those of the structural design. This is more likely for severe exposure conditions, which require a minimum compressive strength of 5000 psi. In some cases, SCMs may be required, which may extend setting time and early-age strength, and result in modifications to construction schedule. For these reasons, cooperation with engineers experienced with concrete materials and mixture proportioning, and with concrete suppliers, is recommended.

4.5.1 *Example 1: Interior suspended slab not exposed to moisture or freezing and thawing*—Consider the design of a cast-in-place, nonprestressed slab in a multistory office building. It is located in a climate zone with frequent freezing-and-thawing cycles; however, the slab will be constructed during summer and the temperatures at night during construction are expected to remain above 40 to 45°F. It is desirable for the slab to quickly gain strength to meet the construction schedule. For this reason, calcium chloride was proposed as an accelerating admixture. The required minimum compressive strength, from structural analysis, is 4000 psi. The slab is 7 in. thick with top and bottom mats of No. 5 bars spaced at 8 in. What additional information should be specified for the slab concrete to meet durability requirements?

Answer: The first step is to assign exposure classes within every exposure category to each structural member or group of members. Once exposure classes are assigned, the Code guides the LDP to satisfy the durability requirements. The step-by-step instructions are as follows:

Step description/ action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F0** (concrete not exposed to freezing-and-thawing cycles) **S0** (soil not in contact with concrete; low and injurious sulfate attack is not a concern) **W0** (there are no specific requirements for low permeability) **C0** (concrete dry or protected from moisture)	Table 19.3.1.1
Assign required minimum compressive strength	**2500 psi** (based on F0)	Table 19.3.2.1
Assign maximum *w/cm*	**Not limited** (based on all exposure classes)	Table 19.3.2.1
Assign minimum concrete cover	**0.75 in.** (not exposed to weather, slabs..., No. 11 bars and smaller)	Table 20.6.1.3.1
Assign nominal maximum size of aggregate	**2 in.** (3/4 x 3 in. clear bar spacing – top and bottom mat, or 1/3 x 7 in. – slab thickness); **use 1 in.** as readily available	26.4.2.1(a)(4)
Assign required air content	**Not air entrained**	Table 19.3.2.1
Assign limits on cementitious materials	**No limits**	Table 19.3.2.1
Assign limits on calcium chloride admixture	**No restriction** (based on S0) [Note: chloride ions from the admixture will significantly affect measured chloride ion content in concrete.]	Table 19.3.2.1
Assign maximum water-soluble chloride ion (Cl^-) content in concrete, percent by weight of cement	**1.00** (based on C0, water-soluble chloride-ion content from all concrete ingredients determined by ASTM C1218/C1218M at age between 28 and 42 days)	Table 19.3.2.1

4.5.2 *Example 2: Balcony slab exposed to moisture and freezing and thawing*—An LDP designs a cast-in-place, nonprestressed balcony slab in a multistory office building, located in a climate zone with frequent freezing-and-thawing cycles. It is anticipated that the balconies will be exposed to moisture, but not deicing salts. The required minimum compressive strength, from structural analysis, is 4000 psi. The balcony slabs are 6 in. thick with top mat of No. 4 bars spaced at 6 in. What additional information is needed for balcony concrete to meet durability requirements?

Answer: Durability requirements are met once the most rigorous requirements of the Code are satisfied. The first step is to assign exposure classes within every exposure category to each structural member or group of members. Once exposure classes are assigned, the code guides the LDP to set the minimum durability requirements. The step-by-step instructions are as follows:

Step description/ action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F2** (concrete exposed to freezing-and-thawing cycles with frequent exposure to water) **S0** (soil not in contact with concrete; low and injurious sulfate attack is not a concern) **W0** (there are no specific requirements for low permeability) **C1** (concrete exposed to moisture but not to an external source of chlorides)	Table 19.3.1.1
Assign required minimum compressive strength	**4500 psi** (based on F2); because 4500 psi is greater than design strength of 4000 psi, the 4500 psi governs	Table 19.3.2.1
Assign maximum *w/cm*	**0.45** (based on F2)	Table 19.3.2.1
Assign minimum concrete cover	**1.5 in.** (exposed to weather, No. 5 bar and smaller)	Table 20.6.1.3.1
Assign nominal maximum size of aggregate	**2 in.** (1/3 x 6-in. – slab thickness, 3/4 x 6-in. clear bar spacing – top mat bars); **use 1 in.** as readily available	26.4.2.1(a)(4)
Assign required air content	**6% ± 1.5%** (for 1 in. aggregate and F2 class) [Note: 1 in. aggregate can be substituted with 3/4 in. aggregate with no air content change]	Table 19.3.3.1 and Section R26.4.2.1(a)(5)
Assign limits on cementitious materials	**No limits**	Table 19.3.2.1
Assign maximum water-soluble chloride ion (Cl^-) content in concrete, percent by weight of cement	**0.30** (water-soluble chloride ion content from all concrete ingredients determined by ASTM C1218/1218M at age between 28 and 42 days)	Table 19.3.2.1
Provide guidance on cold weather construction	Consult ASTM C94/C94M, ACI 306R, and ACI 301 for guidance on temperature limits for concrete delivered in cold weather.	Section 26.5.4.1

4.5.3 *Example 3: Wall foundation exposed to sulfate soil and deicing salts while in service*—An LDP designs a cast-in-place, nonprestressed foundation wall of a partially underground parking structure. The structure is located in a northern climate zone with frequent freezing-and-thawing cycles, high sulfate soil content (6 percent SO_4^{2-} by mass) and exposure to deicing salts are anticipated as a runoff from the nearby streets and a sidewalk. It is desirable for the foundation wall to quickly gain strength to reduce possible frost damage and to meet the construction schedule. The required minimum compressive strength, from structural analysis, is 4000 psi. The foundation wall is 8 in. thick with inside face and outside face mats of No. 4 bars spaced at 12 in. What additional information should be specified for foundation wall concrete to meet durability requirements?

Answer: The first step is to assign exposure classes within every exposure category to each structural member or group of members. Once exposure classes are assigned, the code guides the LDP to set the minimum durability requirements. The step-by-step instructions are as follows:

Step description/ Action item	Selection and discussion	Code reference
Assign exposure classes within each exposure category	**F3** (concrete exposed to freezing-and-thawing cycles with frequent exposure to water and exposure to deicing chemicals) **S3** (structural concrete members in direct contact with soluble sulfates in soil or water) **W1** (concrete in contact with water and low permeability is required) **C2** (concrete exposed to moisture and an external source of chlorides from deicing chemicals)	Table 19.3.1.1
Assign minimum compressive strength	**5000 psi** (based on F3 and C2); because 5000 psi is greater than design strength of 4000 psi, 5000 psi governs	Table 19.3.2.1
Assign maximum *w/cm*	**0.40** (based on F3 and C2)	Table 19.3.2.1
Assign minimum concrete cover	**2.0 in.** – outside face of wall (1.5 in. cover is listed in Table 20.6.1.3.1 for exposure to weather or in contact with ground for No. 5 bar and smaller; cover increased to 2.0 in. based on 20.6.1.4.1) **3/4 in.** – inside face of wall (side of the wall not exposed to weather or in contact with ground)	Table 20.6.1.3.1 20.6.1.4.1
Assign nominal maximum size of aggregate	**1.5 in.** (1/5 x 8 in. – wall thickness, 3/4 x 3-1/4 in. clear bar spacing – between interior and exterior mats of reinforcing steel); **use 1.5 in.**	Section 26.4.2.1(a)(4)
Assign required air content	**5.5% ± 1.5%** (for 1.5 in. aggregate and F3 class) [Notes: 1. Changing to lower nominal maximum aggregate size will require higher air content; 2. Air content reduction of 1% (to 4.5% ± 1.5%) is allowable if concrete compressive strength exceeds 5000 psi; refer to 19.3.3.3]	Table 19.3.3.1
Assign limits on cementitious materials	Limits in accordance with Table 26.4.2.2(b) Cement combinations (for Class S3 in Table 19.3.2.1) must be tested in accordance with ASTM C1012/C1012M and meet the maximum expansion requirement of 0.10% (Class S3); check Table 26.4.2.2(c)	Table 19.3.2.1 Table 26.4.2.2(b) Table 26.4.2.2(c)
Assign limits on calcium chloride admixture	**Not permitted** (based on S2 and C2)	Table 19.3.2.1
Assign maximum water-soluble chloride-ion (Cl^-) content in concrete, percent by weight of cement	**0.15** (based on C2, water-soluble chloride ion content from all concrete ingredients determined by ASTM C1218/C1218M at age between 28 and 42 days)	Table 19.3.2.1

REFERENCES

American Concrete Institute

ACI 201.2R-08—Guide to Durable Concrete

ACI 212.3R-10—Report on Chemical Admixtures for Concrete

ACI 222R-01—Protection of Metals in Concrete Against Corrosion

ACI 222.3R-11—Guide to Design and Construction Practices to Mitigate Corrosion of Reinforcement in Concrete Structures

ACI 301-10—Specification for Structural Concrete

ACI 306R-10—Guide to Cold Weather Concreting

ACI 334.1R-92—Concrete Shell Structures-Practice and Commentary

ACI 350—Please correct document number and add title here and correct reference in body of text (Section 4.3.3)

ACI 372R-13—Guide to Design and Construction of Circular Wire- and Strand-Wrapped Prestressed Concrete Structures

ASTM International

ASTM C94/C94M-15—Standard Specification for Ready-Mixed Concrete

ASTM C1012/C1012M-13—Standard Test Method for Length Change of Hydraulic-Cement Mortars Exposed to a Sulfate Solution

ASTM C1218/C1218M-99—Standard Test Method for Water-Soluble Chloride in Mortar and Concrete

ASTM C150/C150M-12—Standard Specification for Portland Cement

ASTM D516-11—Standard Test Method for Sulfate Ion in Water

ASTM C685/C685M-14—Standard Specification for Concrete Made by Volumetric Batching and Continuous Mixing

ASTM C1580-09-Standard Test Method for Water-Soluble Sulfate in Soil

ASTM D4130-15—Standard Test Method for Sulfate Ion in Brackish Water, Seawater, and Brines

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CHAPTER 5—ONE-WAY SLABS

5.1—Introduction

A one-way slab is generally used in buildings with vertical supports (columns or walls) that are unevenly spaced, creating a long span in one direction and a short span in the perpendicular direction. One-way slabs typically span in the short direction and are supported by beams in the long direction. During preliminary design, the designer determines the loads and spans, reinforcement type (post-tensioned [PT] or nonprestressed), and slab thickness. The designer determines the concrete strength based on experience and the Code's exposure and durability provisions.

This chapter discusses cast-in-place, nonprestressed, and PT slabs. The Code allows for either bonded or unbonded tendons in a PT slab. Because bonded tendons are not usually used in slabs in the United States, this chapter will address PT slabs with unbonded tendons.

At times, the design of a one-way slab will require point load considerations, such as wheel loads in parking garages. These result in local shear forces on the slab, requiring verification of the slab's punching shear strength. Punching shear is addressed in Chapter 6 for two-way slabs in this Handbook.

For relatively small slab openings, trim bars can be used to limit crack widths caused by geometric stress concentrations. For larger openings, a local increase in slab thickness, as well as additional reinforcement, may be necessary to provide adequate serviceability and strength.

5.2—Analysis

ACI 318 allows for the designer to use any analysis procedure that satisfies equilibrium and geometric compatibility, as long as design strength and serviceability requirements are met. The Code includes a simplified method of analysis for one-way slabs that relies on coefficients to calculate moments and shears.

5.3—Service limits

5.3.1 *Minimum thickness*—For nonprestressed slabs, the Code allows the designer for slabs not supporting or attached to partitions or other construction likely to be damaged by large deflection to either calculate deflections or simply satisfy a minimum slab thickness (Section 7.3.1, ACI 318-14). In the case where loads are heavy, nonuniform, or deflection is a concern, calculations should verify that short- and long-term deflections are within the Code limits (Section 24.2.2, ACI 318-14).

The Code does not provide a minimum thickness-to-span ratio for PT two-way slabs, but Table 9.3 of *The Post-Tensioning Manual* (Post-Tensioning Institute (PTI) 2006), lists span-to-depth ratios for different members that have been found from experience to provide satisfactory structural performance.

5.3.2 *Deflections*—For nonprestressed slabs that are thinner than the ACI 318 minimum, or if the slab resists a very heavy live load, superimposed dead load, or both, and for PT slabs as well, the designer calculates deflections. The calculated deflections must not exceed the limits given in Section 24.2, ACI 318-14. Deflections can be calculated by any appropriate method, such as classical equations or software results.

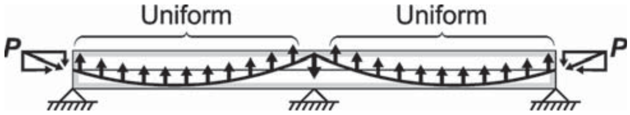


Fig. 5.3.3—Load balancing concept.

Note that the spacing of slab reinforcing bar to limit crack width, timing of form removal, concrete quality, timing of construction loads, and other construction variables all could affect the actual deflection. These variables should be considered when assessing the accuracy of deflection calculations. In addition, creep over time will increase the immediate deflections.

Typically, with a PT slab, slab deflections are usually small. If the designer limits the maximum net concrete tensile stress to below cracking stress under service loads, deflection calculations can consider the gross slab properties.

5.3.3 *Concrete service stress*—Nonprestressed slabs are designed for strength without reference to a concrete pseudo-service flexural stress limit.

For PT slabs, the analysis of concrete flexural tension stresses is a critical part of the design. In Section 8.3.4.1 of ACI 318-14, the concrete flexural tensile stress in negative moment areas at columns in PT slab is limited to $6\sqrt{f_c'}$. At positive moment sections, Section 8.6.2.3 of ACI 318-14 requires slab reinforcing bar if the concrete tensile stress exceeds $2\sqrt{f_c'}$. These service tensile flexural stress limits are below the concrete cracking stress of $7.5\sqrt{f_c'}$, thus having the effect of limiting deflections. In addition, Section 8.6.2.1 of ACI 318-14 requires a PT slab's axial compressive stress in both directions, due to PT, to be at least 125 psi.

Before the slab flexural stresses in a design strip can be calculated, the tendon profile should be defined. Both profile and tendon force are directly related to slab forces and moments created by PT. A common approach to calculate PT slab moments is the use of the "load balancing" concept. Tendons are typically placed in a parabolic profile such that the tendon is at the minimum cover requirements at midspan and over supports; this maximizes the parabolic drape. Anchors are typically placed at mid-depth at the slab edge (Fig. 5.3.3).

The tendon exerts a uniform upward force along its length that counteracts a portion of the gravity loads, usually 60 to 80 percent of the slab self-weight according to Libby (1990); hence, the term "load balancing." The load effect from the prestressing force in the tendon is then combined with the load effect of the gravity loads to determine net concrete stresses.

To achieve Code stress limits, the designer can use an iterative or direct approach. In the iterative approach, the tendon profile is defined and the tendon force is assumed. The analysis is executed, flexural stresses are calculated, and the designer then adjusts the profile or force or both, depending on results and design constraints.

In the direct approach, the designer determines the highest tensile stress permitted, then rearranges equations so that the analysis calculates the tendon force required to achieve the stress limit.

The Code does not impose a minimum concrete compressive stress due to PT, but majority of engineers use 125 psi as a general minimum for cast-in-place slabs. For slabs exposed to aggressive environments, engineers usually design slabs with an increased minimum concrete compressive stress.

5.4—Required strength

The design of one-way slabs are typically controlled by moment strength, not concrete stress or shear strength. Assuming a uniform load, the designer calculates the unit (usually a 1 ft width) factored slab moments. The required area of flexural reinforcement over a unit slab width is calculated with the same assumptions as a beam.

5.4.1 *Calculation of required moment strength*—For nonprestressed reinforced slabs, a quick way to calculate a slab's gravity design moments (if the slab meets the specified geometric and load conditions) is by the moment coefficients in Section 6.5 of ACI 318-14. Chapter 6 of ACI 318-14 permits other, more exact analysis methods.

For PT slabs, effects of reactions induced by prestressing (secondary moments) should be added to the factored gravity moments per Section 5.3.11 of ACI 318-14 to calculate M_u. The slab's secondary moments are a result of the beam's vertical restraint of the slab against the PT "load" at each support. Because the PT force and drape are determined during the service stress checks, secondary moments can be quickly calculated by the "load-balancing" analysis concept.

A simple method for calculating the secondary moment is to subtract the tendon force multiplied by the tendon eccentricity (distance from the neutral axis) from the total balance moment, expressed mathematically as $M_2 = M_{bal} - Pe$.

The critical locations to calculate M_u along the span are usually at the support and midspan. Section 7.4.2.1 of ACI 318-14 allows M_u to be calculated at the face of support rather than the support centerline.

5.4.2 *Calculation of required shear strength*—Assuming a uniform load, the designer calculates the unit (usually a 1 ft width) factored slab shear force by either the coefficient method or more exact calculations.

5.5—Design strength

One-way slabs must have adequate one-way shear strength and moment strength in all design strips.

5.5.1 *Calculation of design moment strength*—The required area of flexural reinforcement for a nonprestressed and PT unit slab width are calculated with the same assumptions as for a beam, Chapter 7, of this Handbook.

Table 5.6.1a—$A_{s,min}$ for nonprestressed one-way slabs (Table 7.6.1.1, ACI 318-14)

Reinforcement type	f_y, psi	$A_{s,min}$, in.^{2}	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\dfrac{0.0018\times 60{,}000}{f_y}A_g$
			$0.0014A_g$

Table 5.6.1b—Maximum spacing of bonded reinforcement in nonprestressed and Class C prestressed (Table 24.3.2 one-way slabs and beams, ACI 318-14, partial)

Lesser of:	$15\left(\dfrac{40{,}000}{f_s}\right)-2.5c_c$
	$12(40{,}000/f_s)$

5.5.2 *Calculation of design shear strength*—Discussion for nominal one-way shear strength is the same as for a beam, Chapter 7, of this Handbook.

5.6—Flexure reinforcement detailing

The Code requires a minimum area of flexural reinforcement be placed in tension regions to ensure that the slab deformation and crack widths are limited when the slab's cracking strength is exceeded. If more than the minimum area is required by analysis, that reinforcement area must be provided. Reinforcement in one way slabs is usually uniformly spaced, unless there is a significant point load or opening.

5.6.1 *Nonprestressed reinforced slab – Flexural reinforcement area and placing*—For nonprestressed slabs, the minimum flexural bar area, $A_{s,min}$, is given in Table 7.6.1.1 of ACI 318-14 (Table 5.6.1a of this Handbook).

The maximum spacing of flexural bars is given in Table 24.3.2 of ACI 318-14 (Table 5.6.1b of this Handbook). Because f_s is usually taken as 40,000 psi, the maximum spacing will not exceed 12 in.

The bar termination rules in Section 7.7.3 of ACI 318-14 cover general conditions that apply to beams, but because one-way slab bars are usually spaced close to the maximum, bars generally cannot be terminated without violating the maximum spacing. This usually results in all bottom bars extending full length into the beams.

5.6.2 *Post-tensioned slab – Flexural tendon area and placing*—For PT one-way slabs, the Code does not have a limit for minimum tendon area or a minimum compressive stress due to PT. This is consistent with the flexible approach on service stresses. The Code limits the maximum tendon spacing to 8h or 5 ft.

5.6.3 *Post-tensioned slab – Flexural reinforcing bar area and placing*—The Code requires the reinforcing bar area, $A_{s,min}$, to be placed close to the slab face at the bottom at midspan and the top at the support. For one-way PT slabs $A_{s,min}$ = $0.004A_{ct}$. Because the one-way slab strip is rectangular, A_{ct} = $0.5A_g$. This minimum is independent of service stress level.

The maximum spacing of reinforcing bar in a PT one-way slab is the lesser of 3*h* and 18 in.

If the slab design moment strength is fully satisfied by the tendons alone, the termination length of $A_{s,min}$ bars for bottom bars is (a) and for top bars is (b):

(a) At least $\ell_n/3$ in positive moment areas and be centered in those areas

(b) At least $\ell_n/6$ on each side of the face of support

The termination length for bars that are required for strength are the same as for nonprestressed slabs.

5.6.4 *Temperature and shrinkage reinforcement and placing*—Shrinkage and temperature (S&T) slab reinforcement is required and could be either reinforcing bar or tendons placed perpendicular to flexural reinforcement.

If the designer uses reinforcing bar, the minimum area of temperature and shrinkage Grade 60 bar is $0.0018A_g$. If the designer uses tendons, the minimum slab effective compression force due to temperature and shrinkage tendons is 100 psi.

The purpose of this reinforcement is to restrain the size and spacing of slab cracks, which can occur due to volume variations caused by temperature changes and slab shrinkage over time. In addition, if the slab is restrained against movement, the Code requires the designer to provide reinforcement that accounts for the resulting tension stress in the slab.

Authored references

Libby, J., 1990, *Modern Prestressed Concrete: Design Principles and Construction Meth*ods, fourth edition, Springer, 871 pp.

Post-Tensioning Institute (PTI), 2006, *Post-Tensioning Manual,* sixth edition, PTI TAB.1-06, 354 pp.

5.7—Examples

One-way Slab Example 1*: Nonprestressed one-way slab—*

Design and detail the second story of the seven story building. The one-way slab consists of five spans of 14 ft each. The slab is supported by 18 in. beams. A 6 ft cantilever extends at the left end of the slab (Fig. E1.1).

Given:

Load—

Service live load L = 100 psf

Concrete—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

Geometry—

Span length: 14 ft

Beam width: 18 in.

Column dimensions: 24 in. x 24 in.

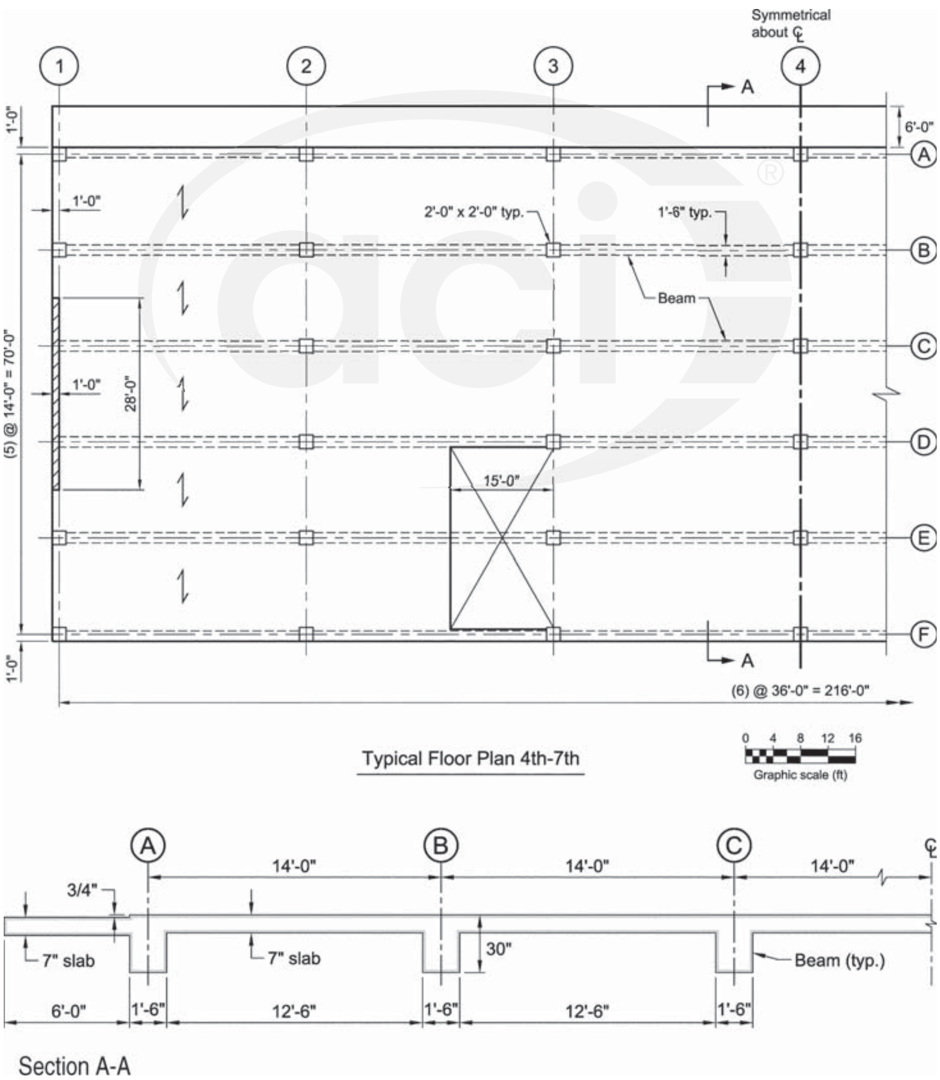


Fig. E1.1—Plan of five-span one-way slab.

ACI 318-14	Discussion	Calculation
Step 1: Geometry		
7.3.1.1	Determine slab thickness using ratios from Table 7.3.1.1. Determine cantilever thickness:	$h \geq \frac{\ell}{24} = \frac{(14\text{ ft})(12\text{ in./ft})}{24} = 7\text{ in.}$ $h \geq \frac{\ell}{10} = \frac{(6\text{ ft})(12\text{ in./ft})}{10} = 7.2\text{ in., say, 7 in.}$ Because the slab and cantilever satisfy ACI 318-14 span-to-depth ratios (Table 7.3.1.1), the designer does not need to check deflections unless the slab is supporting breakable partitions.
	Note: Architectural requirements specify a 3/4 in. step at the cantilever. Detail to maintain 7 in. slab thickness.	
	<u>Self-weight</u> Slab:	$w_s = (7\text{ in.}/12\text{ in./ft})(150\text{ lb/ft}^3) = 87.5\text{ psf}$
Step 2: Loads and load patterns		
5.3.1	The service live load is 100 psf in assembly areas and corridors per Table 4-1 in ASCE 7-10. For cantilever use 100 psf. To account for weights from ceilings, partitions, HVAC systems, etc., add 15 psf as miscellaneous dead load. $U = 1.4D$ (5.3.1a) $U = 1.2D + 1.6L$ (5.3.1b) The slab resists gravity only and is not part of a lateral force-resisting system, except to act as a diaphragm.	The required strength equations to be considered are: $U = 1.4\ (87.5\text{ psf} + 15\text{ psf}) = 143.5\text{ psf}$ $U = 1.2\ (102.5\text{ psf}) + 1.6\ (100\text{ psf})$ $= 123\text{ psf} + 160\text{ psf} = 283\text{ psf}$ **Controls**
6.4.2	Both ASCE 7 and ACI provide guidance for addressing live load patterns. Either approach is acceptable. ACI 318 allows the use of the following two patterns, Fig. E1.2: Factored dead load is applied on all spans and factored live load is applied as follows: (a) Maximum positive M_u near midspan occurs with factored live load on the span and on alternate spans. (b) Maximum negative M_u at a support occurs with factored live load on adjacent spans only.	

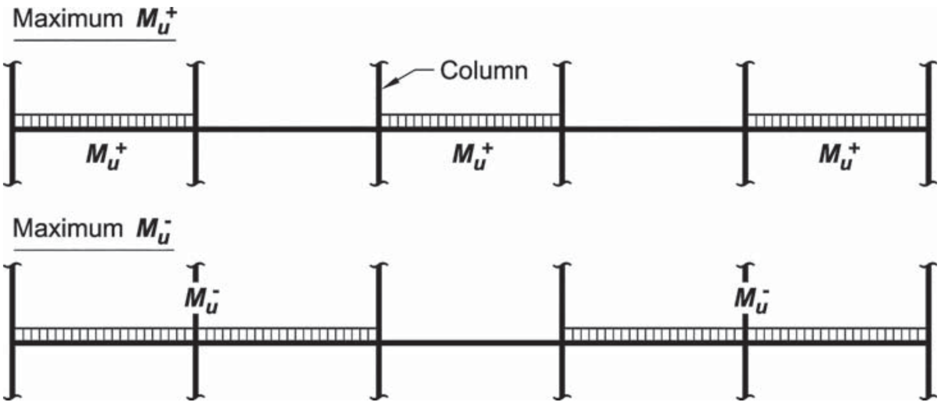


Fig. E1.2—Live load loading pattern.

Step 3: Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of ACI 318-14. The designer determines the durability classes. Please refer to Chapter 4 of this design Handbook for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture must be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 (ACI 318-14) requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.
7.2.2.2	The reinforcement must satisfy Chapter 20 of ACI 318-14. The designer determines the grade of bar and if the reinforcing bar should be coated by epoxy or galvanized, or both.	By specifying the reinforcing bar grade and any coatings, and that the reinforcing bar must be in accordance with ACI 301-10, Chapter 20 requirements are satisfied. In this case, assume Grade 60 bar and no coatings.
Step 4: Slab analysis		
6.3	Because the building relies on the building's other members to resist lateral loads, the slab qualifies for braced frame assumptions, as discussed in the commentary.	Modeling assumptions: Assume an effective moment of inertia for the entire length of the slab. Ignore torsional stiffness of beams. Only the slab at this level is considered.
6.6	The analysis should be consistent with the overall assumptions about the role of the slab within the building system. Because the lateral force-resisting-system only relies on the slab to transmit axial forces, a first-order analysis is adequate.	Analysis approach: The connection to the beams is monolithic; however, when the slab is fully loaded, flexural cracking will soften the joint.
Step 5: Required moment strength		
7.4.2	The slab's negative design moments are taken at the face of support as is permitted by the Code (Fig. E1.3).	

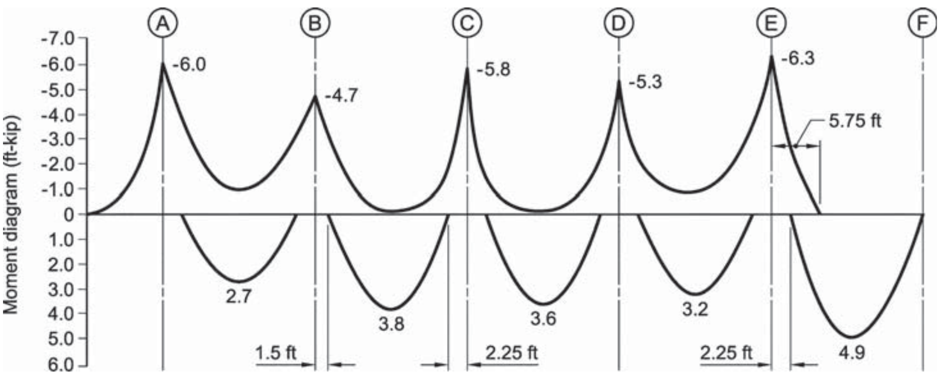


Fig. E1.3—Moment envelope.

The negative moment at the centerline of the end right support is 0.0 ft-kip.

The maximum positive moment in the end span, EF, is 4.9 ft-kip. The inflection points for positive moments are 0.0 ft from the exterior support centerline and 2.25 ft from the first interior support centerline, column line (CL) E.

The maximum negative moment at the face of the first interior support from the right end (CL E) is 6.3 ft-kip. The negative moment's right inflection point is 5.75 ft from the support centerline. On the left side and for the full length of the slab, there is no inflection point.

The maximum positive moment in the interior span, CL BC, is 3.8 ft-kip. The inflection points for positive moments are 1.5 ft from the first interior support centerline and 2.25 ft from the second interior support centerline. Because of pattern loading, a small negative moment can exist across all spans with the exception of the last span.

The maximum negative moment at the exterior left support, CL A, is 6 ft-kip because of the cantilevered slab.

Table 1.1—Maximum moments at supports and midspans

Required strength	Location from left to right along the span					
	Exterior support	First midspan	Second support	Second midspan	Third support	Third midspan
M_u, ft-kip	–6.0	+2.7	–4.7	+3.8	–5.8	+3.6

Continue:

Required strength	Location from left to right along the span				
	Fourth support	Fourth midspan	Fifth support	Fifth midspan	End support
M_u, ft-kip	–5.3	+3.3	–6.3	+4.4	0

Step 6: Required shear strength

7.4.3.1 The slab's maximum shear is taken at the support centerline for simplicity. The maximum shear under all conditions is 2.4 kip (Fig E1.4).

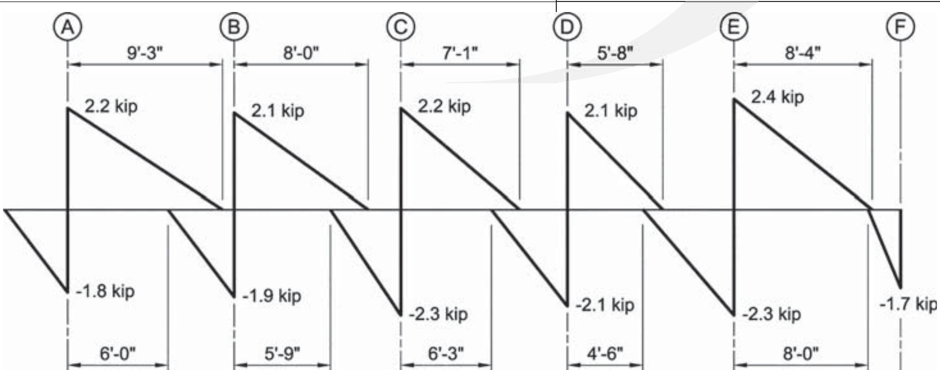


Fig. E1.4—Shear envelope.

Step 7: Design moment strength		
7.5.1 7.5.2 7.3.3.1 21.2.1(a)	The two common strength inequalities for one-way slabs, moment and shear, are noted in Section 7.5.1.1. The one-way slab chapter refers to Section 22.3 for calculation of flexural strength. To ensure a ductile failure usually the Code requires slabs to be designed such that the steel strain at ultimate strength exceeds 0.004 in./in. In most reinforced slabs, such as this example, reinforcing bar strain is not a controlling issue. The design assumption is that slabs will be tensioned controlled, $\phi = 0.9$. This assumption will be checked later.	
22.2.2.1	Determine the effective depth assuming No.5 bars and 0.75 in. cover (Fig. E1.5):	$d_{estimated} = t - cover - d_b/2$ $d_{estimated} = 7\text{ in.} - 0.75\text{ in.} - 0.625\text{ in.}/2$  *Fig. E1.5—Effective depth.*

7.7.1.1 20.6.1.3.1	One row of reinforcement $d = t - \text{cover} - d_b/2$	d = 7 in. – 0.75 in. –0.625 in./2 = 6.18 in., say, 6.0 in.
22.2.2.1	The concrete compressive strain at nominal moment strength is calculated at: $\varepsilon_{cu} = 0.003$	
22.2.2.2	The tensile strength of concrete in flexure is a variable property and is approximately 10 to 15 percent of the concrete compressive strength. ACI 318 neglects the concrete tensile strength to calculate nominal strength.	
22.2.2.3	Determine the equivalent concrete compressive stress at nominal strength: The concrete compressive stress distribution is inelastic at high stress. The Code permits any stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Rather than tests, the Code allows the use of an equivalent rectangular compressive stress distribution of $0.85f_c'$ with a depth of:	
22.2.2.4.1	$a = \beta_1 c$, where β_1 is a function of concrete compressive strength and is obtained from Table 22.2.2.4.3.	
22.2.2.4.3	For $f_c' \leq 5000$ psi:	$\beta_1 = 0.85 - \dfrac{0.05(5000\text{ psi} - 4000\text{ psi})}{1000\text{ psi}} = 0.8$
22.2.1.1	Find the equivalent concrete compressive depth, ***a***, by equating the compression force to the tension force within the beam cross section: $C = T$ $0.85f_c'ba = A_sf_y$ Effective width: 12 in.	$0.85(5000\text{ psi})(b)(a) = A_s(60{,}000\text{ psi})$ $a = \dfrac{A_s(60{,}000\text{ psi})}{0.85(5000\text{ psi})(12\text{ in.})} = 1.176A_s$

One-Way Slabs

	The slab is designed for the maximum flexural moments obtained from the approximate method above.	
7.5.1.1	The first interior support will be designed for the larger of the two moments. The beam's design strength must be at least the required strength at each section along its length: $\phi M_n \geq M_u$ $\phi V_n \geq V_u$	
	Calculate the required reinforcement area: $\phi M_n \geq M_u = \phi A_s f_y \left(d - \frac{a}{2}\right)$ A No. 5 bar has a d_b = 0.625 in. and an A_s = 0.31 in.^{2}	Maximum positive moment: $4.9 \text{ ft-kip} \leq (0.9)(60 \text{ ksi})A_s\left(6.0 \text{ in.} - \frac{1.176A_s}{2}\right)$ $A^+_{s'req'd}$ = 0.18 in.^{2}/ft Use No. 4 at 12 in. on center or No. 5 at 18 in. center bottom. Try No.5 at 18 in. on center $A_{s,provd.}$ = (0.31 in.^{2}/ft)(12 in./18 in.) = 0.21 in.^{2}/ft $A_{s,provd.}$ = 21 in.^{2}/ft > $A^+_{s,req'd}$ = 0.18 in.^{2}/ft **OK** Maximum negative moment: $6.3 \text{ ft-kip} \leq (0.9)(60 \text{ ksi})A_s\left(6.0 \text{ in.} - \frac{1.176A_s}{2}\right)$ $A^-_{s,req'd}$ = 0.24 in.^{2}/ft Use No. 5 at 12 in. on center top $A_{s,provd.}$ = 31 in.^{2}/ft > $A^-_{s,req'd}$ =0.24 in.^{2}/ft **OK**
	Check if the calculated strain exceeds 0.005 in./in. (tension controlled). Form similar triangles (Fig. E1.6). $a = \frac{A_s f_y}{0.85 f_c' b}$ and $c = a/\beta_1$ where β_1 = 0.8 for f_c' = 5000 psi $\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c)$	Top reinforcement $a = 1.176A_s$ = (1.176)(0.31 in.^{2}) = 0.36 in. c = 0.36/0.8 = 0.46 in. $\varepsilon_t = \frac{0.003}{0.46 \text{ in.}}(6 \text{ in.} - 0.46 \text{ in.}) = 0.036 \geq 0.005$ **OK**

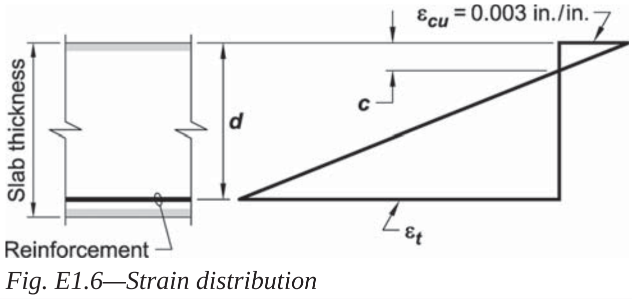


Fig. E1.6—Strain distribution

Step 8: Design shear strength		
7.5.3	Assuming a one-way slab won't contain shear reinforcement, V_n is equal to V_c. Assuming negligible axial force, the Code provides the following expression,	
22.5.5.1 21.2.1	$V_c = 2\sqrt{f_c'}bd$ Shear strength reduction factor:	$V_c = 2\sqrt{5000\text{ psi}}(12\text{ in.})(6\text{ in.}) = 10{,}180\text{ lb/ft} \cong 10\text{ kip/ft}$ $\phi = 0.75$ $\phi V_c = (0.85)(10{,}000\text{ lb}) = 8500\text{ lb} > 2400\text{ lb}$ **OK** This exceeds the maximum V_u (2.4 kip/ft); therefore, no shear reinforcement is required.
Step 9: Minimum flexural reinforcement		
7.6.1	Check if design reinforcement exceeds the minimum required by the Code.	$A_{s,min} = 0.0018 \times 6 \times 12 = 0.13\text{ in.}^2\text{/ft}$ At all critical sections, the required A_s is greater than the minimum.
Step 10: Shrinkage and temperature reinforcement		
7.6.4 24.4.3.2	For one-way with Grade 60 bars, the minimum area of shrinkage and temperature (S+T) bars is $0.0018A_g$. The maximum spacing of S+T reinforcing bar is the lesser of $3h$ and 18 in.	S+T steel area = $0.0018 \times 12 \times 7 = 0.15\text{ in.}^2$ Based on S+T steel area, solutions are No. 4 at 16 in. or No. 5 at 18 in.; use No. 4 at 16 in. placed atop and perpendicular to the bottom flexure reinforcement.
Step 11: Minimum and maximum spacing of flexural reinforcement		
7.7.2.1 25.2.1	The minimum spacing between bars must not be less than the greatest of: (a) 1 in. (b) d_b (c) $4/3d_{agg}$ Assume 1 in. maximum aggregate size.	(a) 1 in. (b) 0.625 in. (c) (4/3)(1 in.) = 1.33 in. **Controls**
7.7.2.2 24.3.2 24.3.2.1 7.7.2.3	For reinforcement closest to the tension face, the spacing between reinforcement is the lesser of (a) and (b): (a) $12(40{,}000/f_s)$ (b) $15(40{,}000/f_s) - 2.5c_c$ $f_s = 2/3f_y = 40{,}000\text{ psi}$ The maximum spacing of deformed reinforcement is the lesser of $3h$ and 18 in.	(a) 12(40,000/40,000) = 12 in. **Controls** (b) 15(40,000/40,000) – 2.5(0.75 in.) = 13.1 in. 3(7 in.) = 21 in. > 18 in. Therefore, Section 24.3.2 controls; 12 in.

One-Way Slabs

Step 12: Select reinforcing bar size and spacing		
	Based on the above requirement, use No. 5 bars. Spacing on top and bottom bars is 12 in. Note that there is no point of zero negative moment along all spans except the last bay, so continue the top bars across all spans. Also, No. 4 bars can be used instead of No. 5. While this solution is slightly conservative (No. 5 versus No. 4 bars), the engineer may desire consistent spacing and reinforcing bar use for easier installation and inspection.	
Step 13: Top reinforcing bar length at the exterior support		
	The top bars have to satisfy the following provisions:	<u>Inflection points</u> The inflection point for negative moment at end span is 5.0 ft from support centerline.
7.7.3.3	Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	<u>Bar cutoffs</u> Extend bars beyond the inflection point at least: d = 6 in. or (12)(0.625 in.) = 7.5 in. Therefore, use 7.5 in.
7.7.3.8.4	At least one-third the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	33 percent of the bars to extend beyond the inflection point at least (14 ft – 1.5 ft)(12)/16 = 9.4 in. > $12d_b$ = 7.5 in. > d = 6 in. Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Section 7.7.3.3 check) can be cut off in the tension zone.
Step 14: Development and splice lengths		
7.7.1.2 25.4.2.3	ACI provides two equations for calculating development length; simplified and detailed. In this example, the detailed equation is used: $$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\right)d_b$$ where ψ_t = bar location; not more than 12 in. of fresh concrete below horizontal reinforcement ψ_e = coating factor; uncoated ψ_s = bar size factor; No. 7 and larger However, the expression: $\frac{c_b+K_{tr}}{d_b}$ must not be taken greater than 2.5.	The development length of a No. 5 black bar in a 7 in. slab with 0.75 in. cover is: $$\ell_d = \left(\frac{3}{40}\frac{60{,}000\text{ psi}}{(1.0)\sqrt{5000\text{ psi}}}\frac{(1.0)(1.0)(0.8)}{1.7\text{ in.}}\right)(0.625\text{ in.})$$ $= 19$ in. ψ_t = 1.0, because not more than 12 in. of concrete is placed below bars. ψ_e = 1.0, because bars are uncoated ψ_s = 0.8, because bars are smaller than No. 7 $\frac{1.06\text{ in.} + 0}{0.625\text{ in.}} = 1.7$ in.

7.7.1.3 25.5 25.5.1.1 25.5.2.1	<u>Splice</u> The maximum bar size is No. 5, therefore, splicing is permitted. Tension lap splice length, ℓ_{st}, for deformed bars in tension must be the greater of: $1.3\ell_d$ and 12 in.	ℓ_{st} = (1.3)(19 in.) = 24.7 in.; use 36 in.
Step 15: Bottom reinforcing bar length along first span		
 7.7.3.3	The bottom bars have to satisfy the following provisions: Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	 <u>Inflection points</u> The inflection points for positive moments are 0 ft from exterior support centerline at CL F and 2.1 ft from the first interior support centerline CL E.
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure.	This condition is satisfied along at any section along the beam span.
7.7.3.5	Flexural tensile reinforcement must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied. $V_u \leq (2/3)\phi V_n$ at the cutoff point. Note that (b) and (c) do not apply.	 2400 lb < (2/3)(10,800 lb) = 7200 lb **OK**
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	
7.7.3.8.3	At points of inflection, d_b for positive moment tensile reinforcement shall be limited such that ℓ_d for that reinforcement satisfies condition (b), because end reinforcement is not confined by a compressive reaction. $\ell_d \leq M_n/V_u + \ell_a$ ℓ_a is the greater of d and $12d_b$ = 7.5 in. $M_n = A_s f_y \left(d - \frac{a}{2}\right)$ The elastic analysis indicates that V_u at inflection point is 1800 lb.	<u>Check if bar size is adequate</u> M_n for an 7 in. slab with No. 5 at 12 in., 0.75 in. cover is: M_n = (0.31 in.^{2}/ft)(60,000 psi)(6 in. – 18 in.) = 108,252 in.-lb ≅ 108,000 in.-lb $\ell_d = 19\text{ in.} \leq \frac{108{,}000\text{ in.-lb}}{1800\text{ lb}} + 7.5\text{ in.} = 67.5\text{ in.}$ Therefore, No. 5 bar is **OK**
Step 16: Bottom bar length		
	The bar cut offs that are implicitly permitted from the Code provisions because of reduced required strength along the span do not apply before the inflection points for this slab, because if any bars were cut off, the maximum reinforcing bar spacing would be violated. Because all bottom bars extend past the tensile zone, Section 7.7.3.5 does not apply. All bottom bars need to extend at least 7 in. (refer to Section 7.7.3.3) beyond the positive moment inflection points.	The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support. Because the cut off locations are close to the supports and for field placing simplicity, extend all bars 6 in. into both supports.

Step 17: Slab detailing

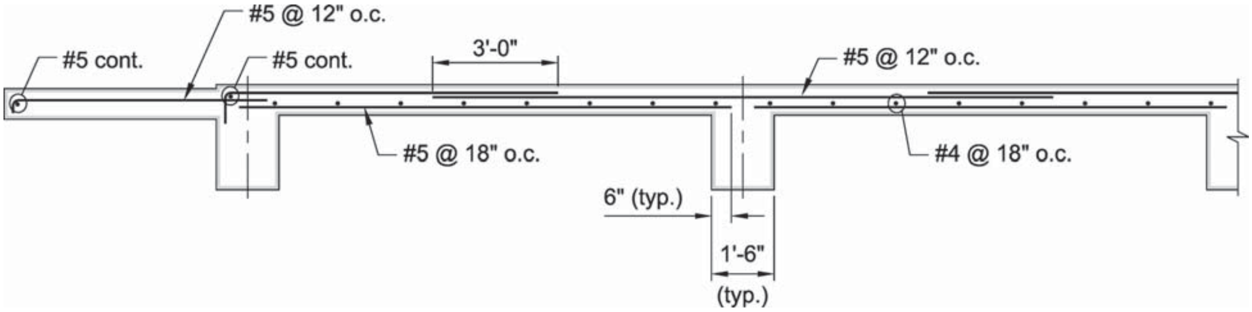


Fig. E1.7—One-way slab reinforcement.

One-way Slab Example 2: *Assembly loading—*

Design and detail a one-way nonprestressed reinforced concrete slab both for service conditions and factored loads. The one-way slab spans 20 ft-0 in. and is supported by 12 in. thick walls on the exterior, and 12 in. wide beams on the interior.

Given:

Load—

Live load L = 100 psf

Concrete unit weight γ_s = 150 lb/ft^3

Geometry—

Span = 20 ft

Slab thickness t = 9 in.

Material properties—

f_c' = 5000 psi (normalweight concrete)

f_y = 60,000 psi

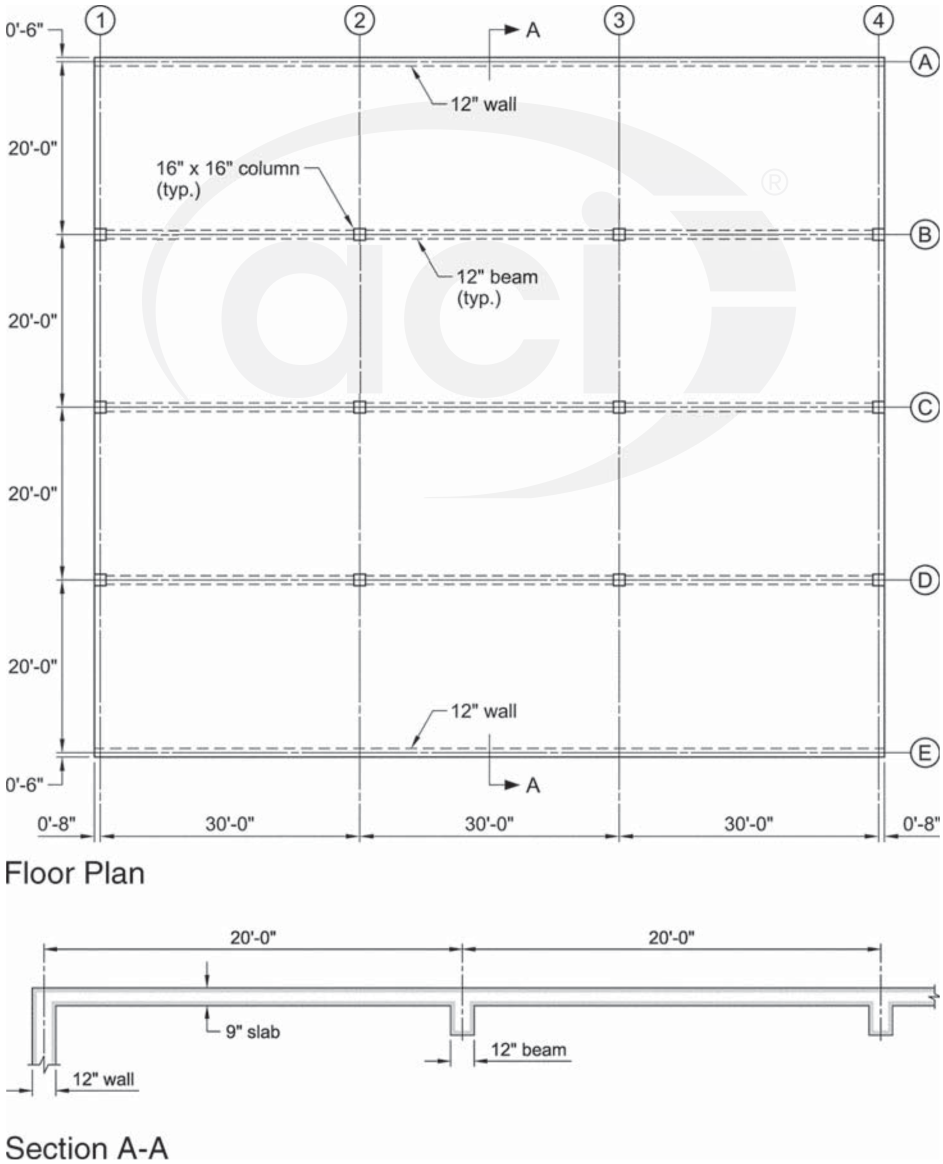
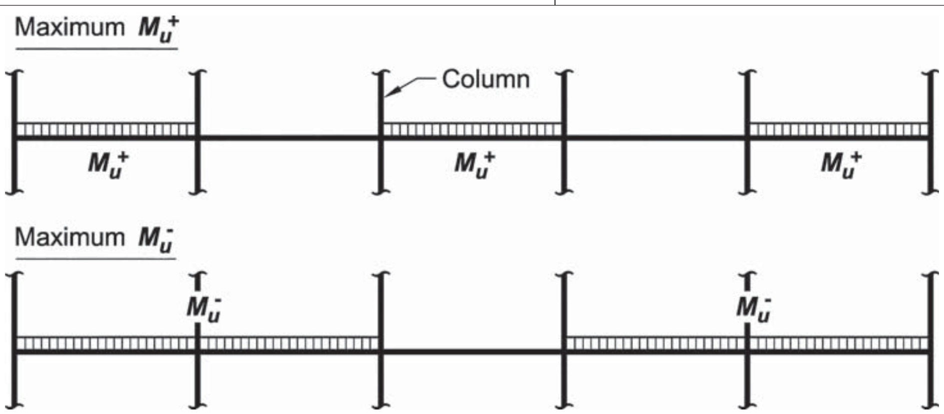


Fig. E2.1—One-way slab framing plan.

ACI 318-14	Discussion	Calculation
Step 1: Geometry		
7.3.1.1	The specified slab thickness is 9 in. Since the slab satisfies the ACI 318-14 span-to-depth ratios (Table 7.3.1.1), the designer does not need to check deflections unless supporting or attached to partitions or other construction likely to be damaged by large deflections.	$h \geq \frac{\ell}{27} = \frac{(20\text{ ft})(12\text{ in./ft})}{27} = 8.89$ in., say, 9 in. This ratio is less than the table value for "both ends continuous," so deflections are not required to be checked.
Step 2: Loads and load patterns		
5.3.1	For hotel lobbies, the live load is assembly occupancy; the design live load is 100 psf per Table 4-1 in ASCE 7-10. A 9 in. slab is a 112 psf dead load. To account for loads due to ceilings, partitions, HVAC systems, etc., add 10 psf as miscellaneous dead load. $U = 1.4D$ (5.3.1a) $U = 1.2D + 1.6L$ (5.3.1b) The slab resists gravity only and is not part of a lateral force-resisting system, except to act as a diaphragm. Both ASCE 7 and ACI provide guidance for addressing live load patterns. Either approach is acceptable. ACI 318 allows the use of the following two patterns, Fig. E2.2:	The required strength equations to be considered are: $U = 1.4(122) = 171$ psf $U = 1.2(122) + 1.6(100) = 146 + 160 = 306$ psf **Controls**
6.4.2	Factored dead load is applied on all spans and factored live load is applied as follows: (a) Maximum positive M_u near midspan occurs with factored live load on the span and on alternate spans. (b) Maximum negative M_u at a support occurs with factored live load on adjacent spans only.	
	 *Fig. E2.2—Live load loading pattern.*	

Step 3: Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements of ACI 318-14. The designer determines the durability classes. Please refer to Chapter 4 of this Handbook for an in-depth discussion of the categories and classes. ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications. There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	By specifying that the concrete mixture must be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied. Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5000 psi.
7.2.2.2	The reinforcement must satisfy Chapter 20 of ACI 318-14. The designer determines the grade of bar and if the reinforcing bar should be coated by epoxy or galvanized, or both.	By specifying the reinforcing bar grade and any coatings, and that the reinforcing bar must be in accordance with ACI 301-10, Chapter 20 requirements are satisfied. In this case, assume Grade 60 bar and no coatings.
Step 4: Slab analysis		
6.3	Because the building relies on the building's other members to resist lateral loads, the slab qualifies for braced frame assumptions, as discussed in the commentary.	Modeling assumptions: Apply the effective moment of inertia for the entire length of the slab. Ignore torsional stiffness of beams. Only the slab at this level is considered.
6.6	The analysis should be consistent with the overall assumptions about the role of the slab within the building system. Because the lateral force-resisting system only relies on the slab to transmit axial forces, a first order analysis is adequate. <u>Analysis approach:</u> The connection to the wall is monolithic; however, when the slab is fully loaded, flexural cracking will soften the joint. Rather than attempting to estimate an appropriate level of softening, the slab is simply modeled twice: To simulate effect of cracking, reduce flexural stiffness by increasing the length of support. In this example, support lengths are increased to 100 ft long columns. Assume fully connected slab to exterior wall, with uncracked moment of inertia, modelled by a 10 ft, 12 in. x 12. in. columns, and the middle three supports are slender, 100 ft columns. Note: The moments resulting from analysis maximize the moments and may be used to design the exterior wall.	
Step 5: Required moment strength		
7.4.2	The slab's negative design moments are taken at the face of support, as is permitted by the Code.	

One-Way Slabs

Analysis (a):

Note: The moment at the exterior support is near zero (refer to Fig. E2.3):

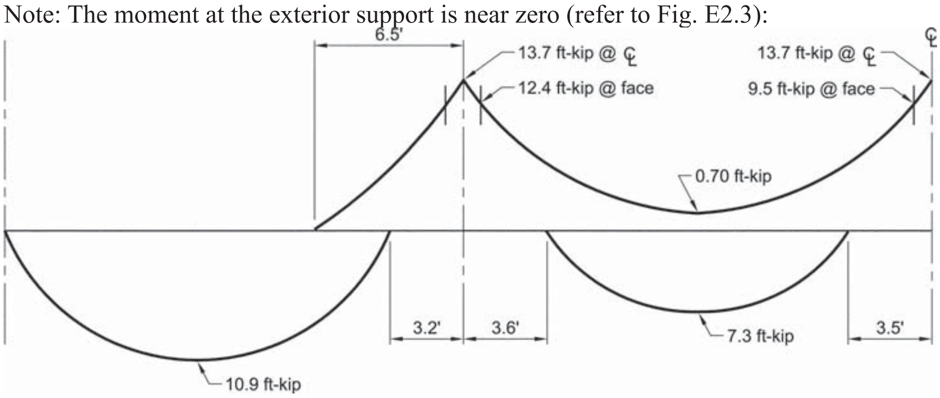


Fig. E2.3—Moment envelope.

The negative moment at the centerline of the exterior support is 0.0 ft-kip

The maximum positive moment in the end span is 10.9 ft-kip. The inflection points for positive moments are 0.0 ft from the exterior support centerline and 3.2 ft from the first interior support centerline.

The maximum negative moment at the face of the first interior support is 12.4 ft-kip. The negative moment's left inflection point is 6.5 ft from the support centerline. On the right side, under the pinned-at-wall assumption, there is no inflection point. Because of pattern loading, a small negative moment can exist across the span.

The maximum positive moment in the interior span is 7.3 ft-kip. The inflection points for positive moments are 3.6 ft from the first interior support centerline and 3.6 ft from the second interior support centerline.

The maximum negative moment at the face of the second interior support is 9.5 ft-kip. On the left side, under the pinned-at-wall assumption, there is no inflection point.

Because of pattern loading, a small negative moment can exist across the span.

Table 2.1—maximum moment for hinged end condition (Approach (a))

Required strength	Location from left to right along the span				
	Exterior support	First midspan	Second support	Second midspan	Middle support
M_u, ft-kip	0.0	+10.9	–12.4	+7.3	–9.5

Analysis (b) (end support is monolithic with wall)

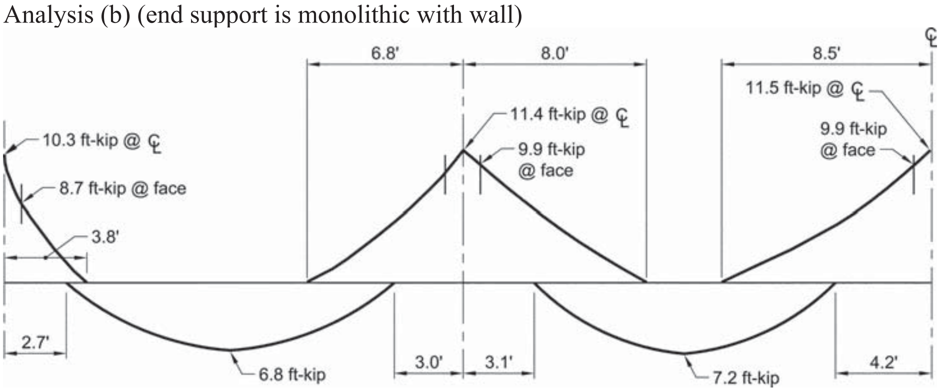


Fig. E2.4—Moment envelope with fixed end condition.

	The negative moment at the face of the exterior support is 8.7 ft-kip. The negative moment's inflection point is 3.8 ft from the support centerline. The maximum positive moment in the end span is 6.8 ft-kip. The inflection points for positive moments are 2.7 ft from the exterior support centerline and 3.0 ft from the first interior support centerline. The maximum negative moment at the face of the first interior support is 9.9 ft-kip. The negative moment's left inflection point is 6.8 ft from the support centerline and the right inflection point is 8.0 ft from the support centerline. The maximum positive moment in the interior span is 7.2 ft-kip. The inflection points for positive moments are 3.1 ft from the first interior support centerline and 4.2 ft from the second interior support centerline. The maximum negative moment at the face of the second interior support is 9.9 ft-kip. The negative moment's left inflection point is 8.5 ft from the support centerline and the right inflection point is 8.5 ft from the support centerline. Following are the maximum moments from a combination of Analysis (a) and (b) (conservative approach).

Table 2.2—Maximum moment for continuity condition between slab on wall (Approach (b))

Required strength	Location from left to right along the span				
	Exterior support	First midspan	Second support	Second midspan	Middle support
M_u, ft-kip	–8.7	+6.8	–9.9	+7.2	–9.9

Step 6: Required shear strength		
7.4.3.1	The slab's maximum shear is taken at the support centerline for simplicity. The maximum shear under any condition is 3.4 kips.	
Step 7: Design moment strength		
7.5.1 21.2.1	The two common strength inequalities for one-way slabs, moment and shear, are noted in Section 7.5.1.1 (ACI 318-14). The flexural strength reduction factor in Section 7.5.1.2 is assumed to be 0.9, which will be checked later.	
7.5.2	The one-way slab chapter refers to Section 22.3 (ACI 318-14) for calculation of flexural strength.	

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22.2.1 22.2.2.2 22.2.2.4.1	Chapter 22 (ACI 318-14) provides the design assumptions for reinforced concrete members. Generate the minimum area of steel for the required moment. $A_s f_y = 0.85 f_c'(b)a$ The effective depth, d, is the overall slab height minus the cover (3/4 in.) minus half the bar diameter (for a single layer of reinforcing bar). Assuming a No. 5 bar, therefore, $d = 9\text{ in.} - 0.75\text{ in.} - 0.625\text{ in}/2$ $= 7.9\text{ in.}$	To calculate A_s in terms of the depth of the compression block, a, set the section's concrete compressive strength equal to steel tensile strength: $T = C$ $A_s = \dfrac{0.85(5000\text{ psi})(12\text{ in.})a}{60{,}000\text{ psi}} = 0.85a$ To calculate the minimum required A_s, set the design strength moment equal to the required strength moment. $\phi A_s f_y(d - a/2) = M_u$ $0.9A_s(60{,}000\text{ psi})\left(7.9\text{ in.} - \dfrac{A_s}{2(0.85)}\right) = M_u$
	Table 2.3 following shows the required area of steel corresponding to the maximum moments from a conservative combination of Analysis (a) and (b).	

Table 2.3—Summary of maximum moment of Approaches (a) and (b)

	Location from left to right along the span				
	Exterior support	First midspan	Second support	Second midspan	Middle support
M_u, ft-kip	–8.7	+10.9	–12.4	+7.3	–9.9
Req'd A_s, in.^{2} per foot	0.26	0.32	0.37	0.22	0.28

           22.2.2.1	To ensure a ductile failure mode, the steel strain at ultimate strength must be at least 0.004 in./in. For usual reinforced slabs, such as this example, bar strain does not usually control the design. A strain diagram is drawn (Fig. E2.5). Maximum strain at the extreme concrete compression fiber is assumed equal to: $\varepsilon_{cu} = 0.003$ in./in.	To calculate reinforcing bar strain, begin with force equilibrium within the section: $T = C$ $A_s f_y = 0.85 f_c' ba$ where $b = 12$ in./ft; $f_y = 60{,}000$ psi; and the slab's maximum reinforcement is $A_s = 0.37$ in.^{2} From above calculations: $A_s = 0.85a$ or $a = A_s/0.85$ Therefore, $a = 0.44$ in. where $a = \beta_1 c$ and $\beta_1 = 0.80$ for f_c' of 5,000 psi; so $c = 0.55$ in. From similar triangles (Fig. E2.5): $\varepsilon_t = \dfrac{0.003(7.9\text{ in.} - 0.55\text{ in.})}{(0.55\text{ in.})} = 0.040 \geq 0.004$ Therefore, the assumption of $\phi = 0.9$ is correct.

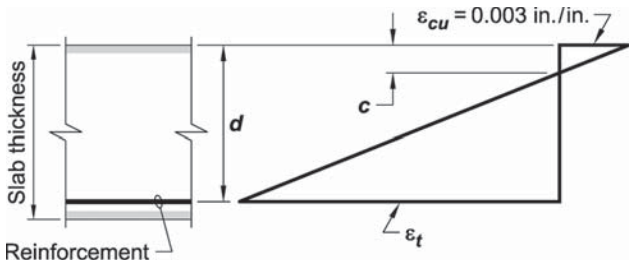

Fig. E2.5—Strain distribution.

Step 8: Design shear strength		
7.5.3	Assuming a one way slab won't contain shear reinforcement, V_n is equal to V_c. Assuming negligible axial force, the Code provides a simple expression:	
22.5.5.1	$V_c = 2\sqrt{f_c'}bd$	$V_c = 2\sqrt{5000\text{ psi}}(12\text{ in.})(7.9\text{ in.}) = 13,400\text{ lb}$
21.2.1	Shear strength reduction factor:	$\phi = 0.75$ $\phi V_c = (0.75)(13,400\text{ lb}) = 10,055\text{ lb} > 3400\text{ lb}$ OK This exceeds the maximum V_u (3.4 kip/ft); therefore, no shear reinforcement is required.
Step 9: Minimum flexural reinforcement		
7.6.1	Check if design reinforcement exceeds the minimum required reinforcement by Code.	$A_{s,min}$ = 0.0018 x 9 in. x 12 in. = 0.20 in.^{2}/ft. At all critical sections, the required A_s is greater than the minimum.
Step 10: Shrinkage and temperature reinforcement		
7.6.4 24.4.3.2	For one-way slabs with grade 60 bars, the minimum area of shrinkage and temperature (S+T) bars is 0.0018 A_g. The maximum spacing of S+T reinforcing bar is the lesser of 3h and 18 in.	S+T steel area = 0.0018 x 12 in. x 9 in. = 0.20 in.^{2} Based on S+T steel area, solutions are No. 4 at 12 in. or No. 5 at 18 in.; use No. 5 at 18 in. placed atop and perpendicular to the bottom flexure reinforcement.
Step 11: Minimum and maximum spacing of flexural reinforcement		
7.7.2.1 25.2.1	The minimum spacing between bars must not be less than the greatest of: (a) 1 in. (b) d_b (c) 4/3d_{agg} Assume 1 in. maximum aggregate size.	(a) 1 in. (b) 0.625 in. (c) (4/3)(1 in.) = 1.33 in. **Controls**

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7.7.2.2 24.3.2	For reinforcement closest to the tension face, the spacing between reinforcement is the lesser of (a) and (b): (a) $12(40{,}000/f_s)$ (b) $15(40{,}000/f_s) – 2.5c_c$	 (a) 12(40,000/40,000) = 12 in. **Controls** (b) 15(40,000/40,000) – 2.5(0.75 in.) = 13.1 in.
24.3.2.1	$f_s = 2/3f_y$ = 40,000 psi	
7.7.2.3	The maximum spacing of deformed reinforcement is the lesser of 3h and 18 in.	 3(7 in.) = 21 in. > 18 in. Therefore, Section 24.3.2 controls; 12 in.
Step 12: Select reinforcing bar size and spacing		

Table 2.4—Bar spacing

Bar size	Location from left to right along the span				
	Exterior support	First midspan	Second support	Second midspan	Middle support
No. 4 at spacing, in.	9	7	6	10	8
No. 5 at spacing, in.	12	11	10	12	12
No. 6 at spacing, in.	12	12	12	12	12

Refer to Fig. E2.6 for provided bar spacing.

	Based on the above, use No. 5 bars. Spacing of top bars at exterior support is 12 in., interior supports is 10 in. Note that there is no point of zero negative moment along the second and third span, so continue the top bars across both spans. While this solution is slightly conservative (10 in. versus 12 in. spacing), the engineer may desire consistent spacing for easier installation and inspection.	

Step 13: Top reinforcing bar length at the exterior support		
	The top bars have to satisfy the following provisions:	Inflection points The inflection point for negative moment is 3.8 ft from support centerline.
7.7.3.3	Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	Bar cutoffs Extend bars beyond the inflection point at least: d = 7.9 in. or (12)(0.625 in.) = 7.5 in.; Therefore, use 8 in. ~ 7.9 in.
7.7.3.8.4	At least one-third the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	33 percent of the bars to extend beyond the inflection point at least (19 ft × 12 in./ft)/16 = 15 in. > d = 8 in. > $12d_b$ = 7.5 in. Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Section 7.7.3.3 of ACI 318-14) can be cut off in the tension zone.
	Bars at the wall connection It is assumed that the wall is placed several days before the first floor slab. Because the wall and the slab will be firmly connected, the wall will tend to restrain the slab from shrinking as it cures. Many designers place extra reinforcement along the slab edge, parallel to the wall, to limit widths of possible cracks due to this restraint.	Solution The top bar length is 6 in. (wall beyond centerline) plus 3.8 ft (inflection) plus an extension of either 8 in. or 15 in. Because the two cut off locations are close together, use a 15 in. extension for all bars. A practical length for top bars is: 3.8 ft + 0.5 ft + 1.25 ft = 5.55 ft, say, 6 ft.
Step 14: Development and splice lengths		
7.7.1.2 25.4.2.3	ACI provides two equations for calculating development length; simplified and detailed. In this example, the detailed equation is used: $\ell_d = \left(\dfrac{3}{40}\dfrac{f_y}{\lambda\sqrt{f_c'}}\dfrac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b+K_{tr}}{d_b}\right)}\right)d_b$ where ψ_t = bar location; not more than 12 in. of fresh concrete below horizontal reinforcement ψ_e = coating factor; uncoated ψ_s = bar size factor; No. 7 and larger But the expression: $\dfrac{c_b+K_{tr}}{d_b}$ must not be taken greater than 2.5.	The development length of a No. 5 black bar in an 7 in. slab with 0.75 in. cover is: $\ell_d = \left(\dfrac{3}{40}\dfrac{60,000\text{ psi}}{(1.0)\sqrt{5000\text{ psi}}}\dfrac{(1.0)(1.0)(0.8)}{1.7\text{ in.}}\right)(0.625\text{ in.})$ $= 19$ in. ψ_t = 1.0, because not more than 12 in. of concrete is placed below bars. ψ_e = 1.0, because bars are uncoated ψ_s = 0.8, because bras are smaller than No. 7 $\dfrac{1.06\text{ in.}+0}{0.625\text{ in.}} = 1.7$ in.
7.7.1.3 25.5 25.5.1.1 25.5.2.1	Splice The maximum bar size is No. 5, therefore, splicing is permitted. Tension lap splice length, ℓ_{st}, for deformed bars in tension must be the greater of: $1.3\ell_d$ and 12 in.	ℓ_{st} = (1.3)(19 in.) = 24.7 in.; use 36 in.

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Step 15: Bottom reinforcing bar length along first span		
	The bottom bars have to satisfy the following provisions:	
7.7.3.3	Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	Inflection points The inflection points for positive moments are 0.0 ft from the exterior support centerline (analysis (a)) and 3.0 ft from the first interior support centerline (analysis (b)).
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure.	This condition is satisfied along at any section along the beam span.
7.7.3.5	Flexural tensile reinforcement shall not be terminated in a tensile zone unless (a), (b), or (c) is satisfied. (a) $V_u \leq (2/3)\phi V_n$ at the cutoff point Note that (b) and (c) do not apply.	
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	
7.7.3.8.3	At points of inflection, d_b for positive moment tensile reinforcement must be limited such that ℓ_d for that reinforcement satisfies condition (b), because end reinforcement is not confined by a compressive reaction. $\ell_d \leq M_n/V_u + \ell_a$ $M_n = A_s f_y \left(d - \frac{a}{2}\right)$ The elastic analysis indicates that V_u at inflection point is 2800 lb. The term ℓ_a is 8 in.	Check if bar size is adequate M_n for an 9 in. slab with No. 5 at 12 in., 0.75 in. cover is: M_n = (0.31 in.^{2})(60,000 psi)(7.9 in. – 0.4 in.) = 140,000 in.-lb $\ell_d \leq \frac{140{,}000\text{ lb}}{2800\text{ lb}} + 8\text{ in.} = 58\text{ in.} > 19\text{ in.}$ Therefore, No. 5 bar is **OK**
Step 16: First span bottom bar length		
	The bar cut offs that are implicitly permitted from prior Code provisions because of reduced required strength along the span do not apply before the inflection points for this slab, because if any bars were cut off, the maximum reinforcing bar spacing would be violated. Because all bottom bars extend past the tensile zone, Section 7.7.3.5 does not apply. All bottom bars need to extend at least 7 in. (refer to Section 7.7.3.3) beyond the positive moment inflection points.	The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support. Because the cut off location is close to the right support and for field placing simplicity, extend all bars 6 in. into both supports.

Step 17: Interior span bottom reinforcing bar lengths		
	<u>Inflection points</u> The inflection points for positive moments are 3.6 ft from the left support centerline and 4.2 ft from the right support centerline.	Create a partial length bar that is symmetrical within the span, so assume both inflection points are 3.6'. The minimum length is 20 ft – 3.6 ft – 3.6 ft + (2 ft)(0.5) = 13.8 ft, say, 14 ft 0 in.
	<u>Bar cutoffs</u> Similar to the first interior span, all bottom bars must extend at least 8 in. past inflection points. The Code requires at least 25 percent of bottom bars be full length, extending 6 in. into the support.	In a repeating pattern, use 3 No. 5 at 14 ft long and 1 No. 5 at 21 ft long.
Step 18: Top reinforcing bar length at the middle support		
	<u>Inflection points</u> There are no inflection points over either span that frame into the middle support.	The required reinforcing bar is No. 5 at 12 in. in the middle support. Because the top bar from the first support is No. 5 at 10 in., extend the No. 5 at 10 in. top over the middle support for simplicity.
	<u>Bar cutoffs</u> Because the top bars will be continuous, no bars are cut off.	
Step 19: Detailing		

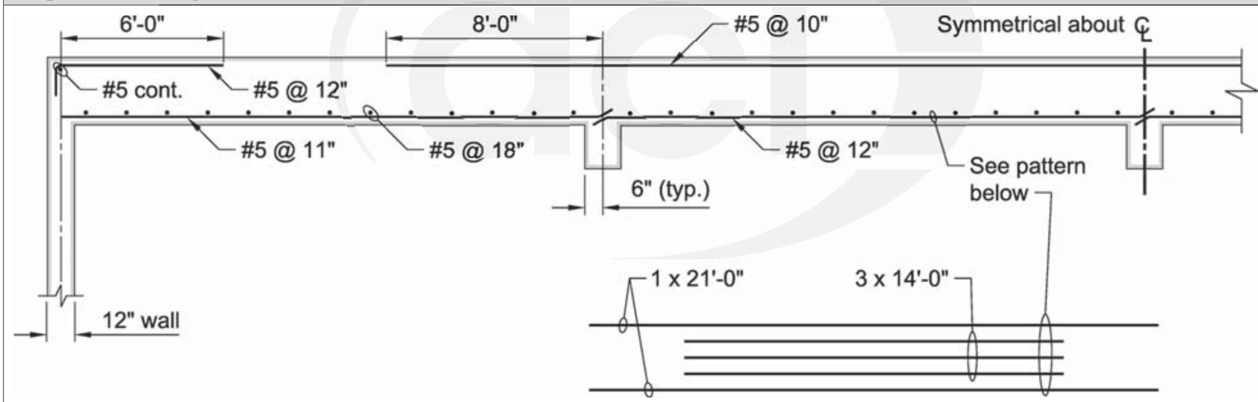

Fig. E2.6—Slab reinforcement detailing.

One-way Slab Example 3: *One-way slab post-tensioned – Hotel loading*

There are four spans of 20 ft-0 in. each, with a 3 ft-0 in. cantilever balcony at each end. The slab is supported by 12 in. walls on the exterior, and 12 in. wide beams on the interior (Fig. E3.1). This example will illustrate the design and detailing of a one-way post-tensioned (PT) slab, both for service conditions and factored loads.

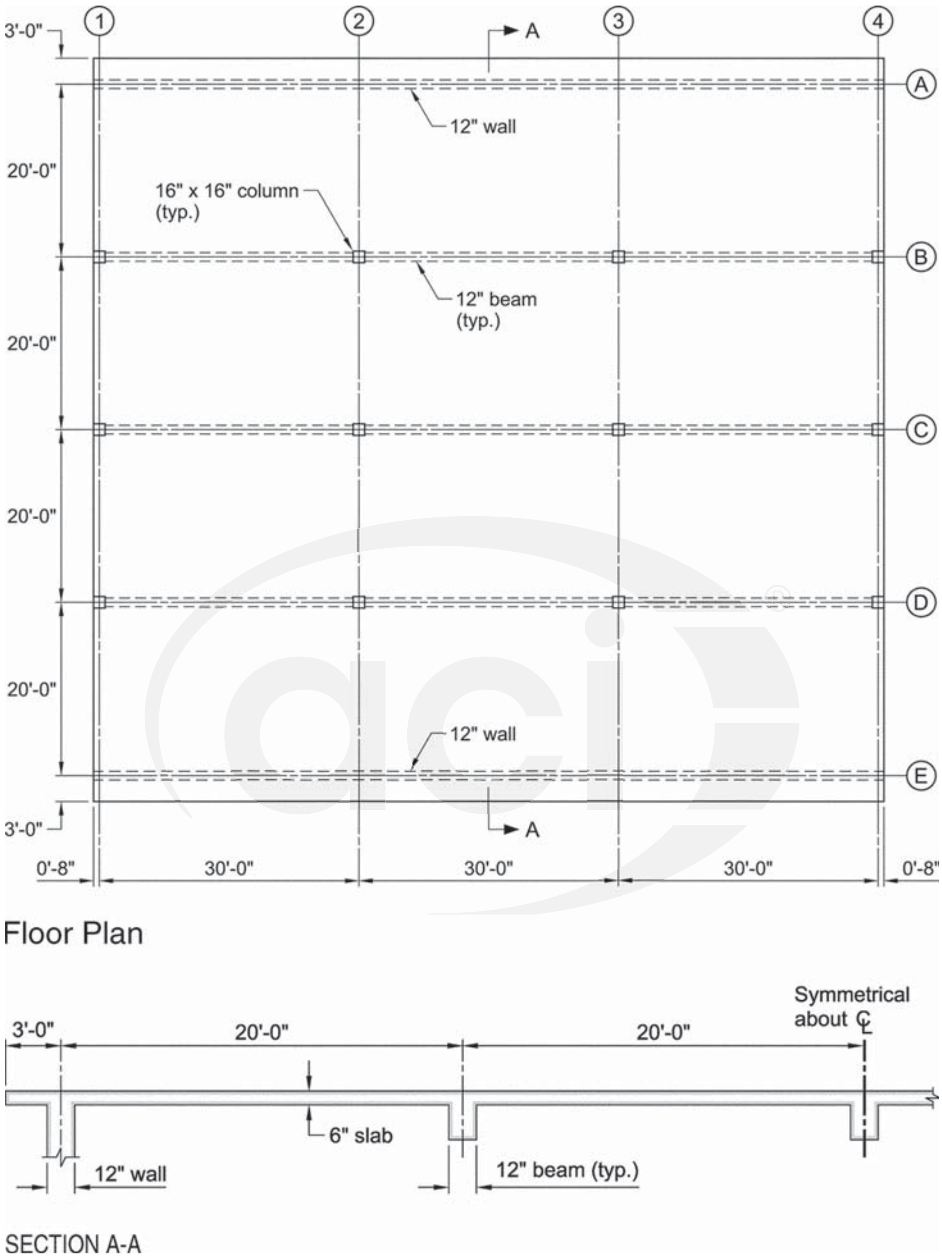

Fig. E3.1—One-way slab.

ACI 318-14	Discussion	Calculation
Step 1: Geometry		
7.3.2	The ACI 318-14 span-to-depth ratios do not apply to PT slabs. The *Post-Tensioning Manual*, 2006, sixth edition Chapter 9, Table 9.3, suggests a ratio limit of $\ell/48$. For this example, this ratio gives a slab thickness of	(20 ft)(12 in./ft)/48 = 5.0 in. This example uses a 6 in. thick slab.
Step 2: Loads and load patterns		
7.4.1.1	For hotel occupancy, the design live load is 40 psf per Table 4-1 in ASCE 7-10. A 6 in. slab is a 75 psf dead load. To account for weights from ceilings, partitions, HVAC systems, etc., add 10 psf as miscellaneous dead load. The slab resists gravity only and is not part of a lateral-force-resisting system, except to act as a diaphragm.	D = 75 psf + 10 psf = 85 psf The required strength equations to be considered are:
5.3.1	$U = 1.4D$ $U = 1.2D+1.6L$	U = 1.4(85) = 119 psf U = 1.2(85) + 1.6(40) = 102 + 64 = 166 psf **Controls**
7.4.1.2	Both ASCE 7 and ACI provide guidance for addressing live load patterns. Either approach is acceptable. ACI 318 allows the design to use the following two patterns (Fig. E3.2):	
6.4.2	Factored dead load is applied on all spans and factored live load is applied as follows: (a) Maximum positive M_u^+ near midspan occurs with factored L on the span and on alternate spans. (b) Maximum negative M_u^- at a support occurs with factored L on adjacent spans only.	

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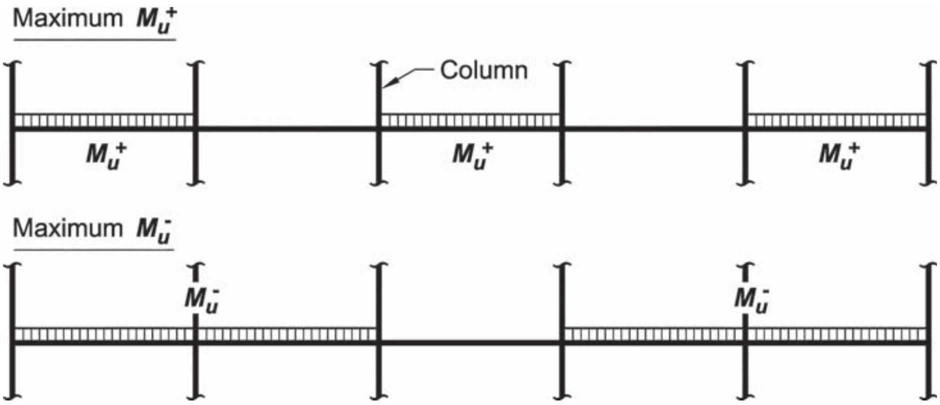


Fig. E3.2—Live load pattern used in elastic analysis of slab for unbalanced loads.

Step 3: Concrete and steel material requirements		
7.2.2.1	The mixture proportion must satisfy the durability requirements of Chapter 19 and structural strength requirements. The designer determines the durability classes. Please refer to Chapter 4 of this design Handbook for an in-depth discussion of the Categories and Classes.	
	ACI 301 is a reference specification that is coordinated with ACI 318. ACI encourages referencing ACI 301 into job specifications.	By specifying that the concrete mixture shall be in accordance with ACI 301-10 and providing the exposure classes, Chapter 19 requirements are satisfied.
	There are several mixture options within ACI 301, such as admixtures and pozzolans, which the designer can require, permit, or review if suggested by the contractor.	Based on durability and strength requirements, and experience with local mixtures, the compressive strength of concrete is specified at 28 days to be at least 5,000 psi.
7.2.2.2	The reinforcement must satisfy Chapter 20.	
	In this example, unbonded, 1/2 in. single-strand tendons are assumed.	By specifying the reinforcement shall be in accordance with ACI 301-10, the PT type and strength, and reinforcing bar grade (and any coatings), Chapter 20 requirements are satisfied. In this example, assume grade 60 bar and no coatings.
	The designer determines the grade of bar and if the reinforcement should be coated by epoxy or galvanized, or both.	
20.3	The Code requires strand material to be 270 ksi, low relaxation (ASTM A416a).	
20.3.2.5.1	The U.S. industry usually stresses, or jacks, monostrand to impart a force equal to the least of $0.80f_{pu}$ and $0.9f_y$. immediate and long-term losses will reduce this force.	The jacking force per individual strand is: (270 ksi)(0.8)(0.153 in.^{2}) = 33 kip. This is immediately reduced by seating and friction losses, and elastic shortening of the slab. Long term losses will further reduce the force per strand. Refer to Commentary R20.3.2.6 of ACI 318-14.
20.3.2.5.1	where $f_y = 0.94f_u$ (Table 20.3.2.5.1) $0.80f_{pu}$ controls, the maximum allowed by the Code.	A PT force design value of 26.5 kips per strand is common.

Step 4: Slab analysis		
6.3	Because the building relies on the building's other members to resist lateral loads, the slab qualifies for braced frame assumptions, as discussed in the commentary.	Modeling assumptions: Slab will be designed as Class U. Consequently, use the gross moment of inertia of the slab in the analysis. Assume the supporting beams have no torsional resistance and act as a knife edge support. Only the slab at this level is considered.
6.6 20.6.1.3.2	The analysis performed should be consistent with the overall assumptions about the role of the slab within the building system. Because the lateral force-resisting system only relies on the slab to transmit axial forces, a first order analysis is adequate. Although gravity moments are calculated independent of PT moments, the same model is used for both. The strands profile is chosen to provide maximum resistance to dead load and live load. At the exterior support an eccentricity of 0.25 in. is chosen to balance the cantilever load. At the interior supports and midspans, the maximum possible eccentricity is chosen (1 in. cover)	Analysis approach: To analyze the flexural effects of post-tensioning on the concrete slab under service loads, the tendon drape is assumed to be parabolic with a discontinuity at the support centerline shown as follows, which imparts a uniform uplift over each span when tensioned. The magnitude of the uplift, w_p, or "balanced load," in each span of a prismatic member is calculated as: $w_p = 8Fa/\ell^2$ where F is the effective PT force and a is the tendon drape (average of the two high points minus the low point). In this example the PT force is assumed constant for all spans, but the uplift force varies due to different tendon drapes.

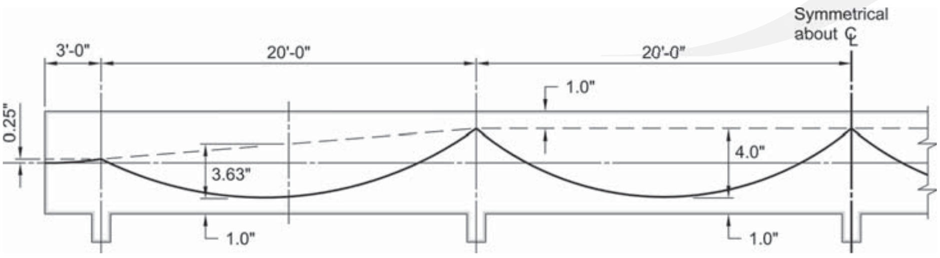


Fig. E3.3—Post-tensioning strand profile.

Step 5: Slab stress limits		
7.3.4.1 19.2.3.1	This example assumes a Class U slab; that is, a slab under full service load with a concrete tension stress not exceeding $7.5\sqrt{f_c'}$. The slab analysis model for the service condition is the same as for the nominal condition.	$7.5\sqrt{5000\text{ psi}} = 530\text{ psi}$
19.2.3.1	To verify that the concrete tensile stresses are less than $7.5\sqrt{5000\text{ psi}}$, the net service moments and tensile stresses at the face of supports are needed. This example assumes two parameters: (a) The PT force provides a F/A slab compressive stress of at least 125 psi (9 kip/ft), and (b) The combination of PT force and strand profile provides an uplift force w_p to balance at least 75 percent of the slab weight, or 56 psf (Fig. E3.4). The basic equation for concrete tensile stress is: $f_t = M/S - F/A$, where M is the net service moment. At the exterior support, the drape is 3.25 in. (Fig. E3.3). The required force is calculated from: $w_p = 8Fa/\ell^2$ where $w_p = 0.056$ kip/ft	$S = (12\text{ in.})(6\text{ in.})^2/6 = 72\text{ in.}^3$(section modulus), $A = (12\text{ in.})(6\text{ in.}) = 72\text{ in.}^2$ (gross slab area per foot). $F = \dfrac{(0.056\text{ kip/ft})(20\text{ ft})^2}{8(3.25\text{ in.})/(12\text{ in./ft})} = 10.3\text{ kip/ft}$

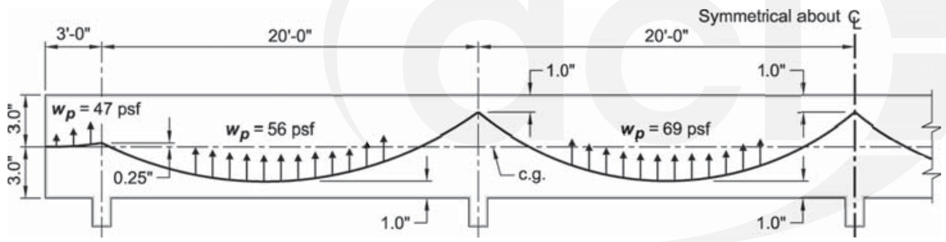


Fig. E3.4—Uplift force due to tendon layout.

7.3.4.2 The service gravity load is 125 psf. The PT uplift is subtracted from the gravity load.

Service loads	Location from left to right along the span		
	Cantilever	**First span**	**Second midspan**
Gravity uniform load, psf	125	125	125
PT uniform uplift, psf	47	56	69
Net "unbalanced load," psf	78	69	56

7.3.4.2	The slab stresses are determined from an elastic analysis using the "net" load, minus the slab's axial compression. Slab axial compression force:	$F/A = (10.3\text{ kip/ft})(1000\text{ lb/kip})\ (6\text{ in.})(12\text{ in./ft}) = 143\text{ psi}.$

7.3.4.2

	Location from left to right along the span			
	Exterior support	First midspan	Interior support	Second midspan
Net service unbalanced moments, ft-kip/ft	0.2	1.5	2.2	1.1
Net tensile stress, psi	33 – 143 = –110	250 – 143 = 107	367 – 143 = 224	183 – 143 = 40

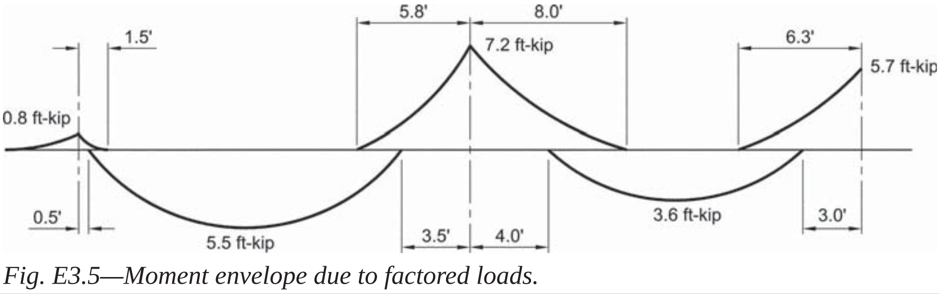
The cantilever moment at the support centerline is:
$w_{net}\ell^2/2 = (0.078\text{ psf})(3\text{ ft})2/2 = 0.351$ kip-ft/ft.

7.4.2.1 ACI 318-14 permits to calculate the design slab moment at the face of the support: 0.2 kip-ft/ft.
The moments for the interior supports are calculated at the faces of interior supports.

7.3.4.2 The aforementioned results show the maximum slab tensile stress (224 psi) calculated for an average PT force of 10.3 kip/ft is less than

19.2.3.1 $7.5\sqrt{f_c'} = 530$ psi.

Therefore, the slab is uncracked and the aforementioned assumption is correct.

Step 6: Deflections		
7.3.2.1	This chapter refers to Section 24.2 of ACI 318-14, "Deflections due to service-level gravity loads," for allowable stiffness approximations to calculate immediate and time-dependent (long term) deflections. Section 24.2.2 provides maximum allowed span-to-deflection ratios. Because slab is a Class U, use I_g:	$I_g = \dfrac{(12\text{ in.})(6\text{ in.})^3}{12} = 216\text{ in.}^4$
24.2.3.8	The balanced portion of the total load is offset by the camber from the prestressing, which results in a zero net deflection. Unbalanced load, however, will result in short- and long-term deflections that must be checked. The following equation, which can be downloaded from the Reinforced Concrete Design Handbook Design Aid – Analysis Tables: https://www.concrete.org/store/productdetail.aspx?ItemID=SP1714DA, will be used to calculate an approximate maximum deflection of the slab span with the largest unbalanced load (69 psf). In general, deflections do not control the design of PT slabs. $\Delta_{max} = 0.0065w\ell^4/EI$ The additional time-dependent deflection is the immediate deflection due to sustained load multiplied by two (refer to Scanlon and Suprenant, 2011, "Estimating Two-Way Slab Deflections," *Concrete International*, V. 33, No. 7, July, pp. 24-29).	$\Delta_{max} = \dfrac{(0.0065)(69\text{ psf})(240\text{ in.})^4}{(4{,}030{,}000\text{ psi})(216\text{ in.}^4)/(12\text{ in./ft})} = 0.14\text{ in.}$ Expressed as a ratio, ℓ/Δ = 240 in./0.14 in. = 1700
24.2.2	Assume that no portion of the live load is sustained. Calculate the immediate deflection based on a total sustained load of 85 psf reduced by the unbalanced load of 56 psf. The long-term multiplier on the immediate deflection is 2, so the ratio is:	(0.14 in.)(85 psf – 56 psf)/(69 psf) = 0.06 in. ℓ/Δ = 240/0.26 in = ℓ/920 The deflection ratios are much less than the limit of ℓ/480, so deflections are satisfied without more detailed calculations.
Step 7: Required moment strength		
7.4.2	The gravity design moments, including pattern loading, are shown in Fig. E3.5:  *Fig. E3.5—Moment envelope due to factored loads.*	

7.4.1.3	The Code requires that moments due to reactions induced by prestressing (secondary moments) be included with a load factor of 1.0. Secondary moments are calculated at each support as: $M_2 = M_{pt} - Fe$. The secondary moment diagram is linear between supports (Fig. E3.6).	
7.4.1.3	The PT secondary moments are:	

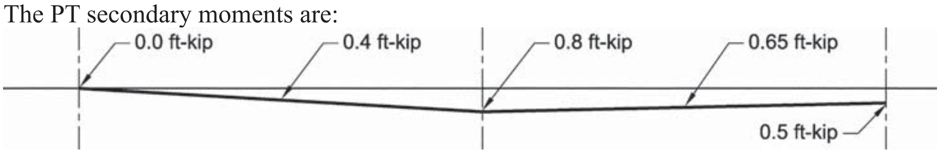


Fig. E3.6—Secondary moments due to post-tensioning.

7.4.2.1 The combined factored and secondary moments, which are the required moment strengths, are shown in Fig. E3.7:

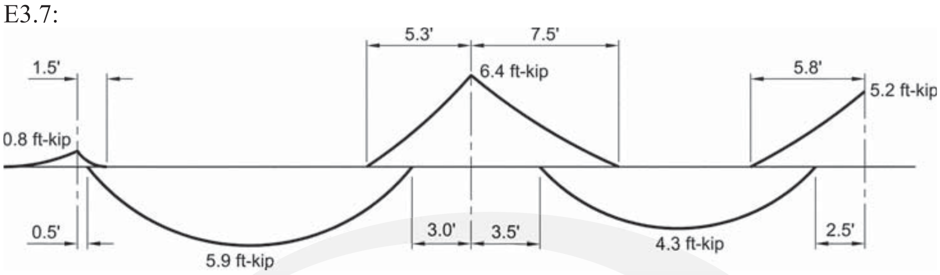


Fig. E3.7—Moment envelope including factored load effect and secondary moments.

Required strength (Gravity only)	Location from left to right along the span				
	Face of exterior support	First midspan	Face of second support	Second midspan	Face of middle support
Factored mom. at face, ft-kip	–0.8	5.5	–7.2	3.6	–5.7
M_2, ft-kip	0.0	0.4	0.8	0.65	0.5
M_u, ft-kip	–0.8	5.9	–6.4	4.3	–5.2

Step 8: Calculate required A_s

7.7.4.2	Check flexural strength considering only PT tendons If the PT tendons alone provide the design strength, $\geq \phi M_n$, then the Code permits the reinforcing bar to be a reduced length. If the PT tendons alone do not provide the design strength, then the reinforcing bar length is required to conform to standard lengths.	The depth of the equivalent stress block, a, is calculated by $a = \dfrac{A_{ps} f_{ps}}{0.85 f_c'(12 \text{ in.-ft})}$ where A_{ps} is the tendon area perfoot of slab.
	7.5.2.1 refers to 22.3 for the calculation of ϕM_n. Section 22.3 refers to 22.2 for calculation of M_n. Section 22.2.4 refers to 20.3.2.4 to calculate f_{ps}. The span-to-depth ratio is 240/6 = 40, so the following equation applies: The reinforcing bar and tendons are usually at the same height at the support and at midspan.	$f_{ps} = f_{se} + 10,000 + \dfrac{f_c'}{300\rho_p}$

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20.3.2.4	Each single unbonded tendon is stressed to the value prescribed by the supplier. Friction losses cause a variation of f_{se} along the tension length, but for design purposes, f_{se} is usually taken as the average value.	The tendon supplier usually calculates f_{se}, and 175,000 psi is a common value. The force per strand is therefore: 175,000 psi × 0.153 in.^{2} = 26,800 lb
	The effective force per foot of slab is 10.3 kip/ft , so the spacing of tendons is:	26.8 kip/10.3 kip × 12 in./ft = 31 in., or 2 ft-7 in.
	The value of A_{ps} is therefore, The value of ρ_p in $A_{ps}/(b \times d_p)$	A_{ps} = (0.153 in.^{2})(12 in./ft)/(31 in.) = 0.059 in.^{2}/ft. ρ_p = 0.059 in.^{2}/ft / 60 in^2 = 0.00098 $f_{ps} = 175{,}000\text{ psi} + 10{,}000\text{ psi} + \frac{5000\text{ psi}}{0.294} = 202{,}000\text{ psi}$
20.3.2.4.1	f_{ps} limit as follows: (a) f_{se} + 30,000 and (b) $f_{py} = 0.9f_{pu}$	(a) 175,000 psi + 30,000 psi = 205,000 psi and (b) (0.9)(270,000 psi) = 243,000 psi So the design value of f_{ps} = 202,000 psi
	The compression block depth is therefore: $a = \frac{A_{ps} f_{ps}}{0.85 f_c'(12\text{ in/ft.})}$	$a = \frac{(0.059\text{ in.}^2)(202{,}000\text{ psi})}{(0.85)(5000\text{ psi})(12\text{ in./ft})} = 0.23\text{ in.}$
22.2.2.4.1	Note that the effective depth is 5 in. at critical locations, except at the exterior joint. Therefore, the Code permits a minimum d of 0.8h, or 4.8 in. For 3/4 in. cover, $\phi M_n = \phi A_{ps} f_{ps}(d - a/2)$	ϕM_n = (0.9)(0.059 in.2)(202,000 psi)(5 in. – 0.12 in.) = 52,129 in.-lb = 4.34 ft-kip/ft.

	Location from left to right along the span				
	Face of exterior support	First midspan	Face of second support	Second midspan	Face of middle support
ϕM_n, only tendons, ft-kip	4.16	4.34	4.34	4.34	4.34
M_u, ft-kip	–0.8	5.9	–5.5	4.3	–4.6

Because almost all the design moments are greater than ϕM_n when considering the tendons alone, standard reinforcement lengths must be used.

Step 9: Minimum flexural reinforcement

7.6.2.3	The minimum area of flexural reinforcing bar per foot is a function of the slab's cross-sectional area.	$A_{s,\,min}$ = 0.004 × 12 in. × 3 in. = 0.15 in.^{2}/ft.

Step 10: Design moment strength

7.5.1	The two common strength inequalities for one-way slabs, moment and shear, are noted in Section 7.5.1.1 of ACI 318-11. The strength reduction factor in Section7.5.1.2 is assumed to be 0.9.	

7.5.2	Determine if supplying the minimum area of reinforcing bar is sufficient to achieve a design strength that exceeds the required strength. Comparing this value with the required moment strength M_u indicates that the minimum reinforcement plus the tendons supply enough tensile reinforcement for slab to resist the factored loads at all locations. $a = \dfrac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c'(12\text{ in./ft})}$	Set the section's concrete compressive strength equal to steel tensile strength, and rearrange for compression block depth a: $a = \dfrac{(0.059\text{ in.}^2)(202{,}000\text{ psi}) + (0.15\text{ in.}^2)(60{,}000\text{ psi})}{(0.85)(5{,}000\text{ psi})(12)}$ $a = 0.41$ in. For 3/4 in. cover: $M_n = \phi\left[A_{ps} f_{ps} + A_s f_y\right]\left(d - \dfrac{a}{2}\right)$ $= 0.9[0.059 \times 202{,}000 + 0.15 \times 60{,}000](5 - 0.21)$ $= 90{,}200$ in.-lb $= 7.5$ ft-kip Therefore, minimum reinforcement provides adequate strength to resist the applied moment. **OK**
Step 11: Design shear strength		
7.5.3.1 21.2.1	The slab's maximum shear is taken at the support centerline for simplicity. Shear strength reduction factor: Note: Shear does not typically control the thickness of one-way post-tensioning slab system.	$V_c = 2\left(\sqrt{5000\text{ psi}}\right)(12\text{ in.})(5\text{ in.}) = 8500\text{ lb}$ $\phi = 0.75$ $\phi V_c = (0.75)(2)\left(\sqrt{5000\text{ psi}}\right)(12\text{ in.})(5\text{ in.}) = 6364\text{ lb}$ The maximum $V_u = 2.0$ kips at the first interior support. Therefore, **OK**.

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Step 12: Shrinkage and temperature reinforcement		
7.6.4.2	To control shrinkage and temperature stresses in the direction to the span, it is typical to use tendons rather than mild reinforcement in one-way post tensioning slabs. To calculate the number and spacing of temperature tendons, the Code allows the designer to consider the effect of beam tendons on the slab. Assuming the beam is 12 in. x 30 in., the concrete cross-sectional area in the beam influence area is: Assume the beam has an effective post-tensioning force of 189 kips, which results in an average compression of: This amount is, therefore, sufficient to meet the Code minimum of 100 psi. The Code also has three spacing requirements which apply: Provide at least one tendon on each side of the beam. If temperature tendon spacing does not exceed 4.5 ft, additional reinforcing bar is not needed; but if temperature tendon spacing exceeds 4.5 ft, supplemental reinforcement is required along the edge of the slab adjacent to tendon anchors. Spacing above 6 ft is prohibited. In this example, temperature tendons, starting at 4 ft from the beam, are specified at 4 ft on center. No supplemental edge reinforcement is needed.	$A_{infl\ area}$ = (6 in.)(20 ft)(12 in./ft) + (12 in.)(24 in.) = 1726 in.^{2} σ = (189,000 lb)/(1726 in.^{2}) = 109 psi.
Step 13: Maximum spacing of flexural reinforcement		
7.7.2.3	The maximum spacing of flexural reinforcing bar in a PT slab is the lesser of 3*h* and 18 in.	The area of flexural reinforcing bar must be at least 0.15 in.^{2}/ft, use No. 4 bar at 16 in. on center, which also satisfies the maximum spacing requirement.

Step 14: Top reinforcing bar length at the exterior support		
7.7.3 7.7.3.3	<u>Reinforcing bar length and details at the exterior support</u> The top bars have to satisfy: Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	<u>Inflection points</u> The inflection point for negative moment is 1.5 ft from exterior support centerline. <u>Bar cutoffs</u> Section 7.7.3.3 requires all bars to extend beyond the inflection point at least d (5 in.) or 12 x 0.5 in; therefore 6 in.
7.7.3.8.4	At least one-third the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	In addition, Section 7.7.3.8.4 requires 33 percent of the bars to extend beyond the inflection point at least (19 ft)(12 in./ft)/16 = 15 in. The top bars at the exterior joint will extend from the end of the cantilever, past the support and into the span. Because the reinforcing bar is at wide spacing, no percentage of bars (as permitted by Section 7.7.3.8) can be cut off in the tension zone.
	<u>Balcony considerations</u> The architect usually specifies a slab recess of about 0.75 in. at the exterior wall of residential units to guard against water intrusion. In addition, the architect usually specifies a balcony slope of about 1/4 in./ft. These two details result in a slab thickness at the edge of 4.5 in. Balcony considerations are discussed in detail by Suprenant in, "Understanding Balcony Drainage," 2004, *Concrete International*, Jan.	<u>Solution</u> The top bar length is 3 ft. (balcony) plus 1.6 ft (inflection) plus an extension of 15 in. A practical length for top bars is 6 ft. <u>Trim bar at the outside edge</u> The PT suppliers usually require two No. 4 continuous "back-up" bars behind the anchorages, about 2 to 3 in. from the edge. These bars can also limit widths of possible cracks due to unexpected restraint, drying shrinkage, or other local issues. At the edge of the balcony, it is recommended to hook the top flexure bars around the continuous edge bars.

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Step 15: Bottom reinforcing bar length along first span		
7.7.3	The bottom bars have to satisfy the following provisions:	
7.7.3.3	Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	
7.7.3.4	Continuing flexural tensile reinforcement must have an embedment length not less than ℓ_d beyond the point where bent or terminated tensile reinforcement is no longer required to resist flexure. $$\ell_d = \left(\frac{3}{40}\frac{f_y}{\lambda\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$	Note: the development length of a No. 4 black bar in an 6 inch slab with 0.75 in cover is: $$\ell_d = \left(\frac{3}{40}\frac{60{,}000\text{ psi}}{1.0\sqrt{5000\text{ psi}}}\frac{1.0\times1.0\times0.8}{\left(\dfrac{1.0+0}{0.5}\right)}\right)0.5 = 13\text{ in.}$$
7.7.3.5	Flexural tensile reinforcement must not be terminated in a tensile zone unless (a), (b), or (c) is satisfied.	$V_u \leq (2/3)\ \phi V_n$ at the cutoff point. V_u = 2000 lb and ϕV_n = 6364 lb (refer to Step 11) 2/3 ϕV_n = 4243 lb > V_u = 2000 lb therefore, **OK** Note that (b) and (c) do not apply.
7.7.3.8.2	At least one-fourth the maximum positive moment reinforcement must extend along the slab bottom into the continuous support a minimum of 6 in.	Extend bottom reinforcement minimum 6 in. into the supports.
7.7.8.3	At points of inflection, d_b for positive moment, tensile reinforcement must be limited such that ℓ_d for that reinforcement satisfies Eq. (11.7.3.3.2) of ACI 318-14. $\ell_d \leq M_n/V_u + \ell_a$ <u>Inflection points</u> The inflection points for positive moments are 0.5 ft from the exterior support centerline and 3.0 ft from the first interior support centerline.	

	<u>Bar cutoffs</u> The bar cutoffs that are implicitly permitted in the aforementioned code provisions do not apply before the inflection points for this slab, because if any bars were cut off, the maximum reinforcing bar spac ing would be violated. Because all bottom bars extend past the tensile zone, Section 7.7.3.5 does not apply. All bottom bars need to extend at least 5 in. (Refer to Section 7.7.3.3) beyond the positive moment inflection points. The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support. <u>Check bar size</u> M_n is: $M_n = [A_s f_y]\left(d - \frac{a}{2}\right) + [A_{ps} f_{ps}]\left(d_p - \frac{a}{2}\right)$ The elastic analysis indicates that V_u at inflection point is 2.0 kips. The term ℓ_a is 5 in. $\ell_d \leq M_n/V_u + \ell_a$	M_n = (0.15 in.^{2})(60,000 psi)(5 in. – 0.21 in.) + [(0.059 in.^{2})(202,000 psi)(4.8 in. – 0.21 in.)] = 97.8 in.-kip $\ell_d \leq \frac{M_n}{V_u} + 5\text{ in.} = \frac{97.8\text{ in.-kip}}{2.0\text{ kip}} + 5 = 54\text{ in.}$ > 13 in.; therefore, No. 4 bar is OK. Because the cut off location is within a foot of the left support, extend all bottom bars 6 in. into left support and then to the edge of the cantilever. These bottom bars will support shrinkage and temperature bars in the balcony. For field placing simplicity, specify all bottom bars in this span also extend 6 in. into the right support.
Step 16: Top reinforcing bar length at the first interior support		
		<u>Inflection points</u> The inflection points for negative moments are 5.3 ft from the support centerline and 7.5 ft from the first interior support centerline.
7.7.3.3	<u>Bar cutoffs</u> All bars must extend beyond the inflection point at least d (5 in.) or 12 × 0.5 in;	Therefore, 6 in.
7.7.3.8.4	In addition, 33 percent of the bars must extend beyond the inflection point at least: However, the top bars at the exterior joint will extend from the end of the cantilever, past the support and into the span. Because the reinforcing bar is at wide spacing, no percentage of bars (as permitted by Section 7.7.3.8) can be cut off in the tension zone.	(19 ft x12 in./ft)/16 = 15 in. Therefore, the top bar length is 5.3 ft (inflection) plus an extension of 15 in. plus 7.5 ft plus 15 in. A practical length for top bars is 16 ft.

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Step 17: Second span bottom reinforcing bar lengths		
		Inflection points The inflection points for positive moments are 3.5 ft from the left support centerline and 2.5 ft from the right interior support centerline. Bar cutoffs The bar cut offs that are implicitly permitted by the Code do not apply before the inflection points for this slab, because if any bars were cut off, the maximum reinforcing bar spacing would be violated. Because all bottom bars extend past the tensile zone, Section 7.7.3.5 doesn't apply. All bottom bars need to extend at least 5 in. (Refer to Section 7.7.3.3) beyond the positive moment inflection points. The Code requires that at least 25 percent of bottom bars be full length, extending 6 in. into the support. Solution The minimum bottom bar length is (20 ft minus 3.5 ft (inflection) plus an extension of 5 in. minus 2.5 ft (inflection) plus 5 in. A practical length for bottom bars is one at 20 ft and three at 16 ft.
Step 18: Top reinforcing bar length at the middle support, CL C		
7.7.3	The top bars have to satisfy the following provisions:	Inflection points The inflection points for negative moments are 5.8 ft from the support centerline on both sides.
7.7.3.3	Reinforcement must extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and $12d_b$, except at supports of simply-supported spans and at free ends of cantilevers.	
7.7.3.8.4	At least one-third the negative moment reinforcement at a support must have an embedment length beyond the point of inflection at least the greatest of d, $12d_b$, and $\ell_n/16$.	Bar cutoffs Section 7.7.3.3 requires all bars to extend beyond the inflection point at least d (5 in.) or 12 x 0.625 in; therefore, 8 in. In addition, Section 7.7.3.8.4 requires 33 percent of the bars to extend beyond the inflection point at least $(19 \times 12)/16 = 15$ in. Because the reinforcing bar is already at maximum spacing, no percentage of bars (as permitted by Section 11.7.3.5 or 7.7.3.8 of ACI 318-14) can be cut off in the tension can be cut off in the tension zone. Solution The top bar length is two times (5.8 ft (inflection) plus an extension of 15 in. at each end) A practical length for top bars is 15 ft.

Step 19: Tendon termination		
	There are requirements both for anchorage zones (the reinforced concrete around the anchorage) and for the anchorages themselves.	
7.7.4.3.1	Post-tensioned anchorage zones must be designed and detailed in accordance with Section 25.9 of ACI 318-14.	The concrete around the anchorage is divided into a local zone and a general zone. For monostrand anchorages, the local zone reinforcement, according to Code, “shall meet the bearing resistance requirements of ACI 423.7.” ACI 423.7 limits the bearing stresses an anchorage can impose on the concrete, unless the monostrand anchorage is tested to perform, as well as those meeting those stresses. All U.S. manufacturers’ supply tested anchorages. For the general zone, Section 25.9.3.1 (a) of ACI 318-14 requires two “back up” bars for monostrand anchorages at the edge of the slab, and Section 25.9.3.2 (b) is not applicable to this example.
7.7.4.3.2	Post-tensioning anchorages and couplers must be designed and detailed in accordance with 25.7.	The information in Section 25.7 of ACI 318-14 provides performance requirements for the design of PT anchorages. These only apply to the anchorage design, so the engineer rarely (if ever) is concerned about Section 25.7.
Step 20: Shrinkage and temperature tendons		
7.7.6.3	There are 4 temperature tendons evenly spaced per span (at 4 ft-0 in. O/C). Because the spacing is less than 4 ft 6 in., no additional edge reinforcement is required by the Code. The commentary recommends placing temperature tendons so that the resultant is within the kern of the slab (middle 2 inches). Anchors are usually attached to the outside forms at mid-height of the slab and longitudinally supported directly by the flexural tendons in such a manner as to meet this recommendation.	
Step 21: Detailing		

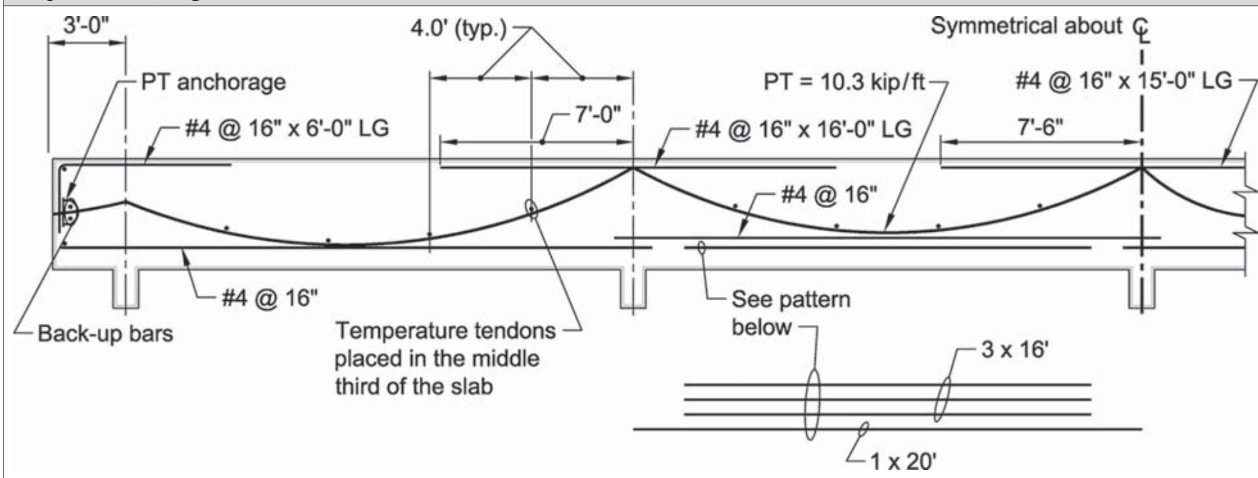

Fig. E3.8—Slab reinforcement.

CHAPTER 6—TWO-WAY SLABS

6.1—Introduction

A two- way slab is usually used in buildings with columns that are approximately evenly spaced, creating a span length in one direction that is within a factor of 2 to the perpendicular direction. Structural concrete two-way slabs, which have been constructed for over 100 years, have taken many forms. The basic premise for these forms is that the slab system transmits the applied loads directly to the supporting columns through internal flexural and shear resistance.

This chapter discusses cast-in-place, nonprestressed, and post-tensioned (PT) slabs. The Code (ACI 318-14) allows for either bonded or unbonded tendons in a PT slab. Because bonded tendons are not usually placed in two-way slabs in the U.S., this chapter only discusses PT slabs with unbonded tendons.

At the preliminary design level, with spans given by the architect, the designer determines the loads, reinforcement type (prestressed or nonprestressed), and slab thickness. The preliminary concrete strength is based on experience and the Code's exposure and durability provisions.

6.2—Analysis

ACI 318-14 allows the designer to use any analysis procedure that satisfies equilibrium and geometric compatibility, as long as design strength and serviceability requirements are met. The Code includes detailed provisions for the Direct Design Method (DDM) and the Equivalent Frame Method (EFM), as well as general provisions for Finite Element Analysis (FEA). The commentary notes that while the analysis of a slab system is important, the design results should not deviate far from common practice, unless it is justified based on the reliability of the calculations used in the analysis.

6.2.1 *Direct Design Method*—The DDM (ACI 318-14, Section 8.10) is a simplified method of analysis that has several geometric and loading limitations. Nonprestressed reinforced flat plates, flat slabs, and waffle slabs can all be designed by this method. The Code does not permit PT slabs to be designed by DDM. The results of the DDM are the approximate magnitude and distribution of slab moments, both along the span and transverse to it. The coefficients that distribute the total static moment in the design panel to the column and middle strips are based on papers by Corley, Jirsa, Sozen, and Siess (Corley et al. 1961; Jirsa et al. 1963, 1969; Corley and Jirsa 1970). The total static moment is determined assuming that the reactions are along the faces of the support perpendicular to the span considered. Once the total static moment is determined, it is then distributed to negative and positive moment areas of the slab. From there, it is further distributed to the column strip and middle strips. The designer uses these moments to calculate the flexural reinforcement area in the direction being designed. The designer needs to perform calculations in both directions to determine two-way slab reinforcement. The DDM also provides the design shear at each column.

6.2.2 *Equivalent Frame Method*—The EFM (ACI 318-14, Section 8.11) can be used for a broader range of slab geometries than are allowed for DDM use, as well as PT slabs. Flat plates, flat slabs, and waffle slabs can all be designed by this method. The EFM assumptions used to calculate the effective stiffness of the slab, torsional beams, and columns at each joint are based on papers by Corley, Jirsa, Sozen, and Siess (Corley et al. 1961; Jirsa et al. 1963, 1969; Corley and Jirsa 1970). The EFM models a three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The original analysis method used with the EFM was the moment distribution method; however, any linear elastic analysis will work. The analysis calculates design moments and shears along the length of the model. For nonprestressed slabs, the EFM uses DDM coefficients to distribute the total moments into column strips and middle strips. For prestressed slabs, the slab strip is from the middle of one bay to the middle of the next bay, and is designed in flexure as a wide, shallow beam.

6.2.3 *Finite Element Method*—A great variety of FEA computer software programs are available, including those that perform static, dynamic, elastic, and inelastic analysis. Any two-way slab geometry can be accommodated. Finite element models could have beam-column elements that model structural framing members along with plane stress elements; plate elements; and shell elements, brick elements, or both, that are used to model the floor slabs, mat foundations, diaphragms, walls, and connections. The model mesh size selected should be capable of determining the structural response in sufficient detail. Any set of reasonable assumptions for member stiffness is allowed.

6.3—Service limits

6.3.1 *Minimum thickness*—For nonprestressed flat plates and flat slabs, the Code allows the designer to either calculate slab deflections or simply satisfy a minimum slab thickness (ACI 318-14, Section 8.3.1). Most flat slab and flat plate designs simply conform to the minimum thickness criteria and, therefore, designers do not usually calculate deflections for nonprestressed reinforced two-way slabs. The Code does not provide a minimum thickness-to-span ratio for PT two-way slabs, but the ratio for usual conditions is in the range of 37 to 45.

6.3.2 *Deflections*—For nonprestressed two-way flat plate or flat slabs that are thinner than the ACI 318 minimum, for slabs that resists a heavy live load, for waffle slabs, and for PT slabs, the designer calculates deflections. Deflections can be calculated by EFM, the FEM, or classical methods. For EFM, the slab system is modelled in both directions, and the calculated deflection at midspan of a panel is the sum of the column strip deflection and the perpendicular middle strip deflection (refer to the crossing beam method (ACI 435R)).

The calculated deflections must not exceed the limits in Section 24.2 of ACI 318-14. For most buildings, the limit of $\ell/480$ for long term deflections usually controls.

Note that the spacing of slab reinforcing bar to limit crack width, timing of form removal, concrete quality, timing of construction loads, and other construction variables all can affect the actual measured deflection. These variables should be considered when assessing the accuracy of deflection calculations. In addition, creep over time will increase the immediate deflections.

Typically, with a PT slab thickness-to-span ratios in the range of 37 to 45, slab deflections are usually within the Code allowable limits. The Code limits the maximum service concrete tensile stress to below cracking stress, so deflection calculations use the gross slab properties.

6.3.3 *Concrete service stress*—Nonprestressed slabs are designed for strength without reference to a pseudo-concrete service flexural stress limit.

For PT slabs, the analysis of concrete flexural tension stresses is a critical part of the design. In ACI 318-14, Section 8.3.4.1, the concrete tensile flexural stress in negative moment areas at columns PT slab is limited to $6\sqrt{f_c'}$. At positive moment sections, Section 8.6.2.3 requires slab reinforcing bar if the concrete tensile stress exceeds $2\sqrt{f_c'}$. This bottom reinforcing bar is often not required. These service tensile flexural stress limits are below the concrete cracking stress of $7.5\sqrt{f_c'}$, thus having the effect of reducing deflections. In addition, Section 8.6.2.1 requires a PT slab's axial compressive stress in both directions due to post-tensioning to be at least 125 psi.

Before the slab flexural stresses in a design strip can be calculated, the tendon profile needs to be defined. The profile and the tendon force are directly related to the slab forces and moments created by the PT. A common approach to calculate PT slab moments is to use the "load balancing" concept, where the profile is usually the maximum practical considering cover requirements, the tendon profile is parabolic, the parabola has an angular "break" at the column centerlines, and that the tendon terminates at middepth at the exterior (refer to Fig 6.3.3).

The load balancing concept assumes the tendon exerts a uniform upward "load" in the parabolic length, and a point load down at the support. These loads are then combined with the gravity loads, and the analysis is performed with a net load. Fig. 6.3.3 shows the commonly used simplification of the tendon profile. The real tendon profile is smooth with reverse parabolas over the interior supports rather than cusps.

To conform to the Code stress limits, the designer can use an iterative approach or a direct approach. In the iterative approach, the tendon profile is defined and the tendon force is assumed. The analysis is executed, flexural stresses are calculated, and the designer then adjusts the profile or force or both, depending on results and design constraints.

In the direct approach, the designer determines the highest tensile stress permitted, then rearranges equations so that the analysis calculates the tendon force needed to achieve the stress limit.

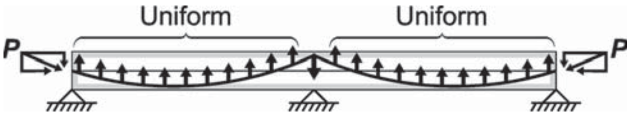

Fig. 6.3.3—Load balancing concept.

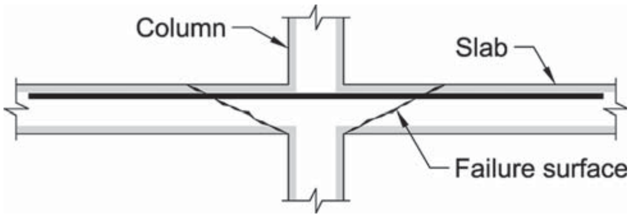

Fig. 6.4.1—Punching shear failure.

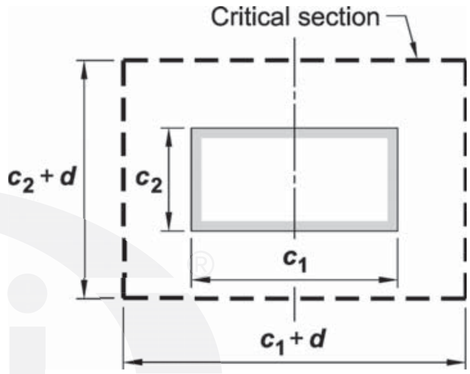

Fig. 6.4.2—Critical section geometry.

6.4—Shear strength

Two-way slabs must have adequate one-way shear strength in each design strip (assuming the slab is a wide, shallow beam) and adequate two-way shear strength at each column. The discussion for the nominal one-way shear strength are the same as provided in Chapter 7 (Beams) of this Handbook and is not reproduced here.

6.4.1 *Punching shear strength*—Two-way shear strength, also called punching shear strength, is considered a critical strength for two-way slabs. ACI 318 calculates nominal punching shear strength based on the slab's concrete strength and shear reinforcement when provided. The effect of the slab's flexural reinforcement on punching shear strength is ignored.

The assumed punching shear failure shape (Fig. 6.4.1) is usually a truncated cone or pyramid-shape surface around the column.

6.4.2 *Critical section*—For geometric simplicity, ACI 318 assumes a critical section. This is a vertical section extended from the column at a distance $d/2$, where d is the slab's effective depth. In Fig. 6.4.2, the critical perimeter is $b_o = 2[(c_1 + d) + (c_2 + d)]$. The critical section to calculate concrete shear stress is b_od.

6.4.3 *Calculation of nominal shear strength*—In ACI 318, punching shear strength limits are given in terms of stress. As shown below, the shear stress limit for a nonprestressed reinforced slab is the least of three expressions:

The Code punching shear strength limit for PT slabs are usually slightly higher than those for nonprestressed rein-

Table 6.4.3—Calculation of v_c for two-way shear (ACI 318-14, Table 22.6.5.2)

	v_c
Least of:	$4\lambda\sqrt{f_c'}$
	$\left(2+\frac{4}{\beta}\right)\lambda\sqrt{f_c'}$
	$\left(2+\frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'}$

Note: β is the ratio of long side to short side of the column and α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

forced slabs as shown in the following Eq. (a) and (b). For PT two-way slabs, the designer can use Eq. (a) and (b) unless the column is closer to a discontinuous edge than four times the slab thickness *h*. For many edge columns, this requires shear strength to be calculated by Table 6.4.3.

For prestressed, two-way members, v_c is permitted to be the lesser of (a) and (b) (ACI 318-14, Eq. 22.6.5.5 (a) and (b):

$$\text{(a)} \quad v_c = \left(3.5\lambda\sqrt{f_c'} + 0.3 f_{pc}\right) + \frac{V_p}{b_o d}$$

$$\text{(b)} \quad v_c = \left(1.5 + \frac{\alpha_s d}{b_o}\right)\lambda\sqrt{f_c'} + 0.3 f_p c + \frac{V_p}{b_o d}$$

where α_s is the same as in Table 6.4.3, the value of f_{pc} is the average of f_{pc} in the two directions, limited to 500 psi, V_p is the vertical component of the effective prestress force crossing the critical section, and the value of $\sqrt{f_c'}$ is limited to 70 psi.

Because of the shallow depth of most PT slabs, many engineers conservatively ignore the $V_p/b_o d$ component when calculating v_c.

The Code also requires the engineer to consider slab openings close to the column. Such openings, which are commonly used for heating, ventilating, and air conditioning (HVAC), and plumbing chases, will reduce the shear strength. The Code requires a portion of b_o enclosed by straight lines projecting from the centroid of the column and tangent to the boundaries of the opening to be considered ineffective (ACI 318-14, Section 22.6.4.3).

6.5—Calculation of required shear strength

The factored punching shear stress at a column, v_u, is the total of two components: 1) direct shear stress, v_{ug}; and 2) shear stresses due to moments transferred from the slab to the column. The two stress diagrams are added and the total is the required shear stress diagram at the critical section.

Direct shear stress v_{ug} is calculated by $v_{ug} = V_u/b_o d$. To calculate the shear stresses due to slab bending, ACI 318 first stipulates that a percentage of the unbalanced slab moment at the column, M_{sc}, is resisted by slab flexure within a limited width over the column. The remaining percentage of M_{sc} is transferred to the slab by eccentricity of shear. The follow two sections of Chapter 8 (ACI 318-14) state that:

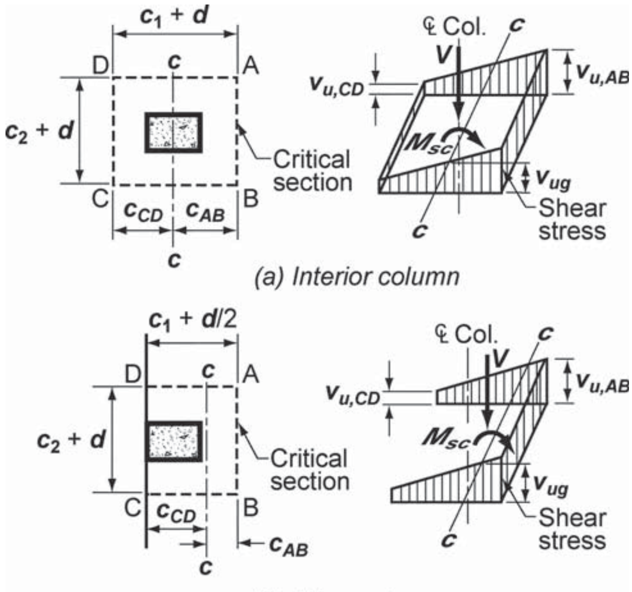

Fig. 6.5—Assumed distribution of shear stress (ACI 318-14, Commentary Section R8.4.4.2.3).

8.4.2.3.2 The fraction of factored slab moment resisted by the column, $\gamma_f M_{sc}$, shall be assumed to be transferred by flexure, where γ_f shall be calculated by:

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

8.4.4.2.2 The fraction of M_{sc} transferred by eccentricity of shear, $\gamma_v M_{sc}$, shall be applied at the centroid of the critical section in accordance with Section 8.4.4.1 (ACI 318-14), where

$$\gamma_v = 1 - \gamma_f$$

Under certain circumstances given in Table 8.4.2.3.4 of ACI 318-14, the value of γ_f can be increased, which then decreases the fraction of M_{sc} required to be transferred by eccentricity of shear.

Note that these modified values do not apply for PT slabs.

The slab shear stresses due to the unbalanced moment transferred to the column by eccentricity of shear is calculated by $\gamma_v M_{sc} c/J_c$, where *c* is the distance from b_0 to the critical section centroid, and J_c is the polar moment of inertia of the critical section about its centroidal axis. When v_{ug} is added, the total shear stress diagram is shown by Fig. 6.5.

If the maximum total factored shear stress does not exceed the design shear stress, the slab's concrete shear strength is adequate. If the maximum total factored shear stress exceeds the design shear stress, the slab thickness near the column can be increased by using, for example, shear capitals (ACI 318-14, Section 8.2.5) or shear reinforcement can be added.

At times, the design of a two-way slab requires point loads to be considered, such as wheel loads in parking garages.

Two-Way Slabs

Table 6.6a—Maximum v_c for two-way members with shear reinforcement (ACI 318-14, Table 22.6.6.1)

Type of shear reinforcement	Maximum v_c at critical sections defined ACI 318-14, Section 22.6.4.1	Maximum v_c at critical section defined in ACI 318-14, Section22.6.4.2
Stirrups	$2\lambda\sqrt{f_c'}$	$2\lambda\sqrt{f_c'}$
Headed shear stud reinforcement	$3\lambda\sqrt{f_c'}$	$2\lambda\sqrt{f_c'}$

The contribution of shear reinforcement is calculated by: $v_s = A_v f_{yt}/b_o s$ (ACI 318-14, Eq. (22.6.8.2)).

Table 6.6b—Maximum v_u for two-way members with shear reinforcement (ACI 318-14, Table 22.6.6.2)

Type of shear reinforcement	Maximum v_u at critical sections defined in 22.6.4.1
Stirrups	$\phi 6\sqrt{f_c'}$
Headed shear stud reinforcement	$\phi 8\sqrt{f_c'}$

These result in local shear slab stresses, and the slab's punching shear strength in that area needs to be verified.

6.6—Calculation of shear reinforcement

Shear reinforcement can be provided to increase the slab's nominal shear strength close to a column. Assuming the shear reinforcement is uniformly spaced, shear strength is first checked at the first critical section at $d/2$ beyond the column face including the contribution of shear reinforcement. The shear strength is then checked at $d/2$ beyond the outermost peripheral line of shear reinforcement, without the contribution of shear reinforcement. In slabs without shear reinforcement, v_c is usually $4\sqrt{f_c'}$; however, for slab sections with shear reinforcement, the concrete contribution to shear strength is limited to the values in Table 6.6a.

There is an upper limit to a slab's nominal shear strength even with shear reinforcement, as shown in Table 6.6b. The Code states this limit in terms of the maximum factored two-way shear stress, v_u, calculated at a critical section.

Note that the use of stirrups as slab shear reinforcement is limited to slabs with an effective depth d that satisfy (a) and (b):

(a) d is at least 6 in.

(b) d is at least $16d_b$, where d_b is the diameter of the stirrups

The use of shear studs is not limited by the slab thickness, but the studs must fit within the geometric envelop. The overall height of the shear stud assembly needs to be at least the thickness of the slab minus the sum of (a) through (c):

(a) Concrete cover on the top flexural reinforcement

(b) Concrete cover on the base rail

(c) One-half the bar diameter of the flexural tension reinforcement

6.7—Flexural strength

After the designer calculates the factored slab moments, the required area of flexural reinforcement over a slab width is calculated with the same behavior assumptions as a beam.

6.7.1 *Calculation of required moment strength*—There are two calculations for required moment strength for two-way slabs. The first calculation is to determine factored moments over the entire panel in the positive and negative moment areas.

For nonprestressed reinforced slabs, the slab analysis should provide the distribution of panel factored moments to the column strip and middle strip.

For PT slabs, effects of reactions induced by prestressing (secondary moments) need to be included. The slab's secondary moments are a result of the column's vertical restraint of the slab against the PT load at each support. Because the PT force and drape are determined during the service stress checks, secondary moments can be quickly calculated by the load-balancing analysis concept.

A simple way to calculate the secondary moment is to subtract the tendon force times the tendon eccentricity (distance from the NA) from the total balance moment, expressed mathematically as $M_2 = M_{bal} - P \times e$.

The second calculation is to determine $\gamma_f M_{sc}$ at each slab-column joint. The value of M_{sc} is the difference between the design moments on either side of the column.

6.7.2 *Calculation of design moment strength*—In a nonprestressed slab, the required reinforcement area A_s resisting the column and middle strip's negative and positive M_u is usually placed uniformly across each strip. The required reinforcement area A_s resisting $\gamma_f M_{sc}$ must be placed within a width b_{slab}.

For PT slabs, the tendons are banded, which is where all tendons are placed together in a line that follows the column lines in one direction and uniformly distributed in the other. The slab flexural strength calculations for tendons (with f_{ps} determined from Section 20.3.2.4 of ACI 318-14 substituted for f_y in the M_n equation) in the banded direction and in the uniform direction are the same, regardless of the tendon's horizontal location within the slab.

For PT slabs, the reinforcement area A_s resisting the panel's negative M_u is usually placed only at the column region. The A_s plus A_{pt} resisting $\gamma_f M_{sc}$ must be placed within a width b_{slab} per ACI 318-14, Section 8.4.2.3.3. If the panel reinforcement already within b_{slab} is not sufficient, designers usually add only A_s to increase the flexural strength.

For PT slabs, the A_{pt} provided to limit concrete service tensile stresses will usually be sufficient to also resist the panel's positive M_u.

6.8—Shear reinforcement detailing

6.8.1 *Stirrups*—If stirrups are provided to increase shear strength, ACI 318 provides limits on their location and spacing in Table 6.8.1.

The related ACI 318-14 Commentary Fig. R8.7.6d as shown in the following Fig. 6.8.1 of this Handbook, also includes the two critical section locations:

6.8.2 *Shear studs*—If shear studs are provided to increase shear strength, ACI 318-14 provides limits on shear stud locations and spacing in Table 6.8.2.

The related ACI 318-14 Commentary Fig. R8.7.7 as shown as the following Fig. 6.8.2, which also includes the two critical section locations.

6.9—Flexure reinforcement detailing

6.9.1 *Nonprestressed reinforced slab reinforcement area and placing*—The Code requires a minimum area of flexural reinforcement in tension regions, with the area as shown in Table 6.9.1 (ACI 318-14, Table 8.6.1.1). If more than the minimum area is required by analysis, that reinforcement area must be provided.

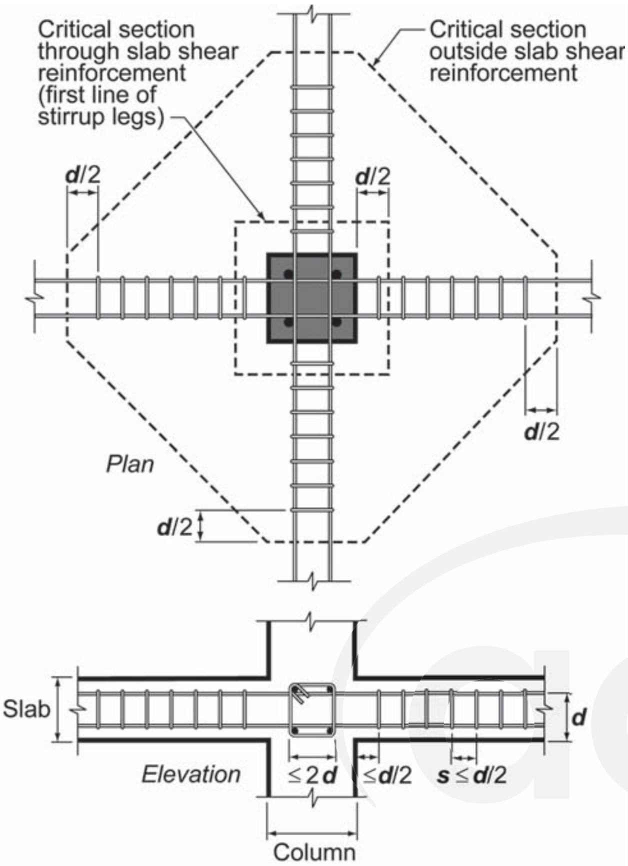

Fig. 6.8.1—Arrangement of stirrup shear reinforcement, interior column (ACI 318-14, Commentary Fig. R8.7.6d).

Table 6.8.1—First stirrup location and spacing limits (ACI 318, Table 8.7.6.3)

Direction of measurement	Description of measurement	Maximum distance or spacing, in.
Perpendicular to column face	Distance from column face to first stirrup	$d/2$
	Spacing between stirrups	$d/2$
Parallel to column face	Spacing between vertical legs of stirrups	$2d$

Two-way slab flexural reinforcement is placed in top and bottom layers. For nonprestressed reinforced two-way slabs without beams, Fig. 6.9.1 (ACI 318-14, Commentary Fig. R8.6.1.1) provides a typical layout of column strip and middle strip top and bottom bars. ACI 318-14, Fig. 8.7.4.1.3(a) provides the minimum reinforcing bar extensions, lap locations, and the minimum A_s at various sections. If the panel geometry is rectangular rather than square, the outer layer is usually placed parallel to the longer span.

6.9.2 *Corners*—Corner restraint, created by walls or stiff beams, induces slab moments in the diagonal direction and perpendicular to the diagonal. These moments are in addition to the calculated flexural moments. Additional reinforcement per ACI 318-14, Section 8.7.3, is required for this condition.

6.9.3 *Post-tensioned slab – Reinforcing bar area and placing*—Over each column region, the Code requires an area of flexural reinforcing bar of at least $0.0075A_{cf}$ in each direction, placed within $1.5h$ of the outside of the column. The Code also requires reinforcing bar in positive moment areas if the calculated service tensile flexural stress in other areas (usually midspan at the bottom) exceeds $2\sqrt{f_c'}$ or if required for strength. Bottom bar placement is at the discretion of the designer.

The Code allows for a reduced top and bottom minimum bar lengths if the design strength, calculated with only the PT tendons, is at least the required design strength. The top bars must extend at least $\ell/6$ on each side of the column. The bottom bars (if needed) must be at least $\ell/3$ and be centered at the maximum moment. The shorter lengths often control under typical spans and loadings. If the sectional strength using only the area of PT is insufficient to satisfy design strength, then the minimum top and bottom bar lengths are the same as a nonprestressed reinforced slab.

6.9.4 *Post-tensioned slab – Tendon area and placing*—A minimum of 125 psi axial compression in each direction is required in a PT two-way slab. Post-tensioned tendons are usually placed in two orthogonal directions. In this configuration, the Code allows banding of tendons in one direction and in the other direction the tendon spacing is uniform across the design panel, within the spacing limits of $8h$ and 5 ft. This layout is predominant in the U.S. The Code also requires at least two tendons to be placed within the column

Table 6.8.2—Shear stud location and spacing limits (ACI 318-14, Table 8.7.7.1.2)

Direction of measurement	Description of measurement	Condition		Maximum distance or spacing, in.
Perpendicular to column face	Distance from column face to first peripheral line of shear studs	All		$d/2$
	Constant spacing between peripheral lines of shear studs	Nonprestressed slab with	$v_u \le \phi 6\sqrt{f_c'}$	$3d/4$
		Nonprestressed slab with	$v_u > \phi 6\sqrt{f_c'}$	$d/2$
		Prestressed slabs conforming to Section 22.6.5.4 of ACI 318-14.		$3d/4$
Parallel to column face	Spacing between adjacent shear studs on peripheral line nearest to column face	All		$2d$

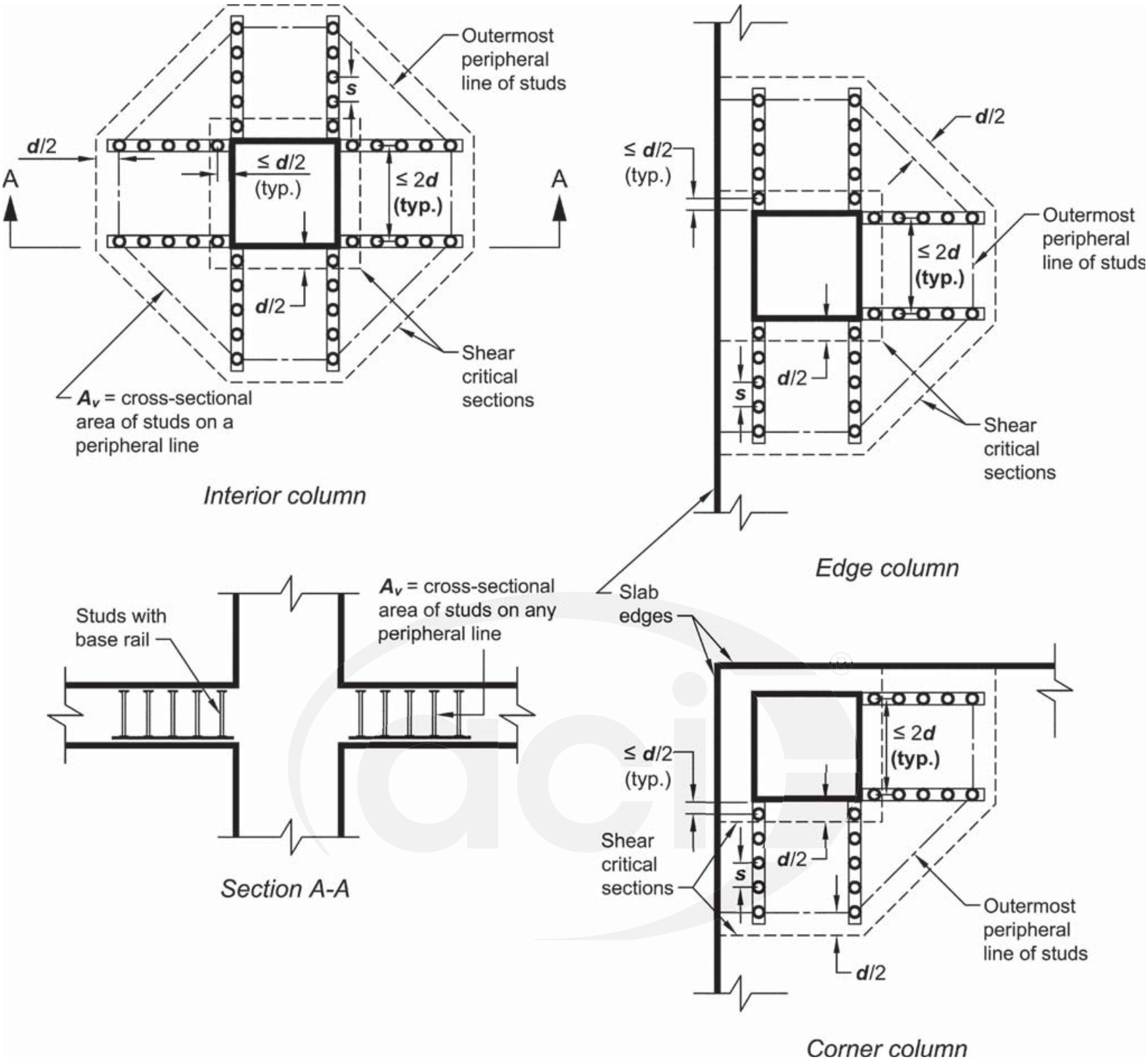

Fig. 6.8.2—Typical arrangements of headed shear stud reinforcement and critical sections (ACI 318-14, Commentary Fig. R8.7.7)

Table 6.9.1—$A_{s,min}$ for nonprestressed two-way slabs

Reinforcement type	f_y, psi	$A_{s,min}$, in.^{2}	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\dfrac{0.0018 \times 60{,}000}{f_y}A_g$
			$0.0014A_g$

reinforcement cage in either direction for overall building integrity.

6.9.5 *Slab openings*—For relatively small slab openings, trim reinforcing bar usually limits crack widths that can be caused by geometric stress concentrations and provides adequate strength. For larger openings, a local increase in slab thickness as well as additional reinforcement may be necessary to provide adequate serviceability and strength.

REFERENCES

American Concrete Institute (ACI)

ACI 435R-95—Control of Deflection in Concrete Structures (Reapproved 2000)

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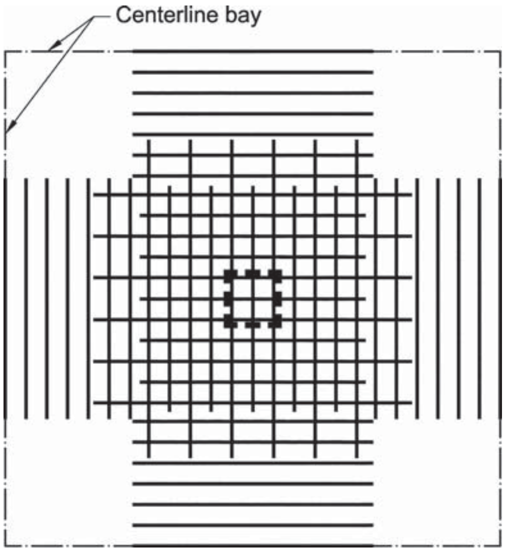

Fig. 6.9.1—Arrangement of minimum reinforcement near the top of a two-way slab (ACI 318-14, Commentary Fig. R8.6.1.1).

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6.10—Examples

Two-way Slab Example 1: *Two-way slab design using direct design method (DDM) – Internal Frame*

This two-way slab is nonprestressed without interior beams between supports. This example designs the internal strip along grid line B. Material properties are selected based on the code requirements of Chapters 5 and 6, engineering judgment, and locally available materials. Lateral loads are resisted by shear walls; therefore, the design is for gravity loads only. Diaphragm design is not considered in this example.

Given:

Uniform loads:

Self-weight dead load is based on concrete density including reinforcement at 150 lb/ft^3

Superimposed dead load D = 0.015 kip/ft^2

Live load L = 0.100 kip/ft^2

Material properties:

f_c' = 5000 psi

f_y = 60,000 psi

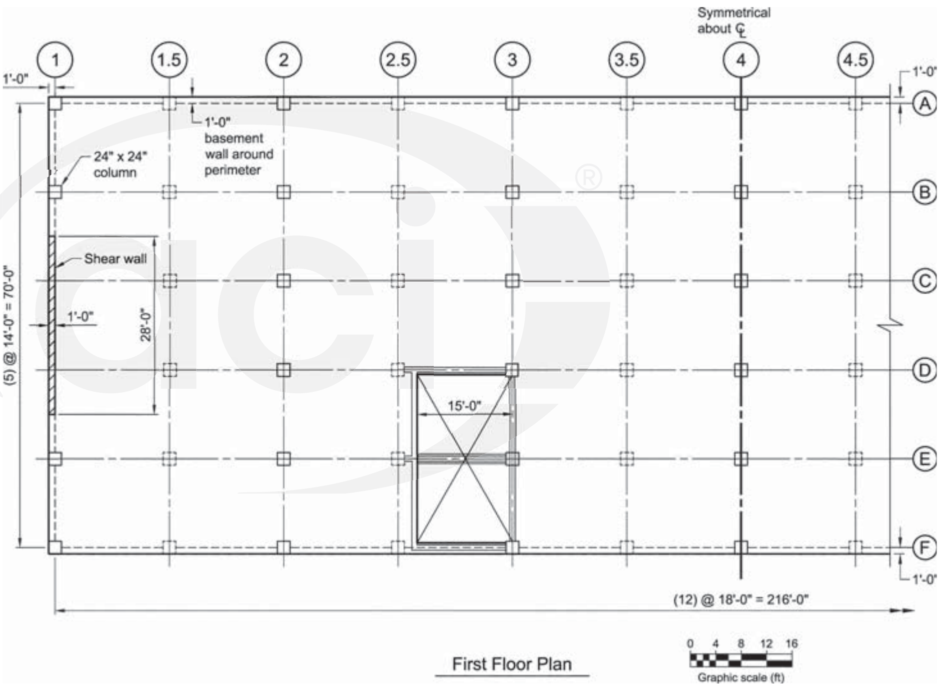


Fig. E1.1—First floor plan

ACI 318-14	Discussion	Calculation
Step 1: Geometry		
8.2.1 8.10 8.10.1.1 8.10.2.1 8.10.2.2 8.10.2.3 8.10.2.4 8.10.2.5 8.10.2.6 8.10.2.7 8.2.4 8.2.5	This slab is designed using the Direct Design Method (DDM) in Section 8.10. The slab geometry satisfies the limits of Sections 8.10.2.1 through 8.10.2.4, which allows the use of DDM. The uniform design loads satisfy the limits of Sections 8.10.2.5 and 8.10.2.6 to allow use of DDM.	There are at least three continuous spans in each direction so Section 8.10.2.1 is satisfied. The successive spans are the same lengths so Section 8.10.2.2 is satisfied. The ratio of the longer to the shorter panel dimension is 1.29 so Section 8.10.2.3 is satisfied. Columns are not offset through the slab so Section 8.10.2.4 is satisfied. All design loads are distributed uniformly and due to gravity only so Section 8.10.2.5 is satisfied. Ratio of unfactored live load to unfactored dead load is approximately 100/102.5 = 0.98. This ratio is less than 2, so Section 8.10.2.6 is satisfied. There are no supporting beams so Section 8.10.2.7 is not applicable. This example does not include drop panels or shear caps so Sections 8.2.4 and 8.2.5 are not applicable.
8.3.1.1	Check the slab thickness for deflection control.	Using Table 8.3.1.1 with f_y = 60,000 psi, without drop panels, and assuming the wall performs as a stiff edge beam, the minimum thickness for the external panel is: $\frac{\ell_n}{33} = \frac{192\text{ in.}}{33} = 5.8\text{ in.}$ The minimum thickness for the internal panels is calculated using the same table and the result is the same as that for the external panel. The remainder of the building is using a slab thickness of 7 in.; therefore, use 7 in. The slightly thicker than necessary slab aids with both deflections and shear strength.
8.3.1.3	No concrete floor finish is placed monolithically with the slab or composite with the floor slab.	
8.3.2	Calculated deflections are not required because the slab thickness-to-span ratio satisfies Section 8.3.2.1 of ACI 318-14.	
Step 2: Load and load patterns		
8.4.1.1	The load factors are provided in Table 5.3.1 of ACI 318-14.	The load combination that controls is 1.2D + 1.6L. Because Section 8.10.2.6 is satisfied in Step 1, pattern loading need not be checked.

Two-Way Slabs

Step 3: Initial two-way shear check		
	Before performing detailed calculations, it is often beneficial to perform an approximate punching shear check. This check should reduce the probability of having to repeat the calculations shown in this example. This check uses the following limits on the ratio of the design shear strength to the effects of shear stress based on direct shear stress alone ($\phi v_n/v_{ug}$): For interior columns: $\phi v_n/v_{ug} \geq 1.2$ For edge columns: $\phi v_n/v_{ug} \geq 1.6$ For corner columns: $\phi v_n/v_{ug} \geq 2.0$ If these ratios are not exceeded, it is possible that the slab will not satisfy two-way shear strength requirements. The design slab could be thickened, drop panels added, or other options for adding two-way shear strength may be considered.	For $\phi \boldsymbol{v_n}$, the calculations here are discussed in Step 10 more fully: $v_n = 4\sqrt{f_c'} = \frac{4\sqrt{5000}}{1000}$ ksi = 0.283 ksi $v_n = \left(2 + \frac{4}{\beta}\right)\sqrt{f_c'} = \frac{6\sqrt{5000}}{1000}$ ksi = 0.424 ksi $v_n = \left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = \frac{3.89\sqrt{5000}}{1000}$ ksi = 0.275 ksi* *controls $\phi v_n = 0.75 \times 0.275$ ksi = 0.206 ksi For v_{ug}, the calculations here are discussed in Step 7 more fully: $v_{ug} = \frac{V_u}{b_o d}$ $V_u = \left(14\text{ ft} \times 18\text{ ft} - \frac{29.6\text{ in.} \times 29.6\text{ in.}}{144}\right) \times \frac{283}{1000}\frac{\text{kip}}{\text{ft}^2}$ $V_u = 70$ kip $v_{ug} = \frac{70\text{ kip}}{118.4\text{ in.} \times 5.6\text{ in.}} = 0.106$ ksi $\phi v_n/v_{ug} = 0.206/0.106 = 1.94 \geq 1.2 \therefore$ proceed. Note that due to the basement wall supporting the exterior perimeter of the slab, the punching shear for the edge and corner columns will not need to be checked in this example.
Step 4: Analysis – Direct design method moment determination		
8.4.1.3 8.4.1.4 8.4.1.5 8.4.1.6 8.10.3.1	The geometry of the design is shown in Fig. E1.2.	The design strip is bounded by the panel center line on each side of the column line and consists of a column strip and two half-middle strips.

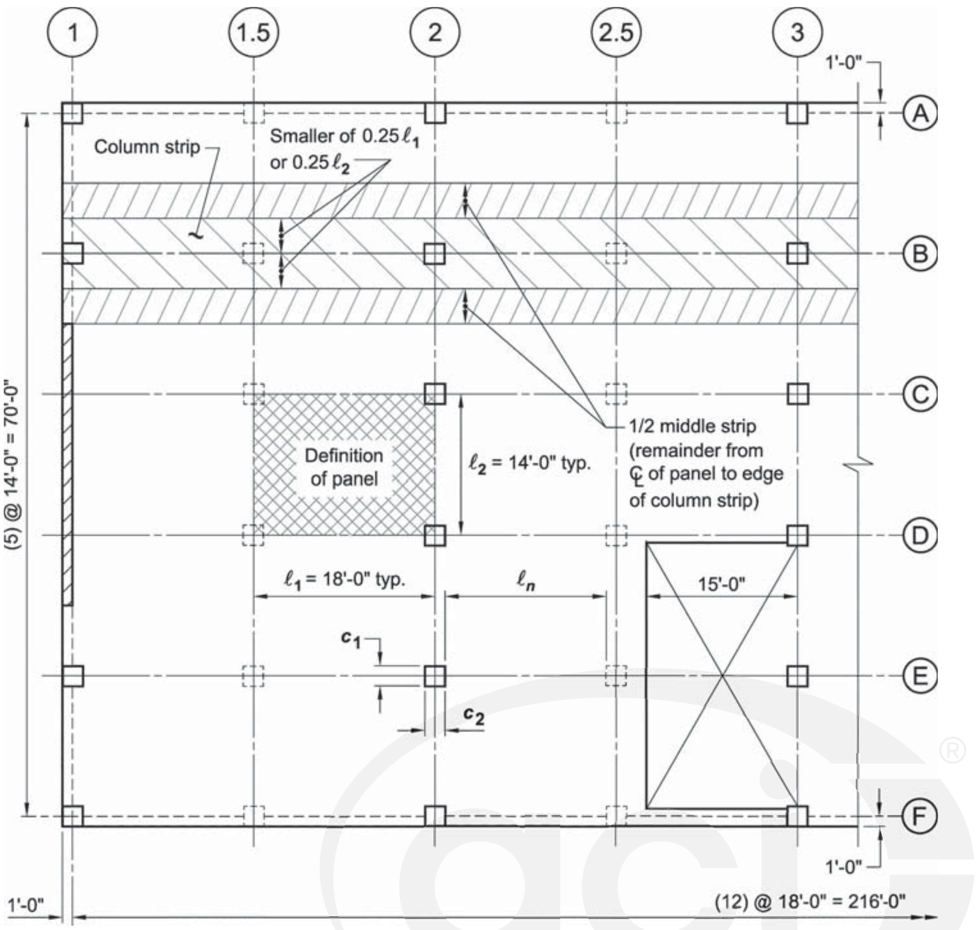
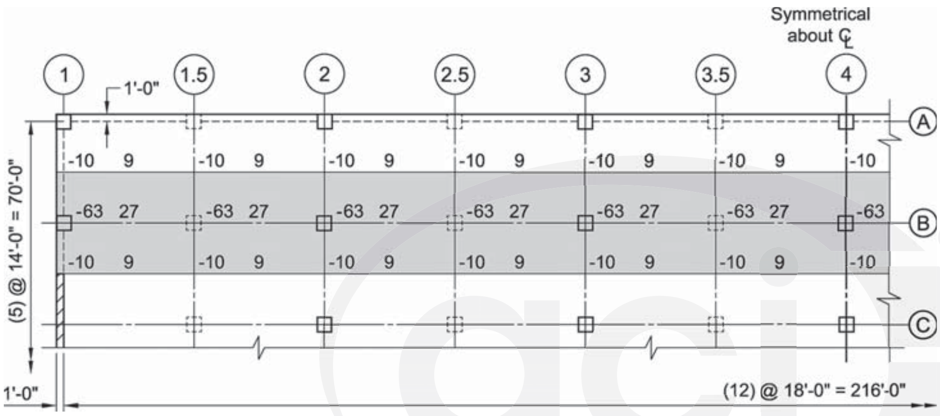


Fig. E1.2—Geometry definitions of panels and strips

8.4.1.9	The lateral loads in this building are assumed to be resisted by shear walls with the slabs only acting as diaphragms between the shear walls. The slab is assumed not to contribute flexural resistance to lateral loads.	Because all lateral loads are assumed to be resisted by shear walls, the slab's flexural analysis is not combined with the lateral load analysis.
8.4.2, 8.4.2.1, 8.4.2.2	The slab's factored moments are calculated using the DDM of Section 8.10.	Section 8.10 provides the DDM to determine the slab's factored moments.
8.10.1, 8.10.2	The slab is eligible for design by DDM as shown in Step 1.	
8.10.3.1, 8.10.3.2.1, 8.10.3.2.2, 8.10.3.2.3	Refer to Fig. E1.2 for slab span lengths in both directions and column dimensions in plan.	
8.10.3.2	The DDM calculates a total panel M_o and then uses coefficients to determine maximum positive and negative design moments. In this example, all spans have $\ell_n = 16$ ft. If spans vary, M_o must be calculated for each span length.	$M_o = \frac{q_u \ell_2 \ell_n^2}{8}$ <u>Long span</u> $\ell_n = 16$ ft $\ell_2 = 14$ ft $q_u = 1.2 * (DL_{super} + DL_{slab\ sw}) + 1.6 \times LL = 283$ psf $M_o = \frac{q_u \ell_2 \ell_n^2}{8} = 127$ ft-kip
8.10.4	Distribute M_o in the end span, from A to B.	

8.10.4.1 8.10.4.2 8.10.4.3 8.10.4.4 8.10.4.5	Table 8.10.4.2 gives the M_o distribution coefficients for the slab panel. In Table 8.10.4.2, this example uses the fully restrained column of the table. The reason for this is that the combined member of the wall and column is much stiffer than the slab and little rotation is expected at the slab-to-wall connection. Section 8.10.4.3 gives the option of modifying the factored moments by up to 10 percent, but that allowance is not used in this example. Section 8.10.4.4 indicates the negative moments are at the face of the supporting columns. Section 8.10.4.5 requires that the greater value of the two interior negative moments at the first interior column controls the design of the slab.	In Table 8.10.4.2, for the exterior edge being fully restrained: Negative M_u at face of exterior column = $0.65M_o$ = 83 ft-kip Maximum positive $M_u = 0.35M_o$ = 45 ft-kip Negative M_u at face of first interior column = $0.65M_o$ = 83 ft-kip
8.10.5	Proportion the total panel factored moments from 8.10.4 to the column and middle strips for the end span, from A to B.	
8.10.5.1	After distributing the total panel negative and positive M_u as described earlier in Section 8.10.4, Table 8.10.5.1 then proportions the interior negative M_u assumed to be resisted by the column strip.	In Table 8.10.5.1, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_{f1} = 0. Therefore, the top line of the table controls: $M_{u,int.neg,\,cs}$ = 0.75 × 83 ft-kip = 63 ft-kip
8.10.5.2	After distributing the total panel M_u as described earlier in Section 8.10.4, Table 8.10.5.2 then proportions the exterior negative M_u assumed to be resisted by the column strip.	In Table 8.10.5.2, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_{f1} = 0. Assuming the wall behaves as a beam, C is calculated to determine β_t using Eq. (8.10.5.2(a) and (b)). $\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$ $C = \left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3}$ $x = 10$ in. $y = 120$ in. $C = 37{,}900$ in.^{4} $E_{cb} = E_{cs}$ $I_s = \frac{bh^3}{12} = \frac{168\text{ in.}\times(7\text{ in.})^3}{12} = 4802\text{ in.}^4$ $\beta_t = \frac{37{,}900}{2\times 4802} = 3.9$
8.10.5.5	After distributing the total panel M_u as described earlier in Section 8.10.4, Table 8.10.5.5 then proportions the positive M_u assumed to be resisted by the column strip.	In Table 8.10.5.5, ℓ_2/ℓ_1 = 14/18 = 0.778 and α_{f1} = 0. Therefore, the top line of the table controls. $M_{u,\,pos.\,cs}$ = 0.60 × 45 ft-kip = 27 ft-kip
8.10.6	The total panel M_u from 8.10.4 is distributed into column strip moments and middle strip moments. The middle strip M_u is the portion of the total panel M_u not resisted by the column strip.	Determine the amounts distributed to the middle strips. Subtract the amounts distributed to the column strips in Section 8.10.5 from the panel M_u calculated in Section 8.10.4. $M_{u,\,int.\,neg,\,ms}$ = 83 ft-kip – 63 ft-kip = 20 ft-kip $M_{u,\,ext.\,neg,\,ms}$ = 83 ft-kip – 63 ft-kip = 20 ft-kip $M_{u,\,pos.,\,ms}$ = 45 ft-kip – 27 ft-kip = 18 ft-kip

8.10.7.3	The gravity load moment transferred between slab and edge column by eccentricity of shear is $0.3M_o$.	If there was no wall supporting the exterior edge of the slab, this moment would be used to calculate the two-way shear in the slab at the exterior column in 8.5. However, because of the wall, two-way shear does not apply to the design at the exterior column.
8.10	Repeat the M_u calculations for the interior span.	The results are shown for interior panels, with the same negative M_u at either end of the panel. $M_{u,\,neg,\,cs}$ = 63 ft-kip $M_{u,\,neg,\,ms}$ = 20 ft-kip $M_{u,\,pos,\,cs}$ = 27 ft-kip $M_{u,\,pos,\,ms}$ = 18 ft-kip Refer to Fig. E1.3 for final distribution along this column line. The middle strip moments are split into two half-middle strips, one on either side of the column strip.
 *Fig. E1.3—Final moment distribution*		
Step 5: Required strength – Factored slab moment resisted by the column		
8.4.2.3 8.4.2.3.1 8.4.2.3.2 8.4.2.3.3 8.4.2.3.4 8.4.2.3.5 8.4.2.3.6	Slab negative moments at a column can be unbalanced; that is, different on either side of the column. This difference in slab moments, M_{sc}, must be transferred into the column, usually by a combination of flexure or shear. Eq. (8.4.2.3.2) calculates a factor that determines the fraction of M_{sc} transferred by flexure. In this example, the permitted modifications to this factor are not used.	The columns are square, so $b_1/b_2 = 1$. $\gamma_f = \dfrac{1}{1+\dfrac{2}{3}\sqrt{\dfrac{b_1}{b_2}}} = 0.6$
8.4.2.3.3, 8.4.2.3.5	The effective slab width to resist $\gamma_f M_{sc}$ is the width of the column plus $1.5h$ of the slab on either side of the column. Section 8.4.2.3.5 requires sufficient reinforcement within the effective slab width to resist $\gamma_f M_{sc}$.	This concentration of reinforcement within the effective slab width is considered during the detailing of the column slab joint in Section 8.5. Figure E1.4 shows undistributed total panel moments. The moment diagram is symmetric about the axis of the column in the center of the building (108 ft). Note that using this moment diagram will result in a net zero M_{sc}. The DDM uses an artificial unbalanced load condition in Section 8.10.7 to avoid an unconservative design for two-way shear.

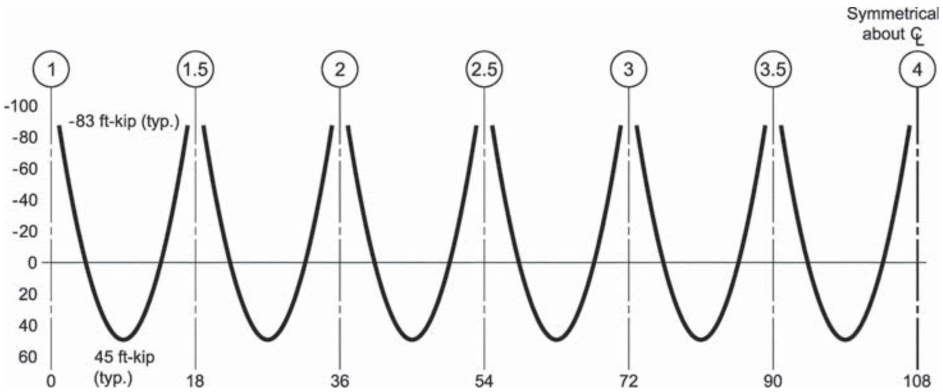


Fig. E1.4—Total panel moments

8.10.7 8.10.7.1 8.10.7.2	M_{sc} to satisfy the DDM provisions at an interior column is calculated by Eq. (8.10.7.2): $M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2]$ where the ′ indicates the shorter span. When the spans are the same as in our example, the ′ simply indicates the next span.	At an interior column: $M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2]$ $M_{sc} = 0.07[(0.123\text{ kip/ft}^2 + 0.5(0.160\text{ kip/ ft}^2))$ $14\text{ ft}(16\text{ ft})^2 - 0.123\text{ kip/ft}^2(14\text{ ft})(16\text{ ft}^2)] = 20.1\text{ ft-kip}$
8.10.7.3	M_{sc} to satisfy the DDM provisions at an exterior column is calculated by Section 8.10.7.3: $M_{sc} = 0.3M_o$	At an exterior column: $M_{sc} = 0.3M_o$ $M_{sc} = 0.3(127\text{ ft-kip}) = 38.1\text{ ft-kip}$
8.4.2.3.2, 8.4.4.2.2	M_{sc} is required to be transferred through both flexure and two-way shear into the column. The two-way shear calculations are discussed in Step 7. The amount of steel required to transfer $\gamma_f M_{sc}$ into the column via flexure is determined. The flexural reinforcement determined in later steps is allowed to be used to meet the required A_s in this step. Therefore, at step 13, the required reinforcement from this step will be checked.	At interior columns: $\gamma_f = 0.6$ $M_{sc} = 20.1\text{ ft-kip}$ $\gamma_f M_{sc} = (0.6)(20.1\text{ ft-kip})$ $\gamma_f M_{sc} = 12.1\text{ ft-kip}$ Using the method described in Step 8, the amount of flexural steel required within 1.5h of the column is: $A_s = 0.49\text{ in.}^2/45\text{ in. or }0.13\text{ in.}^2/\text{ft}$ At exterior columns: $\gamma_f = 0.6$ $M_{sc} = 38.1\text{ ft-kip}$ $\gamma_f M_{sc} = (0.6)(38.1\text{ ft-kip})$ $\gamma_f M_{sc} = 22.9\text{ ft-kip}$ Using the method described in Step 8, the amount of flexural steel required within 1.5h of the column is: $A_s = 0.94\text{ in.}^2/45\text{ in. or }0.25\text{ in.}^2/\text{ft}$
Step 6: Required strength — Factored one-way shear		
8.4.3 8.4.3.1 8.4.3.2	One-way shear slab rarely controls over two way shear in the design of a two-way slab, but it must be checked. In this section, V_u is determined. In 8.5, it is verified that the slab shear strength, ϕV_n, is sufficient to resist V_u.	Figure E1.5 shows one-way shears. The shear diagram is symmetric about the axis of the column at the center of the building (108 ft). $V_u = 32\text{ kip}$

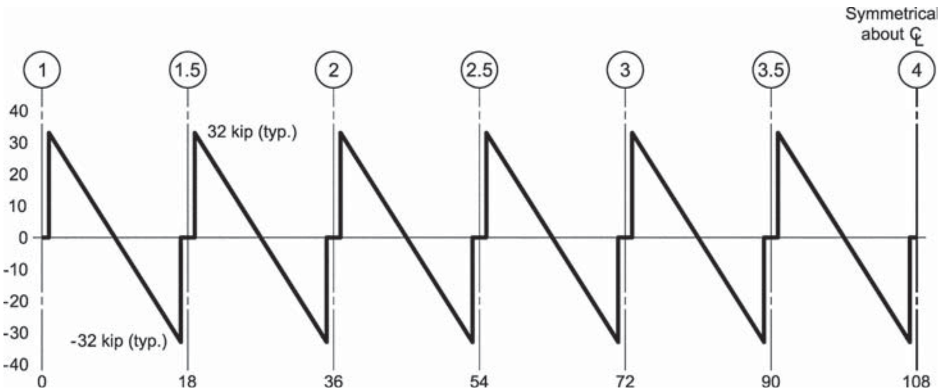
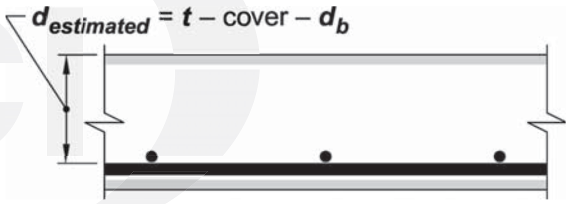


Fig. E1.5—Shear diagram

Step 7: Required strength — Factored two-way shear		
8.3.1.4	Stirrups are not used as shear reinforcement in this example.	
8.4.4, 8.4.4.1, 22.6.4, 20.6.1.3.1	Determine the critical section for two-way shear without shear reinforcement. Calculate b_o at an interior column: $b_o = 2 \times (c_1 + d) + 2 \times (c_2 + d)$ where d is the average effective depth (Fig. E1.6) and this example assumes No. 5 bars when determining d. Cover is assumed to be 0.75 in. per Table 20.6.1.3.1	$b_o = 2 \times (24\text{ in.} + 5.6\text{ in.}) + 2 \times (24\text{ in.} + 5.6\text{ in.})$ $b_o = 118.4$ in.  *Fig. E1.6—Average slab effective depth* Figure E1.7 shows two-way critical sections, b_o, at an interior column.

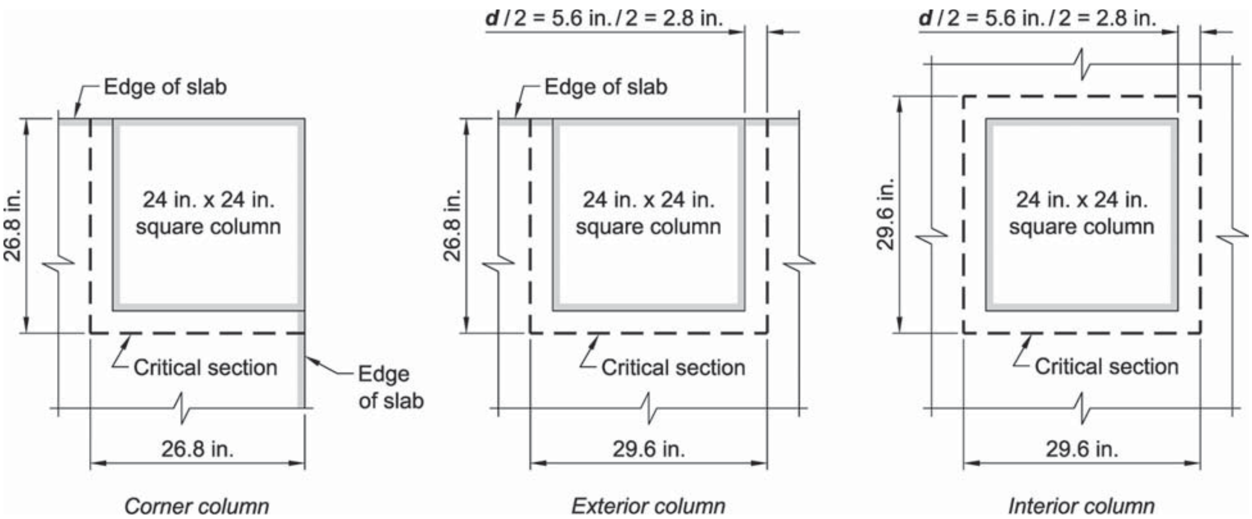


Fig. E1.7—Two-way shear critical section locations

Two-Way Slabs

8.4.4.2 8.4.4.2.1	Determine v_{ug} due to direct slab shear stress.	Calculate the direct shear stress at the interior column with full factored load on all spans: $v_{ug} = \frac{V_u}{b_o d}$ $V_u = \left(14\text{ ft} \times 18\text{ ft} - \frac{29.6\text{ in.} \times 29.6\text{ in.}}{144}\right) \times \frac{283}{1000}\frac{\text{kip}}{\text{ft}^2}$ $V_u = 70\text{ kip}$ $v_{ug} = \frac{70\text{ kip}}{118.4\text{ in.} \times 5.6\text{ in.}} = 0.106\text{ ksi}$
8.4.4.2.1 8.4.4.2.2	Determine the slab shear stress due to moment.	Calculate the shear stress due to moments at an interior column: $\gamma_v = 0.4$ $M_{sc} = 20.1\text{ ft-kip}$ $c_{AB} = 14.8\text{ in.}$ $J_c = 97688\text{ in.}^4$ $\frac{\gamma_v M_{sc} c_{AB}}{J_c} = 0.015\text{ ksi}$
8.4.4.2.3	Calculate v_u by combining the two-way direct shear stress and the stress due to moment transferred to the column via eccentricity of shear.	$v_u = v_{ug} + \frac{\gamma_v M_{sc} c_{AB}}{J_c}$ Calculate the design shear stress at an interior column: $v_u = 0.106\text{ ksi} + 0.015\text{ ksi} = 0.121\text{ ksi}$ Note that these calculations are conservative. M_{sc} assumes that some live load is not present to produce unbalanced moments, but v_{ug} assumes that full live load and dead load are present.

Step 8: Design strength—Reinforcement required to resist factored moments		
8.5.1 8.5.1.1 8.5.1.2 8.5.2 8.5.2.1 8.4.2.3.5	There are many methods available to determine the flexural reinforcement required at all sections within the span in each direction. To determine the amount of flexural steel required, this example solves the following quadratic equation: $\phi M_n = \phi\left(A_s f_y\left(d - \frac{a}{2}\right)\right)$ $\phi M_n = \phi\left(A_s f_y\left(d - \frac{A_s f_y}{2 \times 0.85 b f_c'}\right)\right)$ $\omega = \frac{A_s f_y}{b d f_c'}$ $\phi M_n = \phi(b d f_c' \omega(d - 0.59\omega d))$ $\phi M_n = \phi(b d^2 f_c' \omega(1 - 0.59\omega))$ Set $\phi M_n = M_u$ and solve for ω.	ϕ is assumed to be 0.9 for flexure as the slab is lightly reinforced. Using the moments shown in Fig. E1.8 and E1.9 for the column strip and middle strip, respectively, to determine the reinforcement required at each location. Reinforcement in an exterior panel Column strip at the columns: M_u = 63 ft-kip Solving the quadratic equation gives $\omega = 0.0664 \therefore$ $A_s = \frac{\omega b d f_c'}{f_y}$ $A_s = \frac{(0.0664)(84\text{ in.})(5.6\text{ in.})(5000\text{ psi})}{60{,}000\text{ psi}}$ $A_s = 2.61\text{ in.}^2$ Column strip at midspan: M_u = 27 ft-kip Solving the quadratic equation gives: $\omega = 0.0278 \therefore$ $A_s = \frac{\omega b d f_c'}{f_y}$ $A_s = \frac{(0.0278)(84\text{ in.})(5.6\text{ in.})(5000\text{ psi})}{60{,}000\text{ psi}}$ $A_s = 1.09\text{ in.}^2$ Using the same method, the following can be found: <u>Exterior Panels:</u> Column strip at column line: $A_s = 2.61\text{ in.}^2$ Middle strip at column line: $A_s = 0.81\text{ in.}^2$ Column strip at midspan: $A_s = 1.09\text{ in.}^2$ Middle strip at midspan: $A_s = 0.73\text{ in.}^2$