Binary Search Trees

Data Structures and Algorithms in Java {Y2014, E6, Wiley}

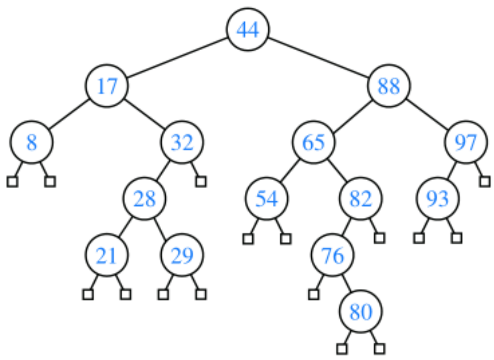


Figure 11.1: A binary search tree with integer keys. We omit the display of associated values in this chapter, since they are not relevant to the order of entries within a search tree.

Searching

Data Structures and Algorithms in Java {Y2014, E6, Wiley}

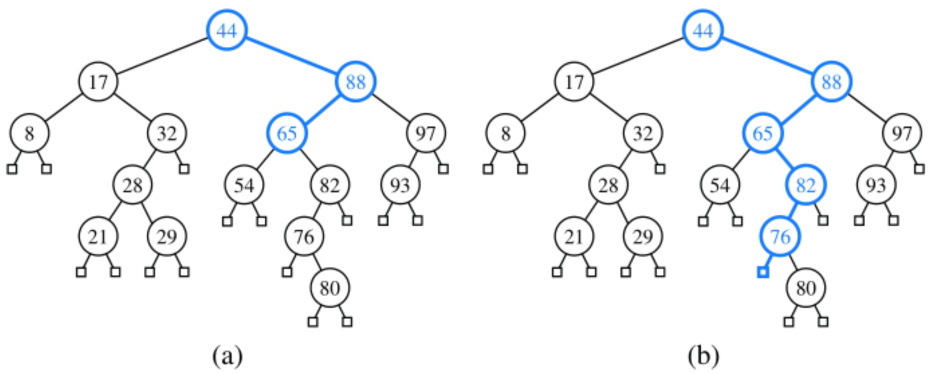


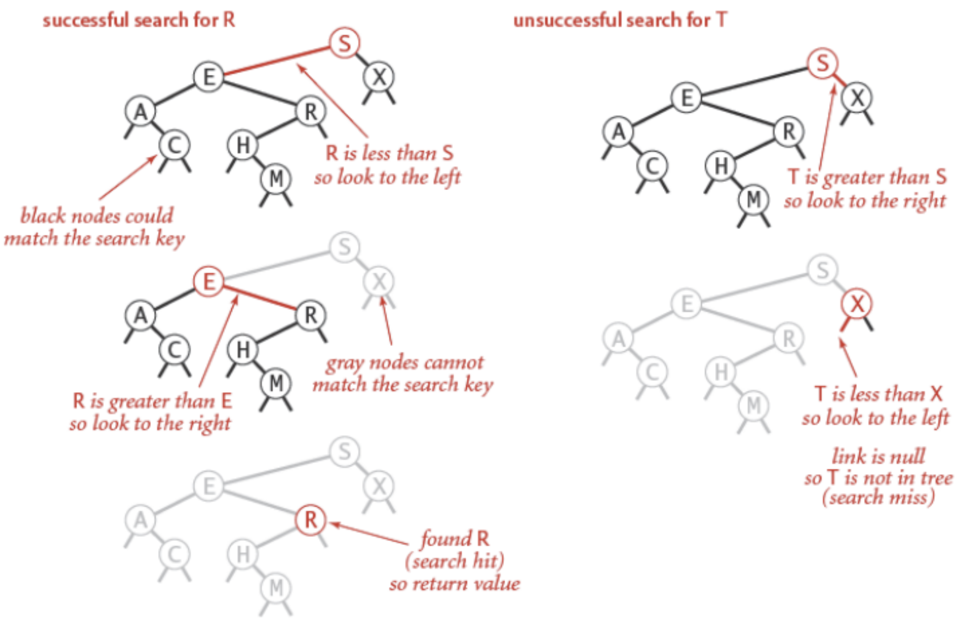
Figure 11.2: (a) A successful search for key 65 in a binary search tree; (b) an unsuccessful search for key 68 that terminates at the leaf to the left of the key 76.

Algorithm TreeSearch(p, k):
  if p is external **then**
    return p
  else if $k ==$ key(p) **then**
    return p
  else if $k <$ key(p) **then**
    return TreeSearch(left(p), k)
  else {we know that $k >$ key(p)}
    return TreeSearch(right(p), k)

Data Structures Using C {Y2018, E2}

```
searchElement (TREE, VAL)

Step 1: IF TREE->DATA = VAL OR TREE = NULL
            Return TREE
        ELSE
         IF VAL < TREE->DATA
           Return searchElement(TREE->LEFT, VAL)
         ELSE
           Return searchElement(TREE->RIGHT, VAL)
         [END OF IF]
        [END OF IF]
Step 2: END
```

Algorithms {Y2011, E4}



Search hit (left) and search miss (right) in a BST

```
// Return value associated with key in the subtree rooted at x;
// return null if key not present in subtree rooted at x.
Value get(Node x, Key key)
{
    if (x ==null) return null;
    int cmp = key.compareTo(x.key);
    if (cmp < 0) return get(x.left, key);
    else if (cmp > 0) return get(x.right, key);
    else return x.val;
}
```

Essential Algorithms - A Practical Approach to Computer Algorithms Using Python and C# {Y2019, E2}

```
// Find a node with a given target value.

BinaryNode: FindNode(Key: target)
    // If we've found the target value, return this node.
    If (target == Value) Then Return <this node>
```

```
        // See if the desired value is in the left or right subtree.
        If (target < Value) Then
            // Search the left subtree.
            If (LeftChild == null) Then Return null
            Return LeftChild.FindNode(target)
        Else
            // Search the right subtree.
            If (RightChild == null) Then Return null
            Return RightChild.FindNode(target)
        End If
    End FindNode
```

Insertion

Data Structures and Algorithms in Java {Y2014, E6, Wiley}

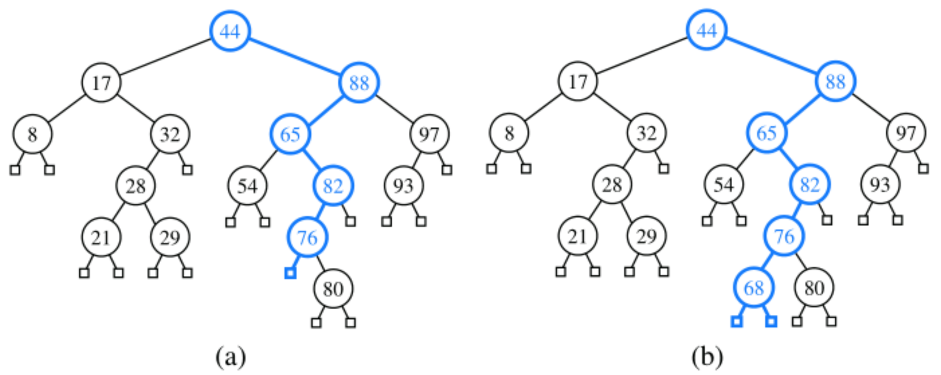


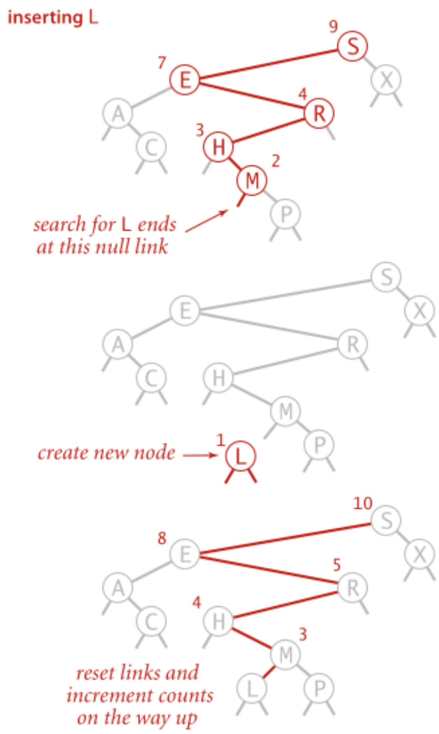
Figure 11.4: Insertion of an entry with key 68 into the search tree of Figure 11.2. Finding the position to insert is shown in (a), and the resulting tree is shown in (b).

Algorithm TreeInsert(k, v):
    Input: A search key k to be associated with value v
    p = TreeSearch(root( ), k)
    if k == key(p) **then**
        Change p's value to (v)
    else
        expandExternal(p, (k, v))

Data Structures Using C {Y2018, E2}

```
Insert (TREE, VAL)

Step 1: IF TREE = NULL
              Allocate memory for TREE
              SET TREE -> DATA = VAL
              SET TREE -> LEFT = TREE -> RIGHT = NULL
        ELSE
              IF VAL < TREE -> DATA
                    Insert(TREE -> LEFT, VAL)
              ELSE
                    Insert(TREE -> RIGHT, VAL)
              [END OF IF]
        [END OF IF]
Step 2: END
```

Algorithms {Y2011, E4}



Insertion into a BST

```
private Node put(Node x, Key key, Value val)
{
    //Change key's value to val if key in subtree rooted at x.
    //Otherwise, add new node to subtree associating key with val.
    if (x ==null) return new Node(key, val, 1);
    int cmp = key.compareTo(x.key);
    if (cmp < 0) x.left put(x.left, key, val);
    else if (cmp > 0) x.right = put(x.right, key, val);
    else x.val =val;
    x.N = size(x.left) + size(x.right) + 1;
    return x;
}
```

Essential Algorithms - A Practical Approach to Computer Algorithms Using Python and C# {Y2019, E2}

```
// Add a node to this node's sorted subtree.
AddNode(Data: new_value)
    // See if this value is smaller than ours.
    If (new_value < Value) Then
        // The new value is smaller. Add it to the left subtree.
        If (LeftChild == null) LeftChild = New BinaryNode(new_value)
        Else LeftChild.AddNode(new_value)
    Else
        // The new value is not smaller. Add it to the right subtree.
        If (RightChild == null) RightChild = New BinaryNode(new_value)
        Else RightChild.AddNode(new_value)
    End If
End AddNode
```

Deletion

Data Structures and Algorithms in Java {Y2014, E6, Wiley}

predecessor => the entry having the greatest key that is strictly less than that of position p.
located in the right-most internal position of the left subtree of position p

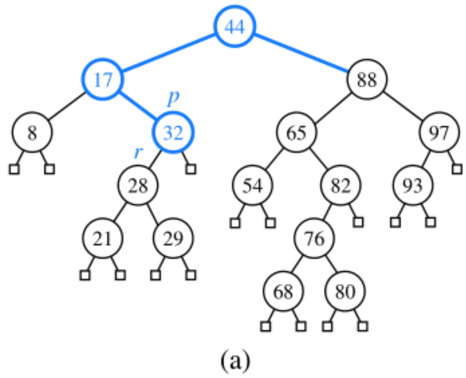


Figure 11.5: Deletion from the binary search tree of Figure 11.4b, where the entry to delete (with key 32) is stored at a position p with one child r: (a) before the deletion; (b) after the deletion.

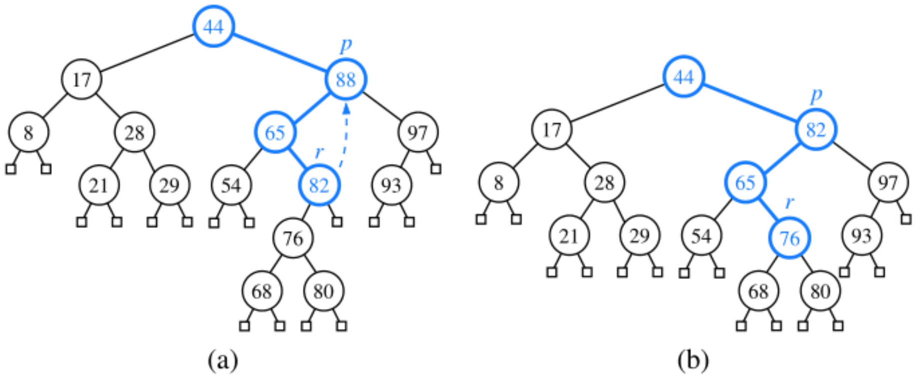


Figure 11.6: Deletion from the binary search tree of Figure 11.5b, where the entry to delete (with key 88) is stored at a position p with two children, and replaced by its predecessor r: (a) before the deletion; (b) after the deletion.

Data Structures Using C {Y2018, E2}

```
Delete (TREE, VAL)

Step 1: IF TREE = NULL
          Write "VAL not found in the tree"
        ELSE IF VAL < TREE -> DATA
          Delete(TREE->LEFT, VAL)
        ELSE IF VAL > TREE -> DATA
          Delete(TREE -> RIGHT, VAL)
        ELSE IF TREE -> LEFT AND TREE -> RIGHT
          SET TEMP = findLargestNode(TREE -> LEFT)
          SET TREE -> DATA = TEMP -> DATA
          Delete(TREE -> LEFT, TEMP -> DATA)
        ELSE
          SET TEMP = TREE
         IF TREE -> LEFT = NULL AND TREE -> RIGHT = NULL
              SET TREE = NULL
          ELSE IF TREE -> LEFT != NULL
               SET TREE = TREE -> LEFT
          ELSE
               SET TREE = TREE -> RIGHT
          [END OF IF]
          FREE TEMP
        [END OF IF]
Step 2: END
```

Finding the Smallest Node in a Binary Search Tree

Data Structures Using C {Y2018, E2}

```
findSmallestElement(TREE)

Step 1: IF TREE = NULL OR TREE -> LEFT = NULL
            Returen TREE
        ELSE
            Return findSmallestElement(TREE -> LEFT)
        [END OF IF]
Step 2: END
```

Finding the Largest Node in a Binary Search Tree

Data Structures Using C {Y2018, E2}

```
findLargestElement(TREE)

Step 1: IF TREE = NULL OR TREE -> RIGHT = NULL
           Return TREE
        ELSE
          Return findLargestElement(TREE -> RIGHT)
        [END OF IF]
Step 2: END
```

Lowest Common Ancestors

Essential Algorithms - A Practical Approach to Computer Algorithms Using Python and C# {Y2019, E2}

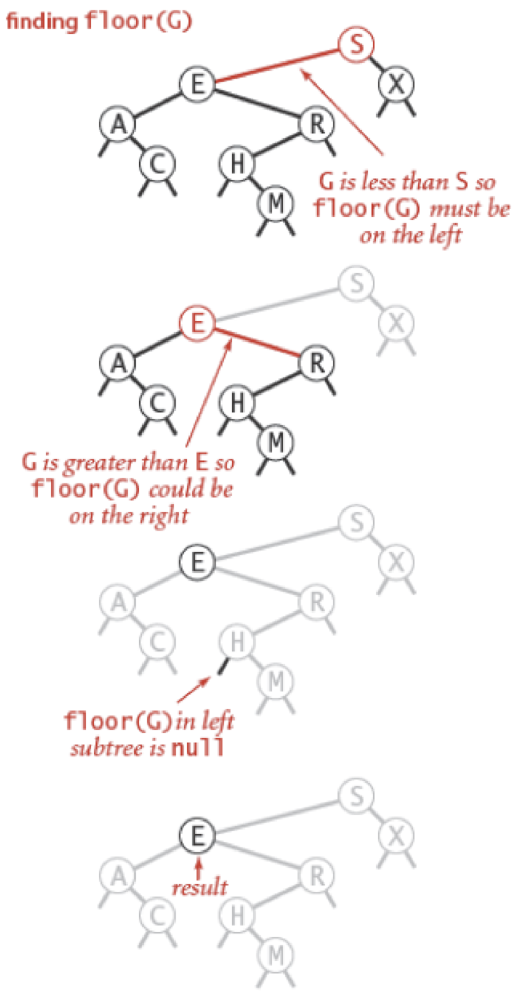
```
// Find the LCA for the two nodes.
TreeNode: FindLcaSortedTree(Integer: value1, Integer: value2)
    // See if both nodes belong down the same child branch.
    If ((value1 < Value) && (value2 < Value)) Then
        Return LeftChild.FindLca(value1, value2)
    End If
    If ((value1 > Value) && (value2 > Value)) Then
        Return RightChild.FindLca(value1, value2)
    End If

    // This is the LCA.
    Return <this node>
End FindLcaSortedTree
```

Floor

Algorithms {Y2011, E4}

Floor(k) ➞ the largest key less than or equal to k.

- If a given key k is less than the key at the root , then the floor of k must be in the left subtree
- If k is greater than the key at the root and if there is a key smaller than or equal to k in the right subtree then the floor of k is in the right subtree
- if not (or if k is equal to the key at the root), then the key at the root is the floor of k.



Computing the floor function

Data Structures and Algorithms in Java {Y2014, E6, Wiley}

```
/** Returns the entry with greatest key less than or equal to given key (if any).
*/
public Entry<K,V> floorEntry(K key) throws IllegalArgumentException
{
    checkKey(key); // may throw IllegalArgumentException
    Position<Entry<K,V>> p = treeSearch(root(), key);
    if (isInternal(p)) return p.getElement(); // exact match
    while (!isRoot(p))
    {
        if (p == right(parent(p)))
            return parent(p).getElement(); // parent has next lesser key
        else
            p = parent(p);
    }
    return null
}
```

Ceiling

Ceiling(k) ➞ the smaller key greater than or equal to k.

- If a given key k is greater than the key at the root , then the ceiling of k must be in the right subtree
- If k is less than the key at the root and if there is a key greater than or equal to k in the left subtree then the ceiling of k is in the left subtree
- if not (or if k is equal to the key at the root), then the key at the root is the ceiling of k.

GetRange

GetRange ➞ returns to a client all the nodes in a specified range

Algorithms {Y2011, E4}

```
public Iterable<Key> keys(Key lo , Key hi)
{
    Queue<Key> queue = new Queue<Key>();
    keys(root, queue, lo, hi);
    return queue;
}

private void keys(Node x, Queue<Key> queue, Key lo, Key hi)
{
    // skip the recursive calls for subtrees that cannot contain keys in the
range
    if (x ==null ) return;
    int cmplo = lo.compareTo(x.key);
    int cmphi = hi.compareTo(x.key);
    //ignore x.left if lo > x.key
    if (cmplo < 0) keys(x.left, queue, lo , hi);    // lo < x.key
    if (cmplo <= 0 && cmphi >= 0) queue.enqueue(x.key); //x.key in [lo, hi]
    //ignore x.right if hi < x.key
```

```
    if (cmphi > 0) keys(x.right, queue, lo , hi);    //hi < x.key
}
```

Min

- If the left link of the root is null, the smallest key in a is the key at the root;
- if the left link is not null, the smallest key in the BST is the smallest key in the subtree rooted at the node referenced by the left link

Algorithms {Y2011, E4}

```
private Node min(Node x)
{
    if (x. left == null) return x;
    return min(x.left)
}
```

Data Structures Using C {Y2018, E2}

```
findSmallestElement(TREE)

Step 1: IF TREE = NULL OR TREE -> LEFT = NULL
            Returen TREE
        ELSE
            Return findSmallestElement(TREE -> LEFT)
        [END OF IF]
Step 2: END
```

Figure 10.25 Algorithm to find the smallest node in a binary search tree

Max

Data Structures Using C {Y2018, E2}

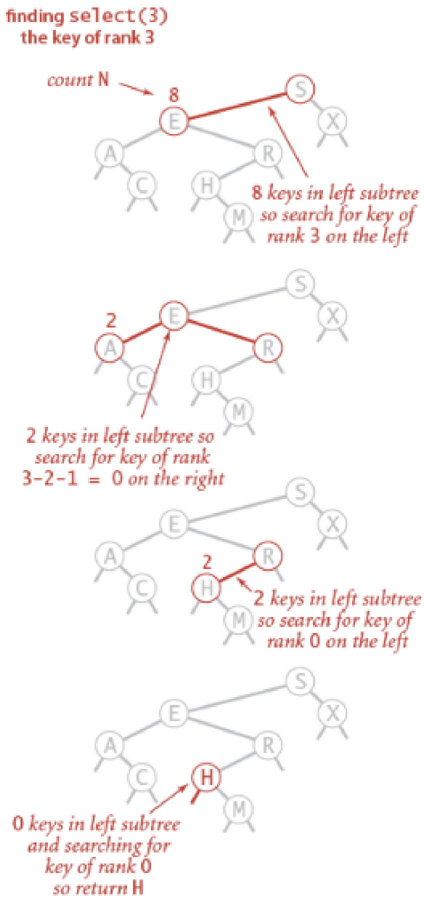
```
findLargestElement(TREE)

Step 1: IF TREE = NULL OR TREE -> RIGHT = NULL
            Return TREE
        ELSE
          Return findLargestElement(TREE -> RIGHT)
        [END OF IF]
Step 2: END
```

Figure 10.26 Algorithm to find the largest node in a binary search tree

GetNode

GetNode(n) ➞ the key such that precisely n other keys in the BST are smaller

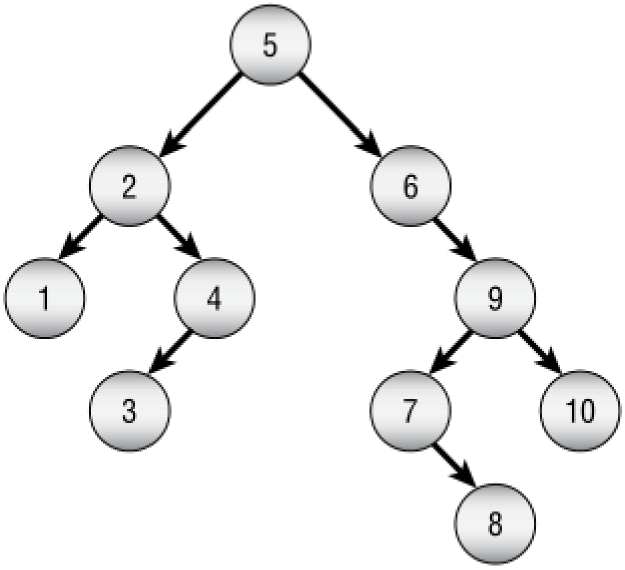
- If the number of keys **t** in the left subtree is larger than **n**, we look (recursively) for the key of rank **n** in the left subtree;
- if **t** is equal to **n**, we return the key at the root;
- if **t** is smaller thank, we look (recursively) for the key of rank **n - t - 1** in the right subtree.



Selection in a BST

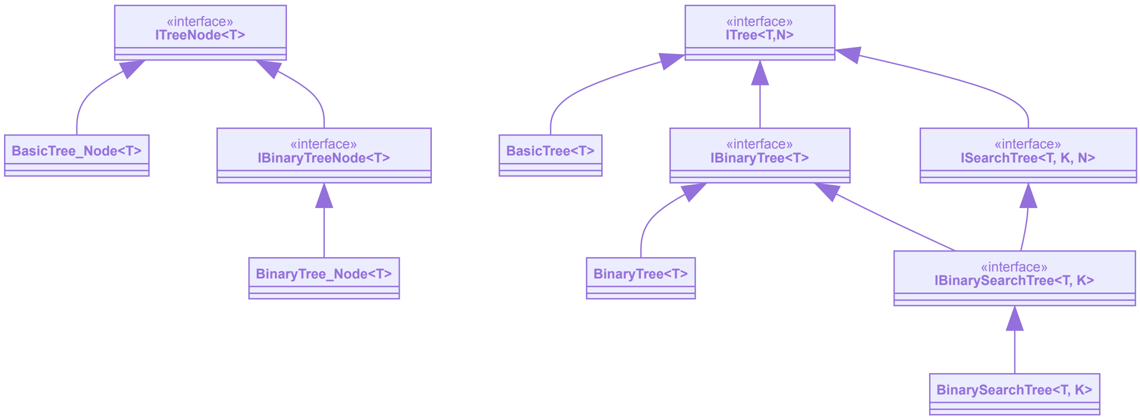
```
Node select(Node x, int k)
{
    //Return Node containing key of rank k.
    if (x ==null ) return null;
    int t = size(x.left);
    if (t > k) return select(x. left , k);
    else if (t < k) return select(x.right, k -t-1 );
    else return x;
}
```

Successor




```
Successor(node)
    if node.RightChild == null
        p = node.Parent
        while p != null and p.RightChild == node
            node == p
            p = p.Parent
        end while
        node = p;
    else
        node = node.RightChild

        while node.LeftChild != null
            node = node.LeftChild
    end if

    return node
END
```

Impl



```
public interface ITree<out T, N> : IEnumerable<T>
        where N : ITreeNode<T>
{
    N Root { get; }
    IEnumerable<N> Nodes { get; }
    IEnumerable<N> Leaves { get; }
    bool IsEmpty { get; }
    IEnumerable<N> Enumerate(TraversalOrder order);
    int GetCount();
    IEnumerable<N> GetPath(N node);
}
```

```
public interface IBinaryTree<out T, N> : ITree<T, N>
    where N : IBinaryTreeNode<T>
{}
```

```
public interface ISearchTree<T, K, N>: ITree<T, N>
    where N: ITreeNode<T>
{
    Comparison<K> KeyCompare {get;}
    Func<T, K> KeySelect {get;}
    int Count {get;}
    IEnumerable<K> Keys {get;}
    bool ContainsKey(K key);
    N Min();
    N Max();
        //perform badly
    //int GetIndex(K key);
```

```
    //N GetNode(int ndx);
    N Ceiling(K Key);
    N Floor(K key);
    N Locate(K key);
    IEnumerable<N> GetRange(K loKey, K hiKey);
    N FirstCommonAcestor(N node0, N node1);
}
```

```
public interface IBinarySearchTree<T, K>: IBinaryTree<T, IBinaryTreeNode<T>>,
ISearchTree<T, K, IBinaryTreeNode<T>>
{}
```

```
public class BinarySearchTree<T, K>: IBinarySearchTree<T, K>
{
    public BinarySearchTree(Func<T, k> selector  null, Comparison<T> comp =
null);

    public Comparison<K> KeyCompare {get;}
    public Func<T, K> KeySelect {get;}
    public IBinaryTreeNode<T> Root { get; }

    public int Count {get;}
    int IBinaryTree<T, IBinaryTreeNode<T>>.GetCount();

    public bool IsEmpty { get; }
    public IEnumerable<K> Keys {get;}
    public IEnumerable<IBinaryTreeNode<T>> Nodes { get; }
    public IEnumerable<IBinaryTreeNode<T>> Leaves { get; }

    public IEnumerable<IBinaryTreeNode<T>> Enumerate(TraversalOrder order);

    public IEnumerable<IBinaryTreeNode<T>> GetPath(N node);
    public IEnumerable<IBinaryTreeNode<T>> GetRange(K loKey, K hiKey);
    /*  pre:
        loKey <= hiKey
    */

    public bool ContainsKey(K key);
    public IBinaryTreeNode<T> Locate(K key);

    public IBinaryTreeNode<T> Min();
    /*  pre:
        !IsEmpty
    */

    public IBinaryTreeNode<T> Max();
    /*  pre:
        !IsEmpty
    */

    public IBinaryTreeNode<T> Ceiling(K Key);
    public IBinaryTreeNode<T> Floor(K key);
```

```
    public IBinaryTreeNode<T> FirstCommonAcestor(IBinaryTreeNode<T> n0,
IBinaryTreeNode<T> n1);
    /*  pre:
        no != null
        Contains(no)
        n1 != null
        Contains(n1)
    */

    public IBinaryTreeNode<T> Add(T item);
    /*  pre:
        !ContainsKey(KeySelector(item))
    */

    public bool TryAdd(T item, out IBinaryTreeNode<T> node );
    public void Remove(IBinaryTreeNode<T> node);
    /*  pre:
        Trees.Contains(this, node)
    */
    public void ReplaceItem(IBinaryTreeNode<T> node, T item);
    /* pre:
        node != null
        KeySelector(node.Item) == KeySelector(item)
    */
}
```