Experimental Study of Diffraction by a Thin Cone

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A problem of diffraction by elongated body of revolution is studied. A direct diffraction experiment is used to measure the diffracted field on the surface of a thin cone. The experiment is performed using MLS (Maximum Length Sequence) method. The incident field falls from different directions. The wavelengths are small comparatively to the length of the cone. A boundary integral equation is used to describe the field theoretically. The integral equation is solved numerically by iterations. The results of the experiment are calculated with the results of the calculation.

1 Introduction

Diffraction by a thin cone attracts considerable attention of researchers. Several different approaches exist to developing both asymptotics of the diffracted field and the diffracted field itself. First, there is a traditional asymptotic approach based on ray representation [1]. Second, there is an approach based on the parabolic equation method [2]. Third, there is an approach based on the boundary integral equation method for the parabolic equation in Cartezian coordinates [3]. Also, there is an approach based on the Smyshlyaev's formula [4], and an approach based on Kontorovich-Lebedev integral representation [5]. All these methods are mathematically complicated, and the question about which one works better is still open.

In this work, a direct diffraction experiment is used to measure the field diffracted by a cone. The experiment is performed using MLS technique [6]. We put our receiver to the surface of the cone. After that the cone is irradiated by a point source from different directions so that the receiver is enlightened or shadowed. The parabolic equation of diffraction theory is used as the governing equation. The boundary integral equation developed in [7] is used to calculate the diffracted field being compared to the results of the experiment. The equation is of Volterra type, so it is solved by iterations.

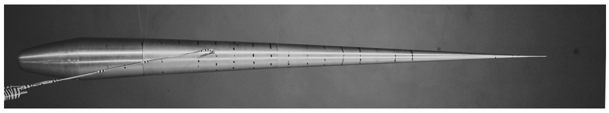

Figure 1: Photo of the duralumin cone with the mic on its surface.

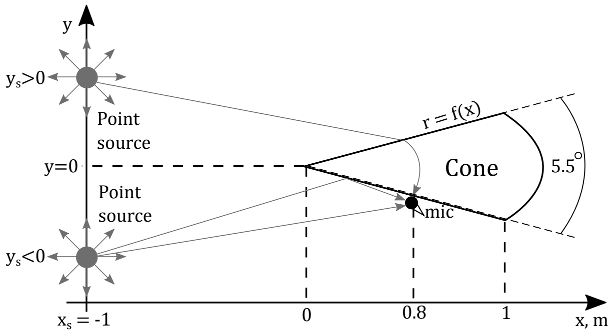

Figure 2: Experiment scheme, view from above.

2 Experiment

2.1 Description

A narrow (the angle is $\alpha = 5.5°$) duralumin cone (see Fig. 1) is hanged in free space. A point-sized microphone is placed on the surface of the cone. The cone is irradiated by a point source from different directions so the receiver can be shadowed or enlightened (see Fig. 2). A miniature armature Knowles driver is used as a source and a well-calibrated no-name microphone is used as a receiver.

To conduct the experiment, MLS method is used [6]. The cone is irradiated by a pseudo-random signal and then impulse response of the acoustical path is calculated through calculating correlation of the MLS signal with the output signal.

The input MLS signal is correlated with the measured signal (see Fig. 3) and impulse response is calculated. The procedure is identical to that described in [6]. From impulse response a region of interest corresponding to diffraction by the cone is cropped. After that the frequency dependence of this region is calculated and compared to theory.

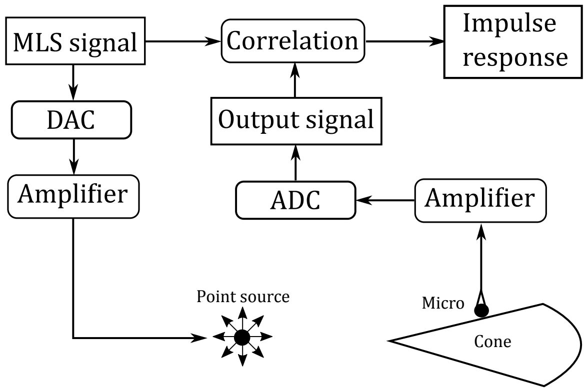

Figure 3: Experimental data processing scheme.

3 Theoretical description

3.1 Problem statement

The acoustical field is described using Helmholtz equation:

$$\Delta \tilde{u} + k^2 \tilde{u} = 0. \tag{1}$$

Our cone is very narrow (its vertex angle is much smaller than 1 radian) so most of the waves are going to be scattered under small angles. This means that the parabolic equation of diffraction theory can be used to describe the field on the surface of the cone so the field on the surface of the cone can be represented as follows:

$$\tilde{u}(x, r, \varphi) = \exp\{ikx\}\, u(x, r, \varphi), \tag{2}$$

where u is a slow function of x, compared to the exponential factor. Substituting (2) in (1) and neglecting the term with the second x-derivative, get the approximate equation for u:

$$\left(\frac{\partial}{\partial x} + \frac{1}{2ik} \Delta_\perp \right) u = 0, \tag{3}$$

which is the parabolic equation of the diffraction theory (PETD).

As we are using a point source, its field will be equal to Green's function of the parabolic equation. Let us denote the points of space by $\vec{r} = (x, r, \varphi)$ and the point source be located at $\vec{r_s} = (x_s, r_s, \varphi_s)$. Thus the field of the source will be the solution of

$$\left(\frac{\partial}{\partial x} + \frac{1}{2ik} \Delta_\perp \right) g(\vec{r}, \vec{r_s}) = \delta(\vec{r} - \vec{r_s}), \tag{4}$$

where the operator in the left part acts on the components of $\vec{r}$, δ is the Dirac's delta-function. The solution should obey the initial condition, i. e. it should be equal to zero for $x < x_s$.

One can check that the Green's function for $x > x_s$ will be

$$g(\vec{r}, \vec{r_s}) = \frac{k}{2\pi i (x - x_s)} \exp\left[\frac{ik}{2} \frac{(\Delta r)^2}{x - x_s} \right], \tag{5}$$

where Δr is the distance between the projections of $\vec{r}$ and $\vec{r_s}$ onto the transversal plane:

$$(\Delta r)^2 = r^2 + r_s^2 - 2 r r_s \cos(\varphi - \varphi_s). \tag{6}$$

3.2 Calculating the diffracted field

In [7] a boundary equation is derived for the field diffracted by a body of revolution. The equation is applied to the bodies having their curvature dependent on x like $r = f(x)$. We are going to apply the results of [7] for our conical case.

Let U be the full field on the surface, U^{in} be the incident field on the surface. Both of these fields depend only on x and φ. If x_*, φ_* are coordinates of the observation point, X is such that $f(x < X) = 0$ (for the case of the cone X is the coordinate of its vertex), then

$$U(x_*, \varphi_*) = \int_0^{2\pi} \int_X^{x_*} K(x_*, \varphi_*, x, \varphi) U(x, \varphi) dx d\varphi + 2U^{\text{in}}(x_*, \varphi_*), \tag{7}$$

$$K(x_*, \varphi_*, x, \varphi) = \frac{ikf(x)}{2\pi} \times \left[\frac{\dot{f}(x)}{x_* - x} + \frac{f(x) - f(x_*)\cos(\varphi - \varphi_*)}{(x_* - x)^2} \right] \times \exp\left\{ \frac{ik}{2} \frac{f^2(x_*) + f^2(x) - 2f(x_*)f(x)\cos(\varphi - \varphi_*)}{x_* - x} \right\}, \tag{8}$$

For our case $f(x)$ is a linear function of x, $r = f(x) = \tan(\alpha/2)x \approx x\alpha/2$, where α is the angle at the vertex of our cone.

Note that equation (7) is of Volterra type with respect to variable x, and is of difference kernel type with respect to the variable φ. This means that it can be solved using iterations with respect to x.

Let us discretize the field $U(x, \varphi)$. We need to define linear shape functions $N_i(x)$ such that:

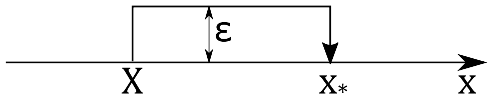

Figure 4: Integration contour for (4).

$$N_i(x) = \begin{cases} 1, & x = x_i, \\ 0, & x \neq x_i. \end{cases} \quad (9)$$

Then

$$U(x, \varphi) = \sum_i U_i N_i(x). \quad (10)$$

We substitute (10) to (7) and get

$$U(x_*, \varphi_*) \approx \sum_i U_i \int_0^{2\pi} \int_X^{x_*} K(x_*, \varphi_*, x, \varphi) N_i(x) dx d\varphi + 2U^{\rm in}(x_*, \varphi_*). \quad (11)$$

Now, (7) becomes a discretized matrix equation and $K(x_*, x)$ becomes a lower-triangle matrix. $K(x_*, x)$ has a pole at $x = x_*$ we need to deal with to calculate the integral in (11). To do this we slightly ($\varepsilon \ll X$) shift the integration contour to the upper complex half-plane $x \to x + i\varepsilon$, see Fig. 4.

Equation (11) can be solved by iterations with respect to x:

$$\begin{aligned} U(x) &= \sum_{n=0}^{\infty} U^{(\rm n)}(x), \\ U^{(0)}(x) &= 2U^{\rm in}(x), \\ U^{(n+1)}(x_*) &= \int_X^{x_*} K(x_*, x) U^{(\rm n)}(x) dx, \quad n > 0 \end{aligned} \quad (12)$$

We calculate the solution for each frequency ω and finally we get $U(\omega)$ for the field in observation point (see Fig. 2).

4 Experimental results and modeling

4.1 Axial incidence

For axial incidence, one can notice that the field fades like $1/x$ which is just fading due to the spherical front of the incident field. This means that the incident field doesn't feel the presence of the obstacle and thus is not diffracted.

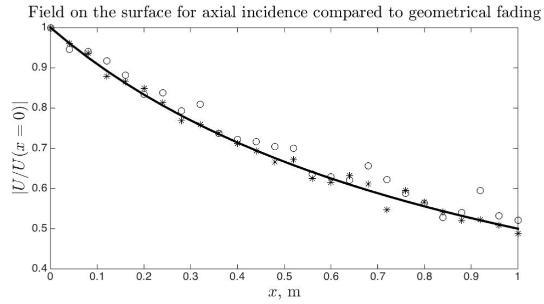

Figure 5: Measured field for different x for $f = 2000$ Hz (circles) and for $f = 3000$ Hz (stars) compared to geometrical fading $\sim 1/(x+1)$.

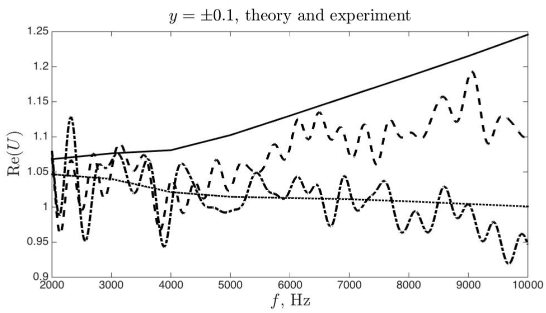

Figure 6: Comparison of experiment and theory for $y = \pm 0.1$. Theory (solid line) and experiment (dashed line) for $y = -0.1$, theory (dotted line) and experiment (dash-dotted line) for $y = 0.1$.

I have difficulties with this part. The length of the Fresnel zone on the surface of the cone for 1000 Hz is approximately 17 cm, so there are several of them on our cone. I don't know how to explain the absence of diffraction under axial incidence.

4.2 Non-axial incidence

The experimental results are compared to modeling results for $y_s = -0.5, -0.4, ..., 0.4, 0.5$ (see Fig. 6, 7, 8). The field in the figures is represented in terms of the incident field, 1 corresponds to $U^{\rm in}(x)$.

The experimental results highly depend on the phase shift which is technically the distance between the point source and the microphone. The distances in our experiments were measured with error about ± 1 cm, so we attenuated the phase shift during the calculations by hand within $\Delta = 1$ cm

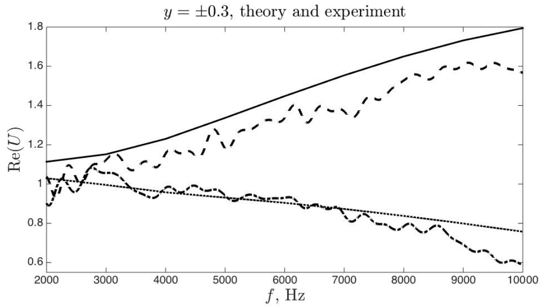

Figure 7: Comparison of experiment and theory for $y = \pm 0.3$. Theory (solid line) and experiment (dashed line) for $y = -0.3$, theory (dotted line) and experiment (dash-dotted line) for $y = 0.3$.

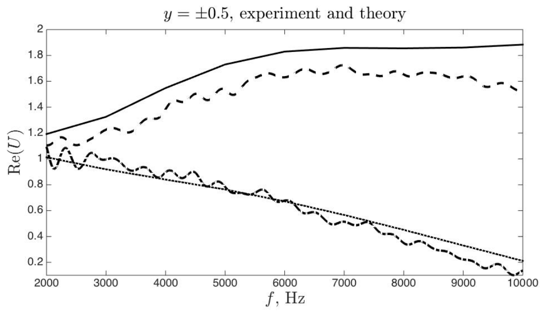

Figure 8: Comparison of experiment and theory for $y = \pm 0.5$. Theory (solid line) and experiment (dashed line) for $y = -0.5$, theory (dotted line) and experiment (dash-dotted line) for $y = 0.5$.

via multiplying the field by $\exp(ik\Delta)$.

What we see for the cases of negative y when the source becomes enlightened is that the field tends to 2. This is pretty much what we expect because the field should show the reflection from a hard wall and thus be the incident field doubled. For the case of positive y the field on the mic is that transmitted along the cone. Its amplitude fades while the frequency grows as the high-frequency field feels the obstacle.

5 Conclusion

In this work, a direct diffraction experiment was held on diffraction by a narrow cone with the use of a point source. The results of the experiment for axial incidence show that there is almost no diffraction due to a big longitudinal size of the Fresnel zone along the cone surface. For non-axial incidence the results agree with the diffraction theory in parabolic approximation. The theoretical results were calculated using iterations.

References

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