Classical

Kronecker-Weber Thm:

All abelian extensions of $\mathbb{Q}$ are $\mathbb{Q}[\underbrace{\zeta_n}_{n^{th}\text{ root of unity}}]$

Analog - Finite order elts in mult. group

Lubin-Tate: Get galois extensions from

Formal group.

<u>Formal Gp</u>. Unital, comm., assoc.

Category of Lie varieties over R $:= \mathcal{C}$

Objs: $\mathbb{A}^n$, $n = 0, 1, 2, \cdots$

Morphisms: $F: \mathbb{A}^n \to \mathbb{A}^1 \in R[x_i]_{i=1}^n$

$\to$ Formal gp = gp object $\in \mathcal{C}$

Q: How many F.G.'s are there?

How to construct?

Lazard's Thm

The functor $\mathcal{F} : R \mapsto \{FG\text{'s over } R\}$

is <u>representable</u>.

$\mathrm{Ring}(L, R)$ where $L = \mathbb{Z}[x_i]_{i=1}^{n}$

Define an <u>iso</u> as $F \xrightarrow{g} G$

$$g(x \underset{F}{+} y) = g(x) \underset{G}{+} g(y)$$

Use in AT

Generalized cohom E with Chern classes

For $\mathbb{C}^1$ complex line bundles

$$\begin{array}{c} \mathbb{C}^1 \\ \downarrow \\ X \end{array}$$

$c_n(X) \in E^{2n}$ w/ $c_n(V \oplus W) = \sum_{i+j=n} c_i V \, c_j W$

<u>Not true</u>: $c_1(L_1 \otimes L_2) = c_1 L_1 + c_1 L_2$

Thm (Quillen)

For general E, exists a FG law F

$$c_1(L_1 \otimes L_2) = F(c_1(L_1), c_1(L_2))$$
$$= c_1(L_1) \underset{F}{+} c_1(L_2)$$

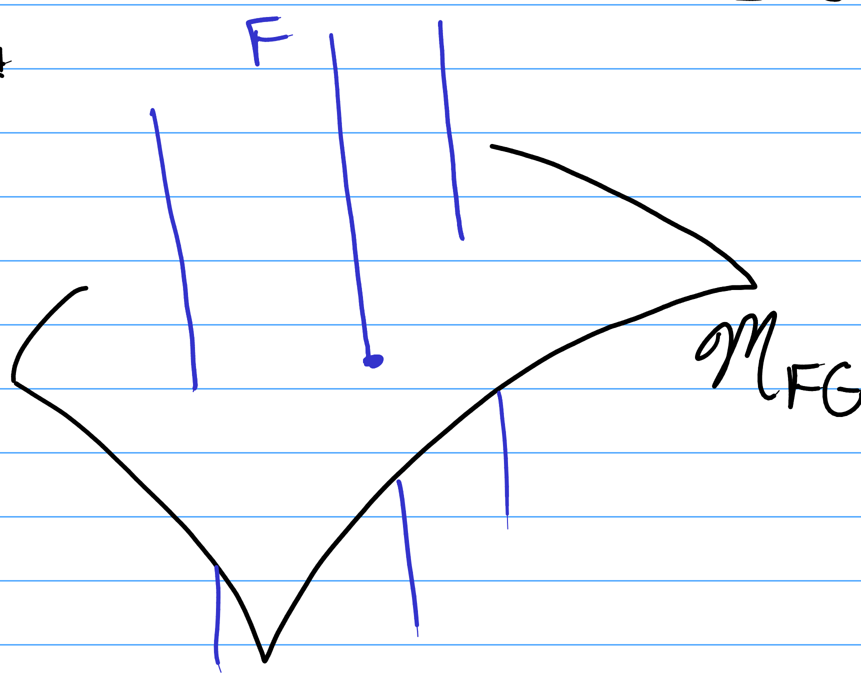
$\longrightarrow$ SS

$$\underbrace{H^s(\mathcal{M}_{FG}, \omega^t)}_{} \longrightarrow \pi_{2t-s} S^0$$

$\mathcal{M}_{FG}$: moduli stack

$$\pi_{2t-s} S^0 = \lim_{n\to\infty} \pi_{2t-s+n} S^n$$

$\omega \in \mathrm{Lie}\, F^*$



Examples of FGs

$G_a = \quad x+y$

$G_m = \quad x+y-xy$
$= 1-(1-x)(1-y)$

Are they isomorphic?

Over $\mathbb{Q}$? Sure, use

$$g(x) = 1-e^{-x}$$

$$\to g(x+y) = g(x)+g(y)-g(x)g(y)$$

(needed denominators)

Over $\mathbb{F}_p$? Use

$$g : G_a \to G_m$$

$$g(x+y) = (x+y)^p$$

Height:

$R = K$ a field of char p

$$f: G_1 \longrightarrow G_2$$

then $\exists!$ $g(x)$, $g'(0) \neq 0$,

$$q = p^{h}, \quad \mathbb{1}(x) = g(x^{q})$$

Ex

height $G_a = \infty$

height $G_m = 1$

<u>Thm</u>: K perfect, alg closed $\rightarrow$ height is a complete invariant of FG's over K.

Now try to deform away from char p.

Lubin-Tate deform. spaces

B : a complete local ring w/ maximal ideal $\mathfrak{m}$

A deformation $\Gamma \to B$ is this data

$$\left.\begin{array}{l} G, \qquad \begin{array}{c} B \\ \big\downarrow r \\ K \xrightarrow{\ i\ } B/\mathfrak{m} \end{array} \\[1em] G \xrightarrow{F} i^{*}\mathfrak{m} \end{array}\right\} \quad (G, i, F)$$

$\mathrm{Deform}_{\Gamma}(B)$ ← groupoid

$\downarrow$

$\pi_0 \, \mathrm{Deform}(B)$ ← iso classes

Thm if $n = \text{height } \Gamma$

then $\pi_0 \mathrm{Deform}_\Gamma(B) = \mathfrak{m}^{n-1}$] naturally

→ But want to mod out by isos that don't necessarily cover the identity.

ie $\mathfrak{m}^{n-1} / \mathrm{Aut}\,\Gamma$

$\boxed{\text{Understand hty gps of spheres via moduli space of FG's}}$

Can construct a universal deformation space over $W[[u_1, \cdots u_{n-1}]]$ $\left\{\begin{array}{l}\text{witt vectors} \\ \text{over } k\end{array}\right\}$

→ Ring homomorphisms represent the functor

$E_0 = W[[u_1 \cdots u_{n-1}]]$

even part of $E_* = E_0[u, u^{-1}]$, $\deg u = 2$

Interested in $H^*(S_n; E_0)$

where $\underbrace{\text{Aut}\,\Gamma = S_n \text{ acts on } E_*}$

and $uE_0 = E_{-2}$, sections Lie G

1) Can we explicitly describe this?

$\rightsquigarrow$ Related to H-H-R, for any finite sg?

2) What is Pic(LT-space)

$= H^1(\text{Aut}\,\Gamma; E_0^*)$

Conjecture for height $n=2$, $p>5$,

See projects.

For $n=2$, $H^*(S^n; W) \xrightarrow[\sim]{\Omega} H^*(S_n; E_0)$

iso!

For all p.

Where we're headed

Replace formal fns with "entire p-adic fns".

Explain crystalline period map.