# EXERCISES FOR LIE GROUPS AND LIE ALGEBRAS, MATH 229A

1. Show that a normal discrete subgroup of a connected topological group is central.
2. Show that $\mathrm{GL}_n(\mathbf{C})$ is connected
3. Using quaternions, construct a surjective morphism of Lie groups $\mathrm{SU}(2) \times \mathrm{SU}(2) \to \mathrm{SO}(4)$ and a surjective morphism of Lie groups $\mathrm{SO}(4) \to \mathrm{SO}(3) \times \mathrm{SO}(3)$ that are both local isomorphisms.
4. Show that a Lie group is commutative if and only if its Lie algebra is commutative (i.e. the Lie bracket vanishes).
5. Show that there are two isomorphism classes of 2-dimensional Lie algebras over a field
6. Identify the algebra of real quaternions $\mathbf{H}$ with its standard basis $\{1, i, j, k\}$ with $\mathbf{R}^4$, so that $\mathrm{SO}(4) \subset \mathrm{GL}_4(\mathbf{R})$ acts on $\mathbf{H}$. Recall that the group $U(\mathbf{H})$ of unit quaternions identifies with $\mathrm{SU}(2)$. Show that the morphism $U(\mathbf{H}) \times U(\mathbf{H}) \to \mathrm{GL}_4(\mathbf{R})$, $(a, b) \mapsto (h \mapsto ahb^{-1})$ defines a morphism of Lie groups with image contained in $\mathrm{SO}(4)$. Show that the resulting morphism $\mathrm{SU}(2) \times \mathrm{SU}(2) \to \mathrm{SO}(4)$ is the universal cover of $\mathrm{SO}(4)$.