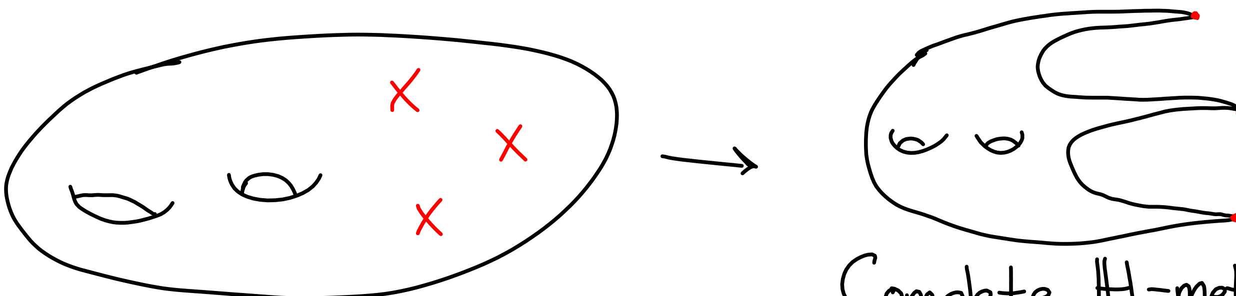
MCGs of ∞-type Σs and

$\curvearrowright$ $\mathbb{H}$-graphs

S connected, oriented, $\chi(S)<0$ ($\Rightarrow \mathbb{H}$ struct)

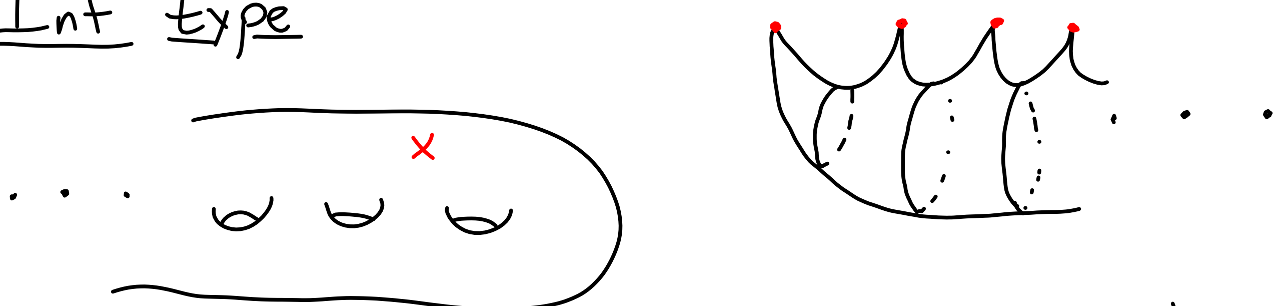
Finite type: $\cong_{Top} S'$ where #punctures $<\infty$

cusps



Complete $\mathbb{H}$-metric

__Inf type__

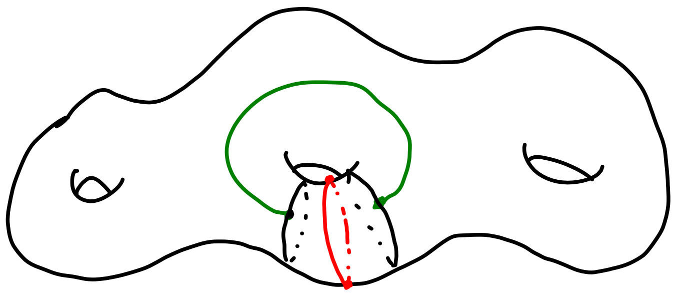


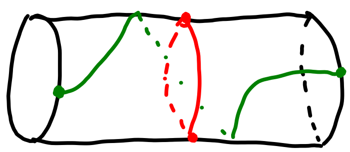
isotopy

__Def__: $MCG(S) := [S,S]^+_{Top} \,/\, f\sim g$

__Ex__: Permute punctures

__Ex__: Dehn Twist $\longrightarrow$ generate MCG!





Nielson-Thurston Classification: For S finite type

$f \in Map(S)$ is either

- Periodic $(f^k = id)$
- Reducible (Preserves a multicurve)
- Pseudo-Anosov
  - ↳ Highly mixing
  - ↳ No power fixes any simple closed curves
  - ↳ Yields a pair of foliations
    - 1 Stretching
    - 1 Shrinking

<u>Def</u>: Curve complex $\mathcal{C}(S)$

- Vertices = $[\text{simple closed curves}]_{\text{isotopy}}$
- Edges = Disjointness

Can make a higher complex

$\longrightarrow$ Graph metric $(\ell(\text{edge}) = 1)$

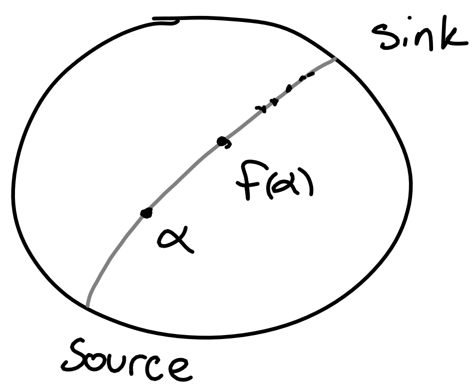
Masur Minsky 99 <u>Thm</u>: $\mathcal{C}(S)$ is ∞-diameter & δ-hyperbolic

<u>Rmk</u> $\mathrm{Map}(S) \curvearrowright \mathcal{C}(S)$ by isometries

f is p.A. iff the action is <u>loxodromic</u>

(Quasi-geodesic axis where it acts by translation)

Schematic:

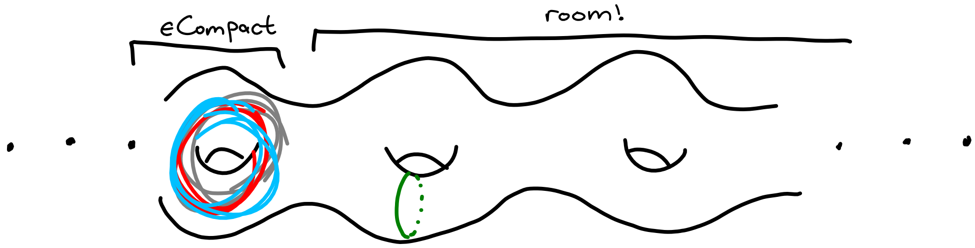


<u>Part 2 - ∞-type</u>

See Danny Caligeri's posts

No classification! No good analog for p.A.

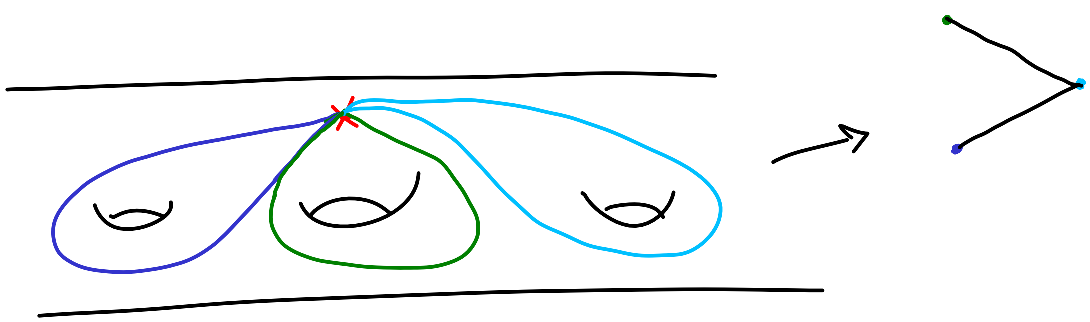
Bad news: $\mathrm{diam}(\mathcal{C}(S)) = 2 < \infty$



$$d(b,r) = d(b,g) + d(g,r) = 2$$

Def: Relative arc graph $\mathcal{A}(S,p)$

- Vertices = s.c.c.'s rel a marked point
- edges = disjointness



Thm $\mathrm{diam}(\mathcal{A}(S,p)) = \infty$, is δ-hyp

Let $\mathrm{Map}(S,p) \subseteq \mathrm{Map}(S)$ fix p.

Q: What are the loxodromic isometries?

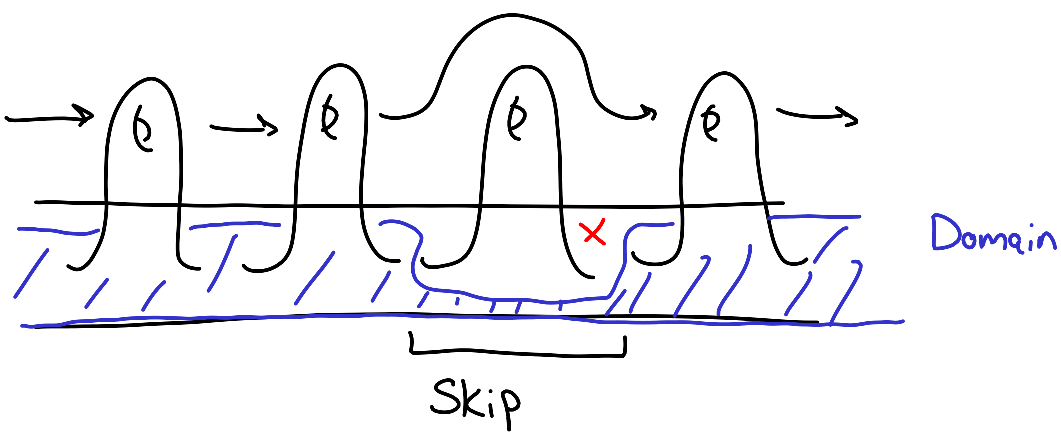
Partial answer: $p \in S' \subseteq S$ of finite type, then

p.A $\Rightarrow$ Loxodromic. But what else?

Q: What about $\underbrace{\underline{\text{intrinsically}}\ \infty\text{-type}}$?

$\hookrightarrow$ Are there even any homeos $\in Map(S,p)$

1) Handle shifts



$\rightsquigarrow$ Leads to train track - Flavor constructions