Differential Equations and Linear Algebra Spring 2014 Notes

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Contents

Chapter 1

Vector Spaces

Next Exam: April 9th?

1.1 Inner Product Spaces (4.11)

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1.1.1 Axioms

4 Axioms of an Inner Product

1. $V_1 \cdot V_1 >= 0$ and $V_1 \cdot V_1$ iff $V_1 = 0$
   Check that the scalar result is positive or zero.
   Show that $\langle A, A \rangle = 0$ forces the coefficients to be zero.
2. $V_1 \cdot V_2 = V_2 \cdot V_1$
3. $(cV_1) \cdot V_2 = c(V_1 \cdot V_2)$
4. $V_1 \cdot (V_2 + V_3) = V_1 V_2 + V_1 V_3$

1.1.2 Orthogonality

$\langle p, q \rangle = 0 \Rightarrow$ Orthogonality.

1.1.3 Examples

1.

Let $A, B.C \in M_2(\mathbb{R})$. Define $\langle A, B \rangle = a_{11}b_{11} + 2a_{12}b12 + 3a_{21}b_{21}$. Does this define an inner product on $M_2(\mathbb{R})$?

2.

Instead, let $\langle A, B \rangle = a_{11} + b_{22}$. Does this define an inner product on $M_2(\mathbb{R})$?

3.

Let $p = a_0 + a_1x + a_2x^2$ and $q = b_0 + b_1x + b_2x^2$.
Define $\langle p, q\rangle = \sum_{i=0}^{2}(i+1)a_ib_i$. Does this define an inner product on P_2?

4.

Let $f, g \in C((-\infty, \infty))$. Define

$$\langle f, g\rangle = \int_a^b f(x)g(x)dx.$$

1.2 Gram