The Hydrogen Spectrum and the Bohr Model

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Physics 215L
Effective Date of Report: May 4, 2014

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I. Introduction

THE purpose of this experiment is to demonstrate that spectra predicted by the Bohr model can be confirmed with macroscopic measurements.

II. Theory

For this experiment, we desire a theoretical expression that will allow us to relate the intensity maxima produced by a diffraction grating to the wavelength of the light source. We start by examining Bohr's postulates, deriving expressions angular momentum, kinetic energy, and potential energy. Applying Newton's Second Law allows us to relate this quantities, giving the desired expression.

Under the Bohr model, an atom emits radiation when an electron transitions from a state of high energy to one of lower energy. Bohr postulated that energy was conserved during this event, giving the expression

$$E_i - E_f = hf \tag{1}$$

where E_i and E_f are the initial and final energies respectively, h is Planck's constant, and f is the frequency of the emitted radiation.

The electron is also considered to be a particle that revolves around the nucleus in this model, from which the angular momentum can be expressed as

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}. \tag{2}$$

Since the velocity $\mathbf{v}$ and the radial vector $\mathbf{r}$ are always perpendicular, this can be simplified to express the magnitude of L as

$$|\mathbf{L}| = mvr. \tag{3}$$

Bohr's postulate then suggests that the only allowed orbits for the electron are those for which the angular momentum is restricted such that $L = n\hbar$. This gives the relationship

$$mvr = n\hbar. \tag{4}$$

Since the kinetic energy of the electron is given by

$$K = \frac{1}{2}mv^2,$$

and the electric potential energy is given by

$$U = -k_e \frac{q_p^2}{r},$$

(where k_e is Coulomb's constant, q_p is the charge of the proton, and r is the distance between the proton and the electron), the total energy of the system can be written as

$$\begin{aligned} E &= K + U \\ E &= \frac{1}{2}mv^2 - k_e \frac{q_p^2}{r}. \end{aligned} \tag{5}$$

Additionally, the only significant force acting on the electron is the Coulombic force, which is given by

$$F = k_e \frac{q_e q_p}{r^2} \tag{6}$$

where q_e is the electron's charge.

Applying Newton's Second Law, we have

$$F = ma.$$

From Equation 6 we have the net force, and noting that the centripetal acceleration is given by $a = v^2/r$ yields

$$\begin{aligned} k_e \frac{q_e q_p}{r^2} &= \frac{mv^2}{r} \\ \Rightarrow v^2 &= k_e \frac{q_e q_p}{rm} \end{aligned} \tag{7}$$

Substituting this result into the expression for kinetic energy,

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}k_e \frac{q_e q_p}{r} \end{aligned}$$

This then gives the total energy of the system, which is

$$\begin{aligned} E &= K + U \\ &= \frac{1}{2}k_e \frac{q_e q_p}{r} - k_e \frac{q_p^2}{r} \\ &= -\frac{1}{2}k_e \frac{q_e q_p}{r} \end{aligned} \tag{8}$$

In order to find an expression for r, we combine the expressions involving v from Equations 4 and 7.

$$\begin{aligned} mvr &= n\hbar \\ \Rightarrow (mvr)^2 &= (n\hbar)^2 \\ \Rightarrow v^2 &= \left(\frac{n\hbar}{rm}\right)^2 \end{aligned} \tag{9}$$

Equating this to v^2 from Equation 7,

$$\left(\frac{n\hbar}{rm}\right)^2 = k_e \frac{q_e q_p}{rm}$$
$$\Rightarrow r = \frac{n^2\hbar^2}{k_e m q_e q_p}. \tag{10}$$

To find the smallest allowed radius, a_0, we set $n = 1$ and find that

$$a_0 = \frac{\hbar^2}{k_e m q_e q_p}. \tag{11}$$

This allows us to simplify the expression for the general radius as

$$r = n^2 a_0. \tag{12}$$

Substituting this into the expression for total energy given in Equation 8 yields

$$E = -\frac{1}{2}k_e \frac{q_e q_p}{n^2 a_0} \tag{13}$$
$$= -\frac{1}{2}k_e \frac{q_e q_p}{a_0}\left(\frac{1}{n^2}\right), \tag{14}$$

and from this expression we can calculate how the energy when transitioning between states. Since $E_i - E_f = hf$, we find that

$$hf = -\frac{1}{2}k_e \frac{q_e q_p}{a_0}\left(\frac{1}{n_i^2}\right) + \frac{1}{2}k_e \frac{q_e q_p}{a_0}\left(\frac{1}{n_f^2}\right)$$
$$= \frac{1}{2}k_e \frac{q_e q_p}{a_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$\Rightarrow f = \frac{k_e q_e q_p}{2ha_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

From $c = f\lambda$, we have $1/\lambda = f/c$ and the above expression to be rewritten as

$$\frac{1}{\lambda} = \frac{k_e q_e q_p}{2hca_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right),$$

and rewriting the identical charges q_p and q_e as e, gives the expression for the Balmer series of the Hydrogen spectrum,

$$\frac{1}{\lambda} = \frac{k_e e^2}{2a_o hc}\left(\frac{1}{n_f^2} - \frac{1}{n^2}\right) \tag{15}$$

where n is the quantum number and

$$k_e = 8.988 \times 10^9 N\dot{m}^2/C^2$$
$$e = 1.602 \times 10^{-19} C$$
$$a_o = .0529 nm$$
$$h = 6.6261 \times 10^{-34} J\dot{s}$$
$$c = 2.998 \times 10^8 m/s.$$

III. Data and Results

A. Part 1

Wavelength of Laser	632.8 nm
Location of Laser on Meter Stick:	50 cm
Distance to First Maximum, Right:	25.35 ± .02 cm
Distance to First Maximum, Left:	22.30 ± .02 cm
Average Distance to First Maximum:	23.83 ± .02 cm
Distance from Grating to Meter Stick:	36.00 cm

Slit Spacing d, given by

$$m\lambda = d\sin\theta$$
$$\Rightarrow d = m\lambda\csc\theta$$
$$\Rightarrow d = m\lambda\frac{\sqrt{D^2 + x^2}}{x^2},$$

yields: d = 1.146 μm

B. Part 2

Distance from Grating to Meter Stick: 29.85 cm

	Red	Cyan	Violet
Distance(cm)	23.60	15.75	13.75
Wavelength(nm)	711.0	267.5	159.9
% Error	8.33%	-45.0%	-63.2

Wavelength is given by

$$m\lambda = d\sin\theta$$
$$\Rightarrow \lambda = \frac{d}{m}\sin\theta$$
$$= \frac{d}{m}\sin\left(\tan^{-1}\left(\frac{x}{D}\right)\right)$$

IV. Results

Color	**Intensity**	**Wavelength(Theory) (nm)**	**Wavelength(Meas) (nm)**	**%Error**
Violet	15	410.0	417.5	1.9
Blue-Violet	30	434.0	442.9	-2.1
Blue-Green	80	486.1	489.7	0.74
Red	300	656.2	649.5	-1.0

Color	**Intensity**	**Wavelength(Theory) (nm)**	**Wavelength(Meas) (nm)**	**%Error**
UV	500	388.8	383.7	-1.3
Blue	200	447.1	439.6	-1.7
Green	100	501.5	509.1	1.5
Orange	500	587.5	579.3	-1.4
Red	100	667.8	661.2	-0.99
IR	200	706.5	697.9	-1.2

Color	**Intensity**	**Wavelength(Theory) (nm)**	**Wavelength(Meas) (nm)**	**%Error**
UV	2800	365	369.6	1.26
Violet	1800	404.6	405.99	0.34
Blue-Violet	4000	435.8	435.2	-0.14
Blue - Green	90	491.6	NA	NA
Yellow	1100	546	541.92	-0.75
Yellow-Orange	280	579	572.08	-1.20
Orange-Red	1000	614.9	691.18	12.41