Logistic Sequencses

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Abstract

This purpose of this report is to study the behavior of the logistic sequences based on varying initial conditions. In particular, the logistic difference equation will be examined, which is defined by the equation

$$P_{n+1} = kP_n(1 - P_n)$$

in which $P(n)$ represents the size of the population after n generations as a fraction of the maximum population size. As such, $P(n)$ is given upper and lower bounds such that $0 \leq P(n) \leq 1$, and P_0 represents the ratio of the initial population to its potential maximum.

The variable k represents a constant of proportionality, and its presence denotes the fact that the size of any generation $n + 1$ will be related to the size of the previous generation n.

In order to determine how the initial conditions affect the steady state values of this function as $n \to \infty$, the effect of altering the values of k and P_0 will be examined.

1 $1 < k < 3$

For the first several trials, initial values of P_0 were set to .1 and k was varied between 1 and 3.

1.1 $k = 1$

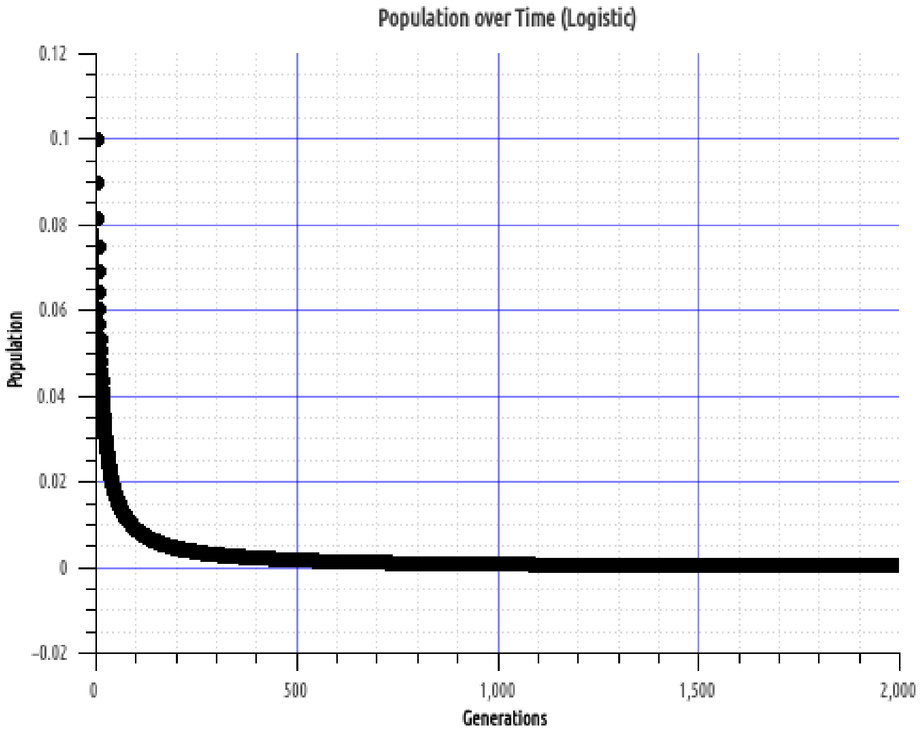


Figure 1: $P_0 = \frac{1}{2}, k = 1$

For these conditions, the sequence is monotonically decreasing, and appears to converge to 0 as $\lim n \to \infty$. Similar graphs for $0 < k < 1$ all converge in a similar fashion to values less than P_0.

1.2 $k = 2$

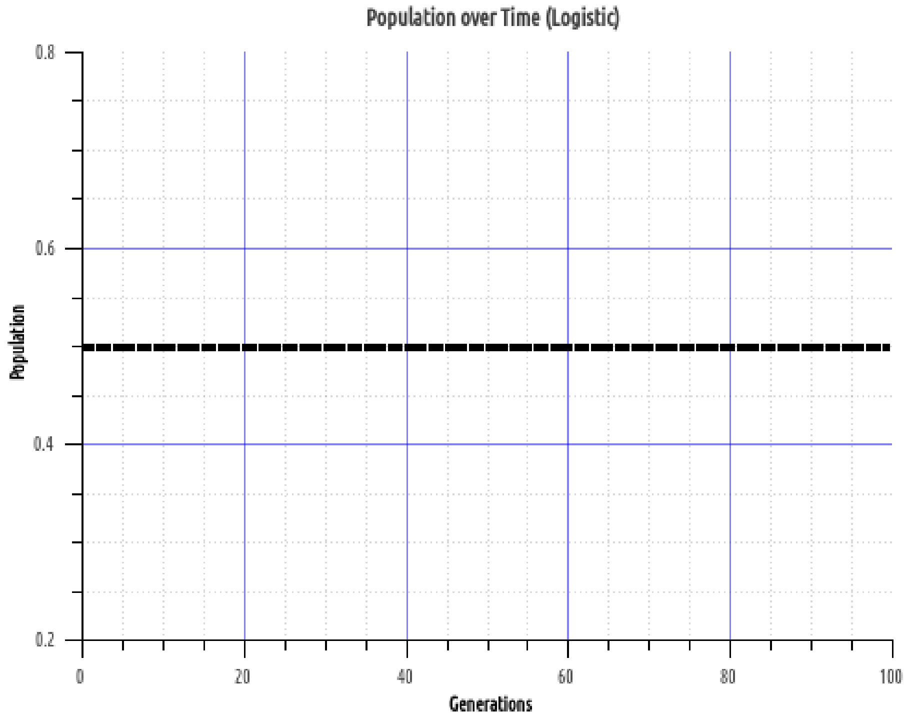


Figure 2: $P_0 = \frac{1}{2}, k = 2$

In this case, the sequence does not deviate from its initial value, and its long term steady-state value is equal to its initial value.

1.3 $k = 2.5$

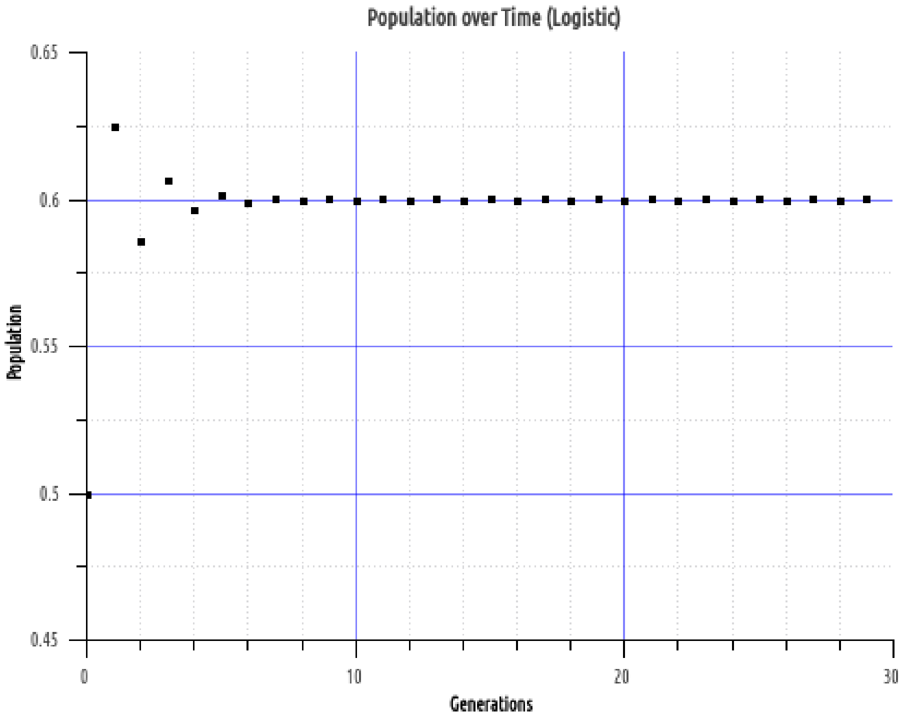


Figure 3: $P_0 = \frac{1}{2}, k = 2.5$

For values of k greater than 2, the sequence oscillates around a value and as $\lim n \to \infty$ appears to converge to some limit L.

1.4 Conclusion

In all cases, the sequence appears to converge to some definite value.
For $1 < k < 2$, the sequences monotonically decrease and converge to some value lower than P_0.
For $2 < k < 3$, it becomes an alternating series and tends to oscillate around a limiting value greater than P_0. This oscillation becomes more pronounced as $k \to 3$.
For all $1 < k < 3$, the choice of P_0 does not change the value of the limit, while small differences in k have large effects on what the sequence converges to.

2 $3 < k < 3.4$

2.1 $k = 3$

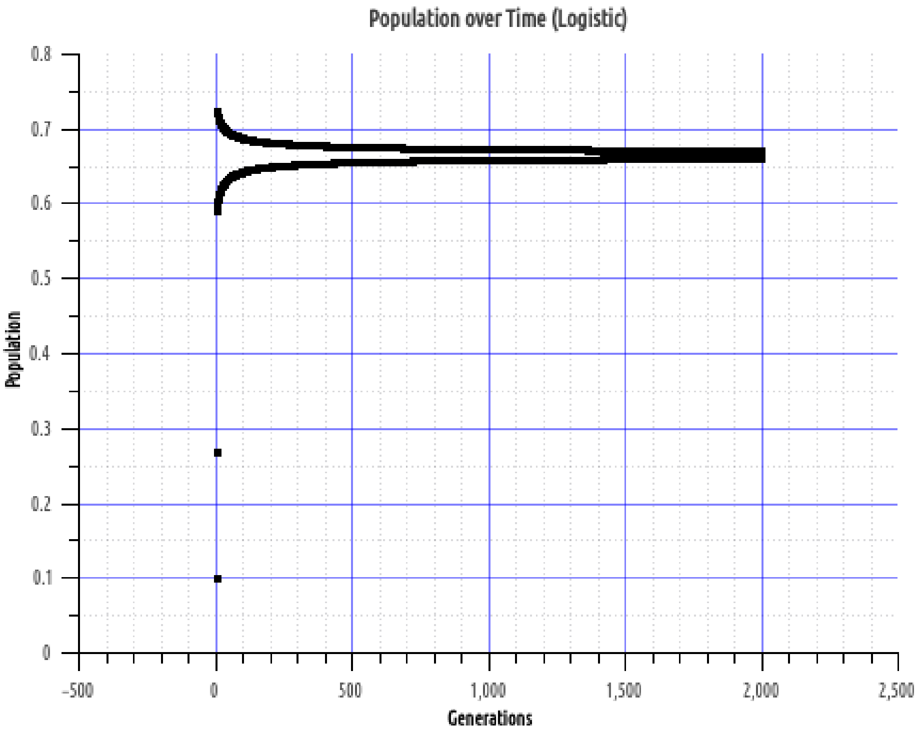


Figure 4: $P_0 = \frac{1}{2}, k = 3$

For $k = 3$, the sequence shows similar behavior as $2 < k < 3$ and alternates around a limiting value.

2.2 $k = 3.2$

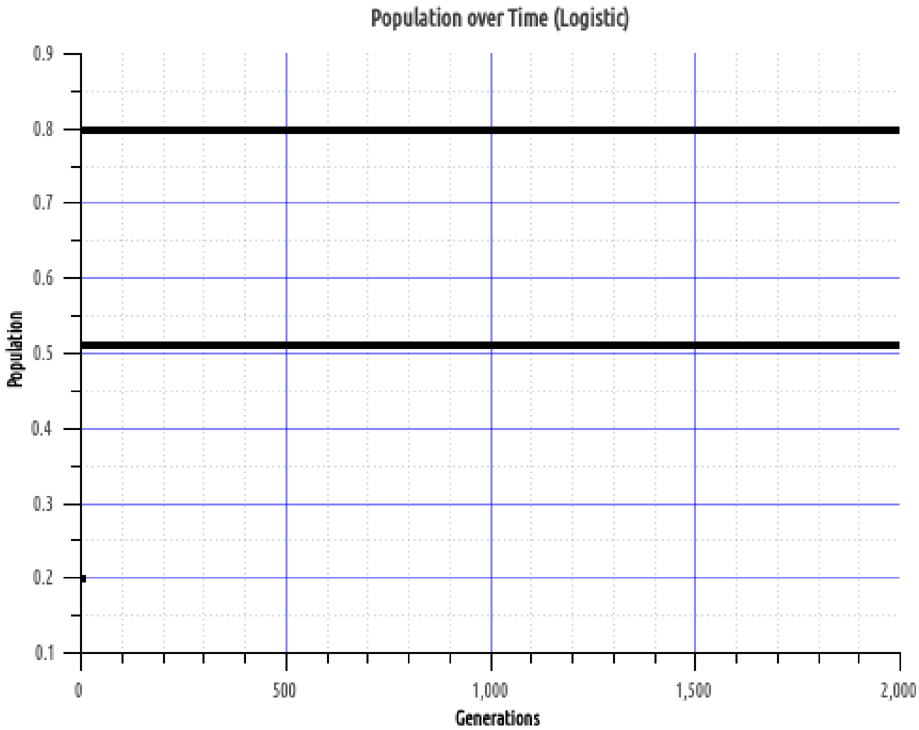


Figure 5: $P_0 = \frac{1}{2}, k = 3.2$

Slightly increasing k leads to drastic changes in behavior. The sequence now oscillates between two distinct values and does not approach any definite limit.

2.3 $k = 3.4$

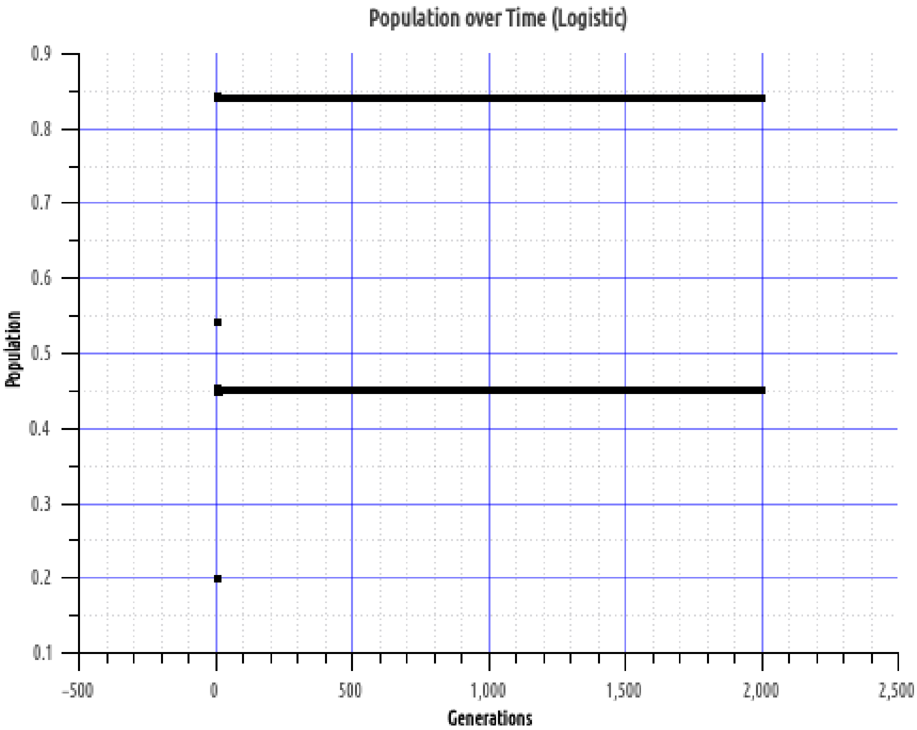


Figure 6: $P_0 = \frac{1}{2}, k = 3.4$

Increasing k to 3.4 produces an even more pronounced effect in which the limiting values are further separated from eachother.

2.4 Conclusion

At some value of k between 3 and 3.2, the sequence no longer converges to a single value, and rather begins to oscillate between two values. This effect becomes more pronounced as k is increased.

3 $3.4 < k < 3.5$

3.1 $k = 3.45$

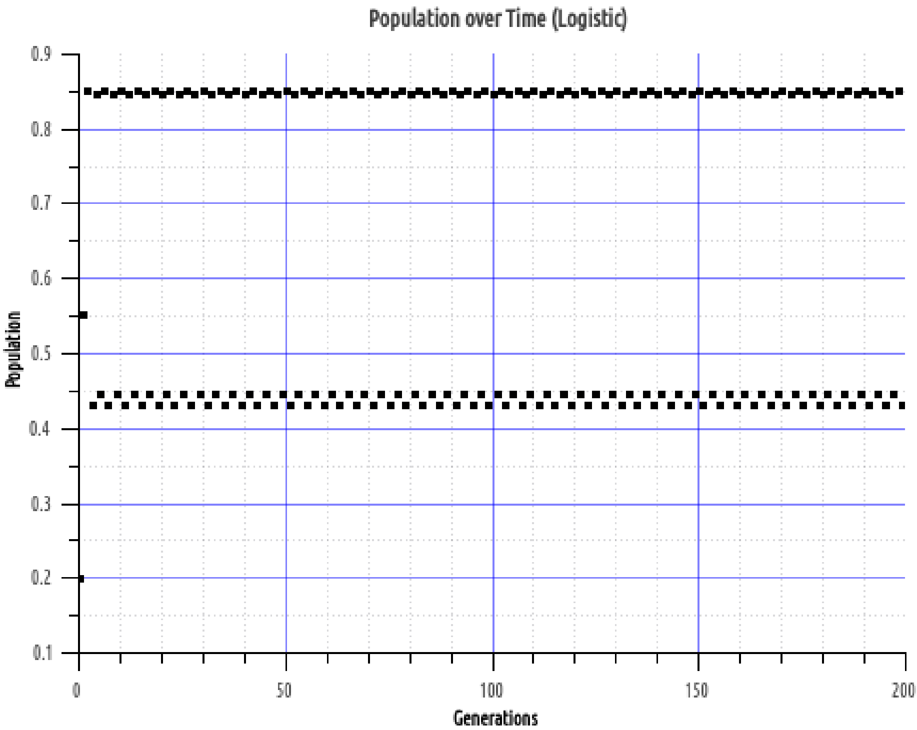


Figure 7: $P_0 = \frac{1}{2}, k = 3.45$

Increasing k beyond 3.4 causes each distinct limiting value to repeat the same alternating behavior, resulting in two lines that themselves have values that oscillate around them.

3.2 $k = 3.5$

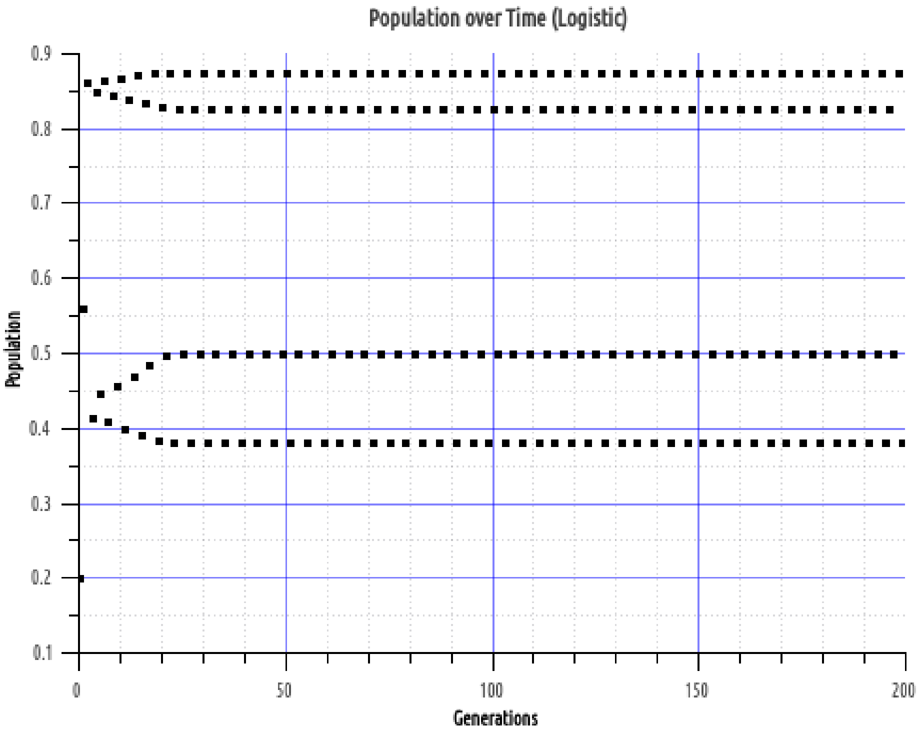
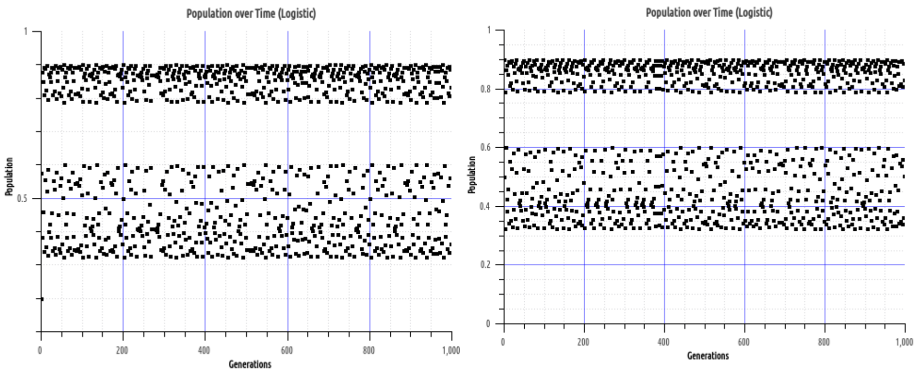


Figure 8: $P_0 = \frac{1}{2}, k = 3.5$

Further increases to k lead to a more prounounced effect.

4 $3.6 < k < 4$

4.1 $k = 3.6, P_0 = .5$ compared to $P_0 = .5001$



(a) $k = 3.6, P_0 = .5$

(b) $k = 3.6, P_0 = .5001$

At $k = 3.6$, the values begin to chaotically oscillate. They are still centered around the previous limiting values, but the distribution around these values is much wider.

Altering the value of P_0 by a value as small as .0001 causes drastic changes in the behavior of the graph and leads to an entirely different distribution of points.

4.2 Conclusion

The behavior of the sequence in this interval becomes more chaotic as $k \to 4$, causing the sequence to repeatedly bifurcate and oscillate between many values.

5 Overall Behavior

Overall, the behavior of this sequence tends to converge for certain values of k, but becomes increasingly chaotic as k is increased. Beyond a certain value, the population does not stabilize to any one limiting value, but rather cycles between many values. The number of values through which the population cycles increases as k is increased. Thus, for large values of k, the patterns would be effectively random.