Modern Physics - Waves, Optics

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Contents

Chapter 1

Geometric Optics

1.1   Reflection

1. Specular Reflection

   Reflection from a smooth surface that reflects rays parallel to each other. Occurs as long as the surface variations are much smaller than λ.

2. Diffuse Reflection

   Reflection from a rough surface that reflects rays in various directions.

In general, the angle of reflection will equal to angle of incidence, or

$$\theta_1' = \theta_1$$
$$\text{(Reflection Angle = Incident Angle)}$$

1.2   Refraction

1.2.1   Flat Refracting Surfaces

See Figure 1.1. The equation generally used is

$$\frac{n_1}{p} = \frac{-n_2}{q}$$

Where $-q$ denotes the distance from the surface to the image. The image formed is also always on the same side of the surface as the object itself.

1.2.2   Equations

1. Index of Refraction:

$$n = \frac{c}{v_{\text{Medium}}} \text{ or } vn = c$$

   Defined as the ratio between the speed of light in a vaccum and in a medium. v is always less than c, so $n > 1$ for all substances. This also implies $\lambda_n < \lambda$, or that the wavelength of light in a medium is always lower than its wavelength in a vacuum.
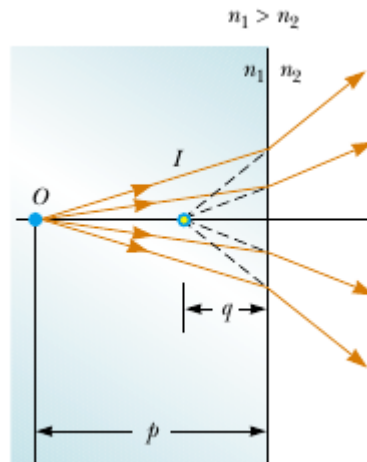
Figure 1.1: Image formed by a flat refracting surface.

2. Snell's Law of Refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$\lambda_1 n_1 = \lambda_2 n_2$$

As light travels from one medium to another, its frequency does not change, but its wavelength <u>does</u>.

3. Total internal reflection:

$$\sin \theta_{\text{crit}} = \frac{n_2}{n_1} \qquad\qquad (\text{for } n_1 > n_2)$$

Occurs when light travels from a medium with a high n to one with a lower n, so $n_2/n_1 \leq 1$. Can be in degrees or radians, just use the conversion

$$\frac{\text{radians}}{2\pi} = \frac{\text{degrees}}{360°}$$

Chapter 2

Image Formation: Mirrors, Lenses

2.1 Flat Mirrors

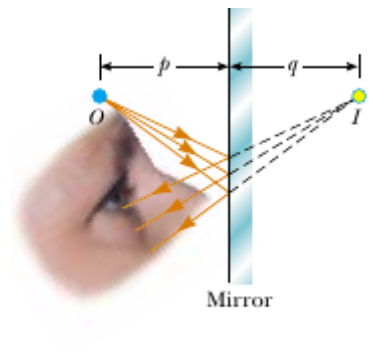
Figure 2.1: Conventions

Conventions

- O: Object
- I: Image of object
- p: Actual distance of object
- q: Distance of Image

2.2 Conic Mirrors

To determine image location, only three principle rays are essential:

1. From the top of the object, parallel to the principal axis.
   (Reflected along line with F. Concave: Towards, Convex: Away)

2. From the top of the object, through the focal point f.
   (Always reflected parallel to principal axis.)

3. From the top of the object, through the center of curvature C.
   (Always reflected back upon itself)
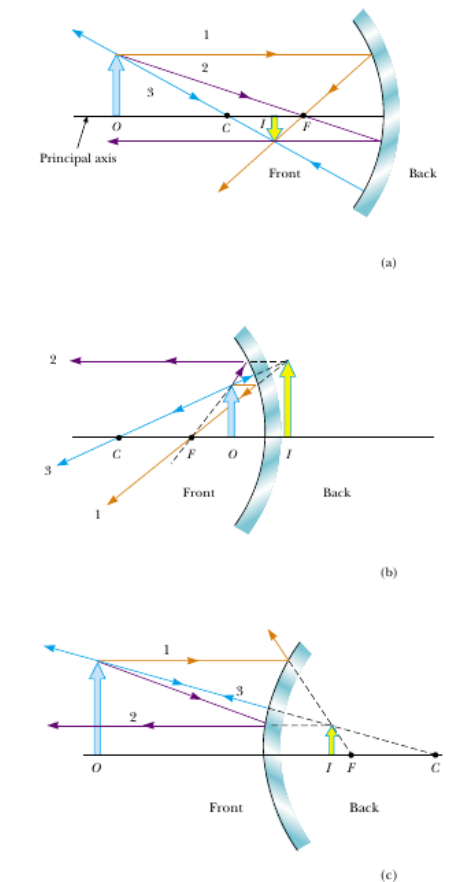
The intersection of these rays locates the image.

Figure 2.2: Cases for object/mirror placement

2.2.1 Concave Mirrors

The center of curvature C is located on the principle axis. All paraxial rays (ones that diverge from the principle axis by only a small angle) converge on I, so objects placed farther away than C (on the principle axis) produce a **real** image in front of the mirror. If rays emanating from O instead tend to diverge from

the principal axis, spherical aberration is introduced and the image becomes blurred.

Always has a **positive** focal length – that is, F is in front of the mirror. **3 possible cases:**

1. $O > C > F$

   If object is *farther* than C, image is **real, inverted,** and **reduced** in size.

2. $C > O > F$

   If the object is *between* O and F, its image is **real, inverted, and enlarged**.

3. $C > F > O$

   If the object is closer than the focal length, the image is **virtual, upright, and enlarged**.

2.2.2 Convex Mirrors

Always has **negative** focal length, where F is located behind the mirror. Images are always **virtual, upright,** and **reduced** in size.

Table 36.1

Sign Conventions for Mirrors		
Quantity	**Positive When**	**Negative When**
Object location (p)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

Figure 2.3: Sign Conventions for Mirrors

2.3 Lenses

Table 36.3

Sign Conventions for Thin Lenses		
Quantity	**Positive When**	**Negative When**
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

Figure 2.4: Sign conventions for lenses

The Lens-Maker's Equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Table 36.2

Sign Conventions for Refracting Surfaces		
Quantity	Positive When	Negative When
Object location (p)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (q)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface

Figure 2.5: Sign conventions for refracting surfaces.

Convention summary:
The signs R indicate whether the corresponding surfaces are convex or concave. If R_1 is positive the first surface is convex, and if R_1 is negative the surface is concave.
The signs are reversed for the back surface of the lens: if R_2 is positive the surface is concave, and if R_2 is negative the surface is convex.
Comparing Image/Object Rates of Change
The thin lens equation can be implicitly differentiated with respect to time to yield this relationship.

$$\frac{d}{dt}\left(\frac{1}{p} + \frac{1}{q} = \frac{1}{f}\right) \rightarrow \left(\frac{-1}{p_0^2}\right)p' + \left(\frac{-1}{q_0^2}\right)q' = 0$$

2.4   Equations

1. Definition of Magnification:

$$M = \frac{h'}{h}$$

2. Magnification in a Concave Mirror:

$$M = \frac{-q}{p}$$

   $M < 1$ indicates that the image is smaller than the object. $-M$ indicates image is inverted.

3. The Mirror Equation:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}(\text{ where} f = \frac{R}{2})$$

   Depends only on the curvature of the mirror, and not its material. $+q$ denotes an image on the front side of the mirror (real), and vice-versa.

A dedicated sports car enthusiast polishes the inside and outside surfaces of a hubcap that is a thin section of a sphere. When she looks into one side of the hubcap, she sees an image of her face 42.0 cm in back of the hubcap. She then flips the hubcap over and sees another image of her face 10.0 cm in back of the hubcap.

(a) How far is her face from the hubcap?

[____] ✔ cm

(b) What is the radius of curvature of the hubcap?
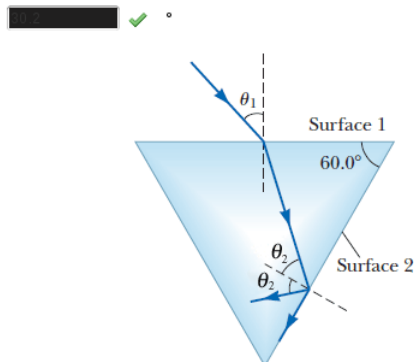
[____] ✔ cm

Figure 2.6: Convex/Concave Mirrors

2.5 Homework / Review

Hints: Results in two equations in two unknowns. Near image is convex, far image is concave. F is negative for convex mirrors.

Solutions: 16.2 cm, 52,5 cm.

The light beam in the figure below strikes surface 2 at the critical angle. Determine the angle of incidence θ_1. (Let $\theta_2 = 40.8°$.)

[30.2] ✔ °

2.

Figure 2.7: Determine angle of incidence, given the critical angle inside the prism.

Hints: The sum of interior angles is always 180°. Solve for ratio of indices of refraction. Use Snell's Law.

Solution: 30.2°.

3. The accommodation limits for a nearsighted person's eyes are 20.0 cm and 81.0 cm. When he wears his glasses, he can see faraway objects clearly. At what minimum distance is he able to see objects clearly?

   Solution: Using the lens equation and take q to be the far limit and $p = \infty$ to find f. Then, take q to be the near limit and solve for p. (26.2 cm)

4. Two converging lenses having focal lengths of $f_1 = 12.5$ cm and $f_2 = 18.0$ cm are placed a distance $d = 52.5$ cm apart as shown in the figure. The image due to light passing through both lenses is to be located between the lenses at the position $x = 30.5$ cm indicated.

   At what value of p should an object be placed to produced this image?

   What is the magnification of the final image?

   What is the orientation of the image? Is it real or virtual?

Two converging lenses having focal lengths of f_1 = 12.5 cm and f_2 = 18.0 cm are placed a distance d = 52.5 cm apart as shown in the figure below. The image due to light passing through both lenses is to be located between the lenses at the position x = 30.5 cm indicated.
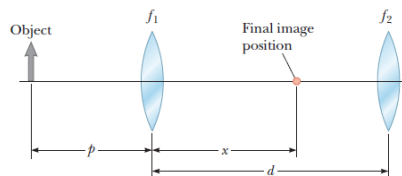

Figure 2.8: Two lenses in series

Solution: Start with the right lens, noting that it must produce a virtual image at I. From this, solve for p_2 to find the location of the virtual object.

The distance q_1 of the real image produced by the first lens is related to p_2, as $q_1 = d - p_2$. This can be used to solve for p_1.

Note sign conventions! q_2 is negative, f is positive in both cases since the lenses are converging, and p is positive in both cases.

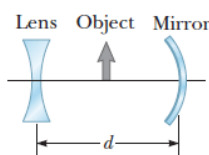p_1 = 17.7 cm, M_T = -5.4, the image is enlarged, inverted, and virtual.

Figure 2.9: Lens/Mirror System

5. The object in the figure below is midway between the lens and the mirror, which are separated by a distance d = 24.5 cm. The magnitude of the mirror's radius of curvature is 19.5 cm, and the lens has a focal length of -16.8 cm.

   Where is the final image formed by this system?

   Is the image real or virtual? What is its orientation and total magnification?

Solution: Start with the mirror. Since p_1 and f_1 are known, solve for q_1. Use this as a virtual object for the lens. q_1 = 47.775 cm, $p_2 = q_1 - d = $ 23.275, $q_2 = -60.38$. Note sign conventions: all quantities for mirror are positive, while f_2 and p_2 are negative.

So I is 35.9 cm to the right of the mirror, M_T = 10.1, and the image is virtual and upright.