Lab 1

The Digital Multi Meter: Voltage and Current Dividers

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Prepared for:

Professor Pattengale

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Engineering 17L - Circuit Theory Lab Section

Tuesday 2:00 - 5:05

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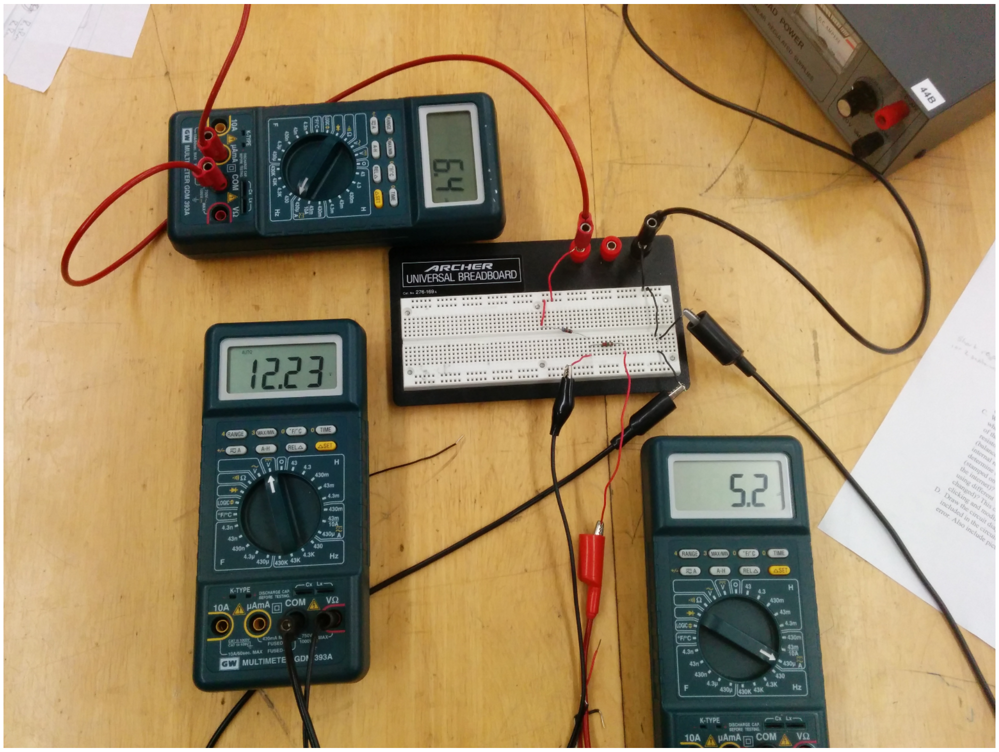


Introduction / Abstract

A common method of measuring the various parameters of a circuit involves the use of a Digital Multimeter, or a DMM. While these are often modeled as ideal devices, the act of introducing them into a circuit in order to measure voltage, current, or resistance fundamentally changes the behavior of the circuit.

When in ammeter mode, the DMM is placed in series with an element to measure the current through it. Ideally, the ammeter would function as a short circuit, and have an effective resistance of zero. However, the ammeter itself has a small amount of internal resistance, which is additive in series with other circuit elements. Similarly, an ideal voltmeter in parallel with a circuit element would function as an open circuit and have infinite resistance, while real meters often have very high but finite resistance.

The purpose of this lab is to investigate how voltages and currents are divided among circuit elements, as well as how the addition of a DMM affects a physical circuit in comparison to the theoretical circuit model.

Theory

Digital multimeters (DMMs) are essentially small computers comprised of many different internal elements designed to read input data and display a corresponding output. This is all done using electronic components, all of which will inherently have some sort of internal resistance. For this reason, it is impossible for a digital multi-meter to have zero resistance when measuring current. In a similar vein, an ideal meter would act as an open circuit, but this is not possible since some charge must pass through the elements to produce a reading. Consequently, it is impossible for a meter to have infinite resistance. Both of these qualities will have an influence on the circuit being measured, resulting in values different than the same circuit sans measuring apparatus.

When manufacturing DMMs, manufacturers produce them to have resistance qualities that will have a minimal effect on what they measure, the goal being an impact that is less than the uncertainty in the data.

All meters will have an impact on a circuit when wired into them to take measurements. The resistance of the meter is usually provided by the manufacturer. Like most precision measuring instruments, it is necessary to calibrate a meter on occasion. The actual resistance of a meter will vary depending on the load within the circuit. Finding the internal resistance cannot be done by simply hooking one meter to another, it must be done by hooking a meter into a circuit, measuring other elements in the circuit and deducing the resulting resistance.

1. Voltage Divider

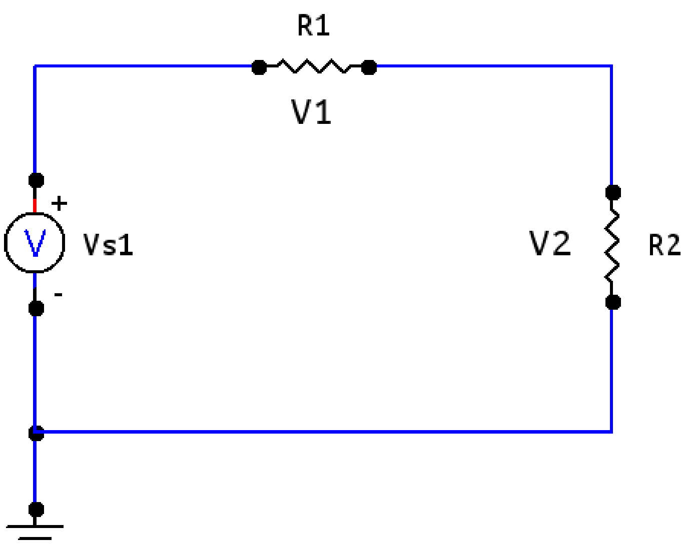


Figure 1: Voltage Divider

$$R_{eq} = \sum_{1}^{n} R_n = R_1 + R_2$$

$$V = iR$$

$$\rightarrow i = \frac{V}{R}$$

$$i_{Vs1} = i_{R1} = i_{R2} ... = i_{Rn}$$

$$\rightarrow \frac{V_{s1}}{R_{eq}} = \frac{V_{R1}}{R_1} = \frac{V_{R2}}{R_2} ... = \frac{V_{Rn}}{R_n}$$

$$\rightarrow \frac{V_{Rn}}{R_n} = \frac{V_{s1}}{R_{eq}}$$

$$\rightarrow V_{Rn} = \left(\frac{R_n}{R_{eq}}\right) V_{s1}$$

To derive an equation for the voltage drop over a single resistor in a series, we make use of Ohm's Law ($V = iR$) and the fact that the current is equal through each resistor in series.

Because all three elements (that is, the voltage source and the two resistors) are on the same path, it is necessary for the current to pass through each element. Solving Ohm's Law for current gives $i = V/R$, showing that if the currents are all equal, the ratio V_n / R_n must also be equal for each element *n*.

For the voltage source V_{s1} , the equivalent resistance is simply the sum of the individual resistor values. Setting the ratios of V_{s1} / R_{eq} and V_n / R_n equal to each other then yields an expression for the voltage drop across any resistor R_n . Plugging in the known values from Figure 1 gives an expression for the voltage drops across each resistor in terms of V_{s1} , R_1, and R_2 , which is summarized in Equations (1) and (2).

Equation **(1)**: V_1, Voltage Across R_1

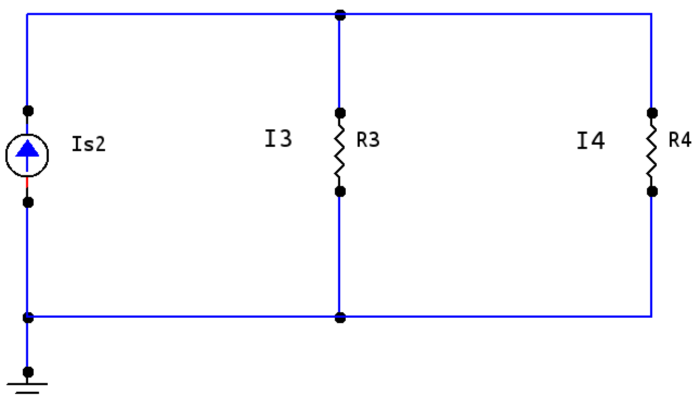
$$V_{R1} = \left(\frac{R_1}{R_1 + R_2} \right) V_{s1}$$

Equation **(2)**: V_2, Voltage Across R_2

$$V_{R2} = \left(\frac{R_2}{R_1 + R_2} \right) V_{s1}$$

2. Current Divider

Figure 2: Current Divider



$$V = iR$$

$$V_{s2} = V_{R3} = V_{R4}$$

$$\rightarrow i_{s2} \times \mathrm{R}_{eq} = i_{R3} \times \mathrm{R}_3 = i_{R4} \times \mathrm{R}_4 \ldots = i_{Rn} \times R_n$$

$$i_{s2} \times \mathrm{R}_{eq} = i_{Rn} \times R_n$$

$$\rightarrow i_{Rn} = i_{s2} \left(\frac{\mathrm{R}_{eq}}{R_n} \right)$$

In order to derive an expression for the current through a single resistor in a network of parallel resistors, we use Ohm's Law and the fact that the voltage across parallel resistors is equal.

$$\frac{1}{\mathrm{R}_{eq}} = \frac{1}{R_3} + \frac{1}{R_4} \rightarrow \mathrm{R}_{eq} = \frac{R_3 R_4}{R_3 + R_4}$$

Because both branches are wired in parallel to the current source, the top and bottom branches are at the same potential and the voltage across each element must be equal. Using Ohm's Law, if the voltages are equal, the product of $i_n R_n$ for each element *n* must also be equal.

Since $i_{s2} \mathrm{R}_{eq}$ must be equal to $i_n R_n$, solving this equation for i_n and substituting in values for R_n and R_{eq} from Figure 2 yields Equations (3) and (4).

Equation **(3)**: i_3, Current Across R_3

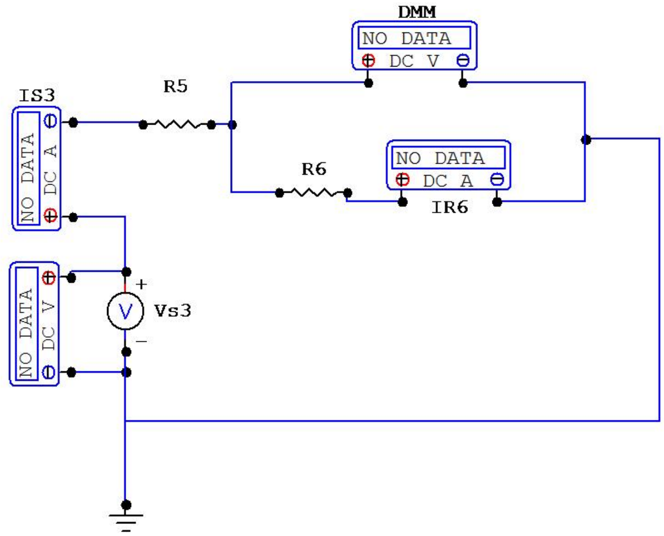
$$i_3 = \left(\frac{R_4}{R_3 + R_4} \right) i_{s2}$$

Equation **(4)**: i_4, Current Across R_4

$$i_4 = \left(\frac{R_3}{R_3 + R_4} \right) i_{s2}$$

3. Determining Resistance of a DMM

Figure 3: Modeling a DMM as a Resistor



In order to find the internal resistance of a DMM, a circuit such as the one in Figure 3 can be used in which the DMM is modeled as a resistor in parallel with another resistor of a known value. This information, along with the measurements of the source current and the amount of current that flows through the branch of the known resistor, is enough to determine the unknown resistance.

From the current division equation, it is known that the current flowing through one of two parallel branches is given by the equation

$$i_{R6} = i_{s3}\left(\frac{R_{eq}}{R_6}\right)$$

Applying the rules for the resistors in parallel, the equivalent resistance is given by the following:

$$\frac{1}{R_{eq}} = \frac{1}{R_n} + \frac{1}{R_6} = \frac{R_n + R_6}{R_n R_6} \rightarrow R_{eq} = \frac{R_n R_6}{R_n + R_6}.$$

where **R_m** is the unknown resistance of the voltmeter. Plugging the equivalent resistance into the current division equation yields

$$i_{R6} = i_{s3}\left(\frac{1}{R_6}\right)\left(\frac{R_m R_6}{R_m + R_6}\right) \rightarrow (R_m + R_6)\, i_{R6} = R_m i_{s3} \rightarrow R_6 i_{R6} = R_m i_{s3} - R_m i_{R6}$$

from which an expression for the unknown voltmeter resistance **R_m** can be derived.

Equation **(5)**: Voltmeter Resistance

$$\rightarrow R_m = R_6\left(\frac{i_{R6}}{i_{s3} - i_{R6}}\right)$$

Equipment List

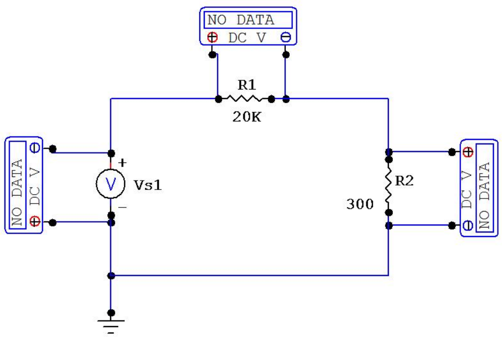
Nomenclature	Manufacturer	Model/Serial #	Function
Assorted Discrete Resistors	R.S.R Electronics, Inc.	300Ω, 20kΩ, 30kΩ, 220kΩ, 2.2MΩ	Circuit components used to model loads.
Breadboard	Global Specialties	Model: Proto board PB-6	Foundation of circuit
Circuit Maker	MicroCode Engineering, Inc.	Student Ver 6.2	Calculating theoretical values of circuit variables.
DC Power Supply	Elenco-Precision	Model: XP-581	Source of power to circuits.
Digital Multimeter	GW Instek Multi Meter	Model: GDM 393A	Measuring voltage and current of circuit elements.
Wire Jumper Kit	Jameco Electronics	Model: JE10	Circuit components
Wire Stripper/Cutter	K.Miller Tool Co.	Model: 102	Trimming wire and stripping insulation.

Procedure

I. **Voltage** *(Circuit I)*

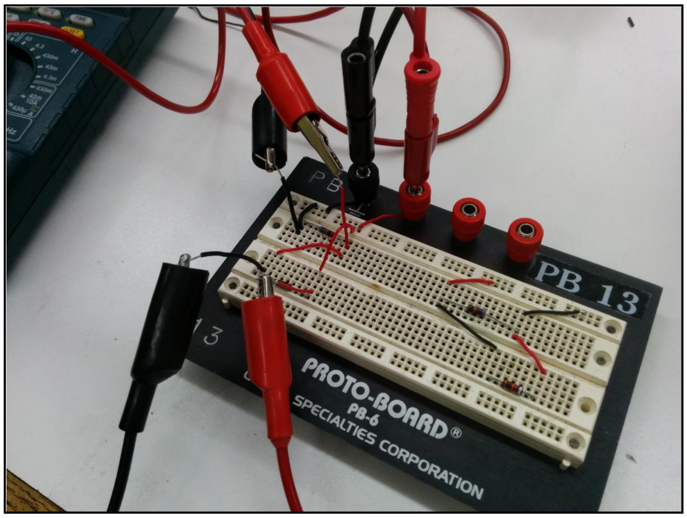
1. Circuit I was constructed with $\mathbf{R_1 = 20k\Omega}$ and $\mathbf{R_2 = 300\Omega}$.

Figure 4: Schematic, Circuit I



2. Three DMMs in voltage mode were connected to the circuit:
   - $\mathbf{V_s}$: Parallel to the power supply terminals.
   - $\mathbf{V_1}$: Parallel to $\mathbf{R_1}$.
   - $\mathbf{V_2}$: Parallel to $\mathbf{R_2}$.

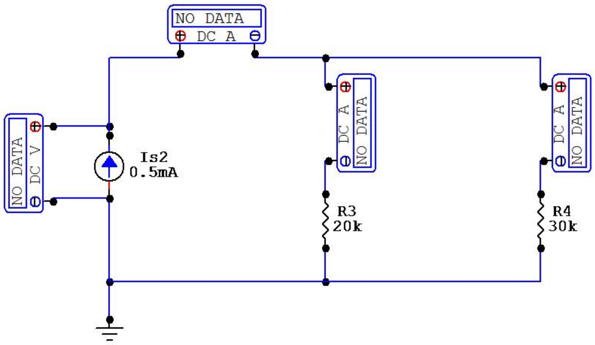
Figure 5: Circuit I (left) and Circuit II (right), Constructed



3. The voltage of the power supply was adjusted to **5V**.
4. Measurements for $\mathbf{V_s}$**,** $\mathbf{V_1}$**,** and $\mathbf{V_2}$ were taken and recorded.
5. We adjusted the power supply to **10V** and **15V**, and measurements were repeated at each voltage.
6. The resistances of $\mathbf{R_1}$ and $\mathbf{R_2}$ were measured by a DMM and recorded.

II. Current *(Circuit II)*

Figure 6: Schematic, Circuit II



1. Circuit II was constructed on a breadboard with $\mathbf{R_3 = 20k\Omega}$ and $\mathbf{R_4} = \mathbf{30k\Omega}$.
2. Three DMMs in ammeter mode were wired into the circuit to measure the current from the source and the current through each resistor's branch.
3. A constant current source of **5.0mA** was connected to the terminals of the breadboard and wired into the appropriate busses.

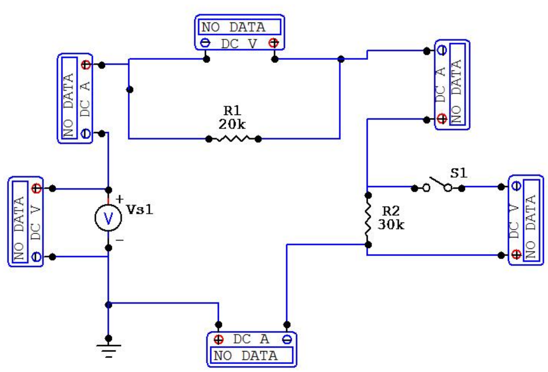
4. Measurements for all three currents were taken and recorded.
5. These measurements were repeated for $\mathbf{I_{s2}}$ = **1.0mA** and **1.5mA**.

III. Meter Loading

b) DMM Input Impedance *(Circuit I)*

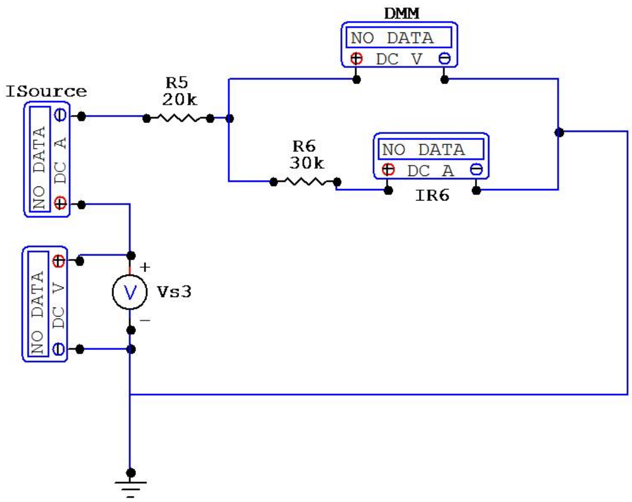
1. Circuit I was reconstructed on a breadboard with $\mathbf{R_1}$ = **20kΩ** and $\mathbf{R_2}$ = **30kΩ**.
2. Using banana-to-alligator clip leads, a constant **5V** source was wired directly into a bus on the breadboard by attaching the leads to short lengths of wire.
3. A DMM as a voltmeter was wired parallel to $\mathbf{R_1}$.
4. The voltage drop was measured and recorded.
5. A second DMM was then wired parallel to $\mathbf{R_2}$.
6. The voltages across $\mathbf{R_1}$ and $\mathbf{R_2}$ were measured a second time and recorded.
7. Steps 3-5 were repeated with $\mathbf{R_1}$ **= 220kΩ** and **2.2MΩ**.

Figure 7: Circuit I, Reconstructed To Measure Meter Loading



c) Voltmeter Resistance *(Circuit III)*

Figure 8: Schematic, Circuit III



1. Circuit I was disassembled and Circuit III was constructed on the same breadboard with
   R$_5$ = 20kΩ and **R$_6$** = **30kΩ**.
2. A DMM was wired in parallel with **R$_5$** in order to approximate its resistance.
3. A second resistor **R$_6$** was wired in series with the **R$_5$**/ Voltmeter parallel combination.
4. An DMM in ammeter mode was wired in series with the voltage source to measure the source current.
5. A second ammeter was wired into the same branch as the voltmeter to measure the current through it.

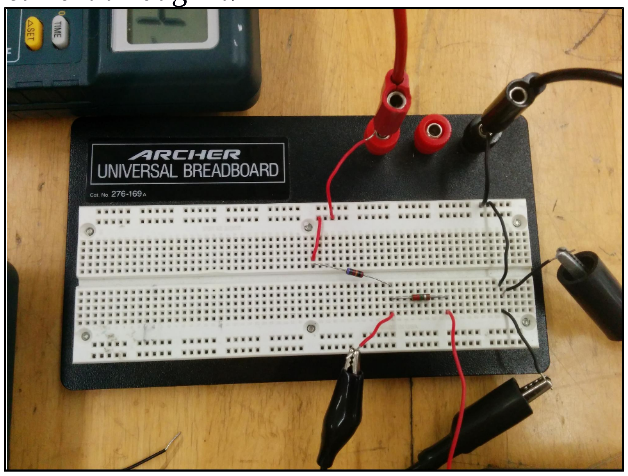


Figure 9: Circuit III Constructed

6. A constant **12V** source was wired directly into the breadboard.
7. The following measurements were taken and recorded:
   - The voltage displayed by the power source
   - The reading of the voltmeter being measured.
   - The current through the voltmeter.

8. All measurements were repeated with **R_6 = 220kΩ** and **2.2MΩ**.

Results

Table 1: Data Collection - Circuit I, Voltage Divider

V_{s1} (V)	$V_{20k\Omega}$ (V) [Measured]	$V_{20k\Omega}$ (V) [Theoretical]	% Error	$V_{300\Omega}$ (mV) [Measured]	$V_{300\Omega}$ (mV) [Theoretical]	% Error	$V_{20k\Omega}$ / $V_{300\Omega}$ [Measured]	$V_{20k\Omega}$ / $V_{300\Omega}$ [Theoretical]	% Error
5.00	4.43	4.926	-10%	72.6	73.91	-1.77%	61.0	66.6	-8.4%
10.03	9.33	9.882	-5.6%	145.7	148.3	-1.75%	64.0	66.6	-3.9%
15.04	14.24	14.82	-3.9%	218.6	222.3	-1.66%	65.1	66.7	-2.4%

Table 1 contains data collected as the source voltage was varied in the voltage divider circuit. Theoretical values were obtained by modeling the same circuit in Circuit Maker, running a simulation, and collecting data with DMMs at an internal resistance of **100MΩ**. As described in Equation **(1)**, the expected voltage ratios were generally proportional to the ratio of resistances. However, error analysis suggests that circuits may deviate from this model at relatively low voltages.

Table 2: Data Collection - Circuit II, Current Divider

V_{s2} (V)	I_{s2} (mA)	$I_{20k\Omega}$ (µA) [Measured]	$I_{20k\Omega}$ (µA) [Theoretical]	% Error	$I_{30k\Omega}$ (µA) [Measured]	$I_{30k\Omega}$ (µA) [Theoretical]	%Error	$I_{20k\Omega}$ / $I_{300k\Omega}$ [Measured]	$I_{20k\Omega}$ / $I_{30k\Omega}$ [Theoretical]	%Error
6	0.5	282.7	300	-5.8%	195.8	200	-2.1%	1.44	1.5	-4%
9	0.75	423.5	449.9	-5.9%	293.5	300	-2.2%	1.44	1.5	-4%
12	1.00	590	599.9	-1.7%	390.9	400	-2.3%	1.51	1.5	0.6%
15.3	1.27	750	761.9	-1.6%	510	507.9	0.4%	1.47	1.5	-2%

Table 2 contains data collected as the source current was varied in the current divider circuit. Similar to the voltage divider, the measured values were compared to values modeled by Circuit Maker, which is used for theoretical values. The results agree with Equation **(2)** in the Theory section, which suggested that the ratio of currents through each resistor would be inversely proportional to the ratio of the resistor values. Error analysis again suggests that this model becomes less accurate at low current levels.

Table 3: Data Collection - Circuit I, Effect of Adding Voltmeter

R_2 (Ω)	V_{s1}	V_1 (mV) [Switch Open]	V_1(mV) [Switch Closed]	% Change	V_2 (V)
30k	5.02 V	2033	2036	+0.15%	2.984

220k	5.02 V	428.4	436	+1.7%	4.58
2.2M	5.02 V	43.5	53.4	+23%	4.97

The data in Table 3 was taken from the circuit modeled in Figure 7, in which the effects of adding a voltmeter to a circuit were examined. A voltmeter was wired parallel to one resistor, giving a value for $\mathbf{V_1}$. A second voltmeter was then wired parallel to a second resistor, and $\mathbf{V_1}$ was read again. The percent change is a reflection of how drastically the initial reading was altered with the addition of the second voltmeter as a new circuit element. Analysis shows that the associated error becomes more pronounced when other elements in the circuit have relatively high resistance values.

Table 4: Data Collection - Circuit III, Determining Resistance of Voltmeter

R_6 (Ω)	V_{s3} (V)	V_{Meter} (V)	I_{s3} (µA)	I_{R6} (µA)	Calculated Meter Resistance (Ω)
30k	12.13	7.30	238.2	237.2	7.1M
220k	12.3	11.25	52.6	51.4	9.4M
2.2M	12.3	12.23	6.4	5.2	9.5M

Average Measured Resistance of DMM:	**8.69M Ω**
Documented Resistance of DMM:	**≥10M Ω**
Percent Difference:	**-13%**

The data in Table 4 was taken as the resistor $\mathbf{R_6}$ in Circuit III (see Figure 8 and Figure 9) was varied to increasing values. The current from the source was measured, as well as the current through the branch of the known resistor, and the estimated voltmeter resistance was calculated from Equation **(5)** in the Theory section. As this equation depended on differences in current on the order of 10^{-6}, increasing the resistance in the known branch tended to decrease the amount of error in the calculation and provided a better approximation to the manufacturer's documented value.

Conclusion

Part 1

For the first portion of this experiment we were asked to measure the voltage across two resistors in series and find the ratio of the two voltages (Figure 4). It was predicted that the ratio would not change since the voltage is determined by the linear relationship $V = iR$ and the current is the same through resistors in series. Therefore the ratio of the voltages will be roughly determined by the ratio of the resistors and will remain constant.

$$V = iR \rightarrow$$
$$V_{R1} = I_{s1}R_2$$
$$V_{R2} = I_{s1}R_2$$
$$\rightarrow \frac{V_1}{V_2} = \frac{I_{s1}R_1}{I_{s1}R_2}$$
$$\rightarrow \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

This prediction was found to be correct, as the ratio of measured voltages differed by less than 8.4% from the ratio of the resistances, indicating that the ratio of the voltages of resistors in series can be modeled by the ratio of their resistances.

Looking at data from Table 1, it appears that a majority of error comes from the 20kΩ resistor. Since both resistors are connected to the same power supply we know that the current through **R_1** is equal to the current through **R_2** which means our measurement of voltage should be inaccurate for both resistors if the inaccuracy were coming from the power supply. However, we see that **R_1** has a maximum error of -10% while **R_2** has a maximum error of -1.77% which points to the resistor being the source of error.

The error associated with **R_2**, and a portion of the error with **R_1**, could be a result of less than perfect circuitry in our equipment. For example, if the internal resistance of our supply were to be accounted for as well as the resistance of the proto board, we may be able to reduce the error even further. Also, we are not using an ideal voltmeter, as we found in Part 3 of this lab, which means that our meter is allowing some current to pass through it and not through the resistor and thereby lowering our measured voltage. This explanation would be consistent with our data as each of our measured voltages were lower than the theoretical. It should also be noted that when setting up our circuit we used a 300 Ohm resistor instead of the scripted 30k Ohm resistor. While this deviation may have affected how our meter calibrated itself, it should only affect the value of our ratio as we have determined that the ratio of the voltages can be modeled by the ratio of the resistors within acceptable error.

$$\left(\frac{V_1}{V_2}\right)_{theoretical} = \frac{4.926V}{0.07391V} = 66.65$$
$$\frac{R_1}{R_2} = \frac{20k\Omega}{300\Omega} = 66.67$$
$$\%diff = \frac{\left(\frac{V_1}{V_2}\right)_{theoretical} - \frac{R_1}{R_2}}{\left(\frac{V_1}{V_2}\right)_{theoretical}} \times 100\%$$
$$\rightarrow \%diff = \frac{66.65 - 66.67}{66.65} \times 100\%$$
$$\rightarrow \%diff = -0.03\%$$

Despite our error we were still able to verify the equation for the voltage divider and gain an understanding of some of our systems strengths and weaknesses.

Part 2

For the second part of this lab we measured the current though two resistors in parallel and then found the ratio of the currents *(Figure 6)*.

$$V = iR \rightarrow$$
$$I_{R3} = \frac{V_{s2}}{R_3}$$
$$I_{R4} = \frac{V_{s2}}{R_4}$$
$$\rightarrow \frac{I_{R3}}{I_{R4}} = \frac{V_{s2}/R_3}{V_{s2}/R_4}$$
$$\rightarrow \frac{I_{R3}}{I_{R4}} = \frac{R_4}{R_3}$$

We predicted that the ratio would remain constant as we increased the current since the current is defined by the ratio $i = V/R$, where R and V are constants for resistors in parallel, thereby making the ratio of current strictly dependent upon the ratio of the resistors.

Again we found this prediction to be correct, as the ratio of the currents varied by less than -4%. As in Part 1 it appears that the 20kΩ resistor played a large part in contributing to our error, with a maximum error of -5.9%. However, in contrast to Part 1, **R₂** also had a relatively large percent error with a maximum error of -2.3%. Since we used a different resistor and had a lower percent error in Part 1, it would appear that the 30kΩ resistor is less accurate than its 300Ω counterpart.

$$\left(\frac{I_{R3}}{I_{R4}}\right)_{theoretical} = \frac{300\mu A}{200\mu A} = 1.5$$
$$\frac{R_4}{R_3} = \frac{30k\Omega}{20k\Omega} = 1.5$$

However, we did change our equipment setup entirely, with the exception of the proto board. This would lead us to look more closely at factors such as the internal resistance of our power supply, the imperfect nature of our meters, and the possible resistivity of our busses. The measured data tends to be lower than the theoretical values which would be consistent with losses in our system due to internal resistance in the power supply, busses, and ammeter. This experiment still has room to improve when it comes to accuracy, but we were still able to clearly validate the equation for the current divider despite the errors.

$$\%diff = \frac{\left(\frac{I_{R3}}{I_{R4}}\right)_{theoretical} - \frac{R_4}{R_3}}{\left(\frac{I_{R3}}{I_{R4}}\right)_{theoretical}} \times 100\%$$
$$\rightarrow \%diff = \frac{1.5 - 1.5}{1.5} \times 100\%$$
$$\rightarrow \%diff = 0.0\%$$

Part 3

The effects of the meter on the circuit are demonstrated clearly in part III. This setup also demonstrated how the impacts of the meter can vary with different native resistance values in the circuit. Changing the value of a resistor demonstrates how significantly the physical results can deviate from the theoretical. In part III b it is shown that as the resistance of R_2 increases, the DVM has a bigger influence on the circuit, increasingly varying the voltage across R_1. This is due to the fact that as resistance increases in R_2 there is less of a disparity between this and the resistance of the meter. The result is more of the current will flow through the meter, decreasing the overall resistance of the portion of the circuit containing these elements, in turn lowering the voltage drop across them, leading to a larger voltage drop in R_1. When a resistors value is many magnitudes smaller than that of a DVM wired in parallel, the resistance contributed to the circuit by the meter will be negligible. This means that DVMs can produce more accurate measurements in circuits with a low total resistance than in those with higher resistance.

Part III c of this lab calculates the resistance of the meter. For all values of R_2 used, the calculated resistance of the meter was in the MΩ range with an average resistance of 8.7 MΩ. The manufacturer's resistance value for this meter is 10 MΩ, a difference of 13%. The calculated values of the 220kΩ and 2.2MΩ resistors are much closer to the given value than the value found using the 20kΩ resistor. This indicates there is likely error in the measurements taken or the setup used for this resistor. It is possible this can be caused by outside electromagnetic interference in the circuit which has a greater impact in low current circuits (more on this later). It makes sense that all the calculated value will be lower than the true values since the calculation did not take into account the resistance in the wires, voltage source or breadboard.

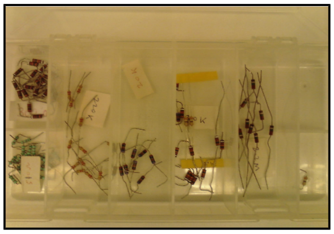
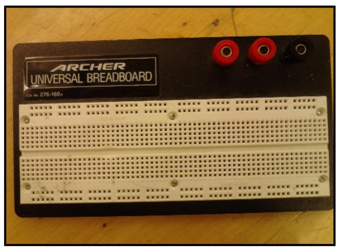
OTHER CONCLUSIONS

The meters are not the only source of deviation in the experimental values from the theoretical ones. Other factors that will change the results are the resistance in the wires/ leads. The lead resistance can be accounted for in some models of multi-meter wherein a function exists that meters leads can be touched together and this resistance will be subtract from the resistance measurement of the circuit element. Knowing the resistance of the wiring in a circuit can be used to create a more accurate model where this value is included and thereby establish a more accurate theoretical value. It is also possible for there to be damage to a wire inside the shielding or within a proto-board that will be unseen to the person setting up a circuit, but can have an effect on the total resistance of a circuit.

One of the biggest factors in gathering accurate data is the interference that can be caused by crossed or moving wires via electromagnetic interference. Although the effects of this are usually small, the impact can be noticeable, especially when dealing with circuits that already have a low amount of current. It is not uncommon for the last digit to fluctuate when taking a measurement, reducing the precision of said measurement (Part III C).

Appendix

Section A: Equipment Photographs

Components	Photographs	Components (cont.)	Photographs (cont)
Assorted Discrete Resistors		Digital Multi Meter	
Breadboard		Wire Stripper/Cutter	
DC Power Supply			

Section B: References

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