RLC Circuits

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Physics 210L
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***Abstract*—**
THE purpose of this experiment...

CONTENTS

I. THEORY

The following expressions were used in the analysis of this circuit:

$$\omega = 2\pi f$$
$$\omega_0 = \sqrt{\frac{1}{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$X_L = \frac{V_L}{I}$$
$$X_C = \frac{V_C}{I}$$
$$Z = \frac{V_s}{I}$$
$$L = \frac{X_L}{2\pi f}$$
$$C = \frac{1}{2\pi f X_C}$$
$$\theta = 360^\circ f \Delta t$$
$$P = I_{\text{rms}} V_{\text{rms}} \cos\theta$$
$$Q = \frac{\omega_0 L}{R}$$
$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\Delta\omega}$$

Δf is the width of the resonance peak between the points where $V = \frac{1}{\sqrt{2}} V_{\text{max}}$

II. METHODOLOGY

1) A simple RLC series circuit was constructed. DMMs were wired to measure ΔV_L, and ΔV_C, the voltages of the inductor L and the capacitor C respectively.
2) An ammeter was wired in series with the previous elements to measure I, and an oscilloscope was wired to measure ΔV_s and ΔV_R, the voltages across the source and the resistor R respectively.
3) The resistance of R was measured in order to determine phase difference between the source voltage V_s and the current I. Δt between these two signals was recorded.
4) The potential difference of the supply was set to a constant 2.0 V (rms), and the ammeter was set to the 430 mA scale.
5) The expected resonance frequency f_0 was calculated, and a range of frequencies symmetric about f_0 were chosen for measurement.
6) For each frequency f, the following quantities were measured:
   a) I_{rms}
   b) V_L
   c) V_C

   Extra data points were taken at frequencies approaching f_0.
7) The data was then placed into a spreadsheet to calculate various variables, which were used in the analysis section.

III. RESULTS

Resistance:	$\underline{R = .96\Omega}$
Inductance:	$\underline{L = 2.64\text{mH}}$
Capacitance:	$\underline{C = 9.20\mu\text{F}}$

A. Average Inductance/Capacitance Values

$\underline{L_{\text{avg}} = 2.532 \pm .002 \text{ mH}}$
$\underline{C_{\text{avg}} = 8.833 \pm .009\ \mu\text{F}}$

B. Impedance vs. Frequency

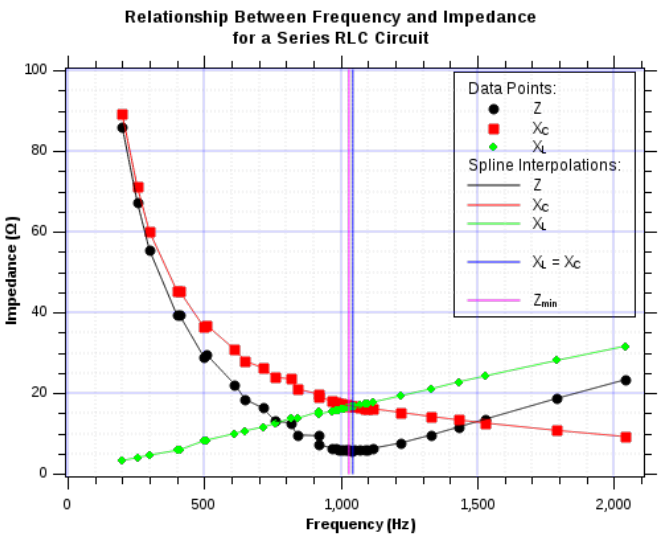

Figure 1. Plot of X_L, X_C, and Z vs. f. The resonant frequency is determined from the intersection of the X_C and X_L curves, and the resistance of the circuit is given at the minimum of the Z curve.

From the plotted data, we have
$$\underline{Z_{\text{min}} = R_{\text{meas}} = 5.72\Omega}$$
$$\underline{f_{\text{res,meas}} = 1044 \text{ Hz}}$$

Why is R_{meas} not the same as the value of the resistor used in the circuit?

C. Current vs. Frequency

The data points were fitted to the function

$$I_{\text{rms}}(\omega) = \frac{V_{\text{rms}}\omega}{\sqrt{(R\omega)^2 + L^2(\omega^2 - \omega_0^2)^2}} \tag{1}$$

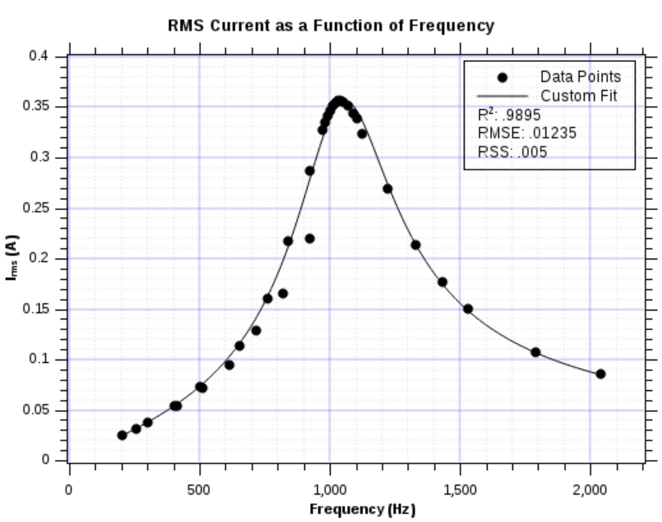

Figure 2. Plot of I_{rms} vs. f. V_{rms} and L_{avg} were used as constants, while R and ω_0 were left as fitting parameters.

From the custom fit described above, the values obtained from the fitting parameters were:
$$\underline{R_{\text{fit}} = 5.80 \pm .06\Omega}$$
$$\underline{f_{\text{res, fit}} = 1059 \pm 5 \text{ Hz}}$$

Why is R (meas) not the same value of the resistor used in the circuit?

This indicates that there was resistance in the circuit that was unaccounted for. This could arise from resistance in the inductor coils, but is very likely to be largely attributed to the internal resistance of the power supply.

D. Difference between measured and fitted values

What is the percent difference between R (measured) and R (fit)? Is R (measured) within the uncertainty of R (fit)?

<u>Percent Difference: 0.87%</u>

The uncertainty in R (fit) was $.06\Omega$ – however, the values differed by $.08\Omega$. While the two values are generally in good agreement, there are slight errors introduced in the interpolation used to find Z (min) on the graph, as well as a certain amount of random error introduced by transient voltage fluctuations when the circuit is near resonance.

E. Theoretical Resonant Frequency

The resonant frequency, calculated from the mean values of L and C from step 1 is given by

$$f_{\text{res,theor}} = \frac{1}{2\pi\sqrt{L_{\text{avg}}C_{\text{avg}}}}. \tag{2}$$

So the theoretical resonant frequency is:
$$\underline{f_{0(\text{theor})} = 1064 \text{ Hz}}$$

Comparing this to the values determined in Steps 2 and 3 above, we have

<u>% Difference, Step 2: 1.9%</u>
<u>% Difference, Step 3: 0.47%</u>

F. Voltage vs Frequency

What does this graph tell you?

Several things are apparent from the graph – the most immediate characteristic of the series RLC circuit is that in the limit of low frequencies, the voltage on the capacitor V_C approaches the source voltage. This agrees with our theoretical understanding of the behavior of a capacitor, which would tend to act as a short circuit at low frequencies. This effect can also be recovered by considering the equation for the impedance of the capacitor, $X_C = 1/2\pi fC$, which predicts that the impedance approaches infinity as the frequency approaches zero. An infinite impedance is effectively a short, necessarily implying that no current will flow and the capacitor will be forced to remain at the potential difference of the source. This is reflected in the graph, as the low-frequency limit of the capacitor voltage approaches the source voltage.

A symmetric but opposite is observed in the inductor, for which the voltage V_L approaches zero in the low-frequency

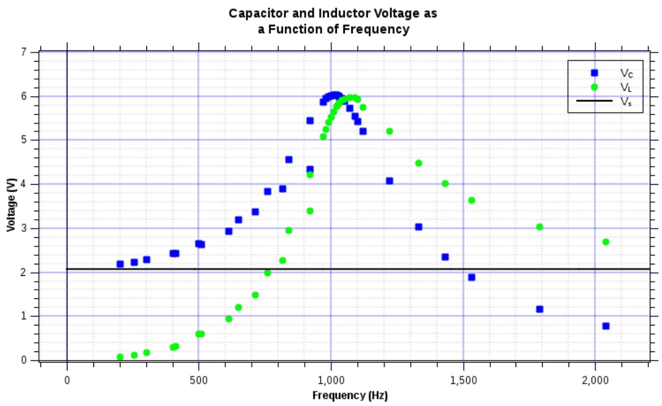

Figure 3. Plot of V_L and V_C vs f, extrapolated to zero frequency.

limit. From the impedance equation $X_L = 2\pi f L$, we see that in this limit, the impedance approaches zero – effectively creating an shunt in the circuit. This causes the inductor to behave like a wire, which agrees with our theoretical model of inductance.

In the limit of high frequencies, the behavior of the inductor and the capacitor switch roles. Both voltages peak symmetrically about the resonant frequency, and the capacitor voltage then approaches zero while the inductor voltage approaches the source voltage. These effects can be seen by considering the limit as f approaches infinity in the previously mentioned inductance equations, and noting that the capacitor's impedance tends to zero while the inductor's impedance tends to infinity. This leads to the capacitor now behaving like a wire, and the inductor like a shunt, which again agrees with our theoretical model of impedance.

The last important characteristic of this graph shows that the maximum voltage is delivered at the resonant frequency. This occurs midway between the maximums the two individual curves, and represents the point at which maximum power is delivered.

G. Phase Angle vs. Frequency

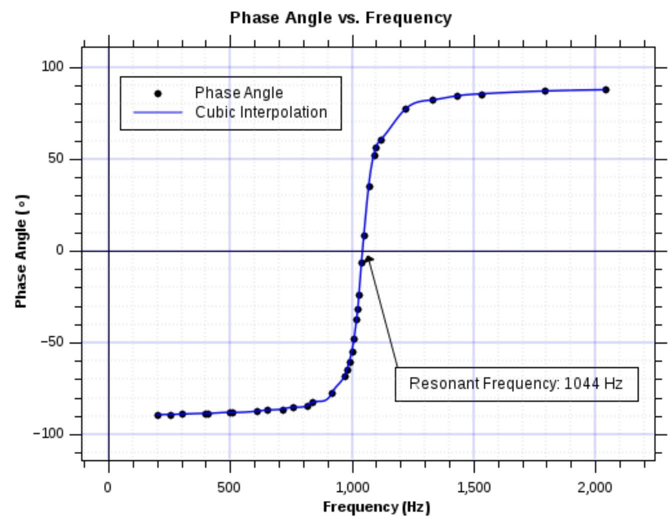

Figure 4. Plot of phase angle ($360° f \Delta t$) vs. f. The y-intercepts represents the frequency at which the current and the voltage are in phase, and corresponds to the circuit's resonant frequency.

Does this curve agree with theory? Explain.

H. Power vs. Frequency

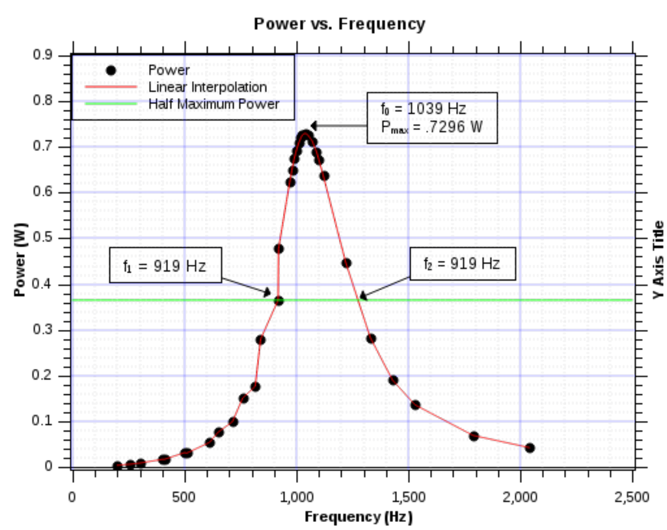

Figure 5. Plot of P vs f, used to determine the full-width at half-max in order to calculate Q.

Bandwidth: Δf = 111.5 Hz

I. Quality Factor

The calculated quality factor is given by

$$Q_{\text{calc}} = \frac{f_{\text{max, meas}}}{\Delta f} \tag{3}$$

Calculated Quality Factor: $\underline{Q_{\text{calc}} = 8.966 \pm d}$
The theoretical Quality factor is given by

$$Q = \frac{\omega_0 L}{R_{\text{meas}}}, \tag{4}$$

where ω_0 is given by $2\pi f_{\text{res,meas}}$ from Step 2 and L is L_{avg} from Step 1.
Theoretical Quality Factor: $\underline{Q = 2.904}$

IV. Analysis and Conclusion