Electric Power and Impedance Matching

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Abstract—
THE purpose of this experiment is to use Kirchoff's Voltage Law to determine the internal resistance and the open circuit voltage of a power supply by examining the effects of varying load resistances on current and voltage measurements over the load. These measurements will also be used to determine the resistance at which maximum power is delivered to a load via impedance matching.

CONTENTS

I. THEORY

Kirchoff's Voltage Law states that the sum of the potential differences in a closed loop is equal to zero, such that

$$\sum V_{\text{Loop}} = 0.$$

For the circuit shown, the potential difference V_R across the load resistor can be found using this equation. Summing the voltages and rewriting them using $V = iR$ (according to Ohm's Law) yields

$$\begin{aligned} \epsilon - V_r - V_R = 0 \Rightarrow \\ V_R = \epsilon - V_r \Rightarrow \\ V_R = \epsilon - ir \end{aligned} \tag{1}$$

where the current i is identical throughout the circuit, as its elements are wired in series, and r is the power supply's internal voltage.

The power released by the resistor R can also be expressed in terms of the known quantities ϵ, r, and R. From the power equation, it is known that $P = iV$. Because the current i is uniform everywhere, it can be related to the source voltage and the equivalent resistance of the entire circuit. Taking the source voltage to as ϵ and the equivalent resistance as $r + R$ and substituting it into the power equation yields

$$\begin{aligned} P = iV, V = iR \Rightarrow P = \frac{i^2}{R} \Rightarrow \\ P_R = \left(\frac{\epsilon}{r+R}\right)^2 R \Rightarrow \\ P_R = \epsilon^2 \frac{R}{(r+R)^2}. \end{aligned} \tag{2}$$

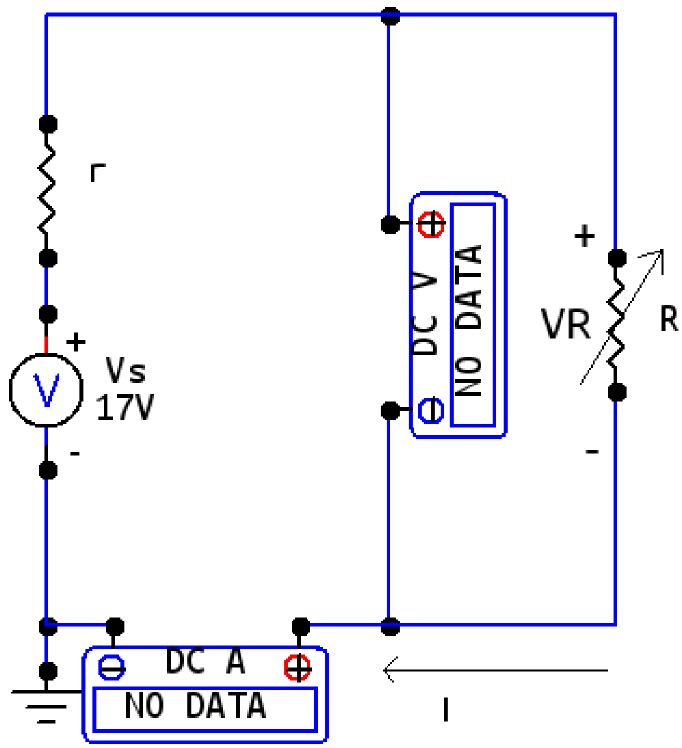

Figure 1. Diagram of circuit used in this experiment. R represents a variable load resistance, while r simulates the internal resistance of the power supply. I is the uniform current that flows through all three components of the circuit.

Calculating the point at which maximum power transfer occurs requires taking the derivative of Equation 2 with respect to the load resistance. This results in

$$\frac{\partial P}{\partial R} = \epsilon^2 \left(\frac{(r + R^2) - 2R(R+r)}{(R+r)^4} \right).$$

Maximizing the power transferred to the load resistor requires setting this derivative equal to zero, which occurs when

$$r + R^2 = 2R(R+r).$$

Solving this expression for R then forces the following condition to be true in order to maximize the power delivered to the load resistor R:

$$R = r. \tag{3}$$

II. METHODOLOGY

1) The circuit was constructed as shown in Figure 1, where r was a known resistor and R was a variable resistor box.

2) Digital voltmeters and ammeters were wired to measure V_R and i.
3) The power supply was set to 17.00 V, and its actual terminal voltage was measured.
4) The known resistor r was measured with an ohmmeter.
5) The resistance of the variable load was incremented, and at each point data was taken for the voltage and current through R.
6) Extra data points were taken at resistances approaching that of the known resistor.

III. Data

Table I
CIRCUIT MEASUREMENTS AND CALCULATED VALUES.

Box Setting R (Ω)	Voltage (V)	Current (mA)	R_{Calc} (Ω)	Power (W)
0	0.004	30.24	0.13	0.00012
10	0.3014	29.69	10.15	0.00895
20	0.574	28.45	20.18	0.01633
30	0.845	28.00	30.18	0.02366
40	1.108	27.60	40.14	0.03058
50	1.365	27.19	50.20	0.03711
100	2.531	25.25	100.24	0.06391
200	4.41	22.03	200.18	0.09715
250	5.18	20.69	250.36	0.10717
300	5.86	19.50	300.51	0.11427
350	6.46	18.44	350.33	0.11912
400	7.00	17.50	400.00	0.12250
450	7.50	16.67	449.91	0.12503
475	7.72	16.24	475.37	0.12537
500	7.94	15.86	500.63	0.12593
510	8.02	15.72	510.18	0.12607
520	8.11	15.57	520.87	0.12627
530	8.19	15.43	530.78	0.12637
540	8.27	15.29	540.88	0.12645
545	8.30	15.22	545.34	0.12633
550	8.34	15.15	550.50	0.12635
555	8.38	15.09	555.33	0.12645
560	8.42	15.02	560.59	0.12647
570	8.50	14.89	570.85	0.12657
580	8.57	14.76	580.62	0.12649
590	8.64	14.63	590.57	0.12640
600	8.71	14.51	600.28	0.12638
700	9.37	13.37	700.82	0.12528
800	9.93	12.39	801.45	0.12303
850	10.17	11.95	851.05	0.12153
875	10.29	11.74	876.49	0.12080
900	10.40	11.55	900.43	0.12012
950	10.62	11.17	950.76	0.11863
975	10.73	10.99	976.34	0.11792
1000	10.82	10.81	1000.93	0.11696

$$r = \underline{0.549\ \text{k}\Omega}$$
$$\epsilon_{\text{Meas}} = \underline{17.02\ \text{V}}$$

A. *Linear Fit of* V_R *vs.* i

$$\text{Internal Resistance } r = \underline{(571 \pm 2)\Omega}$$
$$\text{Source EMF } \mu_{\text{Theory}} = \underline{(16.99 \pm .04)\text{V}}$$

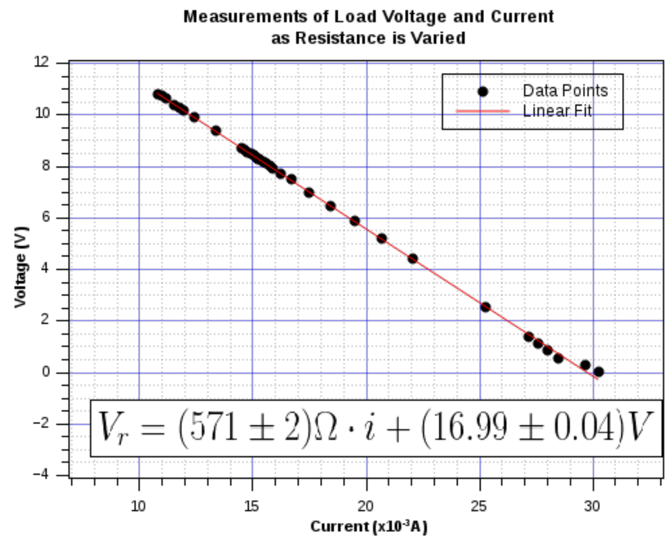

Figure 2. Plot of V_R vs. i for the load resistor R.

$$\%\ \text{Difference in } \epsilon = \underline{0.2\%}$$
$$\%\ \text{Difference in } r = \underline{3.9\%}$$

B. *Polynomial Fit of* P_R *vs.* R_{Calc}

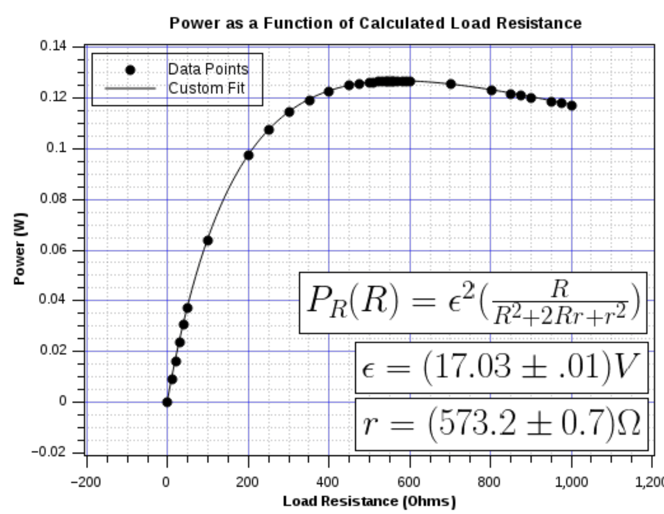

Figure 3. Plot of power (from $P = iV$) vs. Calculated Load Resistance (from $R = \frac{V_R}{i}$), fitted to the expression given in Equation 2.

$$r = \underline{(573.2 \pm 0.7)\Omega}$$
$$\%\ \text{Difference} = \underline{0.38\%}$$

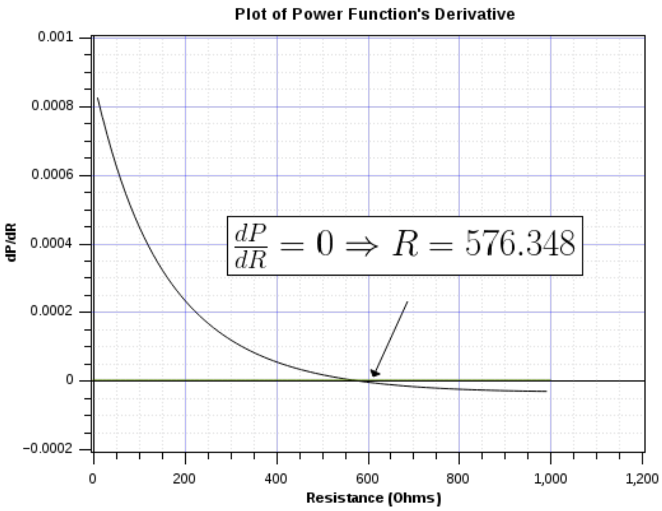

Figure 4. Plot of the derivative of P vs. $R_{\text{Calculated}}$ and its intersection with 0, giving the resistance at which the maximum power is delivered.

IV. Analysis

1) *How does the resistance box value compare with $R_{Calculated}$ as the current in the circuit increases?*
   The deviation between the resistance box value and the calculated resistance remains relatively constant, regardless of the current in the circuit. This represents a source of systematic error in this experiment on the order of approximately 1 Ω. In many cases, the calculated value is within .15 Ω of the box value, which is equal to the amount of resistance that is calculated be in the box when it is set to zero ohms. It is possible that this represents a certain amount of internal resistance in the box's wiring that is added to the value read from the dials.
2) *How can you determine the value for $R_{Calculated}$ that gives $P_{R\text{-}Max}$?*
   Equation 2 shows that the maximum power will be delivered to the resistor R when its resistance is equal to the internal resistance r. Since r was measured to be .549 kΩ, more data points were collected in the 500-600 Ω range.
3) *What are the possible discrepancies between the values of r obtained from the two function fits?*
   The first value is derived from measured quantities, which introduces a certain amount of uncertainty from the measurements and from the curve fit itself. The second value is given by a function that is taken from two derived values, which introduces rounding error. It is also fit to a non-linear function, which tends to be more difficult to model accurately. This in turn produces a slightly different value for r.