MAKEMEAQUAL UNIVERSITY
Department of Mathematics

PhD Qualifying Examination

in

Mathematics

March 11, 2021

0.1 Question 1 (UGA 0 #0)

Let X denote the quotient space formed from the sphere S^2 by identifying two distinct points.
Compute the fundamental group and the homology groups of X.

0.2 Question 2 (UGA 0 #0)

Let L be the union of the z-axis and the unit circle in the xy-plane. Compute $\pi_1(\mathbb{R}^3 \setminus L, *)$.

0.3 Question 3 (UGA 0 #0)

Let C be cylinder. Let I and J be disjoint closed intervals contained in ∂C.
What is the Euler characteristic of the surface S obtained by identifying I and J?
Can all surface with nonempty boundary and with this Euler characteristic be obtained from this construction?

0.4 Question 4 (UGA 0 #0)

Let X be the topological space obtained as the quotient of the sphere $S^2 = \left\{ \mathbf{x} \in \mathbb{R}^3 \;\middle|\; \|\mathbf{x}\| = 1 \right\}$ under the equivalence relation $\mathbf{x} \sim -\mathbf{x}$ for $\mathbf{x}$ in the equatorial circle, i.e. for $\mathbf{x} = (x_1, x_2, 0)$. Calculate $H_*(X; \mathbb{Z})$ from a CW complex description of X.

0.5 Question 5 (UGA 0 #0)

Give a self-contained proof that the zeroth homology $H_0(X)$ is isomorphic to $\mathbb{Z}$ for every path-connected space X.

0.6 Question 6 (UGA 0 #0)

Compute the fundamental group, using any technique you like, of $\mathbb{RP}^2 \# \mathbb{RP}^2 \# \mathbb{RP}^2$.

0.7 Question 7 (UGA 0 #0)

a. Show that any finite index subgroup of a finitely generated free group is free. State clearly any facts you use about the fundamental groups of graphs.

b. Prove that if N is a nontrivial normal subgroup of infinite index in a finitely generated free group F , then N is not finitely generated.

0.8 Question 8 (UGA 0 #0)

Let A and B be circles bounding disjoint disks in the plane $z = 0$ in $\mathbb{R}^3$. Let X be the subset of the upper half-space of $\mathbb{R}^3$ that is the union of the plane $z = 0$ and a (topological) cylinder that intersects the plane in $\partial C = A \cup B$.
Compute $H_*(X)$ using the Mayer–Vietoris sequence.

0.9 Question 9 (UGA 0 #0)

Does there exist a map of degree 2013 from $S^2 \to S^2$.

0.10 Question 10 (UGA 0 #0)

Let M and N be finite CW complexes.

a. Describe a cellular structure of $M \times N$ in terms of the cellular structures of M and N.

b. Show that the Euler characteristic of $M \times N$ is the product of the Euler characteristics of M and N.