

# Problem Set 1

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November 9, 2019

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## 1 Problem 6

### 1.1 Part 1

Let  $M = S^2$  as a smooth manifold, and consider a vector field on  $M$ ,

$$X : M \rightarrow TM$$

We want to show that there is a point  $p \in M$  such that  $X(p) = 0$ .

Every vector field on a compact manifold without boundary is complete, and since  $S^2$  is compact with  $\partial S^2 = \emptyset$ ,  $X$  is necessarily a complete vector field.

Thus every integral curve of  $X$  exists for all time, yielding a well-defined flow

$$\phi : M \times \mathbb{R} \rightarrow M$$

and thus a one-parameter family

$$\phi_t : M \rightarrow M \in \text{Diff}(M, M).$$

In particular,  $\phi_0 = \text{id}_M$ , and  $\phi_1 \in \text{Diff}(M, M)$ . Moreover  $\phi_0$  is homotopic to  $\phi_1$  via the homotopy

$$\begin{aligned} H : M \times I &\rightarrow M \\ (p, t) &\mapsto \phi_t(p). \end{aligned}$$

We can now apply the Lefschetz fixed-point theorem to  $\phi_0$  and  $\phi_1$ . For an arbitrary map  $f : M \rightarrow M$ , we have

$$\Lambda(f) = \sum \text{Tr} \left( f_* \Big|_{H_k(\ )} \right)$$