

# Title

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# 1 | Tuesday, September 15

## 1.1 Setup

- $(M, \omega)$  a symplectic manifold,  $H \in ?$  a Hamiltonian,  $X_H$  its ?
- $\int_{S^2} u^* \omega = \sigma_1$  where  $u \in C^\infty(S^2, W)$ .
- $\langle c_1(TW), \pi_2(TW) \rangle = 0$ ?
- $C_k(H) := \mathbb{Z}/2\mathbb{Z}[S]$  where  $S$  is the set of periodic orbits of  $X_H$  of Maslov index  $k$ .
- $x, y$  critical points of  $\mathcal{A}_H$  with  $\mathcal{M}(x, y)$  the moduli space of contractible solutions of finite energy connecting  $x, y$ .

## 1.2 Review Last Time

- $\mathbb{R} \curvearrowright \mathcal{M}_{x,y}$ , so we quotient to define  $\mathcal{L}(x, y) := \mathcal{M}_{x,y}/\mathbb{R}$  with the quotient topology.
- Topology defined by when sequences converge:

$$\tilde{u}_n \xrightarrow{n \rightarrow \infty} \tilde{u} \iff \exists \{s_n\} \subseteq \mathbb{R} \text{ such that } u_n(s_n + s, \cdot) \xrightarrow{n \rightarrow \infty} u(s, \cdot).$$

**Proposition 1.1(?)**.

$\mathcal{L}(x, y)$  is Hausdorff.

- Want to show  $\mathcal{L}(x, y)$  is a compact 0-dimensional manifold.
- Have a differential

$$\begin{aligned}\partial : C_k(H) &\longrightarrow C_{k-1}(H) \\ \partial(x) &= \sum_{\text{Ind}(y)=k-1} n(x, y)y.\end{aligned}$$

with  $n(x, y)$  the number (mod 2) of trajectories of grad  $\mathcal{A}_H$  connecting  $x, y$ , i.e solutions to the Floer equation.

- Want to prove that the following is a 1-dimensional manifold:

$$M := \bar{\mathcal{L}}(x, z) = \mathcal{L}(x, z) \cup_{\mu(y)=\mu(x)+1} \mathcal{L}(x, y) \times \mathcal{L}(y, z).$$

and show that  $M$  is compact with  $\partial M$  equal to the last union.

- Last time: closure of space of trajectories connecting  $x, y$  contains “broken” trajectories.
- Last time: toward proving that  $M$  is compact

### 1.3 Upcoming

- Wanted to compactify  $\mathcal{L}(x, y)$ , needed to go to space of broken trajectories.
- Main theorem of chapter 9: 9.2.1.

#### **Theorem 1.2(9.2.1).**

Let  $(H, J)$  be a regular pair with  $H$  nondegenerate.

Let  $x, z$  be two periodic trajectories of  $H$  such that  $\mu(x) = \mu(z) + 2$ .

Then  $\bar{\mathcal{L}}(x, y)$  is a compact 1-manifold with boundary satisfying

$$\partial \bar{\mathcal{L}}(x, y) = \bigcup_{\mu(x) < \mu(y) < \mu(z)} \mathcal{L}(x, y) \times \mathcal{L}(y, z).$$

As a corollary,  $\partial^2 = 0$ .

- Know  $\bar{\mathcal{L}}(x, y)$  is compact and  $\mathcal{L}(x, y)$  is a 1-manifold
- Now suffices to study in a neighborhood of boundary points (“gluing theorem”)

Three steps to gluing theorem:

1. Pre-gluing: Get a function  $w_p$  which interpolates between  $u$  and  $v$  (not exactly a solution itself, but will be approximated by one later).
2. Constructing  $\psi$  a “true solution” from  $w_p$  using the Newton-Picard method. We’ll have

$$\psi(p) = \exp_{w_p}(\gamma(p)) \quad \gamma(p) \in W^{1,p}(w_p^* TW) = T_{w_p} \mathcal{P}(x, z).$$

- 3.