Linearization Continued

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# Linearization Continued Section 8.4 Follow-Up

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Linearization Continued

The Floer equation is given by

$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \operatorname{grad} H_t(u) = 0.$$

– We fixed a solution and lifted it to a sphere:

$$u \in C^{\infty}(S^1 \times \mathbb{R}; W) \mapsto \tilde{u} \in C^{\infty}(S^2; W)$$

- We use the assumption: For every  $w \in C^{\infty}(S^2, W)$  there exists a symplectic trivialization of the fiber bundle  $w^*TW$ , i.e.  $\langle c_1(TW), \pi_2(W) \rangle = 0$  where  $c_1$  denotes the first Chern class of the bundle TW.
- We use this trivialize the pullback  $\tilde{u}^*TW$  to obtain an orthonormal unitary frame

$$\{Z_i\}_{i=1}^{2n}\subset T_{u(s,t)}W$$

where

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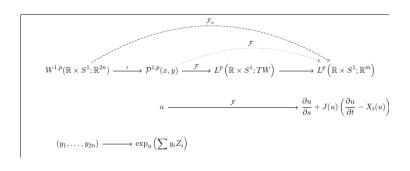
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– We used the chosen frame  $\{Z_i\}$  to define a chart centered at u of  $\mathcal{P}^{1,p}(x,y)$  given by

$$\iota: W^{1,p}\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right) \longrightarrow \mathcal{P}^{1,p}(x,y)$$
$$\mathbf{y} = (y_1, \dots, y_{2n}) \longmapsto \exp_u\left(\sum y_i Z_i\right).$$

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Extract the part that is linear in *Y* and collect terms:

$$(d\mathcal{F})_{u}(Y) = \left(\frac{\partial Y}{\partial s} + J(u)\frac{\partial Y}{\partial t}\right) + \left((dJ)_{u}(Y)\frac{\partial u}{\partial t} - (dJ)_{u}(Y)X_{t} - J(u)(dX_{t})_{u}(Y)\right)$$

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