Math 174, Hw 8

Thursday, 30 November, 2017 10:29 PM

1) • Euler's method:

$$w_i = w_{i-1} + hf(t_{i-1}, w_{i-1})$$
$$w_0 = \alpha$$

• Trapezoid method:

$$w_i = w_{i-1} + \frac{h}{2}\left(f(t_i, w_i) + f(t_{i-1}, w_{i-1})\right)$$

So let

$$\tilde{w}_i = w_{i-1} + h\, f(t_{i-1}, w_{i-1})$$
$$w_i = w_{i-1} + \left(\frac{h}{2}\right)\left(f(t_i, \tilde{w}_i) + f(t_{i-1}, w_{i-1})\right)$$

$$w_0 = y(0) = 1$$

then if $h = 1/2$, $y(1) \approx w_2$ So let $t_i = i \cdot \frac{1}{2} = \{0, \frac{1}{2}, 1, \dots\}$, $f(t,w) = \sin w$

$$\tilde{w}_1 = w_0 + \tfrac{1}{2} f(t_0, w_0) = 1 + (1/2)(\sin 1) \approx 1.420735$$

$f(t_0, w_0) = \sin 1$

$$w_1 = w_0 + (1/4)(f(t_1, \tilde{w}_1) + f(t_0, w_0)) = 1 + (1/4)(\sin(1 + \tfrac{1}{2}\sin 1) + \sin 1) \approx 1.457558242$$

$$\tilde{w}_2 = w_1 + \tfrac{1}{2} f(t_1, w_1) = \underbrace{\tfrac{1}{4}\sin(\sin 1) + \tfrac{1}{4}\sin 1}_{w_1} + \tfrac{1}{2}\underbrace{\sin\left(\tfrac{1}{4}\sin(\sin 1) + \sin 1\right)}_{f(t_1, w_1)} \approx 1.954355$$

$$w_2 = w_1 + \tfrac{1}{4}(f(t_2, \tilde{w}_2) + f(t_1, w_1))$$

$$= 1 + \tfrac{1}{2}\sin 1 + \tfrac{1}{4}\sin\left(\tfrac{1}{4}\sin\sin 1 + \tfrac{1}{4}\sin 1 + \tfrac{1}{2}\sin\left(\tfrac{1}{4}\sin\sin 1 + \sin 1\right)\right) + \tfrac{1}{2}\sin\left(\tfrac{1}{4}\sin\sin 1 + \sin 1\right))$$

$$\boxed{\approx 1.937791701}$$

2) Given $y' = -z$, $y(0) = 1$
$z' = y$, $z(0) = 0$, we have

• Euler's method (for vector-valued functions):

$$\bar{w}_i = \bar{w}_{i-1} + h\mathbf{F}(\bar{w}_{i-1})$$
$$\bar{w}_0 = (y_0, z_0)$$

where

$$\bar{w}_i = \begin{bmatrix} y_i \\ z_i \end{bmatrix}$$

$$\mathbf{F}(\bar{w}_k) = \mathbf{F}\left(\begin{bmatrix} y_k \\ z_k \end{bmatrix}\right) = \begin{bmatrix} -z_k \\ y_k \end{bmatrix}$$

$$\bar{w}_0 = \begin{bmatrix} y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and $h = \frac{1}{2} \rightarrow \bar{X}'(1) \approx \bar{X}_2$

$$\omega_0 = \begin{bmatrix} y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and $h = \frac{1}{4} \to \bar{X}'(1) \approx \bar{X}_4$

So $\bar{\omega}_0 = (1,0)^T$

$$\bar{\omega}_1 = \bar{\omega}_0 + \tfrac{1}{4}F(\bar{\omega}_0)$$
$$= (1,0)^T + \tfrac{1}{4}(-0,1)^T$$
$$= \underline{(1, \tfrac{1}{4})^T}$$

$$\bar{\omega}_2 = \bar{\omega}_1 + \tfrac{1}{4}F(\bar{\omega}_1)$$
$$= (1, \tfrac{1}{4})^T + \tfrac{1}{4}(\tfrac{-1}{4}, 1)$$
$$= \underline{(15/16, 1/2)^T}$$

$$\bar{\omega}_3 = (15/16, 1/2)^T + \tfrac{1}{4}(-1/2, 15/16)$$
$$= (\tfrac{15}{16} - \tfrac{1}{8}, \tfrac{1}{2} + \tfrac{15}{64})^T$$
$$= \underline{(13/16, 47/64)^T}$$

$$\bar{\omega}_4 = (13/16 - \tfrac{1}{4}(47/64), 47/64 + \tfrac{1}{4}(13/16))^T$$
$$= (161/256, 15/16)^T$$
$$\boxed{\approx \begin{bmatrix} .628890625 \\ .9375 \end{bmatrix}}$$

3) $\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ -4 & 0 & 4 & -1 \end{array}\right]$

$-\frac{1}{2}R_1 + R_2 \to R_2$, $R_2 \to 2R_2$
$2R_1 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 0 & 5 & -3 & 3 \\ 0 & -6 & 6 & 1 \end{array}\right]$

$\frac{-2}{5}R_2 + R_3 \to R_3$, $R_3 \to 5R_3$

a) $\boxed{\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 0 & 5 & -3 & 3 \\ 0 & 0 & 12 & 23 \end{array}\right]}$

- $12x_3 = 23 \to x_3 = \underline{23/12}$
- $5x_2 - 3x_3 = 3 \to x_2 = \frac{3}{5}(1 + x_3)$
  $= \frac{3}{5}(\frac{35}{12})$
  $= \underline{7/4}$
- $2x_1 - 3x_2 + x_3 = 1 \to x_1 = \frac{1}{2}(1 - x_3 + 3x_2)$
  $= \frac{1}{2}(1 - 23/12 + 3(7/4))$
  $= \frac{1}{2}(13/3)$
  $= 13/6$

b) $\boxed{\rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13/6 \\ 7/4 \\ 23/12 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 26 \\ 21 \\ 23 \end{bmatrix} \approx \begin{bmatrix} 2.1\overline{6} \\ 1.75 \\ 1.91\overline{6} \end{bmatrix}}$

$M(n)$ = " " mults/divs " " "

4) Let $A(n)$ = total # adds/subs to calculate x_n. And U be $n \times n$.

Then we want to find $A = \sum_{i=1}^{n} A(n)$ For (a)

$M = \sum_{i=1}^{n} M(n)$ For (b)

We have

- $x_n = b_n / u_{n,n}$ so $A(n) = 0$, $M(n) = 1$
- $x_{n-1} = \left(b_{n-1} - \sum_{j=n}^{n} u_{n-1,j}\, x_j\right) / u_{n-1,n-1}$

$= (b_{n-1} - u_{n-1,n} \cdot x_n) / u_{n-1,n-1}$

so $A(n-1) = 1$
$M(n-1) = 2$

In general, for $2 \le i \le n-1$, we have these contributions

$$x_{n-i} = \overbrace{(b_{n-i} - }^{1\text{ subtraction}} \underbrace{\sum_{j=n-(i-1)}^{n} U_{ij}\, x_j}_{i\text{ terms}} \overbrace{) / u_{n-i,n-i}}^{1\text{ division}}$$

$\rightarrow$ i multiplications,
$\rightarrow$ $\underline{i-1}$ additions

So $A(n-i) = 1 + (i-1) = i$
$M(n-i) = i+1$

$$\rightarrow A = \sum_{j=1}^{n} A(j) = \sum_{i=0}^{n-1} A(n-i)$$

$$= \sum_{i=0}^{n-1} i$$

$$= (n-1)((n-1)+1)/2$$

a) $\boxed{\rightarrow A = \frac{1}{2}(n^2 - n)}$

$$\rightarrow M = \sum_{j=1}^{n} M(j) = \sum_{i=0}^{n-1} M(n-i)$$

$$= \sum_{i=0}^{n-1} i+1$$

$$= \sum_{k=1}^{n} k$$

$$= n(n+1)/2$$

b) $\boxed{\rightarrow M = \frac{1}{2}(n^2+n)}$

5) Let A be $n \times n$ & symmetric, so $(A)_{ij} = (A)_{ji}$.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & & \\ a_{12} & a_{22} & a_{23} & & \\ a_{13} & a_{23} & \ddots & & \\ \vdots & \vdots & & & \\ & & & & a_{nn} \end{bmatrix}$$

$-\frac{a_{12}}{a_{11}} R_1 + R_2 \to R_2$, let $c_2 = a_{12}/a_{11}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & c_2 a_{12} + a_{22} & c_2 a_{13} + a_{23} & \cdots & c_2 a_{1n} + a_{2n} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & a_{nn} \end{bmatrix}$$

$-\frac{a_{13}}{a_{11}} R_2 + R_3 \to R_3$, so let $c_3 = \frac{a_{13}}{a_{11}}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & c_2 a_{12} + a_{22} & c_2 a_{13} + a_{23} & \cdots & c_2 a_{1n} + a_{2n} \\ 0 & c_3 a_{12} + a_{23} & c_3 a_{13} + a_{33} & \cdots & c_3 a_{1n} + a_{3n} \\ a_{14} & \vdots & \vdots & \ddots & \vdots \\ \vdots & & & & \\ a_{1n} & a_{2n} & a_{3n} & \cdots & a_{nn} \end{bmatrix}$$

Continuing this way, letting $c_i = a_{1i}/a_{11}$, we obtain

$$B := \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & c_2 a_{12} + a_{22} & c_2 a_{13} + a_{23} & \cdots & c_2 a_{1n} + a_{2n} \\ 0 & c_3 a_{12} + a_{23} & c_3 a_{13} + a_{33} & \cdots & c_3 a_{1n} + a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & c_{n-1} a_{12} + a_{2n} & c_{n-1} a_{13} + a_{3n} & \cdots & c_{n-1} a_{1n} + a_{nn} \end{bmatrix}$$

And so $B(2{:}n, 2{:}n)$ is given by

$$D := \begin{bmatrix} c_2 a_{12} + a_{22} & c_2 a_{13} + a_{23} & \cdots & c_2 a_{1n} + a_{2} \\ c_3 a_{12} + a_{23} & c_3 a_{13} + a_{33} & \cdots & c_3 a_{1n} + a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n-1} a_{12} + a_{2n} & c_{n-1} a_{13} + a_{3n} & \cdots & c_{n-1} a_{1n} + a_{nn} \end{bmatrix}$$

$c_1 a_{13} + a_{23} + a$

So $(D)_{ij} = c_{i+1} a_{1,j+1} + a_{i+1,j+1}$

$= \left(\frac{a_{1,i+1}}{a_{11}}\right) a_{1,j+1} + a_{i+1,j+1}$

regroup

$= a_{1,i+1} \left(\frac{a_{1,j+1}}{a_{11}}\right) + \underline{a_{j+1,i+1}}$ ← since A is symmetric

$= a_{1,i+1} \underline{c_j} + a_{j+1,i+1}$

$= c_j a_{1,i+1} + a_{j+1,i+1}$

$= (D)_{ji}$, proving D is symmetric. ■

6)

a) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, note $A^2 = I$ so $A^{-1} = A$

But any $LU = \begin{bmatrix} l_1 & 0 \\ l_2 & l_3 \end{bmatrix} \begin{bmatrix} u_1 & u_2 \\ 0 & u_3 \end{bmatrix}$

$= \begin{bmatrix} l_1 u_1 & l_1 u_2 \\ l_2 u_1 & l_2 u_2 + l_3 u_3 \end{bmatrix} = A$, then

- $l_1 u_1 = 0 \rightarrow l_1 = 0 \lor u_1 = 0$
  - $\hookrightarrow u_1 = 0 \rightarrow l_2 u_1 = 0 \neq 1$, so $LU \neq A$
  - $\hookrightarrow l_1 = 0 \rightarrow l_1 u_2 = 0 \neq 1$, so $LU \neq A$.

So $A \neq LU$ for any L, U.

$L_{ii} = 1$, then...

b) $\begin{bmatrix} -2 & 0 & 1 & -1 \\ -1 & 2 & 0 & 1 \\ 4 & -1 & -2 & -4 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

$R_2 + \frac{-1}{2} R_1 \rightarrow R_2$, $\underline{L_{21} = \frac{1}{2}}$

$R_3 + 2R_1 \rightarrow R_3$, $\underline{L_{31} = -2}$

$\begin{bmatrix} -2 & 0 & 1 & -1 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 0 & -6 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

$R_3 + \frac{1}{2} R_2 \rightarrow R_3$, $L_{32} = \frac{-1}{2}$

$\begin{bmatrix} -2 & 0 & 1 & -1 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -1/4 & -21/4 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

$R_4 + 8R_3 \rightarrow R_4$, $L_{43} = -8$

b) $$\boxed{A = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ -2 & -1/2 & 1 & 0 \\ 0 & 0 & -8 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} -2 & 0 & 1 & -1 \\ 0 & 2 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{5}{4} & -\frac{21}{4} \\ 0 & 0 & 0 & -42 \end{bmatrix}}_{U}}$$

c) $Ax = b \rightarrow LUx = b$

1) Let $y = Ux$, solve

$Ly = b$

for y.

$$Ly = b \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ -2 & -1/2 & 1 & 0 \\ 0 & 0 & -8 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$\rightarrow$ • $1y_1 + 0 + 0 + 0 = 0 \rightarrow \underline{y_1 = 0}$

• $\frac{1}{2}\cancel{y_1} + y_2 = 1 \rightarrow y_2 = 1$

• $-2\cancel{y_1} - \frac{1}{2}y_2 + y_3 = 2$

$\rightarrow y_3 = 2 + \frac{1}{2}y_2 \rightarrow \underline{y_3 = 5/2}$

• $-8y_3 + y_4 = 3$

$\rightarrow y_4 = 3 + 8y_3 = 3 + 8 \cdot \frac{5}{2}$

$\rightarrow \underline{y_3 = 23}$

$$\text{So } \bar{y} = \begin{bmatrix} 0 \\ 1 \\ 5/2 \\ 23 \end{bmatrix}$$

2) Solve $Ux = y$

$$\rightarrow \begin{bmatrix} -2 & 0 & 1 & -1 \\ 0 & 2 & -1/2 & 3/2 \\ 0 & 0 & -1/4 & -21/4 \\ 0 & 0 & 0 & -42 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 5/2 \\ 23 \end{bmatrix}$$

• $-42x_4 = 23 \rightarrow \underline{x_4 = -23/42}$

• $-\frac{1}{4}x_3 - \frac{21}{4}x_4 = 5/2$

$\rightarrow x_3 = -4\left(\frac{5}{2} + \frac{21}{4} \cdot \frac{-23}{42}\right)$

$\rightarrow \underline{x_3 = 3/2}$

• $2x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 = 1$

$\rightarrow x_2 = \frac{1}{2}\left(1 + \frac{1}{2}x_3 - \frac{3}{2}x_4\right)$

$= \frac{1}{2}\left(1 + \frac{1}{2}\,\frac{3}{2} - \frac{3}{2}\,\frac{-23}{42}\right)$

$\rightarrow \underline{x_2 = 9/7}$

• $-2x_1 + x_3 - x_4 = 0$

$\rightarrow x_1 = \frac{-1}{2}(x_4 - x_3)$

$= \frac{-1}{2}\left(\frac{-23}{42} - \frac{3}{2}\right)$

$\underline{= 43/42}$

$$\rightarrow \bar{x} = \begin{bmatrix} 43/42 \\ 9/7 \\ 3/2 \\ -23/42 \end{bmatrix}$$

7) $i=1 \rightarrow \begin{bmatrix} a_{11} & a_{12} & & & & \bigcirc \\ a_{21} & a_{22} & a_{23} & & & \\ & a_{32} & a_{33} & a_{34} & & \\ & & a_{43} & a_{44} & \ddots & \\ \bigcirc & & & a_{54} & \ddots & \\ & & & & \ddots & \end{bmatrix}$

$R_2 + \frac{-a_{21}}{a_{11}} R_1 \rightarrow R_2$ 1 div, 2 mult, 2 add

$i=2 \rightarrow \begin{bmatrix} a_{11} & a_{12} & & & & \bigcirc \\ 0 & a'_{22} & a'_{23} & & & \\ & a_{32} & a_{33} & a_{34} & & \\ & & a_{43} & a_{44} & \ddots & \\ \bigcirc & & & a_{54} & \ddots & \\ & & & & \ddots & \end{bmatrix}$

$R_3 + {}^{-a_{32}}/_{a'_{22}} R_2 \rightarrow R_3$ 1 div, 2 mult, 2 add

$i \rightarrow \begin{bmatrix} a_{11} & a_{12} & & & & \bigcirc \\ 0 & a'_{22} & a'_{23} & & & \\ & \ddots & \ddots & \ddots & & \\ & & a_{i\,i-1} & a_{ii} & a_{i\,i+1} & \\ \bigcirc & & 0 & a_{i+1\,i} & a_{i+1\,i+1} & a_{i+1\,i+2} \\ & & & \ddots & \ddots & \ddots \end{bmatrix} \longrightarrow$ $i=n-1 \rightarrow \begin{bmatrix} a_{11} & a_{12} & & & \\ 0 & a_{22} & a_{23} & & \\ & 0 & \ddots & \ddots & \\ \bigcirc & & \ddots & a_{n-1\,n-1} & a_{n-1\,n} \\ & & & a_{n\,n-1} & a_{nn} \end{bmatrix} \longrightarrow \begin{bmatrix} a_{11} & a_{12} & & & \\ 0 & a_{22} & a_{23} & & \\ & 0 & \ddots & \ddots & \\ & & \ddots & a_{n-1\,n-1} & a_{n-1\,n} \\ & & & 0 & a_{nn} \end{bmatrix}$, upper triangular.

$\vdots$

$R_{i+1} + \left(\frac{-c_i}{d_i}\right) R_i \rightarrow R_i$, 1 div, 2 mult, 2 add

$R_n + \left(\frac{-a_{n\,n-1}}{a_{n-1\,n-1}}\right) R_{n-1} \rightarrow R_n$

1 div, 2 mult, 2 add

So exactly $n-1$ row operations are done, each

with 3 mult/divs & 2 add/subs

a) This yields $\boxed{\begin{matrix} 3n-3 \text{ mult/divs} \\ 2n-2 \text{ add/subs} \end{matrix}}$

From the above process, we also find that

$$(L)_{ij} = \begin{cases} \frac{a_{ij}}{a_{jj}}, & i=j+1 \\ 1, & i=j \\ 0, & \text{else} \end{cases} = \begin{bmatrix} 1 & 0 & & & \bigcirc \\ d_{21} & 1 & 0 & & \\ & d_{32} & 1 & 0 & \\ & & d_{43} & 1 & \ddots \\ \bigcirc & & & \ddots & 0 \\ & & & d_{n,n-1} & 1 \end{bmatrix} \quad d_{ij} = \frac{a_{ij}}{a_{jj}}$$

and $U = \begin{bmatrix} c_1 & c_2 & & & \bigcirc \\ 0 & c_3 & c_4 & & \\ & 0 & c_5 & c_6 & \\ & & 0 & \ddots & \ddots \\ \bigcirc & & & \ddots & c_{2n-1} \\ & & & 0 & c_{2n} \end{bmatrix}$ for some constants c_i depending on the a_{ij}.

Then solving $L\bar{y}=b$, we have

Then solving $L\bar{y}=b$, we have

- $1y_1 = b_1 \rightarrow \underline{y_1 = b_1}$ (no ops)
- $d_{21} y_1 + 1y_2 = b_2 \rightarrow y_2 = b_2 - d_{21} \cdot y_1$ (1 add, 1 mult)
- $d_{32} y_2 + 1y_3 = b_3 \rightarrow y_3 = b_3 - d_{32} y_2$ (" ")

  $\vdots$
- $d_{n\,n-1} \cdot y_{n-1} + 1y_n = b_n \rightarrow y_n = b_n - d_{n\,n-1} y_{n-1}$ (" ")

$$\boxed{\begin{array}{l} \rightarrow \text{Total: } n-1 \text{ adds/subs} \\ \qquad\qquad n-1 \text{ mults/divs} \end{array}}$$

Now solving $Ux = y$, we have

- $C_{2n} x_n = y_n \rightarrow x_n = y_n / C_{2n}$ (1 div)
- $C_{2n-2} x_{n-1} + C_{2n-1} x_n = y_n \rightarrow x_{n-1} = \dfrac{y_n - C_{2n-1} \cdot x_n}{C_{2n-2}}$ (1 add, 1 mult, 1 div)

  $\vdots$
- $C_1 x_1 + C_2 x_2 = y_2 \rightarrow x_1 = \dfrac{y_2 - C_2 x_2}{C_1}$ (" " ")

So we have $\left.\begin{array}{l} (n-1)+1 \text{ divs} \\ n-1 \text{ mults} \\ n-1 \text{ adds} \end{array}\right]$

$$\boxed{\begin{array}{l} \text{Total: } 2n-1 \text{ mults/divs} \\ \qquad\quad n-1 \text{ adds/subs} \end{array}}$$

8) See attached.

Editor - /home/zack/Downloads/LU.m

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```matlab
function y = LU(n, A)

L = eye(size(A));

for j = 1:n
    % Working on the jth column
    for i = j+1:n
        % Use the leading entry in row j to eliminate
        % the leading entry of all later rows.

        % Store the multipliers separately, so they aren't
        % affected by in-place row operations.
        L(i, j) = A(i, j) / A(j,j);

        % Perform the row operation in place.
        A(i, :) = A(i,:) - L(i,j) * A(j, :);
    end
end

% Now A is U, and zero below the diagonal, so we can
% mix in L by just adding everything below L's diagonal.
y= A + tril(L, -1);
end
```

```
>> LU(10, rand(10))

ans =

    0.1622    0.4505    0.1067    0.4314    0.8530    0.4173    0.7803    0.2348    0.5470    0.9294
    4.8975   -2.1227    0.4396   -1.2022   -3.5556   -1.9939   -3.4315   -0.7967   -2.3826   -3.7759
    1.9189    0.2994   -0.3316   -0.2860   -0.2213    0.6990   -0.2281    0.6092    0.4084   -0.1660
    3.2589    0.2614   -0.9421   -1.0973   -1.5456    0.7648   -1.4566    0.0325   -0.5860   -1.7622
    1.0214    0.1450   -1.9438    0.6168    0.5692    1.2409    0.2521    1.0829    1.6289    0.8092
    3.7118    0.3988   -0.8972    1.1323   -0.2123   -0.2398    0.1025    0.3549    0.4791    0.3806
    1.6215    0.0905    0.3869   -0.3548   -2.2558  -11.0184    1.2570    6.4698    8.2844    4.8009
    4.0330    0.3867    0.6041    0.4761   -1.8838   -6.4210    0.8757   -1.6373   -1.7369   -1.0742
    4.2496    0.8652    1.7298   -0.2291   -0.5907   -0.0837    0.3553    1.4269    0.2136    0.3553
    4.6130    0.7706    0.0924    0.8132    0.5660    6.9492   -0.4332    0.2306   -2.0100    0.8047

>>
```

```
>> LU(20, rand(20))

ans =

  Columns 1 through 15

    0.6312    0.2238    0.2751    0.1379    0.4116    0.7829    0.1079    0.5013    0.8352    0.2815    0.5008    0.4035    0.3037    0.5039    0.9437
    0.5625    0.2477    0.0939    0.1402    0.3711    0.2534    0.1215    0.1497   -0.1474    0.0720    0.2062   -0.1050   -0.1246    0.3633    0.0183
    1.5798   -1.0738    0.1180    0.1150    0.4889   -0.9547    0.0592    0.3665   -0.9253    0.3438    0.3027   -0.4816   -0.4180   -0.0990   -0.7427
    0.3552    2.2836   -0.6994   -0.2442   -0.0537   -0.8761    0.2177    0.5510    0.3890    0.6020   -0.0808    0.0153    0.6021   -0.9312   -0.3192
    1.0337   -0.2047    4.5719    2.1805   -1.9613    6.0040   -0.6387   -2.9790    3.0816   -2.5703   -1.2369    2.0616    0.9806    2.5079    3.1455
    0.9585   -0.6841    6.6887    0.7634    1.4495   -1.6260    1.2406    1.8246    0.8512    1.4130    0.4575   -0.0945    1.4081   -2.1424    0.1915
    0.6135    2.3855   -3.0590   -3.5832   -0.3645    2.5778   -2.7242   -3.1817   -2.1238   -1.6880   -1.7484   -0.0745   -1.7027    2.4197   -2.7385
    0.2253    1.1982    3.0611    0.8855    0.8896    1.5810    0.6652    0.6099   -0.0850    0.0770    1.4132   -0.2285   -1.1165    0.5794    1.2266
    0.0398    2.6312   -1.4489   -0.8301   -0.2085    0.4295    0.0606    0.5189    0.5253    0.1719   -1.0811    1.0444    1.4128   -0.2318   -0.9577
    0.6672    0.9471    4.4899   -0.9967    0.9806    1.4194    0.2194    0.4532    1.5181   -0.3742    0.6440   -1.2546   -1.0327   -0.8948    1.4491
    0.2917    2.2694    5.8987    0.3727    1.8000    3.3842    1.2752    1.5169    0.6257    0.1019   -0.4090    0.3887   -0.7226   -1.0167    0.7962
    1.1499   -0.9514   -1.3564   -3.3088   -0.6883   -0.1304   -0.2544   -0.0812    2.3872   -1.7340   -7.3106   -1.1367   -8.5830   -9.2126    9.8392
    0.5868    3.1462    4.0910    0.0170    1.5819    3.7910    1.4849    0.7065   -0.2985    0.9287    2.2475   -0.0470    1.9248    2.0454   -2.6310
    1.3333    2.0276   -4.5606   -3.3738   -0.8582    1.5800    0.6183    0.1800   -1.5898   -1.7732    2.2122    1.3821    7.5046   -2.4464    4.1417
    1.1632    1.9804    1.5242    1.1214    0.7402    1.8539    0.7397   -1.3553   -2.4085    1.3270    1.7914   -2.7070  -10.4243   -0.3358   -2.8058
    0.9047    2.4855    2.5719    0.4250    1.0881    3.0054    1.0165   -0.6327   -2.3865    0.0882    4.0801   -0.7656   -1.6458   -0.4908   -0.4495
    0.2802    1.2944    2.8443    1.2408    0.8822    1.2992    0.3837   -0.0389    0.1367    1.8921    2.2817   -1.3833   -4.9863   -1.1233   -0.6255
```

```
>> X = LU(100, rand(100))

X =

   1.0e+04 *

  Columns 1 through 15

    0.0000    0.0000    0.0001    0.0001    0.0000    0.0000    0.0001    0.0001    0.0000    0.0000    0.0000    0.0001    0.0000    0.0001    0.0000
    0.0001    0.0001    0.0000   -0.0000    0.0001    0.0001   -0.0001   -0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000    0.0000
    0.0002    0.0000   -0.0001   -0.0001    0.0000    0.0000   -0.0001   -0.0001    0.0001   -0.0001    0.0000   -0.0000    0.0000   -0.0001    0.0000
    0.0004   -0.0001    0.0003   -0.0002   -0.0000   -0.0001    0.0000   -0.0001   -0.0002    0.0001   -0.0002   -0.0003   -0.0002    0.0000   -0.0000
    0.0002   -0.0001    0.0001    0.0001    0.0000    0.0000   -0.0000    0.0000    0.0001    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0002    0.0001    0.0001    0.0000   -0.0062    0.0028   -0.0006    0.0025    0.0056    0.0024    0.0000    0.0016    0.0009    0.0021    0.0025
    0.0001    0.0001    0.0001   -0.0000   -0.0022    0.0000    0.0001    0.0001    0.0001    0.0001   -0.0000   -0.0001    0.0000    0.0001    0.0001
    0.0005   -0.0000    0.0003    0.0002    0.0018   -0.0000   -0.0001    0.0001    0.0002   -0.0000    0.0001    0.0001    0.0001    0.0000    0.0001
    0.0002    0.0001    0.0002    0.0000   -0.0081    0.0001    0.0001    0.0001   -0.0002    0.0001    0.0001    0.0000   -0.0000   -0.0001   -0.0000
    0.0004   -0.0001    0.0002    0.0001    0.0012   -0.0000   -0.0001    0.0001    0.0000    0.0001   -0.0001   -0.0001   -0.0001   -0.0001    0.0000
    0.0003    0.0001    0.0002    0.0000   -0.0136    0.0002    0.0000    0.0000    0.0000   -0.0001    0.0000   -0.0001    0.0001   -0.0002    0.0000
    0.0002    0.0000    0.0001    0.0000    0.0016   -0.0000    0.0000   -0.0000   -0.0000   -0.0002   -0.0005   -0.0008    0.0003   -0.0013    0.0000
    0.0004   -0.0000    0.0003    0.0001   -0.0041    0.0001   -0.0000   -0.0000   -0.0001    0.0001    0.0008   -0.0001   -0.0002   -0.0003   -0.0000
    0.0004   -0.0000    0.0002    0.0001   -0.0034    0.0001   -0.0001    0.0000   -0.0001    0.0003    0.0017   -0.0003    0.0001   -0.0001   -0.0000
```

```
>> X = LU(200, rand(200))

X =

   1.0e+03 *

  Columns 1 through 15

    0.0008    0.0006    0.0001    0.0003    0.0004    0.0008    0.0002    0.0006    0.0001    0.0006    0.0002    0.0003    0.0010    0.0009    0.0007
    0.0010   -0.0005    0.0009    0.0001   -0.0001   -0.0001    0.0007   -0.0005    0.0003   -0.0003    0.0006    0.0006   -0.0008   -0.0006   -0.0004
    0.0011    0.0005    0.0000    0.0004    0.0000   -0.0001    0.0003    0.0001    0.0000   -0.0004   -0.0002   -0.0002    0.0002   -0.0004   -0.0004
    0.0001    0.0001    0.0100   -0.0039    0.0004    0.0017   -0.0022   -0.0009    0.0000    0.0050    0.0024    0.0020   -0.0015    0.0040    0.0041
    0.0008    0.0062    0.0097    0.0011   -0.0004   -0.0010   -0.0006   -0.0006    0.0005   -0.0013    0.0001   -0.0004   -0.0006   -0.0012   -0.0012
    0.0006   -0.0006    0.0655    0.0073    0.0122    0.0057    0.0058    0.0053   -0.0077    0.0058   -0.0018    0.0002    0.0005    0.0113    0.0106
    0.0004   -0.0007    0.0494    0.0053    0.0185    0.0010   -0.0003   -0.0004    0.0016    0.0012    0.0010    0.0008    0.0001   -0.0005   -0.0007
    0.0007   -0.0011    0.0438    0.0047    0.0093    0.0008   -0.0028   -0.0014    0.0047    0.0049    0.0040    0.0038   -0.0007   -0.0012   -0.0016
    0.0004   -0.0004    0.0447    0.0048    0.0081    0.0006   -0.0006   -0.0000    0.0002    0.0023    0.0007    0.0006   -0.0001    0.0006    0.0004
    0.0005   -0.0012    0.0717    0.0077    0.0152    0.0015    0.0014   -0.0001    0.0018   -0.0028   -0.0005   -0.0005   -0.0004   -0.0023   -0.0006
    0.0003   -0.0007    0.0363    0.0038    0.0072    0.0006   -0.0009    0.0002    0.0048    0.0032   -0.0008   -0.0008    0.0014    0.0045    0.0009
    0.0001    0.0001    0.0328    0.0034    0.0047    0.0003    0.0006   -0.0004    0.0013    0.0003   -0.0008    0.0001    0.0009    0.0052    0.0016
    0.0002   -0.0008    0.0577    0.0063    0.0121    0.0011   -0.0019    0.0007    0.0003    0.0001    0.0010    0.0040   -0.0041   -0.0304   -0.0091
```