Title

D. Zack Garza

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1.1 1.a

Proof .
$A \implies B$:

- Suppose $\{a_n\}$ is not bounded above.
- Then any $k \in \mathbb{N}$ is not an upper bound for $\{a_n\}$.
- So choose a subsequence $a_{n_k} > k$, then by order-limit laws,

$$a_{n_k} > k \implies \liminf_{k \longrightarrow \infty} a_{n_k} > \liminf_{k \longrightarrow \infty} k = \infty.$$

> Note that $\lim_{n \longrightarrow \infty} a_n$ need not exist, but liminf/limsup always exist.

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Proof .
$\not A \implies \not B$:

- Suppose $\{a_n\}$ is bounded by M, so $a_n < M$ for all $n \in \mathbb{N}$.
- Then if $\{a_{n_k}\}$ is a subsequence, we have $a_{n_k} \in \{a_n\}$, so $a_{n_k} < M$ for all $k \in \mathbb{N}$.
- But then

$$a_{n_k} < M \implies \limsup_{k \longrightarrow \infty} a_{n_k} \leq M,$$

- Now just note that if $\lim_{k \longrightarrow \infty} a_{n_k}$ exists,

$$\lim_{k \longrightarrow \infty} a_{n_k} \limsup_{k \longrightarrow \infty} a_{n_k} \leq M,$$

so no subsequence can converge to infinity.

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1.2 3.a

Proof (Using definition (i)).

- Suppose $|x_n| \leq M$ for every n.
- Let $\{x_{n_k}\}$ be an arbitrary subsequence, then since $x_{n_k} \in \{x_n\}$ for all k, $|x_{n_k}| \leq M$ for all k.
- By order-limit laws,

$$|x_{n_k}| \leq M \implies \inf_k |x_{n_k}| \leq M.$$

since the

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