

# Problem Set 8

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## 1 Problem 1

### 1.1 Part a

Define a map

$$\begin{aligned}\phi_{\text{ev}} : \text{hom}_{\mathbb{Z}}(\mathbb{Z}_m, A) &\rightarrow A \\ (f : \mathbb{Z}_m \rightarrow A) &\mapsto f(1)\end{aligned}$$

Then noting that  $\phi_{\text{ev}}$  is a homomorphism, forcing  $f(\bar{0}) = 0_A$  (where  $\bar{0} : \mathbb{Z}_m \rightarrow A$  is the zero map), we must have

$$0 = f(0) = f(m) = mf(1),$$

we must have  $mf(1) = 0$  in  $A$ . So

$$\text{im } \phi_{\text{ev}} = \{a \in A \mid ma = 0\} := A[m].$$

It is also the case that

$$\ker \phi_{\text{ev}} = \{f \in \text{hom}_{\mathbb{Z}}(\mathbb{Z}_m, A) \mid f(1) = 0\} = \{\bar{0}\},$$

which follows from the fact that  $\mathbb{Z}_m = \langle 1 \bmod m \rangle$  and  $A = \langle 1_A \rangle$  as  $\mathbb{Z}$ -modules, so if  $f(1 \bmod m) = 0_A$  then

$$f(n \bmod m) = nf(1 \bmod m) = 0$$

and so  $f$  is necessarily the zero map. So  $\ker \phi = \bar{0}$ .

We can then apply the first isomorphism theorem,

$$\frac{\text{hom}_{\mathbb{Z}}(\mathbb{Z}_m, A)}{\ker \phi_{\text{ev}}} \cong \text{im } \phi_{\text{ev}} \implies \text{hom}_{\mathbb{Z}}(\mathbb{Z}_m, A) \cong A[m].$$

## 1.2 Part 2