

Assignment 6: The Fourier Transform

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1 Problem 1

Assuming the hint, we have

$$\lim_{|\xi| \rightarrow \infty} \hat{f}(\xi) = \lim_{\xi' \rightarrow 0} \frac{1}{2} \int_{\mathbb{R}^n} (f(x) - f(x - \xi')) \exp(-2\pi i x \cdot \xi) \, dx$$

But as an immediate consequence, this yields

$$\begin{aligned} |\hat{f}(\xi)| &= \left| \int_{\mathbb{R}^n} (f(x) - f(x - \xi')) \exp(-2\pi i x \cdot \xi) \, dx \right| \\ &\leq \int_{\mathbb{R}^n} |f(x) - f(x - \xi')| |\exp(-2\pi i x \cdot \xi)| \, dx \\ &\leq \int_{\mathbb{R}^n} |f(x) - f(x - \xi')| \, dx \\ &\rightarrow 0, \end{aligned}$$

which follows from continuity in L^1 since $f(x - \xi') \rightarrow f(x)$ as $\xi' \rightarrow 0$.

2 Problem 2

3 Problem 3

4 Problem 4

5 Problem 5

6 Problem 6