Algebraic Topology 2: Smooth Manifolds

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1 Lecture 1

The key point of this class will be a discussion of *smooth structures*. As you may recall, a sensational result of Milnor's exhibited exotic spheres with smooth structures – i.e., a differentiable manifold M which is homeomorphic but *not* diffeomorphic to a sphere.

> Summary of this result: Look at bundles $S^3 \to X \to S^4$, then one can construct some $X \cong S^7 \in \mathbf{Top}$ but $X \ncong S^7 \in \mathbf{Diff}^{\infty}$. There are in fact 7 distinct choices for X.

> It is not known if there are exotic smooth structures on S^4. The Smooth Poincare' conjecture is that these do not exist; this is believed to be false.

The other key point of this course is to show that $X \in \mathbf{Diff}^{\infty} \implies X \hookrightarrow \mathbb{R}^n$ for some n, and is in fact a topological subspace.

A short list of words/topics we hope to describe: - Differentiable manifolds - Local charts - Submanifolds - Projective spaces - Lie groups - Tangent spaces - Vector fields - Cotangent spaces - Differentials of smooth maps - Differential forms - de Rham's theorem