Discussion Notes

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1 Discussion 1

If X is an F_σ set, then

$$X = \bigcup_{i=1}^{\infty} F_i \quad \text{with each } F_i \text{ closed.}$$

If X is a G_δ set, then

$$X = \bigcap_{i=1}^{\infty} G_i \quad \text{with each } G_i \text{ open.}$$

A set A is *nowhere dense* iff $(\overline{A})^\circ = \emptyset$ iff for any interval I, there exists a subinterval S such that $S \bigcap A = \emptyset$. This is a set that is not dense in any nonempty open set. If the closure of a subset of $\mathbb{R}$ contains no open intervals, it will be nowhere dense.

A set A is *meager* or *first category* if it can be written as

$$A = \bigcup_{i \in \mathbb{N}} A_i \quad \text{with each } A_i \text{ nowhere dense}$$

A set A is *null* if for any ε, there exists a cover of A by countably many intervals of total length less than ε, i.e. there exists $\{I_k\}_{j\in\mathbb{N}}$ such that $A \subseteq \bigcup_{j\in\mathbb{N}} I_j$ and $\sum_{j\in\mathbb{N}} \mu(I_j) < \varepsilon$. If A is null, we say $\mu(A) = 0$.

Some facts:

- If $f_n \to f$ and each f_n is continuous, then D_f is meager.
- If $f \in \mathcal{R}(a, b)$ and f is bounded, then D_f is null.
- If f is monotone, then D_f is countable.
- If f is monotone and differentiable on (a, b), then D_f is null.

We define the *oscillation of f* as

$$\omega_f(x) \coloneqq \lim_{\delta \to 0^+} \sup_{y,z \in B_\delta(x)} |f(y) - f(z)|$$

1.1 Uniform Convergence

We say that $f_n \to f$ *converges uniformly on A* if $\|f_n - f\|_\infty = \sup_{x \in A} |f_n(x) - f(x)| \to 0$.