Homework 6

D. Zack Garza

October 23, 2019

Contents

1 Homework Problems **1**

1.1 Problem 1 . . . 1

1.2 Problem 2 . . . 1

1.3 Problem 3 . . . 2

1.3.1 Part 1 . . . 2

1.4 Problem 4 . . . 2

1.5 Problem 5 . . . 2

1.6 Problem 6 . . . 2

2 Qual Problems **2**

2.1 Problem 1 . . . 2

2.2 Problem 2 . . . 2

2.3 Problem 3 . . . 2

1 Homework Problems

1.1 Problem 1

Todo

1.2 Problem 2

We can note that since f has 4 roots, the Galois group G of its splitting field will be a subgroup of S_4. Moreover, G must be a *transitive subgroup* of S_4, i.e. the action of G on the roots of f should be transitive. This reduces the possibilities to $G \cong S^4, A^4, D^4, \mathbb{Z}_4, \mathbb{Z}_2^2$.

Since f has exactly 2 real roots and thus a pair of roots that are complex conjugates, the automorphism given by complex conjugation is an element of G. But this corresponds to a 2-cycle $\tau = (ab)$, and we can then make the following conclusions:

- Not A_4: A_4 contains only even cycles, and τ is odd.
- Not Z_4: This subgroup is generated by a single 4-cycle σ, which up to conjugacy is (1234), and σ^n is not a 2-cycle for any n.
- Not $\mathbb{Z}_2^2$: In order to be transitive, this subgroup must be $\{e, (12)(34), (13)(24), (14)(23)\}$, which does not contain τ.

The only remaining possibilities are S^4 and D^4. $\square$

1.3 Problem 3

1.3.1 Part 1

To see that $\phi(n)$ is even for all $n > 2$, we can write

$$\phi(n) = \phi(\prod_{=1}^{m} p_i^{k_i} = \prod_{=1}^{m} \phi(p_i^{k_i})$$

1.4 Problem 4

1.5 Problem 5

1.6 Problem 6

2 Qual Problems

2.1 Problem 1

2.2 Problem 2

2.3 Problem 3