Summary/Outline
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8.8.3: $\mathrm{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

Summary/Outline
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8.8.3: Ind(L_1) = k^- − k^+
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8.8.5: dim ker F, F^*
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Summary/Outline

Summary/Outline
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8.8.3: Ind(L_1) = k^- − k^+
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8.8.5: dim ker F, F^*
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Outline

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x) - \mu(y)$.

What we have so far:

- Define

$$L : W^{1,p}\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right) \longrightarrow L^p\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t) Y$$

where

$$S : \mathbb{R} \times S^1 \longrightarrow \mathrm{Mat}(2n; \mathbb{R})$$
$$S(s, t) \overset{s \longrightarrow \pm\infty}{\longrightarrow} S^{\pm}(t).$$

Summary/Outline
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8.8.3: Ind(L_1) = k^- − k^+
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8.8.5: dim ker F, F^*
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Outline

- Took $R^{\pm} : I \longrightarrow \mathrm{Sp}(2n; \mathbb{R})$: symplectic paths associated to $S^{\pm}$
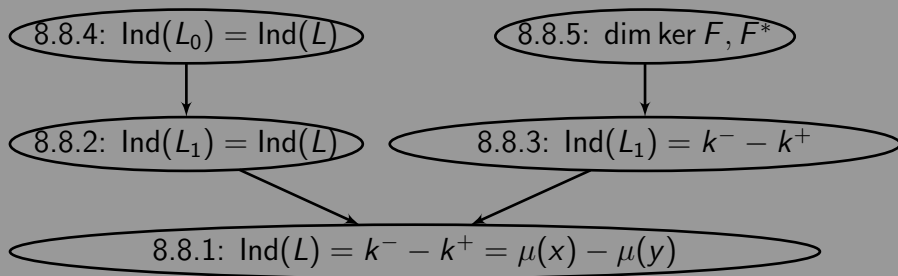
- These paths defined $\mu(x), \mu(y)$

- Section 8.7:

$$R^{\pm} \in \mathcal{S} := \left\{ R(t) \;\middle|\; R(0) = \mathrm{id}, \; \det(R(1) - \mathrm{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\mathrm{Ind}(L) \stackrel{\text{Thm?}}{=} \mu(R^-(t)) - \mu(R^+(t)) = \mu(x) - \mu(y).$$

Summary/Outline
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8.8.3: $\text{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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From Yesterday

- Han proved 8.8.2 and 8.8.4.
  - So we know $\text{Ind}(L) = \text{Ind}(L_1)$
- Today: 8.8.5 and 8.8.3:
  - Computing $\text{Ind}(L_1)$ by computing kernels.

8.8.4: $\text{Ind}(L_0) = \text{Ind}(L)$

8.8.5: dim ker F, F^*

8.8.2: $\text{Ind}(L_1) = \text{Ind}(L)$

8.8.3: $\text{Ind}(L_1) = k^- - k^+$

8.8.1: $\text{Ind}(L) = k^- - k^+ = \mu(x) - \mu(y)$

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8.8.3: $\text{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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$$8.8.3: \ \text{Ind}(L_1) = k^- - k^+$$

Summary/Outline
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8.8.3: $\mathsf{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, $F^\star$
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Setup

Used in the proof of 8.8.3 ($\mathsf{Ind}(L_1) = k^- - k^+$).

$$L : W^{1,p} \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right) \longrightarrow L^p \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t) Y$$

$$L_1 : W^{1,p} \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right) \longrightarrow L^p \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s) Y$$

$$L_1^\star : W^{1,q} \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right) \longrightarrow L^q \left(\mathbb{R} \times S^1; \mathbb{R}^{2n} \right)$$
$$Z \longmapsto -\frac{\partial Z}{\partial s} + J_0 \frac{\partial Z}{\partial t} + S(s)^t Z$$

Summary/Outline
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8.8.3: $\mathsf{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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Setup

Shorthand

$$L = \frac{\partial s}{\partial} + J_0 \frac{\partial t}{\partial +} S(s, t)$$

$$L_1 = \frac{\partial s}{\partial} + J_0 \frac{\partial t}{\partial +} S(s)$$

$$L_1^* = -\frac{\partial s}{\partial} + J_0 \frac{\partial t}{\partial +} S(s)^t$$

.

We have

$$J_0^1 := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \implies J_0 = \begin{bmatrix} J_0^1 & & & \\ & J_0^1 & & \\ & & \ddots & \\ & & & J_0^1 \end{bmatrix} \in \bigoplus_{i=1}^{n} \mathrm{Mat}(2; \mathbb{R}).$$

Summary/Outline
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8.8.3: $\text{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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Summary/Outline
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8.8.3: $\text{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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Summary/Outline
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8.8.3: $\mathsf{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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8.8.3: Ind(L_1) = k^- − k^+
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8.8.5: dim ker F, F^*
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8.8.5: dim ker F, F^*

Summary/Outline
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8.8.3: Ind(L_1) = k^- − k^+
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8.8.5: dim ker F, F^*
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Summary/Outline
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8.8.3: Ind$(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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Summary/Outline
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8.8.3: Ind(L_1) = $k^- - k^+$
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8.8.5: dim ker F, F^*
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Summary/Outline
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8.8.3: $\mathrm{Ind}(L_1) = k^- - k^+$
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8.8.5: dim ker F, F^*
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