Problem Set 1

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1.1 a

On the real plane: a circle of radius 1 centered at $(1, 0)$.

1.2 b

Let $z = x + iy$. Then

$$
\begin{aligned}
|z-1| = 2|z-2| &\iff |z-1|^2 = 4|z-2|^2 \\
&\iff (x-1)^2 - y^2 = 4((x-2)^2 - y^2) \\
&\iff x^2 - \frac{14}{3}x - y^2 = -5 \\
&\iff \left(x - \frac{14}{6}\right) - y^2 = -5 + \left(\frac{14}{6}\right)^2 = \frac{4}{9} \\
&\iff \left(\frac{x - 14/6}{2/3}\right)^2 - \left(\frac{y}{2/3}\right)^2 = 1,
\end{aligned}
$$

which describes a horizontally shifted hyperbola.

1.3 c.

Equivalently, $z\overline{z} = 1 = |z|^2$, so this is the circle $S^1 = \left\{z \in \mathbb{C} \;\middle|\; |z| = 1\right\}$.

1.4 d.

On the real plane: A vertical line passing through $(3, 0)$ and $(3, t)$ for every $t \in \mathbb{R}$.

1.5 e.

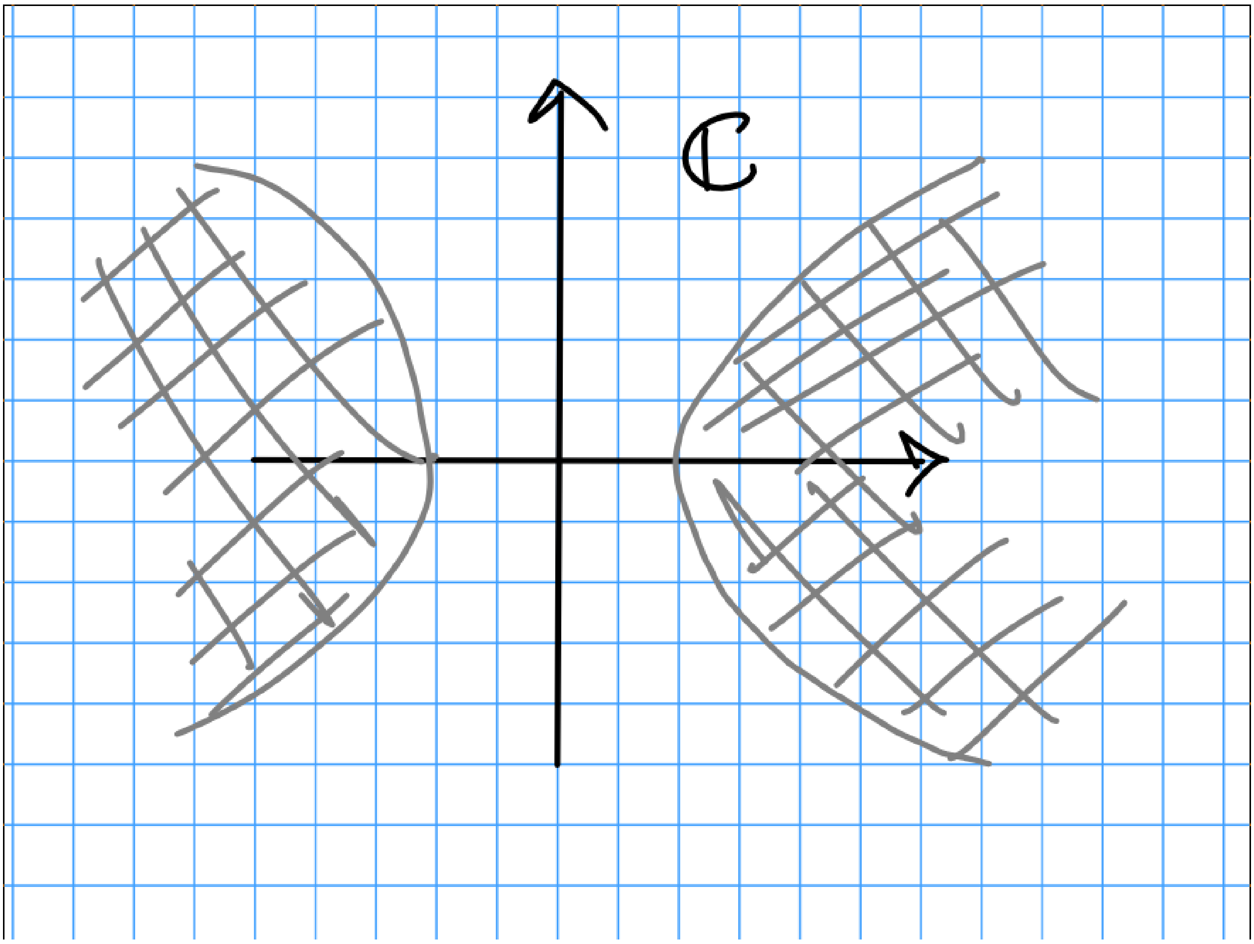
On the real plane: A horizontal line passing through $(0, a)$ and (t, a) for every $t \in \mathbb{R}$.

1.6 f.

On the real plane: A right half-plane $H = \left\{(x, y) \in \mathbb{R}^2 \;\middle|\; x \geq a, y \in \mathbb{R}\right\}$.

1.7 g.

The two regions "inside" the branches of the hyperbola given in b, i.e.



2 2

?

3 3

By part 2, we have

$$|z| \leq 1 \implies |f(z)| = \left|z^3 + 2z + 4\right| \geq |z|^3 + 2|z| + 4 \geq 6,$$

so $f(z) = 0$ is not possible for any z in the unit disk.

4 4

4.1 a

4.2 b

5 5

6 6

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