Title

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1 | Lecture 15: The L-Polynomial

Recall that we had $Z(t) + F(t) + G(t)$:

$$
\begin{aligned}
(q-1)F(t) &= \sum_{0 \leq \deg C \leq 2g-2} q^{\ell(C)} t^{\deg(C)} \\
(q-1)G(t) &= h\left(\frac{q^g t^{2g-1}}{1-qt} - \frac{1}{1-t}\right).
\end{aligned}
$$

Note that $F(t)$ is a polynomial of degree at most $2g-2$, and clearing denominators in $G(t)$ yields a polynomial of degree at most $2g$

Definition 1.0.1 (The L-polynomial)
The L-polynomial is defined as

$$
L(t) := (1-t)(1-qt)Z(t) = (1-t)(1-qt)\sum_{n=0}^{\infty} A_n t^n \in \mathbb{Z}[t].
$$

It turns out that the degree bound of $2g$ is sharp:

Theorem 1.0.2 *(?).*
Let $K/\mathbb{F}_q$ be a function field, then

- $\deg L = 2g$.
- $L(1) = h$
- $L(t) = q^g t^{2g} L\left(\frac{1}{qt}\right)$.
- Writing $L(t) = \sum_{j=1}^{2g} a_j t^j$,
  - $a_0 = 1$ and $a_{2g} = q^g$.
  - For all $0 \leq j \leq g$, we have $a_{2g-j} = q^{g-j} a_j$.
  - $a_1 = |\Sigma(K/\mathbb{F}_q)|$.