Title

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Friday 4th September, 2020

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1 Wednesday, September 02

Recommended exercises:

- 0.9
- 0.5 (easy)
- 0.10

Taken:

- 0.11
- 0.3
- 0.4

Exercise 1.1 (0.5)**.**
Let R_1, R_2 be two k-algebras that are also domains with fraction fields K_i.

Show that $R_1 \otimes_k R_2$ is a domain $\iff$ $K_1 \otimes_k K_2$ is a domain.

Exercise 1.2 (0.9)**.**
Let k be a field and $d \geq 2$ with $4 \nmid d$ and $p \in k[x]$ a polynomial of positive degree.

Factor p in $\bar{k}[x]$ as $\displaystyle\prod_{i=1}^{r}(x-a_i)^{e_i}$, and suppose there is some i such that $d \nmid e_i$. Show that

$$f(x, y) \coloneqq y^d - p(x) \in k[x, y]$$

is geometrically irreducible.

Conclude that

$$ff(k[x, y]/\langle f \rangle).$$

is a regular one-variable function field over k.

Solution:
Recall:

- For L/K,
- A polynomial $f \in k[t_i]$ is *geometrically irreducible* iff $f \in \bar{k}[t_i]$ is irreducible as a polynomial, i.e. if $f = pq \implies p = 1$ or $q = 1$.
- A field extension L/k is *regular* iff any of the following conditions hold:
  - $\kappa(k) = k$ and L/k is separable, where $\kappa(k)$ is the field of elements of L algebraic over k
  - $L \otimes_k \bar{k}$ is a domain or a field.
  - For all L'/k, $L \otimes_k L'$ is a domain.