Algebra

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1 Lecture 1 (Thu 15 Aug 2019)

Definition: A *group* is an ordered pair $(G, \cdot : G \times G \to G)$ where G is a set and $\cdot$ is a binary operation, which satisfies the following axioms:

1. Associativity: $(g_1 g_2) g_3 = g_1 (g_2 g_3)$
2. Identity: $\exists e \in G \ni ge = eg = g$
3. Inverses: $g \in G \implies \exists h \in G \ni gh = gh = e$.

Some examples of groups:

- $(\mathbb{Z}, +)$
- $(\mathbb{Q}, +)$
- $(\mathbb{Q}^\times, \times)$
- $(\mathbb{R}^\times, \times)$
- $(\mathrm{GL}(n, \mathbb{R}), \times) = \{A \in \mathrm{Mat}_n \ni \det(A) \neq 0\}$
- $(S_n, \circ)$

Definition: A subset $S \subseteq G$ is a *subgroup* of G iff

1. $s_1, s_2 \in S \implies s_1 s_2 \in S$
2. $e \in S$
3. $s \in S \implies s^{-1} \in S$

We denote such a subgroup $S \leq G$.

Examples:

- $(\mathbb{Z}, +) \leq (\mathbb{Q}, +)$
- $\mathrm{SL}(n, \mathbb{R}) \leq \mathrm{GL}(n, \mathbb{R})$, where $\mathrm{SL}(n, \mathbb{R}) = \{A \in \mathrm{GL}(n, \mathbb{R}) \ni \det(A) = 1\}$

Definition: A group G is cyclic iff G is generated by a single element.

Exercise: Show $\langle g \rangle = \{g^n \ni n \in \mathbb{Z}\} \cong \bigcap \{H \leq G \ni g \in H\}$