

Problem Set 6

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1 Humphreys 5.3

Let λ be regular, antidominant, and integral, and suppose $M(\lambda)^n \neq 0$ but $M(\lambda)^{n+1} = 0$. In the Jantzen filtration of $M(w \cdot \lambda)$, show that $n = \ell_\lambda(w)$ where ℓ_λ is the length function of the system $(W_{[\lambda]}, \Delta_{[\lambda]})$. Thus there are $\ell(w) + 1$ nonzero layers in this filtration.

Use 0.3(2) to describe $\Phi_{w \cdot \lambda}^+$.

2 Humphreys 7.2

Let $\mathfrak{g} = \mathfrak{sl}(2, \mathbb{C})$ and show that T_λ^μ need not take Verma modules to Verma modules.

For example, let $\lambda = 1$ and $\mu = -3$.

3 Exercise p.108

- Work out the Jantzen filtration sections for $M(w_0 \cdot \lambda)$. List carefully any additional assumptions or facts needed to deduce $M(w_0 \cdot \lambda)^i$ uniquely.
- Continue #4.11 for the case of singular λ , e.g. $(\lambda + \rho, \hat{\alpha}) = 1$. If you didn't deduce the structure of all $M(w \cdot \lambda)$ there, can you complete it now?
- Work out the non-integral case. (There are several different cases to consider.)