Title

D. Zack Garza

January 15, 2020

Contents

1 Fall 2018

1.1 1

Let $f(x) = \frac{1}{x}$. Show that f is uniformly continuous on $(1, \infty)$ but not on $(0, \infty)$.

1.2 2

Let $E \subset \mathbb{R}$ be a Lebesgue measurable set. Show that there is a Borel set $B \subset E$ such that $m(E \setminus B) = 0$.

1.3 3

Suppose $f(x)$ and $xf(x)$ are integrable on $\mathbb{R}$. Define F by

$$F(t) := \int_{-\infty}^{\infty} f(x) \cos(xt) dx$$

Show that

$$F'(t) = -\int_{-\infty}^{\infty} xf(x) \sin(xt) dx.$$

1.4 4

Let $f \in L^1([0,1])$. Prove that

$$\lim_{n\to\infty} \int_0^1 f(x)|\sin nx|dx = \frac{2}{\pi}\int_0^1 f(x)dx$$

Hint: Begin with the case that f is the characteristic function of an interval.

1.5 5

Let $f \geq 0$ be a measurable function on $\mathbb{R}$. Show that

$$\int_{\mathbb{R}} f = \int_0^\infty m(\{x : f(x) > t\})dt$$

1.6 6

Compute the following limit and justify your calculations:

$$\lim_{n\to\infty} \int_1^n \frac{dx}{(1+\frac{x}{n})^n \sqrt[n]{x}}$$