

Title

Contents

1 Monday, October 26

2

1 | Monday, October 26

Problem. (Gathmann 4.13)

Let $f : X \rightarrow Y$ be a morphism of affine varieties and $f^* : A(X) \rightarrow A(Y)$ the induced map on coordinate rings. Determine if the following statements are true or false:

- f is surjective $\iff f^*$ is injective.
- f is injective $\iff f^*$ is surjective.
- If $f : \mathbb{A}^1 \rightarrow \mathbb{A}^1$ is an isomorphism, then f is *affine linear*, i.e. $f(x) = ax + b$ for some $a, b \in k$.
- If $f : \mathbb{A}^2 \rightarrow \mathbb{A}^2$ is an isomorphism, then f is *affine linear*, i.e. $f(x) = Ax + b$ for some $a \in \text{Mat}(2 \times 2, k)$ and $b \in k^2$.

Problem. (Gathmann 4.19)

Which of the following are isomorphic as ringed spaces over \mathbb{C} ?

- $\mathbb{A}^1 \setminus \{1\}$
- $V(x_1^2 + x_2^2) \subset \mathbb{A}^2$
- $V(x_2 - x_1^2, x_3 - x_1^3) \setminus \{0\} \subset \mathbb{A}^3$
- $V(x_1 x_2) \subset \mathbb{A}^2$
- $V(x_2^2 - x_1^3 - x_1^2) \subset \mathbb{A}^2$
- $V(x_1^2 - x_2^2 - 1) \subset \mathbb{A}^2$

Problem. (Gathmann 5.7)

Show that

- Every morphism $f : \mathbb{A}^1 \setminus \{0\} \rightarrow \mathbb{P}^1$ can be extended to a morphism $\hat{f} : \mathbb{A}^1 \rightarrow \mathbb{P}^1$.
- Not every morphism $f : \mathbb{A}^2 \setminus \{0\} \rightarrow \mathbb{P}^1$ can be extended to a morphism $\hat{f} : \mathbb{A}^2 \rightarrow \mathbb{P}^1$.
- Every morphism $\mathbb{P}^1 \rightarrow \mathbb{A}^1$ is constant.

Problem. (Gathmann 5.8)

Show that

- Every isomorphism $f : \mathbb{P}^1 \rightarrow \mathbb{P}^1$ is of the form

$$f(x) = \frac{ax + b}{cx + d} \quad a, b, c, d \in k.$$

where x is an affine coordinate on $\mathbb{A}^1 \subset \mathbb{P}^1$.

- Given three distinct points $a_i \in \mathbb{P}^1$ and three distinct points $b_i \in \mathbb{P}^1$, there is a unique isomorphism $f : \mathbb{P}^1 \rightarrow \mathbb{P}^1$ such that $f(a_i) = b_i$ for all i .

Proposition 1.0.1(?).

There is a bijection

$$\begin{array}{ccc} \{\text{morphisms } X \rightarrow Y\} & \xleftrightarrow{1:1} & \{K\text{-algebra homomorphisms } \mathcal{O}_Y(Y) \rightarrow \mathcal{O}_X(X)\} \\ & & f \longmapsto f^* \end{array}$$

Problem. (Gathmann 5.9)

Does the above bijection hold if

- a. X is an arbitrary prevariety but Y is still affine?
- b. Y is an arbitrary prevariety but X is still affine?