Assignment 6 Qual Problems

D. Zack Garza

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1 Problem 1

1.1 Part (a)

Definition: A field extension L/F is said to be a *splitting field* of a polynomial $f(x)$ if L contains all roots of f and thus decomposes as

$$f(x) = \prod_{i=1}^{n} (x - \alpha_i)^{k_i} \in L[x]$$

where α_i are the distinct roots of f and k_i are the respective multiplicities.

1.2 Part (b)

Let F be a finite field with q elements, where $q = p^k$ is necessarily a prime power, so $F \cong \mathbb{F}_{p^k}$. Then any finite extension of E/F is an F-vector space, and contains $q^n = (p^k)^n = p^{kn}$ elements. Thus $E \cong \mathbb{F}_{p^{kn}}$ Then if $\alpha \in E$, we have $\alpha^{p^{kn}} = \alpha$, so we can define $f(x) := x^{p^{kn}} - x \in F[x]$.