

Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

Intro

Outline

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x) - \mu(y)$.

What we have so far:

- Define

$$\begin{aligned} L : W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) &\longrightarrow L^p(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \\ Y &\longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y \end{aligned}$$

where

$$\begin{aligned} S : \mathbb{R} \times S^1 &\longrightarrow \text{Mat}(2n; \mathbb{R}) \\ S(s, t) &\xrightarrow{s \rightarrow \pm\infty} S^\pm(t). \end{aligned}$$

Outline

- Took $R^\pm : I \longrightarrow \mathrm{Sp}(2n; \mathbb{R})$: symplectic paths associated to S^\pm
- These paths defined $\mu(x), \mu(y)$
- Section 8.7:

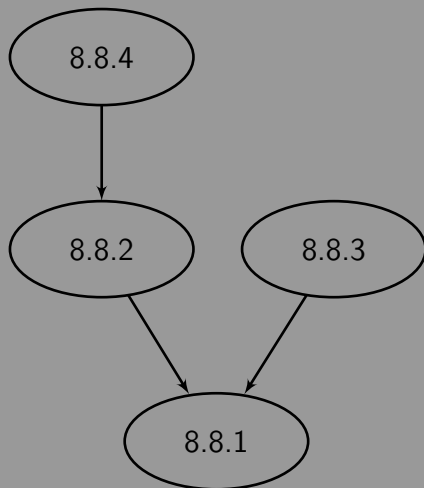
$$R^\pm \in \mathcal{S} := \left\{ R(t) \mid R(0) = \mathrm{id}, \det(R(1) - \mathrm{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\mathrm{Ind}(L) \stackrel{\mathrm{Thm?}}{=} \mu(R^-(t)) - \mu(R^+(t)) = \mu(x) - \mu(y).$$

From Yesterday

– Proved 8.8.2:



8.8.5:

8.8.3:

Section 1

Outline

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Section 2

Outline

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