Linearization Continued

Section 8.4 Follow-Up

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Review

- The Floer equation is given by
$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \operatorname{grad} H_t(u) = 0.$$
- We fixed a solution and lifted it to a sphere:
$$u \in C^\infty(S^1 \times \mathbb{R}; W) \quad \mapsto \quad \tilde{u} \in C^\infty(S^2; W)$$
- We use the assumption:
For every $w \in C^\infty(S^2, W)$ there exists a symplectic trivialization of the fiber bundle $w^* TW$, i.e. $\langle c_1(TW),\ \pi_2(W)\rangle = 0$ where c_1 denotes the first Chern class of the bundle TW.
- We use this trivialize the pullback $\tilde{u}^* TW$ to obtain an orthonormal unitary frame
$$\{Z_i\}_{i=1}^{2n} \subset T_{u(s,t)}W$$
where

Review

- We used the chosen frame $\{Z_i\}$ to define a chart centered at u of $\mathcal{P}^{1,p}(x, y)$ given by

$$\begin{aligned} \iota : W^{1,p}\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right) &\longrightarrow \mathcal{P}^{1,p}(x, y) \\ \mathbf{y} = (y_1, \ldots, y_{2n}) &\longmapsto \exp_u\left(\sum y_i Z_i\right). \end{aligned}$$

Review

$$
\begin{tikzcd}
W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \arrow[r, "\iota"] \arrow[rrr, bend left, dashed, "\mathcal{F}_u"] & \mathcal{P}^{1,p}(x,y) \arrow[r, "\mathcal{F}"] \arrow[rr, bend left, dotted, "\bar{\mathcal{F}}"] & L^p\left(\mathbb{R} \times S^1; TW\right) \arrow[r] & L^p\left(\mathbb{R} \times S^1; \mathbb{R}^m\right)
\end{tikzcd}
$$

$$
u \xrightarrow{\quad \bar{\mathcal{F}} \quad} \frac{\partial u}{\partial s} + J(u)\left(\frac{\partial u}{\partial t} - X_t(u)\right)
$$

$$
(y_1, \ldots, y_{2n}) \longrightarrow \exp_u\left(\sum y_i Z_i\right)
$$

Review

Extract the part that is linear in Y and collect terms:

$$(d\mathcal{F})_u(Y) = \left(\frac{\partial Y}{\partial s} + J(u) \frac{\partial Y}{\partial t} \right) + \left((dJ)_u(Y) \frac{\partial u}{\partial t} - (dJ)_u(Y) X_t - J(u) \left(dX_t \right)_u (Y) \right)$$

.