Problem Set 5

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1 Problem 1

We first make the following definitions:

$$S \coloneqq \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{jk} = \sup \left\{ \sum_{(j,k) \in B} a_{jk} \ni B \subset \mathbb{N}^2,\ |B| < \infty \right\}$$
$$T \coloneqq \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} a_{kj} = \sup \left\{ \sum_{(k,j) \in C} a_{kj} \ni C \subset \mathbb{N}^2,\ |B| < \infty \right\}.$$

We will show that $S = T$ by showing that $S \leq T$ and $T \leq S$.

1.1 $S \leq T$:

Let $B \subset \mathbb{N}^2$ be finite, so $B \subseteq [0, I] \times [0, J] \subset \mathbb{N}^2$. Now letting $R > \max(I, J)$, we can define $C = [0, R]^2$, which satisfies $B \subseteq C \subset \mathbb{N}^2$ and $|C| < \infty$. Moreover, since $a_{jk} \geq 0$ for all pairs (j, k), we have the following inequality:

$$\sum_{(j,k) \in B} a_{jk} < \sum_{(k,j) \in C} a_{jk} \leq \sum_{(k,j) \in C} a_{jk} \leq T,$$

since T is a supremum over *all* such sets C, and the terms of any finite sum can be rearranged.

But since this holds for every B, we this inequality also holds for the supremum of the smaller term by order-limit laws, and so

$$S \coloneqq \sup_B \sum_{(k,j) \in B} a_{jk} \leq T.$$