Problem Set 2

D. Zack Garza

Tuesday 15$^{\text{th}}$ September, 2020

1 | Exercises

Exercise 1.1 (Gathmann 2.17).
Find the irreducible components of

$$V(x - yz, xz - y^2) \subset \mathbb{A}^3/\mathbb{C}.$$

Exercise 1.2 (Gathmann 2.18).
Let $X \subset \mathbb{A}^n$ be an arbitrary subset and show that $V(I(X)) = \overline{X}$.

Exercise 1.3 (Gathmann 2.21).
Let $\{U_i\}_{i \in I} \rightrightarrows X$ be an open cover of a topological space with $U_i \cap U_j \neq \emptyset$ for every i, j.

   a. Show that if U_i is connected for every i then X is connected.

   b. Show that if U_i is irreducible for every i then X is irreducible.

Exercise 1.4 (Gathmann 2.22).
Let $f : X \to Y$ be a continuous map of topological spaces.

   a. Show that if X is connected then $f(X)$ is connected.

   b. Show that if X is irreducible then $f(X)$ is irreducible.

> **Definition 1.0.1** (Ideal Quotient).
> For two ideals $J_1, J_2 \trianglelefteq R$, the *ideal quotient* is defined by
>
> $$J_1 : J_2 := \left\{ f \in R \mid f J_2 \subset J_1 \right\}.$$

Exercise 1.5 (Gathmann 2.23).
Let X be an affine variety.

a. Show that if $Y_1, Y_2 \subset X$ are subvarieties then

$$I(\overline{Y_1 \setminus Y_2}) = I(Y_1) : I(Y_2).$$

b.

Exercise 1.6 (Gathmann 2.24)**.**