Title

D. Zack Garza

Contents

1 Monday, November 09

1.1 Chapter 1

Let k be a field, not necessarily algebraically closed.

Definition 1.1.1 (Algebraic Function Field)**.**
An one variable **algebraic function field** F/K is a field extension F of K which factors as

$$\begin{tikzcd}
F \arrow[dr, "\text{algebraic}"] & \\
& k(x) \arrow[dl, "\text{transcendental}"] \\
k &
\end{tikzcd}$$

where $x \in \bar{k}$ is some element that is not algebraic over k.

Definition 1.1.2 (Field of Constants)**.**
The subfield

$$\tilde{k} \coloneqq \left\{ z \in F \cap K^{\text{alg}} \right\} \leq F,$$

consisting of elements that are algebraic over F is denoted the **field of constants**.

Definition 1.1.3 (Algebraically Closed)**.**
If $\tilde{k} = k$, we say that k is **algebraically closed** in F.

Definition 1.1.4 (Rational Function Field)**.**
An extension F/k is **rational** iff $F = k(y)$ for some $y \in k^{\text{transc}}$ which is transcendental over k.

Definition 1.1.5 (Valuation Ring)**.**
A ring $\mathcal{O} \subseteq F$ is a **valuation ring** for F iff $k \subset \mathcal{O} \subseteq F$ and $z \in F \implies z \in \mathcal{O}$ or $z^{-1} \in \mathcal{O}$.

Definition 1.1.6 (Discrete Valuation Ring)**.**
A ring local R (thus with a unique maximal ideal) which is a PID but not a field is a **discrete valuation ring**.