# Title

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## Contents



# 1 Friday, August 21

## 1.1 Intro and Definitions

**Definition 1.0.1** (Affine Variety)**.**
Let $k = \bar{k}$ be algebraically closed (e.g. $k = \mathbb{C}, \overline{\mathbb{F}_p}$). A variety $V \subseteq k^n$ is an *affine $k$-variety* iff $V$ is the zero set of a collection of polynomials in $k[x_1, \cdots, x_n]$.

Here $\mathbb{A}^n \coloneqq k^n$ with the Zariski topology, so the closed sets are varieties.

**Definition 1.0.2** (Affine Algebraic Group)**.**
An *affine algebraic $k$-group* is an affine variety with the structure of a group, where the multiplication and inversion maps

$$\begin{aligned}\mu : G \times G &\longrightarrow G \\ \iota : G &\longrightarrow G\end{aligned}$$

are continuous.

**Example 1.1.**
$G = \mathbb{G}_a \subseteq k$ the *additive group* of $k$ is defined as $\mathbb{G}_a \coloneqq (k, +)$. We then have a *coordinate ring* $k[\mathbb{G}_a] = k[x]/I = k[x]$.

**Example 1.2.**
$G = \mathrm{GL}(n, k)$, which has coordinate ring $k[x_{ij}, T]/\left\langle \det(x_{ij}) \cdot T = 1 \right\rangle$.

**Example 1.3.**
Setting $n = 1$ above, we have $\mathbb{G}_m \coloneqq \mathrm{GL}(1, k) = (k^\times, \cdot)$. Here the coordinate ring is $k[x, T]/\left\langle xT = 1 \right\rangle$.

**Example 1.4.**
$G = \mathrm{SL}(n,k) \leq \mathrm{GL}(n,k)$, which has coordinate ring $k[G] = k[x_{ij}]/\langle \det(x_{ij}) = 1 \rangle$.

**Definition 1.0.3** (Irreducible)**.**
A variety $V$ is *irreducible* iff $V$ can not be written as $V = \bigcup_{i=1}^{n} V_i$ with each $V_i \subseteq V$ a proper subvariety.

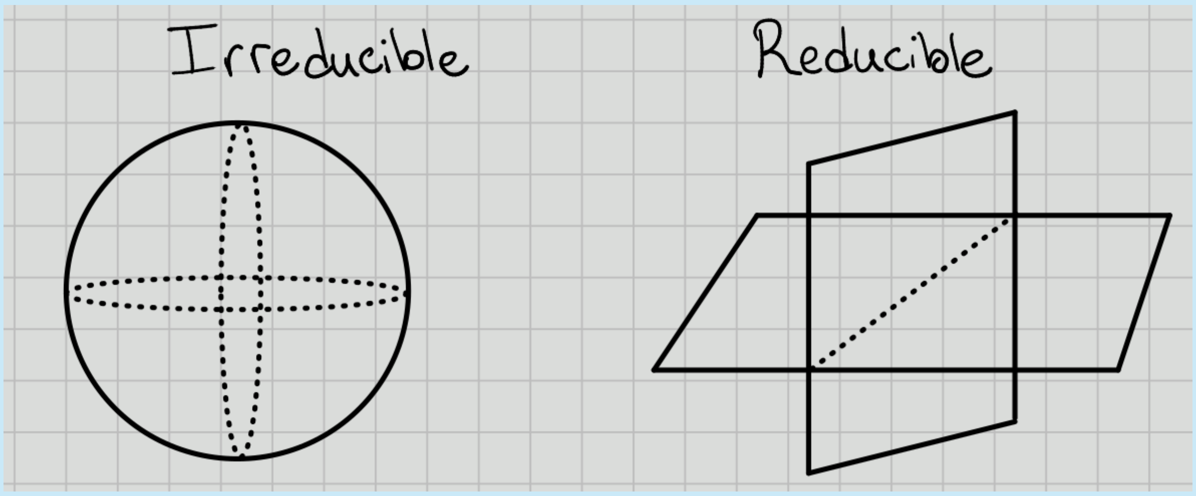


Figure 1: Reducible vs Irreducible

**Proposition 1.1** ***(?).***
There exists a unique irreducible component of $G$ containing the identity $e$. Notation: $G^0$.

**Proposition 1.2** ***(?).***
$G$ is the union of translates of $G^0$, i.e. there is a decomposition

$$G = \coprod_{g \in \Gamma} g \cdot G^0.$$

What is $\Gamma$?

**Proposition 1.3** ***(?).***
One can define solvable and nilpotent algebraic groups in the same way as they are defined for finite groups, i.e. as having a terminating derived or lower central series respectively.

**Proposition 1.4** ***(Existence and Uniqueness of Radical).***