

# Problem Set 5

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October 21, 2019

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## 1 Problem 1

We first make the following definitions:

$$S := \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{jk} = \sup \left\{ \sum_{(j,k) \in B} a_{jk} \mid B \subset \mathbb{N}^2, |B| < \infty \right\}$$

$$T := \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} a_{kj} = \sup \left\{ \sum_{(j,k) \in C} a_{kj} \mid C \subset \mathbb{N}^2, |C| < \infty \right\}.$$

We will show that  $S = T$  by showing that  $S \leq T$  and  $T \leq S$ .

### 1.1 $S \leq T$ :

Let  $B \subset \mathbb{N}^2$  be finite, so  $B \subseteq [0, I] \times [0, J] \subset \mathbb{N}^2$ . Now letting  $R > \max(I, J)$ , we can define  $C = [0, R]^2$ , which satisfies  $B \subseteq C \subset \mathbb{N}^2$  and  $|C| < \infty$ . Moreover, since  $a_{jk} \geq 0$  for all pairs  $(j, k)$ , we have the following inequality:

$$\sum_{(j,k) \in B} a_{jk} < \sum_{(k,j) \in C} a_{jk} \leq \sum_{(k,j) \in C} a_{kj} \leq T,$$

since  $T$  is a supremum over *all* such sets  $C$ , and the terms of any finite sum can be rearranged.

But since this holds for every  $B$ , we this inequality also holds for the supremum of the smaller term by order-limit laws, and so

$$S := \sup_B \sum_{(k,j) \in B} a_{jk} \leq T.$$