

Title

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Questions to look at for next Tuesday:

Exercise 1.1 (?).

Show that the 3 natural coordinate charts on \mathbb{CP}^2 given by e.g. $\varphi_{U_0}([z_0 : z_1 : z_2]) = \left[\frac{z_1}{z_0}, \frac{z_2}{z_0} \right]$ yield a smooth atlas.

Exercise 1.2 (?).

Consider the map

$$\begin{aligned} \pi : \mathbb{CP}^2 &\rightarrow \mathbb{R}^2 \\ [z_0 : z_1 : z_2] &\mapsto \left[\frac{|z_1|^2}{|z_0|^2 + |z_1|^2 + |z_2|^2}, \frac{|z_2|^2}{|z_0|^2 + |z_1|^2 + |z_2|^2} \right]. \end{aligned}$$

Show that π is smooth and $\text{im}\pi = \{p_1, p_2 \geq 0, p_1 + p_2 \leq 1\}$.

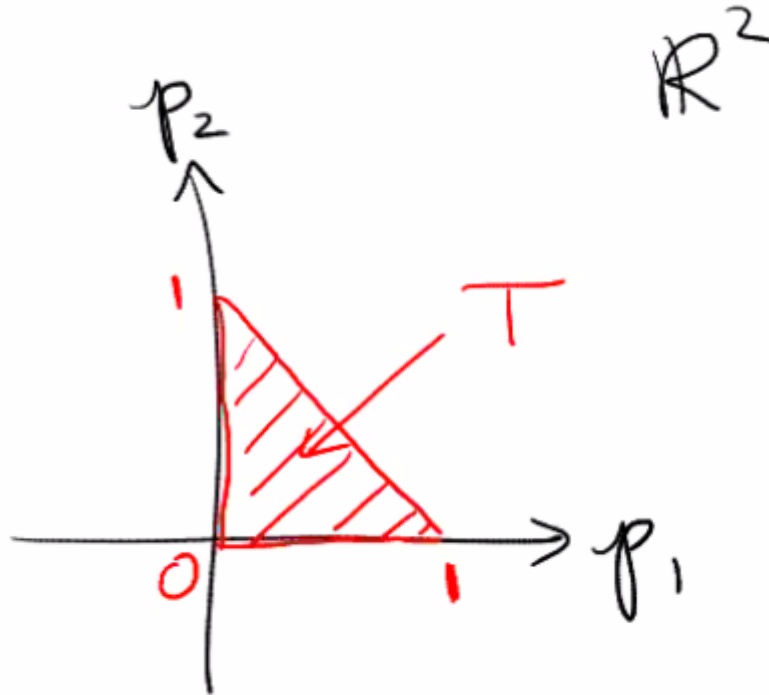


Figure 1: O

Exercise 1.3 (?).

Show that

- If $[p_1, p_2] \in T^\circ$ is in the interior of the above triangle, then $\pi^{-1}(p_1, p_2) \cong S^1 \times S^1$ is diffeomorphic to a torus.
- If the point is on an edge, the fiber is diffeomorphic to S^1 ,
- If the point is on a vertex, the fiber is a single point.

Exercise 1.4 (?).

Find a vector field V on some maximal subset of \mathbb{CP}^2 such that $D\pi(V) = p_1\partial_{p_1} + p_2\partial_{p_2}$ (the radial vector field).

I.e., for all $q \in \mathbb{CP}^2$, we have a map

$$D_1\pi : T_1\mathbb{CP}^2 \rightarrow T_{\pi(q)}\mathbb{R}^2$$

and $V(q) \in T_q\mathbb{CP}^2$, so we want $D_q\pi(V(q)) = p_1\partial_{p_1} + p_2\partial_{p_2}$.

Note that there will be a problem defining V on the fiber over the hypotenuse of T .

Theorem 1.1 (*Collar Neighborhood*).

For all manifolds with boundary X , there exists an open neighborhood N of ∂X which is diffeomorphic to $(-\varepsilon, 0] \times \partial X$.

Proof strategy: