Title

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Questions to look at for next Tuesday:

Exercise 1.1 (?)**.**
Show that the 3 natural coordinate charts on $\mathbb{CP}^2$ given by e.g. $\varphi_{U_0}([z_0 : z_1 : z_2]) = \left[\frac{z_1}{z_0}, \frac{z_2}{z_0}\right]$ yield a smooth atlas.

Exercise 1.2 (?)**.**
Consider the map

$$\begin{aligned} \pi : \mathbb{CP}^2 &\to \mathbb{R}^2 \\ [z_0 : z_1 : z_2] &\mapsto \left[\frac{|z|_1^2}{|z|_0^2 + |z|_1^2 + |z|_2^2}, \frac{|z|_2^2}{|z|_0^2 + |z|_1^2 + |z|_2^2}\right]. \end{aligned}$$

Show that π is smooth and $\text{im}\pi = \{p_1, p_2 \geq 0, p_1 + p_2 \leq 1\}$.

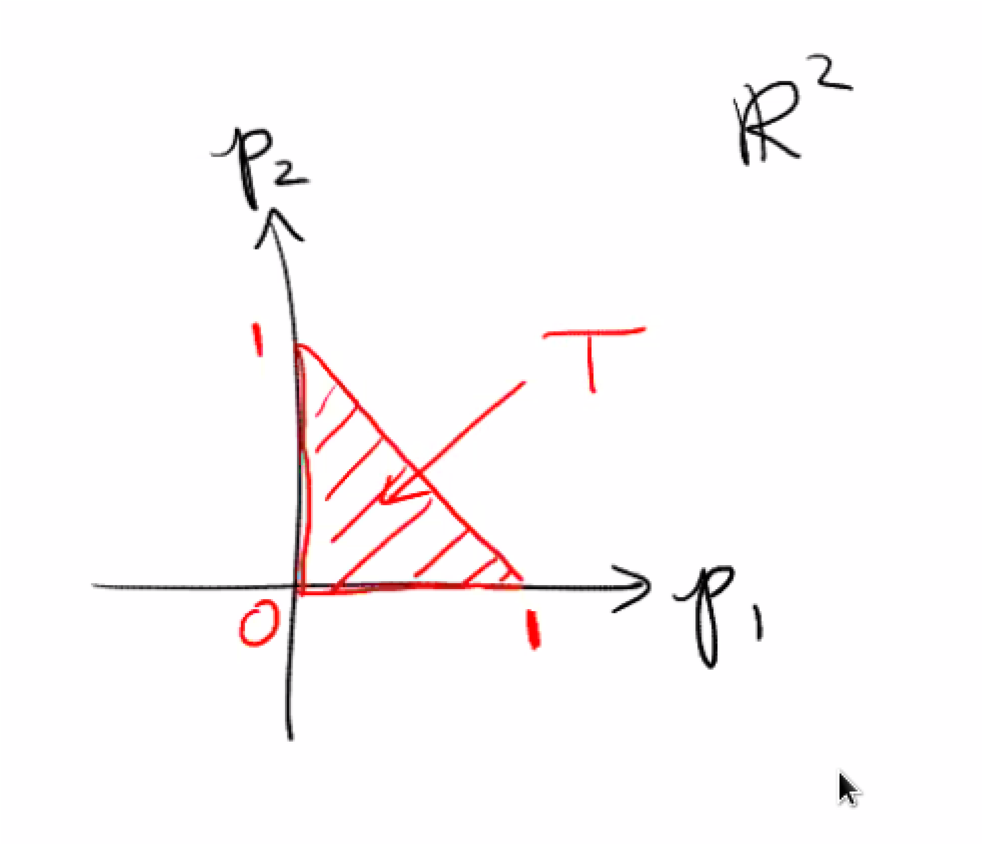

Figure 1: O

Exercise 1.3 (?).
Show that

- If $[p_1, p_2] \in T^\circ$ is in the interior of the above triangle, then $\pi^{-1}(p_1, p_2) \cong S^1 \times S^1$ is diffeomorphic to a torus.
- If the point is on an edge, the fiber is diffeomorphic to S^1,
- If the point is on a vertex, the fiber is a single point.

Exercise 1.4 (?).
Find a vector field V on some maximal subset of $\mathbb{CP}^2$ such that $D\pi(V) = p_1 \partial_{p_1} + p_2 \partial_{p_2}$ (the radial vector field).

I.e., for all $q \in \mathbb{CP}^2$, we have a map

$$D_1\pi : T_1\mathbb{CP}^2 \to T_{\pi(q)}\mathbb{R}^2$$

and $V(q) \in T_q\mathbb{CP}^2$, so we want $D_q\pi(V(q)) = p_1\partial_{p_1} + p_2\partial_{p_2}$.

Note that there will be a problem defining V on the fiber over the hypotenuse of T.

Theorem 1.1 ***(Collar Neighborhood).***
For all manifolds with boundary X, there exists an open neighborhood N of ∂X which is diffeomorphic to $(-\varepsilon, 0] \times \partial X$.

Proof strategy: construct a vector field pointing outward and flow it backward.