Problem Set 5

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1 Problem 1

We first make the following definitions:

$$S := \sum_{j=1}^\infty \sum_{k=1}^\infty a_{jk} = \sup \left\{ \sum_{(j,k) \in B} a_{jk} \ni B \subset \mathbb{N}^2,\ |B| < \infty \right\}$$
$$T := \sum_{k=1}^\infty \sum_{j=1}^\infty a_{kj} = \sup \left\{ \sum_{(j,k) \in C} a_{kj} \ni C \subset \mathbb{N}^2,\ |B| < \infty \right\}.$$

We will show that $S = T$ by showing that $S \leq T$ and $T \leq S$.

Let $B \subset \mathbb{N}^2$ be finite, so $B \subseteq [0, I] \times [0, J] \subset \mathbb{N}^2$. Now letting $R > \max(I, J)$, we can define $C = [0, R]^2$, which satisfies $B \subseteq C \subset \mathbb{N}^2$ and $|C| < \infty$. Moreover, since $a_{jk} \geq 0$ for all pairs (j, k), we have the following inequality:

$$\sum_{(j,k) \in B} a_{jk} < \sum_{(k,j) \in C} a_{jk} \leq \sum_{(k,j) \in C} a_{jk} \leq T,$$

since T is a supremum over *all* such sets C, and the terms of any finite sum can be rearranged.

But since this holds for every B, we this inequality also holds for the supremum of the smaller term by order-limit laws, and so

$$S := \sup_B \sum_{(k,j) \in B} a_{jk} \leq T.$$

An identical argument shows that $T \leq S$, yielding the desired equality.