

Problem Set One

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1 Humphreys 1.1

1.1 a

If $M \in \mathcal{O}$ and $[\lambda] = \lambda + \Lambda_r$ is any coset of $\mathfrak{h}^\vee / \Lambda_r$, let $M^{[\lambda]}$ be the sum of weight spaces M_μ for which $\mu \in [\lambda]$.

Proposition: $M^{[\lambda]}$ is a $U(\mathfrak{g})$ -submodule of M

Proof:

Proposition: M is the direct sum of finitely many submodules of the form $M^{[\lambda]}$.

Proof:

1.2 b

Proposition: The weights of an indecomposable module $M \in \mathcal{O}$ lie in a single coset of $\mathfrak{h}^\vee / \Lambda_r$.

2 Humphreys 1.3*

Proposition: For any $M \in \mathcal{O}$, $M(\lambda)$ satisfies the following property:

$$\mathrm{Hom}_{U(\mathfrak{g})}(M(\lambda), M) = \mathrm{Hom}_{U(\mathfrak{g})}(\mathrm{Ind}_{\mathfrak{b}}^{\mathfrak{g}} \mathbb{C}_\lambda, M) \cong \mathrm{Hom}_{U(\mathfrak{b})}(\mathbb{C}_\lambda, \mathrm{Res}_{\mathfrak{b}}^{\mathfrak{g}} M).$$

Proof:

Noting that $\text{Ind}_{\mathfrak{b}}^{\mathfrak{g}} \mathbb{C}_{\lambda} = U(\mathfrak{g}) \otimes_{U(\mathfrak{b})} \mathbb{C}_{\lambda}$ and \mathfrak{g} -morphisms can always be lifted to $U(\mathfrak{g})$ -morphisms, consider the following two maps:

$$F : \text{hom}_{\mathfrak{g}}(U(\mathfrak{g}) \otimes_{U(\mathfrak{b})} \mathbb{C}_{\lambda}, M) \rightarrow \text{hom}_{\mathfrak{b}}(\mathbb{C}_{\lambda}, M).$$