

Title

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0.1 Exercises

Problem 1.

Let C denote the Cantor set.

1. Show that C contains point that is not an endpoint of one of the removed intervals.
2. Show that C is nowhere dense, meager, and has measure zero.
3. Show that C is uncountable.

Solution 1.

1. First we will characterize the endpoints of the removed intervals. Let C_n be the n th stage of the deleting process used to define the Cantor set; then what remains is a union of intervals:

$$C_n = [0, \frac{1}{3^n}] \cup [\frac{2}{3^n}, \frac{3}{3^n}] \cup \cdots \cup [\frac{3^n - 1}{3^n}, 1],$$

and so the endpoints are precisely the numbers of the form $\frac{k}{3^n}$ where $0 \leq k \leq 3^n$. Moreover, any endpoint appearing in C_n is never removed in any later step, and so all endpoints remaining in C are of this form where we allow $0 \leq n < \infty$.