## Title

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## **Table of Contents**

## **Contents**

Table of Contents	
1 Lecture 09	

Table of Contents

Lecture 09

## 1 | Lecture 09

Last time:

- The Čech-to-derived spectral sequence,
- The Mayer Vietoris LES,
  - Computes the étale cohomology of a scheme using a Zariski open cover.
- Étale cohomology of quasicoherent sheaves,
  - Agrees with Zariski cohomology, first legitimate computation!
  - Use this to compute:
- Étale cohomology of  $\mathbb{F}_p$  in characteristic p.

Last time we had a scheme  $X_{/\mathbb{F}_p}$  and the Artin-Schreier exact sequence of sheaves of  $X_{\text{\'et}}$ :

$$0 \to \mathbb{F}_p \to \mathcal{O}_X^{\text{\'et}} \xrightarrow{t \mapsto t^p - t} \mathcal{O}_X^{\text{\'et}} \to 0.$$

The map appearing here is referred to as the Artin-Schreier map f This works over arbitrary fields of characteristic p, with a modified definition replacing  $t^p$ .

Exercise 1.0.1(?): Check that this is an additive homomorphism of abelian sheaves. This follows from the fact that Frobenius itself is.

**Remark 1.0.2:** From here onward,  $H^i$  will denote  $H^i_{\text{\'et}}$ .

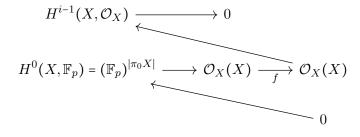
Recall that we had a theorem last time showing that the étale cohomology of quasicoherent sheaves is equivalent to the usual Zariski cohomology. From this we got a long exact sequence:

$$H^{i}(X_{\operatorname{\acute{e}t}}, \underline{\mathbb{F}_{p}}) \xrightarrow{\delta} H^{i}(X, \mathcal{O}_{X}) \xrightarrow{f} H^{i}(X, \mathcal{O}_{X})$$
 $\cdots \longrightarrow H^{i-1}(X, \mathcal{O}_{X})$ 

We don't know how to compute  $H^i(X_{\text{\'et}}, \mathbb{F}_p)$  generally, but the affine case is easy. For X affine,  $H^{>0}(X, \mathcal{O}_X) = 0$ , which in facts holds for any quasicoherent sheave replacing  $\mathcal{O}_X$ , and  $H^0(X, \mathbb{F}_p) = (\mathbb{F}_p)^{|\pi_0 X|}$  where the exponent is the number of connected components of X. So we get an exact sequence

Lecture 09

1 Lecture 09



Lecture 09 4