Problem Sets

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1 1.1

1.1 a

If $M \in \mathcal{O}$ and $[\lambda] = \lambda + \Lambda_r$ is any coset of $\mathfrak{h}^\vee/\Lambda_r$, let $M^{[\lambda]}$ be the sum of weight spaces M_μ for which $\mu \in [\lambda]$. Prove that $M^{[\lambda]}$ is a $U(\mathfrak{g})$-submodule of M and that M is the direct sum of finitely many such submodules.

1.2 b

Deduce that the weights of an indecomposable module $M \in \mathcal{O}$ lie in a single coset of $\mathfrak{h}^\vee/\Lambda_r$.

2 1.3*

Show that $M(\lambda)$ has the following property: for any $M \in \mathcal{O}$,

$$\operatorname{Hom}_{U(\mathfrak{g})}(M(\lambda), M) = \operatorname{Hom}_{U(\mathfrak{g})}\left(\operatorname{Ind}_{\mathfrak{b}}^{\mathfrak{g}} \mathbb{C}_\lambda, M\right) \cong \operatorname{Hom}_{U(\mathfrak{b})}\left(\mathbb{C}_\lambda, \operatorname{Res}_{\mathfrak{b}}^{\mathfrak{g}} M\right).$$