

Algebraic Topology 2: Smooth Manifolds

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August 16, 2019

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The key point of this class will be a discussion of *smooth structures*. As you may recall, a sensational result of Milnor's exhibited exotic spheres with smooth structures – i.e., a differentiable manifold M which is homeomorphic but *not* diffeomorphic to a sphere.

Summary of this result: Look at bundles $S^3 \rightarrow X \rightarrow S^4$, then one can construct some $X \cong S^7 \in \mathbf{Top}$ but $X \not\cong S^7 \in \mathbf{Diff}^\infty$. There are in fact 7 distinct choices for X .

It is not known if there are exotic smooth structures on S^4 . The Smooth Poincaré conjecture is that these do not exist; this is believed to be false.

The other key point of this course is to show that $X \in \mathbf{Diff}^\infty \implies X \hookrightarrow \mathbb{R}^n$ for some n , and is in fact a topological subspace.

A short list of words/topics we hope to describe:

- Differentiable manifolds
- Local charts
- Submanifolds
- Projective spaces
- Lie groups
- Tangent spaces
- Vector fields
- Cotangent spaces
- Differentials of smooth maps
- Differential forms
- de Rham's theorem

We'd like a notion of “convergence” for, say, curves in \mathbb{R}^2 . Consider the following examples.



Note the problematic point on the bottom right, as well as the fact that neither of the usual notions of pointwise or uniform convergence will yield a point on the LHS that converges to the red point on the RHS.



Note the problematic point at the origin.

