

# Title

D. Zack Garza

Wednesday 30<sup>th</sup> September, 2020

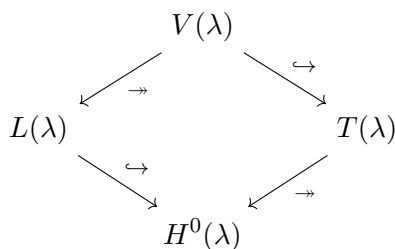
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Recall that we had a dominant weight  $\lambda \in X(T)_+$  with



where we have a module with both a *good* and a *Weyl* filtration.

If  $B \subseteq P \subseteq G$  with  $P$  parabolic and  $M \in \text{Mod}(G)$ , we have a “transfer theorem”: maps

$$H^n(G; M) \xrightarrow{\text{Res}} H^n(P; M) \xrightarrow{\text{Res}} H^n(B; M)$$

induced by restrictions which are isomorphisms.

### Proposition 1.1(?).

Let  $M \in \text{Mod}(P)$  with  $P \supseteq B$ .

- If  $\dim M < \infty$  then  $\dim H^n(P; M) < \infty$ .
- If  $H^j(P; M) \neq 0$  then there exists a weight  $\lambda$  of  $M$  such that  $-\lambda \in \mathbb{N}\Phi^+$  and  $\text{ht}(-\lambda) \geq j$ .

Part (a) is proved in the book, we won't show it here.

*Proof (of part b).*

Suppose  $H^j(P; M) \neq 0$ , then we have an injective resolution  $I_*$  for  $k$ . Tensoring with  $M$  yields an injective resolution for  $M$ ,

$$0 \rightarrow M \rightarrow I_0 \otimes M \rightarrow I_1 \otimes M \rightarrow \cdots$$

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Since  $H^j(B; M) \neq 0$ , we know that the cocycles  $\text{hom}_B(k, I_j \otimes M) \neq 0$  and thus  $\text{hom}_T(k, I_j \otimes M) \neq 0$ . ■