

# Assignment 6 Qual Problems

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## 1 Problem 1

### 1.1 Part (a)

Definition: A field extension  $L/F$  is said to be a *splitting field* of a polynomial  $f(x)$  if  $L$  contains all roots of  $f$  and thus decomposes as

$$f(x) = \prod_{i=1}^n (x - \alpha_i)^{k_i} \in L[x]$$

where  $\alpha_i$  are the distinct roots of  $f$  and  $k_i$  are the respective multiplicities.

### 1.2 Part (b)

Let  $F$  be a finite field with  $q$  elements, where  $q = p^k$  is necessarily a prime power, so  $F \cong \mathbb{F}_{p^k}$ . Every finite degree extension of a finite field is simple, so  $E = F(\alpha)$  for a single primitive element  $\alpha$ . Then  $\alpha^{p^k} = \alpha$ , so we can define  $f(x) := x^{p^k} - x \in F[x]$  and it follows that every power of  $\alpha$  is a root of  $f$ .