Title

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1.1 Intro and Definitions

Definition 1.0.1 (Affine Variety)**.**
Let $k = \bar{k}$ be algebraically closed (e.g. $k = \mathbb{C}, \overline{\mathbb{F}_p}$). A variety $V \subseteq k^n$ is an *affine k-variety* iff V is the zero set of a collection of polynomials in $k[x_1, \cdots, x_n]$.

Here $\mathbb{A}^n \coloneqq k^n$ with the Zariski topology, so the closed sets are varieties.

Definition 1.0.2 (Affine Algebraic Group)**.**
An *affine algebraic k-group* is an affine variety with the structure of a group, where the multiplication and inversion maps

$$\begin{aligned} \mu : G \times G &\longrightarrow G \\ \iota : G &\longrightarrow G \end{aligned}$$

are continuous.

Example 1.1.
$G = \mathbb{G}_a \subseteq k$ the *additive group* of k is defined as $\mathbb{G}_a \coloneqq (k, +)$. We then have a *coordinate ring* $k[\mathbb{G}_a] = k[x]/I = k[x]$.

Example 1.2.
$G = \mathrm{GL}(n, k)$, which has coordinate ring $k[x_{ij}, T] / \left\langle \det(x_{ij}) \cdot T = 1 \right\rangle$.

Example 1.3.
Setting $n = 1$ above, we have $\mathbb{G}_m \coloneqq \mathrm{GL}(1, k) = (k^\times, \cdot)$. Here the coordinate ring is $k[x, T] / \left\langle xT = 1 \right\rangle$.

Example 1.4.
$G = \mathrm{SL}(n,k) \leq \mathrm{GL}(n,k)$, which has coordinate ring $k[G] = k[x_{ij}]/\left\langle \det(x_{ij}) = 1 \right\rangle$.

Definition 1.0.3 (Irreducible)**.**
A variety V is *irreducible* iff V can not be written as $V = \bigcup_{i=1}^{n} V_i$ with each $V_i \subseteq V$ a proper subvariety.

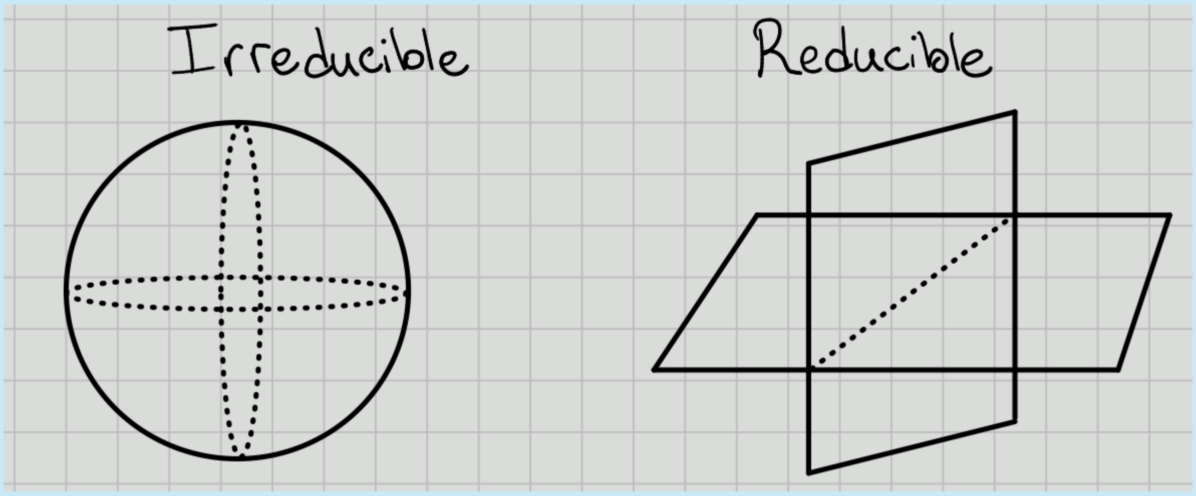

Figure 1: Reducible vs Irreducible

Proposition 1.1 ***(?).***
There exists a unique irreducible component of G containing the identity e. Notation: G^0.

Proposition 1.2 ***(?).***
G is the union of translates of G^0, i.e. there is a decomposition

$$G = \coprod_{g \in \Gamma} g \cdot G^0.$$

What is Γ?

Proposition 1.3 ***(?).***
One can define solvable and nilpotent algebraic groups in the same way as they are defined for