

# Title

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## 1 My Common Mistakes

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$$-x^{-2} \neq \int x^{-1} = \int \frac{1}{x} = \ln x$$

$$\frac{1}{x} \neq \int \ln x = x \ln x - x$$

$$\int x^{-k} = \frac{1}{-k+1} x^{-k+1} \neq \frac{1}{-(k+1)} x^{-(k+1)}$$

$$\text{e.g. } \int x^{-2} = -x^{-1} \neq -\frac{1}{3} x^{-3} \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1 \neq 0$$

$$\frac{\partial}{\partial x} a^x = \frac{\partial}{\partial x} e^{x \ln a} = e^{x \ln a} \ln a = a^x \ln a.$$

Exponentials: when in doubt, write  $a^b = e^{b \ln a}$

$$\frac{\partial}{\partial x} x^{f(x)} = ?$$

$$\sum x^k = \frac{1}{1-x} \neq \frac{1}{1+x} = \sum (-1)^k x^k$$

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	sin	cos	tan
0	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{4}}{2}$	0
$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{1}$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{0}}{2}$	$\infty$

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$(a + b)^2 =$	$a^2 + b^2 + 2ab$
$(a - b)^2 =$	$a^2 + b^2 - 2ab$
$a^2 + b^2 =$	$(a + b)^2 + 2ab$
$a^2 - b^2 =$	$(a + b)(a - b)$
$(a + b)^3 =$	$a^3 + b^3 + 3(a^2b + ab^2)$
$(a - b)^3 =$	$a^3 - b^3 + 3(-a^2b + ab^2)$
$a^3 + b^3 =$	$(a + b)(a^2 + b^2 - ab)$
$a^3 - b^3 =$	$(a - b)(a^2 + b^2 + ab)$
$(\sqrt{a} + \sqrt{b})^2 =$	$a + b + 2\sqrt{ab}$
$(\sqrt{a} - \sqrt{b})^2 =$	$a + b - 2\sqrt{ab}$
$(a + \sqrt{b})(a - \sqrt{b}) =$	$a^2 - b$
$(a + i\sqrt{b})(a - i\sqrt{b}) =$	$a^2 + b$
$(a + b)(a - b) =$	$a^2 - b^2$

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$\frac{\partial f}{\partial x} \Leftarrow$	$f$	$\Rightarrow \int f dx$
$\frac{1}{2\sqrt{x}}$	$\sqrt{x}$	$\frac{2}{3}x^{\frac{3}{2}}$
$nx^{n-1}$	$x^n, n \neq -1$	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$\ln(x)$	$x \ln(x) - x$
$a^x \ln(a)$	$a^x$	$\frac{a^x}{\ln a}$
$\cos(x)$	$\sin(x)$	$-\cos(x)$
$-\sin(x)$	$\cos(x)$	$\sin(x)$
$2 \sec^2(x) \tan(x)$	$\sec^2(x)$	$\tan(x)$
$2 \csc^2(x) \cot(x)$	$\csc^2(x)$	$-\cot(x)$
$\sec^2(x)$	$\tan(x)$	$\ln  \sec(x) $
$\sec(x) \tan(x)$	$\sec(x)$	$\ln  \sec(x) + \tan(x) $
$-\csc(x) \cot(x)$	$\csc(x)$	$\ln  \csc(x) - \cot(x) $
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$	$x \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$	$x \sin^{-1} x + \sqrt{1-x^2}$
$-\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1}(x)$	$x \cos^{-1} x - \sqrt{1-x^2}$
$\frac{1}{\sqrt{x^2+a}}$	$\ln  x + \sqrt{x^2+a} $	.
$-\csc^2(x)$	$\cot(x)$	?
?	$\cos^2(x)$	?
?	$\sin^2(x)$	?
?	$xe^{ax}$	$\frac{1}{a^2}(ax-1)e^{ax}$
?	$e^{ax} \sin(bx)$	$\frac{1}{a^2+b^2}e^{ax}(a \sin bx - b \cos bx)$
?	$e^{ax} \cos(bx)$	$\frac{1}{a^2+b^2}e^{ax}(a \sin bx + b \cos bx)$
?	?	?
$\sqrt{a^2-x^2}$	$\Rightarrow x = a \sin(\theta)$	$dx = a \cos(\theta) d\theta$
$\sqrt{a^2+x^2}$	$\Rightarrow x = a \tan(\theta)$	$dx = a \sec^2(\theta) d\theta$
$\sqrt{x^2-a^2}$	$\Rightarrow x = a \sec(\theta)$	$dx = a \sec(\theta) \tan(\theta) d\theta$

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$$p(y)y' = q(x) \qquad \text{separable}$$

$$y' + p(x)y = q(x) \qquad \text{integrating factor}$$

$$y' = f(x, y), f(tx, ty) = f(x, y) \qquad y = xV(x) \text{ COV reduces to separable}$$

$$y' + p(x)y = q(x)y^n \qquad \text{Bernoulli, divide by } y^n \text{ and COV } u = y^{1-n}$$

$$M(x, y)dx + N(x, y)dy = 0 \qquad M_y = N_x : \varphi(x, y) = c(\varphi_x = M, \varphi_y = N)$$

$$P(D)y = f(x, y) \qquad x^k e^{rx} \text{ for each root}$$

