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1 Chapter 5: Submanifolds

The most important type of manifolds: embedded submanifolds. Most often described as the *level set* of a smooth map, but needs extra conditions. The level sets of constant rank maps are always embedded submanifolds.

Definition 1.0.1 (Embedded Submanifolds).

For $S \subseteq M$ in the subspace topology, with a smooth structure such that the inclusion $S \hookrightarrow M$ is smooth.

Definition 1.0.2 (Embedded Hypersurface).

An embedded submanifold of codimension 1.

Proposition 1.1.

A subset $S \subseteq M$ of codimension zero is an embedded submanifold iff S is an open submanifold.

A way to produce submanifolds: $\{ \text{proposition} \}$ If $F : N \rightarrow M$, then $F(N)$ is a submanifold of M with the subspace topology and a unique smooth structure making F a diffeomorphism onto its image and $F(N) \hookrightarrow M$ an embedding. Thus every embedded submanifold is the image of an embedding, namely the inclusion.