Title

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1.1 Review

Let $k = \bar{k}$, we're setting up correspondences

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\begin{aligned}
\text{Polynomial functions} \quad & \text{Affine space} \\
k[x_1, \cdots, x_n] \quad & \mathbb{A}^n/k \coloneqq \{[a_1, \cdots, a_n] \in k^n\} \\
\text{Maximal ideals } \langle x_1 - a_1, \cdots, x_n - a_n \rangle \quad & \text{Points } [a_1, \cdots, a_n] \in \mathbb{A}^n/k \\
\text{Radical ideals } I \trianglelefteq k[x_1, \cdots, x_n] \quad & \text{Affine varieties } X \subset \mathbb{A}^n/k, \text{ vanishing locii of polynomials} \\
& I \mapsto V(I) \coloneqq \left\{ a \;\middle|\; f(a) = 0 \forall f \in I \right\} \\
I(X) \coloneqq \left\{ f \;\middle|\; f|_X = 0 \right\} \hookleftarrow X & \\
\text{Radical ideals containing } I(X), \text{ i.e. ideals in } A(X) \quad & \text{closed subsets of } X, \text{ i.e. affine subvarieties} \\
A(X) \text{ is a domain} \quad & X \text{ irreducible} \\
A(X) \text{ is not a direct sum} \quad & X \text{ connected} \\
\text{Prime ideals in } A(X) \quad & \text{Irreducible closed subsets of } X \\
\text{Krull dimension } n, \text{ i.e. longest chain of prime ideals is } n \quad & \dim X = n, \text{ the longest chain of irreducible closed subsets is}
\end{aligned}
.
$$

> Recall that we defined the coordinate ring $A(X) \coloneqq k[x_1, \cdots, x_n]/I(X)$, which contained no nilpotents.