Title

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1 Friday February 21st

1.1 Singularities

Recall that there are three types of singularities:

- Removable
- Poles
- Essential

> **Theorem 1.1***(3.2).*
> An isolated singularity z_0 of f is a pole $\iff \lim\limits_{z \longrightarrow z_0} f(z) = \infty$.

> **Theorem 1.2***(3.3, Casorati-Weierstrass).*
> If f is holomorphic in $D_r(z_0) \setminus \{z_0\}$ and has an essential singularity z_0, then there exists a radius r such that $f(D_r(\{z_0\}) \setminus \{z_0\})$ is dense in $\mathbb{C}$.

> *Proof* .
> Proceed by contradiction. Suppose there exists a $w \in \mathbb{C}$ and a $\delta > 0$ such that
>
> $$D_\delta(w) \bigcap f(D_r(\{z_0\}) \setminus \{z_0\}) = \emptyset.$$
>
> If $z \in D_r(w) \setminus z_0$, then $|f(z) - w| > \delta$. Define $g(z) = \dfrac{1}{f(z) - w}$ on $D_r(z_0) \setminus \{z_0\}$; then $|g(z)| < \dfrac{1}{\delta}$. Then $g(z)$ has a removable singularity at $z = z_0$ by theorem 3.1.
> If $g(z_0) \neq 0$, then $f(z) - w$ is holmorphic at z_0, a contradiction. If not, then z_0 is a pole. $\blacksquare$