Title

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Tuesday 15th September, 2020

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1.1 Review

Let $k = \bar{k}$, we're setting up correspondences

Polynomial functions	Affine space
$k[x_1, \cdots, x_n]$	$\mathbb{A}^n/k \coloneqq \{[a_1, \cdots, a_n] \in k^n\}$
Maximal ideals $\langle x_1 - a_1, \cdots, x_n - a_n \rangle$	Points $[a_1, \cdots, a_n] \in \mathbb{A}^n/k$
Radical ideals $I \trianglelefteq k[x_1, \cdots, x_n]$	Affine varieties $X \subset \mathbb{A}^n/k,$ vanishing locii of polynomials
	$I \mapsto V(I) \coloneqq \left\{ a \;\middle\vert\; f(a) = 0 \forall f \in I \right\}$
$I(X) \coloneqq \left\{ f \;\middle\vert\; f\vert_X = 0 \right\} \mapsfrom X$	
Radical ideals containing $I(X)$, i.e. ideals in $A(X)$	closed subsets of X, i.e. affine subvarieties
$A(X)$ is a domain	X irreducible
$A(X)$ is not a direct sum	X connected
Prime ideals in $A(X)$	Irreducible closed subsets of X
Krull dimension n (longest chain of prime ideals)	$\dim X = n$, (longest chain of irreducible closed subsets)
Ring Theory	Geometry/Topology of Affine Varieties.

> Recall that we defined the coordinate ring $A(X) \coloneqq k[x_1, \cdots, x_n]/I(X)$, which contained no nilpotents.

We had some results about dimension

1. $\dim X < \infty$ and $\dim \mathbb{A}^n = n$.
2. $\dim Y + \operatorname{codim}_X Y = \dim X$ when $Y \subset X$ is irreducible.
3. Only over $\bar{k} = k$, $\operatorname{codim}_X V(f) = 1$.

Example 1.1.
Take $V(x^2 + y^2) \subset \mathbb{A}^2/\mathbb{R}$