

# Problem Set One

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## 1 Humphreys 1.1

### 1.1 a

If  $M \in \mathcal{O}$  and  $[\lambda] = \lambda + \Lambda_r$  is any coset of  $\mathfrak{h}^\vee / \Lambda_r$ , let  $M^{[\lambda]}$  be the sum of weight spaces  $M_\mu$  for which  $\mu \in [\lambda]$ .

**Proposition:**  $M^{[\lambda]}$  is a  $U(\mathfrak{g})$ -submodule of  $M$

*Proof:*

Proposition:  $M$  is the direct sum of finitely many submodules of the form  $M^{[\lambda]}$ .

*Proof:*

### 1.2 b

**Proposition:** The weights of an indecomposable module  $M \in \mathcal{O}$  lie in a single coset of  $\mathfrak{h}^\vee / \Lambda_r$ .

## 2 Humphreys 1.3\*

Proposition: For any  $M \in \mathcal{O}$ ,  $M(\lambda)$  satisfies the following property:

$$\mathrm{Hom}_{U(\mathfrak{g})}(M(\lambda), M) = \mathrm{Hom}_{U(\mathfrak{g})}(\mathrm{Ind}_{\mathfrak{b}}^{\mathfrak{g}} \mathbb{C}_\lambda, M) \cong \mathrm{Hom}_{U(\mathfrak{b})}(\mathbb{C}_\lambda, \mathrm{Res}_{\mathfrak{b}}^{\mathfrak{g}} M),$$