MATH 8930 Homework 1 Name _____

This assignment is due Tuesday, July 7, 2020 at the beginning of class.

1. Compute the Jones polynomial of the trefoil knot oriented as in the picture below.

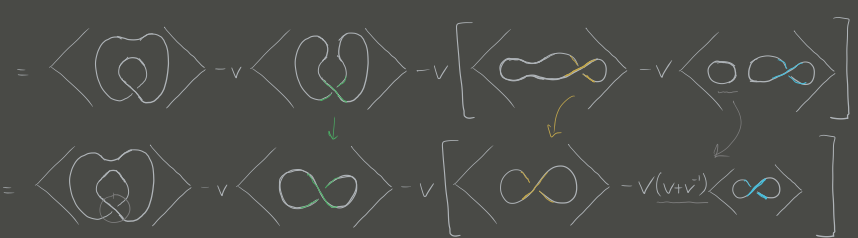
Lemmas

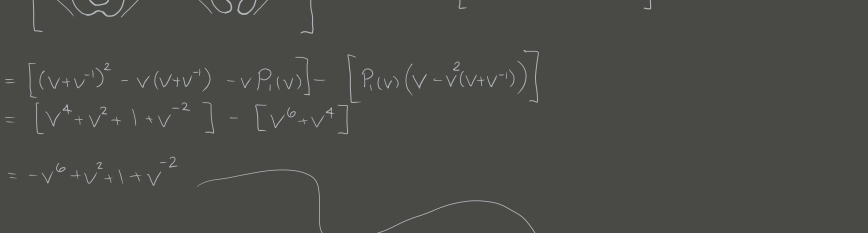
- $\langle \bigcirc \rangle = V + V^{-1}$

- $\langle \bigcirc\bigcirc \rangle = (V+V^{-1})^2 = V^2 + 2 + V^{-2}$

- $\langle \infty \rangle = \langle \asymp \rangle - V \langle \bigcirc\bigcirc \rangle$

$$= (V+V^{-1}) - V(V+V^{-1})^2$$
$$= V + V^{-1} - V(V^2+2+V^{-2})$$
$$= V + V^{-1} - V^3 + 2V + V^{-1}$$
$$= -V^3 - V =: P_1(V)$$

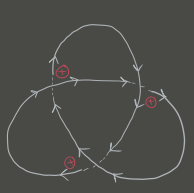




$$= \left[\langle \bigcirc\bigcirc \rangle - V \langle \infty \rangle \right] - V P_1(V) - V \left[P_1(V) - V(V+V^{-1}) P_1(V) \right]$$

$$= \left[(V+V^{-1})^2 - V(V+V^{-1}) - V P_1(V) \right] - \left[P_1(V)\left(V - V^2(V+V^{-1})\right) \right]$$
$$= \left[V^4 + V^2 + 1 + V^{-2} \right] - \left[V^6 + V^4 \right]$$

$$= -V^6 + V^2 + 1 + V^{-2}$$



$\Rightarrow n_+ = 3$, $n_- = 0$ $\Rightarrow$ $J(\text{trefoil}) = (-1)^0 V^{3-2\cdot 0} \langle \text{trefoil} \rangle$

$$= V^3\left(-V^6 + V^2 + 1 + V^{-2}\right)$$

$$= -V^9 + V^5 + V^3 + V.$$