

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): \_\_\_\_\_

Name (print): \_\_\_\_\_

Student Number: \_\_\_\_\_

Instructor's Name: \_\_\_\_\_

Class Time: \_\_\_\_\_

Problem Number	Points Possible	Points Made
1	15	
2	15	
3	12	
4	13	
5	15	
6	15	
Total:	85	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha + \beta) &= \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta), \\ \sin(\alpha + \beta) &= \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta).\end{aligned}$$

1. Let  $\Lambda, \Omega \in \mathbb{R}$  be unknown positive real numbers. Solve the following equations for the independent variable  $x$ .

\_\_\_\_\_ (a) [5 pts]  $\log_{10}(3x + \Omega) = \Lambda$ .

\_\_\_\_\_ (b) [5 pts]  $\Lambda^{x-1} = \Omega$ .

\_\_\_\_\_ (c) [5 pts]  $\frac{1}{\Lambda^x + 1} = \frac{1}{\Omega}$

2. Solve for  $x$  in the following equations:

(a) [5 pts]  $7 \cdot 2^x = 8 \cdot 3^x$

\_\_\_\_\_

(b) [5 pts]  $\log(x^2 + 1) - \log(x + 1) = 2.$

\_\_\_\_\_

(c) [5 pts]  $e^{x^2+1} = e^{2x}.$

\_\_\_\_\_

3. For each description below, determine the formula for the function that matches the description.

\_\_\_\_\_ (a) [6 pts] A function  $W(t)$  that models **exponential growth** where  $W(0) = 4$  and  $W(2) = 6$ .

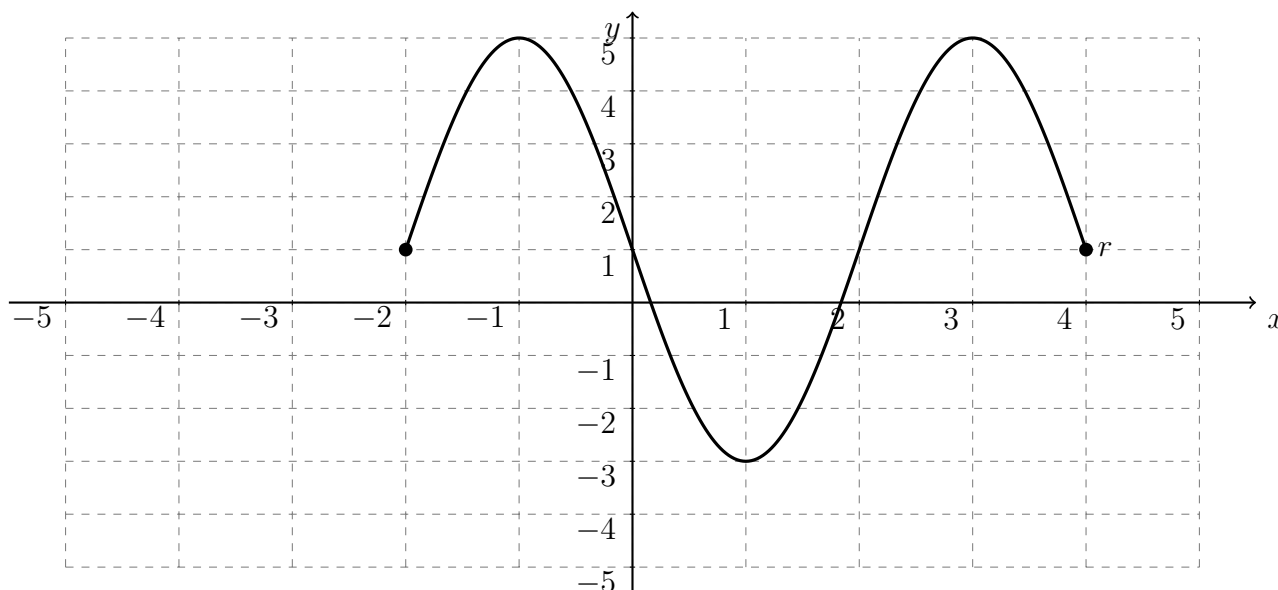
\_\_\_\_\_ (b) [6 pts] The function  $G(x)$  that is the **functional inverse** of the function

$$F(x) = \log_{\alpha}(x).$$

where  $\alpha \in \mathbb{R}$  is an unknown positive real number.

4. Answer each of the following questions relating to **injective** functions.

(a) [6 pts] The graph of a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is shown below.



1. Is this function injective? In at least one sentence, justify why or why not.  
*Hint: you may appeal to a "line test", but you should mention/show a specific line.*
2. What is the domain and range of this function, in interval notation?
3. Determine some new, smaller, restricted domain on which  $f$  is injective. In at least one sentence, justify why  $f$  is injective on this new domain.  
*Hint: you may appeal to a line test again! Your new domain should be some interval contained in the interval you wrote above.*

- (b) [7 pts] Show **using the definition of injectivity** that the following function is injective

\_\_\_\_\_

$$f : \mathbb{R} \rightarrow \mathbb{R}$$
$$x \mapsto \frac{1}{1+x}$$

5. [15 pts] A firm is planning to invest some money into a fund that has an interest rate of 1.1% compounded **once every month**. If they initially invest \$250,000.00 how much money will be in the account after three years?

6. [15 pts] Suppose there is a petri dish filled with bacteria. Suppose the weight of the dish is given by some function  $W(t)$  that models **exponential decay**. At time  $t = 30$  days, it is weighed and found to be  $W(30) = 200$  grams. At time  $t = 50$  days, it is  $W(50) = 150$  grams. How much did it weigh at  $t = 0$  days?



Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): \_\_\_\_\_ Instructor (print): \_\_\_\_\_ Time: \_\_\_\_\_