Problem Set 1

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1.1 Part 1

Let $M = S^2$ as a smooth manifold, and consider a vector field $X : M \to TM$ on M; we want to show that there is a point $p \in M$ such that $X(p) = 0$.

Every vector field on a compact manifold without boundary is complete, and since S^2 is compact with $\partial S^2 = \emptyset$, the vector field X is complete.

Thus every integral curve of X exists for all time, yielding a well-defined flow

$$\phi : M \times \mathbb{R} \to M,$$

and thus a one-parameter family

$$\phi_t : M \to M \in \operatorname{Diff}(M, M).$$

In particular, $\phi_0 = \mathrm{id}_M$, and ϕ_1 is an arbitrary diffeomorphism of M, and moreover ϕ_0 is homotopic to ϕ_1 with homotopy given by

$$H : M \times I \to M(p, t) \mapsto \phi_t(p)$$