# Interesting Topological Spaces in Algebraic Geometry

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  - General type
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- Fake projective planes
- Conics
- Calabi-Yau manifolds
  - Dimension 1: All elliptic curves (up to homeomorphism)
  - Dimension 2: $K3$ surfaces

  - Dimension 3 (threefolds): 500 million +, unknown if infinitely many
  - The bananafold
  - Hyperkähler
- Hurwitz schemes
- Topological galois groups, e.g. $G(\overline{F}/F)$ for $F = \mathbb{Q}, \mathbb{F}_p$.
- Spec $(R)$ for $R$ a DVR (a Sierpinski space)
- Quiver Grassmannians
- Rigid analytic spaces
- Affine line with two origins
- Moduli stack of elliptic curves $\mathcal{M}_{1,1}$.
- Abelian Surface
- Fano Varieties
- Curves: isomorphic to $\mathbb{P}^1$
- Surfaces: Del Pezzo surfaces
- Weighted projective space
- Toric Varieties
- Grassmannian
- Flag Varieties
- Moduli Spaces

Due to Kunihiko Kodaira's classification of complex surfaces, we know that any compact hyperkähler 4-manifold is either a K3 surface or a compact torus T^{4}. (Every Calabi–Yau manifold in 4 (real) dimensions is a hyperkähler manifold, because SU(2) is isomorphic to Sp(1).)

As was discovered by Beauville, the Hilbert scheme of k points on a compact hyperkähler 4-manifold is a hyperkähler manifold of dimension 4k. This gives rise to two series of compact examples: Hilbert schemes of points on a K3 surface and generalized Kummer varieties.