Title

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1 | Lecture 25: Differential Pullback Theorem

This will recover the Riemann-Hurwitz formula by taking degrees.

Lemma 1.0.1 ***(?).***
Let $K/k \subset L/\ell$ be a finite degree extension of function fields, and suppose K/k is regular and L/K is separable. Then ℓ/k and L/ℓ are separable and $L\ell$ is regular.

$$\begin{tikzcd}
 & L & \\
K \arrow[ur, dash, "\text{separable}"] & & \ell \arrow[ul, dash] \\
 & k \arrow[ul, dash, "\text{regular}"] \arrow[ur, dash] &
\end{tikzcd}$$

Link to diagram

Recall some facts/definitions:

- The **adele ring of** K is defined as

$$\mathcal{A}_K := \prod_{v \in \Sigma(K/k)}' K$$

  which is a *restricted direct product*, i.e. each element $\alpha \in \mathcal{A}_K$ has the property that for almost every p, the p-adic valuation of the pth coordinate $v_p(\alpha_p) \geq 0$. There is a diagonal embedding

$$\begin{aligned} K &\hookrightarrow \mathcal{A}_K \\ f &\mapsto (f, f, \cdots). \end{aligned}$$

- For any divisor $D \in \operatorname{Div} K$, define

$$\mathcal{A}_K(D) := \left\{ \alpha \in \mathcal{A}_K \;\middle|\; v_p(\alpha_p) \geq -v_p(D) \ \forall p \right\},$$

  the adelic analog of the Riemann-Roch space.

- A space of linear forms

$$\Omega(D) := \left\{ \omega : \mathcal{A}_K \to A \;\middle|\; \ker \omega \supseteq K + \mathcal{A}_K(D) \right\}$$

  where $D_1 \leq D_2 \implies \Omega_K(D_2) \leq \Omega_K(D_1)$.

- $\Omega_K := \varinjlim D\Omega_K(D)$.