Linearization Continued

Section 8.4 Follow-Up

D. Zack Garza

April 2020

Review

- The Floer equation is given by

$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \text{grad}\, H_t(u) = 0.$$

- We fixed a solution and lifted it to a sphere:

$$u \in C^\infty(S^1 \times \mathbb{R}; W) \quad \mapsto \quad \tilde{u} \in C^\infty(S^2; W)$$

- We use the assumption:
For every $w \in C^\infty(S^2, W)$ there exists a symplectic trivialization of the fiber bundle $w^* TW$, i.e. $\langle c_1(TW),\ \pi_2(W)\rangle = 0$ where c_1 denotes the first Chern class of the bundle TW.

- We use this trivialize the pullback $\tilde{u}^* TW$ to obtain an orthonormal unitary frame

$$\{Z_i\}_{i=1}^{2n} \subset T_{u(s,t)} W$$

where

Review

$$
\begin{tikzcd}
W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \arrow[r, "\iota"] \arrow[rrr, bend left, dashed, "\mathcal{F}_u"] & \mathcal{P}^{1,p}(x,y) \arrow[r, "\mathcal{F}"] \arrow[rr, bend left, dotted, "\bar{\mathcal{F}}"] & L^p\left(\mathbb{R} \times S^1; TW\right) \arrow[r] & L^p\left(\mathbb{R} \times S^1; \mathbb{R}^m\right)
\end{tikzcd}
$$

$$
u \xrightarrow{\quad \bar{\mathcal{F}} \quad} \frac{\partial u}{\partial s} + J(u)\left(\frac{\partial u}{\partial t} - X_t(u)\right)
$$

$$
(y_1, \ldots, y_{2n}) \longrightarrow \exp_u\left(\sum y_i Z_i\right)
$$