Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

Intro

Outline

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x) - \mu(y)$.

What we have so far:

- Define

$$L : W^{1,p}\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right) \longrightarrow L^p\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s,t)Y$$

where

$$S : \mathbb{R} \times S^1 \longrightarrow \mathrm{Mat}(2n; \mathbb{R})$$
$$S(s,t) \overset{s \longrightarrow \pm\infty}{\Longrightarrow} S^{\pm}(t).$$

Outline

- Took $R^{\pm} : I \longrightarrow \mathrm{Sp}(2n;\mathbb{R})$: symplectic paths associated to $S^{\pm}$
- These paths defined $\mu(x), \mu(y)$
- Section 8.7:

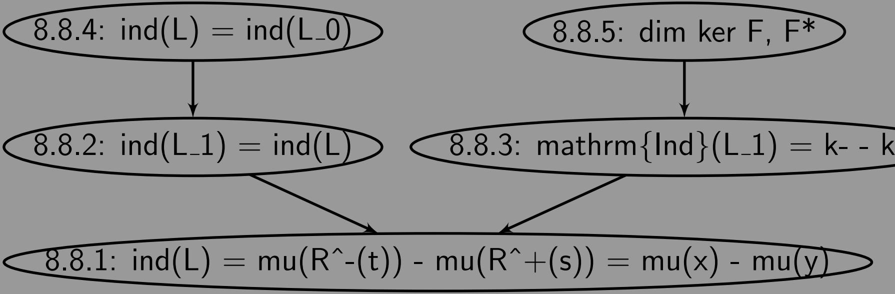
$$R^{\pm} \in \mathcal{S} := \left\{ R(t) \;\middle|\; R(0) = \mathrm{id},\ \det(R(1) - \mathrm{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\mathrm{Ind}(L) \overset{\text{Thm?}}{=} \mu(R^{-}(t)) - \mu(R^{+}(t)) = \mu(x) - \mu(y).$$

From Yesterday

– Proved 8.8.2:



% End of code

Section 1

Outline

asdsdas

Section 2

Outline

asdsadas