

Problem Set 10

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Let ϕ be an n -form. It suffices to show these statements for $n = 2$.

\implies : Suppose ϕ is alternating, then $\phi(b, b) = 0$ for all $b \in B$.

Letting $a, b \in B$ be arbitrary, we then have

$$\begin{aligned}\phi(a + b, a + b) &= \phi(a, a + b) + \phi(b, a + b) \\ &= \phi(a, a) + \phi(a, b) + \phi(b, a) + \phi(b, b) \\ &= \phi(a, b) + \phi(b, a) \\ \implies \phi(a, b) &= -\phi(b, a),\end{aligned}$$

which shows that ϕ is skew-symmetric.

\Leftarrow Suppose ϕ is skew-symmetric, so $\phi(a, b) = -\phi(b, a)$ for all $a, b \in B$.