Manifolds

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These are notes live-tex'd from a graduate course in Smooth Manifolds taught by David Gay at the University of Georgia in Fall 2020. As such, any errors or inaccuracies are almost certainly my own.

D. Zack Garza, Friday 28$^{\text{th}}$ August, 2020
02:20

1 Thursday, August 20

Exercise 1.1.
Show that $\left\{(\mathbb{R}^1, \mathrm{id}), (\mathbb{R}^1, x \mapsto x^3)\right\}$ is *not* a smooth atlas.

Exercise 1.2.
Let $S^1 \coloneqq \left\{(x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + y^2 = 1\right\}$ with charts given by stereographic projection from $(0,1)$ and $(0,-1)$ on $U = S^1 \setminus \{(0,1)\} \longrightarrow \mathbb{R}$ and $V = S^1 \setminus \{(0,-1)\} \longrightarrow \mathbb{R}$.

Show that this gives a smooth atlas.

Exercise 1.3.
Write down a smooth atlas on the unit square.

2 Tuesday, August 25

2.1 Submanifolds

Exercise 2.1.
Prove that charts on a manifold are smooth maps.

> Hint: use the identity smooth structure on $\mathbb{R}^n$.

Exercise 2.2.
Show that open subsets of manifolds are again manifolds in a canonical way.

Exercise 2.3.
Show that S^1 is a manifold.

Example 2.1.
Prove that a submanifold is again a manifold.