Assignment 6: The Fourier Transform

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Assuming the hint, we have

$$\lim_{|\xi| \to \infty} \hat{f}(\xi) = \lim_{\xi' \to 0} \frac{1}{2} \int_{\mathbb{R}^n} (f()) - f(x - \xi')) \exp(-2\pi i x \cdot \xi) \, dx$$

.

But as an immediate consequence, this yields

$$\begin{aligned}
\left|\hat{f}(\xi)\right| &= \left|\int_{\mathbb{R}^n} (f(x) - f(x - \xi')) \exp(-2\pi i x \cdot \xi) \, dx\right| \\
&\leq \int_{\mathbb{R}^n} |f(x) - f(x - \xi')| |\exp(-2\pi i x \cdot \xi)| \, dx \\
&\leq \int_{\mathbb{R}^n} |f(x) - f(x - \xi')| \, dx \\
&\to 0,
\end{aligned}$$

which follows from continuity in L^1 since $f(x - \xi') \to f(x)$ as $\xi' \to 0$.

It thus only remains to show that the hint holds, and that $\xi' \to 0$ as $\xi \to \infty$.

2 Problem 2

3 Problem 3

4 Problem 4

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6 Problem 6