Title

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1 Theorems

1.1 Van Kampen

If $X = U \bigcup V$ where $U, V, U \bigcap V$ are all path-connected then

$$\pi_1(X) = \pi_1 U *_{\pi_1(U \bigcap V)} \pi_1 V,$$

where the amalgamated product can be computed as follows: If we have presentations

$$\begin{aligned}
\pi_1(U, w) &= \left\langle u_1, \cdots, u_k \;\middle|\; \alpha_1, \cdots, \alpha_l \right\rangle \\
\pi_1(V, w) &= \left\langle v_1, \cdots, v_m \;\middle|\; \beta_1, \cdots, \beta_n \right\rangle \\
\pi_1(U \cap V, w) &= \left\langle w_1, \cdots, w_p \;\middle|\; \gamma_1, \cdots, \gamma_q \right\rangle
\end{aligned}$$

then

$$\begin{aligned}
\pi_1(X, w) = &\langle u_1, \cdots, u_k, v_1, \cdots, v_m \rangle \\
&\quad \bmod \left\langle \alpha_1, \cdots, \alpha_l, \beta_1, \cdots, \beta_n, I\left(w_1\right) J\left(w_1\right)^{-1}, \cdots, I\left(w_p\right) J\left(w_p\right)^{-1} \right\rangle \\
= &\frac{\pi_1(U) * \pi_1(B)}{\left\langle \left\{ I(w_i) J(w_i)^{-1} \;\middle|\; 1 \leq i \leq p \right\} \right\rangle}
\end{aligned}$$

where

$$
\begin{aligned}
I &: \pi_1(U \cap V, w) \to \pi_1(U, w) \\
J &: \pi_1(U \cap V, w) \to \pi_1(V, w).
\end{aligned}
$$