Title

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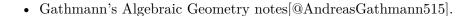
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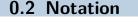
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Prologue







• If a property P is said to hold **locally**, this means that for every point p there is a neighborhood $U_p \ni p$ such that P holds on U_p .

Notation Definition $k[\mathbf{x}] = k[x_1, \cdots, x_n] \\ k(\mathbf{x}) = k(x_1, \cdots, x_n)$ $\mathbf{0.3 \ Polynomial \ ring \ in \ } n \ \mathbf{indeterminates}$ Rational function field in n indeterminates

0.2 Notation 3

$$\left\{ x \in \mathbb{A}^n \mid f(x) \neq 0 \right\} \mid$$

0.4 Summary of Important Concepts

- What is an affine variety?
- What is the coordinate ring of an affine variety?
- What are the constructions $V(\cdot)$ and $I(\cdot)$?
- What is the Nullstellensatz?
- What are the definitions and some examples of:
 - The Zariski topology?
 - Irreducibility?
 - Connectedness?
 - Dimension?
- What is the definition of a presheaf?
 - What are some examples and counterexamples?
- What is the definition of sheaf?
 - What are some examples?
 - What are some presheaves that are not sheaves?
- What is the definition of \mathcal{O}_X , the sheaf of regular functions?
 - How does one compute \mathcal{O}_X for X = D(f) a distinguished open?
- What is a morphism between two affine varieties?
- What is the definition of separatedness?
 - What are some examples of spaces that are and are not separated?
- What is a projective space?
- What is a projective variety?
- What is the projective coordinate ring?
- How does one take the closure of an affine variety X in projective space?
- What is completeness?
 - What are some examples and counterexamples of complete spaces?

0.5 Useful Examples



0.5.1 Varieties

- $V(xy-1) \subseteq \mathbb{A}^2$ a hyperbola
- V(x) a coordinate axis
- V(x-p) a point.

0.5.2 Presheaves / Sheaves

- $C^{\infty}(\cdot, \mathbb{R})$, a sheaf of smooth functions
- $C^0(\cdot,\mathbb{R})$, a sheaf of continuous functions
- $\mathcal{O}_X(\,\cdot\,)$, the sheaf of regular functions on X
- $\underline{\mathbb{R}}(\cdot)$, the constant sheaf associated to \mathbb{R} (locally constant real-valued functions)
- $\operatorname{Hol}(\cdot,\mathbb{C})$, a sheaf of holomorphic functions
- K_p the skyscraper sheaf:

$$K_p(U) \coloneqq \begin{cases} k & p \in U \\ 0 & \text{else.} \end{cases}$$

0.6 The Algebra-Geometry Dictionary

Let $k = \bar{k}$, we're setting up correspondences

| Algebra | Geometry |
|--|--|
| $\frac{1}{k[x_1,\cdots,x_n]}$ | $\mathbb{A}^n_{/k}$ |
| Maximal ideals $\mathfrak{m} = x_1 - p_1, \cdots, x_n - p_n$ | Points $[a_1, \cdots, a_n]$ |
| Radical ideals $J = \sqrt{J} \le k[x_1, \cdots, x_n]$ | V(J) the zero locus |
| Prime ideals $\mathfrak{p} \in \operatorname{Spec}(k[x_1, \cdots, x_n])$ | Irreducible subsets |
| I(S) the ideal of a set | $S \subseteq \mathbb{A}^n$ a subset |
| I + J | $V(I) \cap V(J)$ |
| $\sqrt{I(V) + I(W)}$ | $V \cap W$ |
| $I \cap J, IJ$ | $V(I) \cup V(J)$ |
| $I(V) \cap I(W), \sqrt{I(V)I(W)}$ | $V \cup W$ |
| I(V):I(W) | $\overline{V\setminus W}$ |
| $k[x_1,\cdots,x_n]/I(X)$ | A(X) (Functions on X) |
| A(X) a domain | X is irreducible |
| A(X) indecomposable | X is connected |
| Krull dimension n (chaints of primes) | Topological dimension n (chains of irreducibles) |