# Some thoughts on doing a PhD in topology/geometry

**by Justin Roberts**

**Some thoughts about doing a Phd (in topology/geometry)**

Since it seems likely that I will be asked for advice by students increasingly often, I thought I should try to jot down some thoughts in order to save myself a certain amount of repetition, and also to try to fix what my answers would be. I haven't thought very hard about most of this though, so take it all with a pinch of salt.

**Why should you do a PhD?**

Because you love maths. Of course there are lots of potential reasons for spending time in grad school: you might want to become a research mathematician, or perhaps a teaching mathematician, or just avoid having to make any kind of decision for a few more years. I think in the US it doesn't make much difference to outside career development if you spend 5 years in grad school and then leave to do something completely different. The degree of PhD does seem to be somewhat respected, and the idea of being "overqualified" doesn't seem to be a worry as it seems to be in Britain.

Whatever your goal though, the fact is that trying to prove some theorems and write a thesis is hard work. It demands a certain amount of discipline, isolation, concentration, persistence, and can at times be fairly depressing (you can spend six months stuck on something or find that something you worked very hard to achieve isn't really necessary anyway; someone else may publish your theorem before you, and so on.) The only way it's going to be fun is if you really have an insatiable curiosity and delight in doing maths.

**The first couple of years: lectures and seminars**

In the first couple of years at UCSD you mainly spend time going to lecture courses and passing the quals. Take the quals in topology, algebra, and one of the analysis courses, and get them out of the way as soon as you can. If you are thinking about delaying exams because you need another term or two preparation time, forget it. There's no penalty for taking a qual and getting a mediocre grade and then retaking it later to improve it. Trust yourself!

Besides the basic qual courses, there are three other levels of course available at UCSD. There are what I would call "other basic courses": things like differential geometry, algebraic geometry, geometry and physics, etc. These are all offered on a regular basis (though not necessarily every year) and don't assume much starting background. They would be perfectly suitable as qual courses: don't ask me why they aren't. Then there are "topics courses": usually faculty talking about their special areas, and offered sporadically. These will usually require more background than a first-year would be expected to have. Finally there are seminars, which aren't courses at all. For some reason though they have course numbers and you can register for them on studentlink. I have no idea what the point of this is; it doesn't make any difference to you whether you register or not, but from our point of view you *should* register for any seminars you attend even sporadically, so that the bureaucrats who measure such things can see that our department is actively involving students and doesn't penalise us for teaching under-attended courses.

Recommended courses:

(a) Quals: Topology, Algebra, Real or Complex Analysis

(b) Basic courses: I recommend everyone should learn some differential geometry (either Differential Geometry or Geometry and Physics) and some representation theory (a course called Representation Theory or Lie Groups or Lie Algebras). Algebraic Geometry is not as essential for most topologists but is a great place to learn how "heavy machinery" can be useful in mathematics.

(c) Topics courses: things like Homological algebra, TQFT, Ricci flow, Complex manifolds, are recent examples.

Little initiative is required to potter along quite happily doing qual courses, but you will need at some point to make the transition to becoming a genuine research student, and there are some steps you can take to facilitate this transition.

Firstly, go to lots of courses, at least at the start of each term. You owe it to yourself to find out what the different subjects are about and whether their mathematical "style" pleases you (e.g. are you a calculator or a conceptualiser; do you think with formulae or pictures; how "broad" do you think you are going to be as a mathematician; and so on), plus you get to check out the professors and contemplate your potential compatibility with them as advisors.

Secondly, go to seminars. Many students avoid doing this, probably because they don't have any idea what the announced titles mean, haven't heard of the speakers, think they will be quickly lost, and basically believe seminars are intended for faculty. These are all mistakes!

Going to seminars will gradually make you aware of what is "out there" mathematically: you'll be exposed to lots of different areas of current research in geometry and topology and be able to build up a mental map of what people actually study these days. After a while, seminar titles will start meaning something to you! You will also see lots of "names dropped", and become aware of whose papers you should look at to learn more about given areas. You need to know all this so that you can eventually make an informed decision about what area you want to work on! Seminars are in fact aimed as much at students as at faculty - it would often be more efficient for the faculty to just talk privately to the visiting speaker if hearing the proof of his/her latest theorem was the only point of the visit. It's true that when you first start going to seminars, you'll be mainly lost. (I got almost nothing out of the many seminars on exotic cohomology theories I attended in my first term in Cambridge. But at least by attending loyally I got to meet the faculty and the speakers and feel accepted as "a topologist"). This never entirely disappears! However, all decent speakers attempt to start at a level that everybody can understand, and gradually up the tempo. When in (say) the last fifteen minutes they actually try to explain their proof, it's reasonable to expect that few people unfamiliar with the area are still following all the details, if anything. But the point is that the amount you are able to follow will increase the more you're exposed to seminars - so you may as well start as soon as you can. Moreover, speakers *like* to have people ask questions (so as to feel that people are actually listening to them, and also to have the satisfaction of answering them!) so don't be afraid to interrupt.

Another non-obvious point is that lecture courses, by their nature, are constructed to be as linear as possible: an orderly sequence of definition-theorem-proof is the norm (with, one hopes, some motivation too!). In a seminar you will see "real" maths: one starts from a question, problem, or example; explains what's known about it; describes (sometimes several different attempts at) a solution; and lists the remaining unanswered questions or new problems thrown up by the work. This is organic, living, rough-at-the-edges maths; a seminar typically is closer to a mathematical conversation than to a reading from a textbook.

**To be continued!**

When I have the energy I intend to say a few things about actually doing research, such as: learning how to talk maths: talk to each other as well as to professors

learning to solve problems

learning how to read papers

how to write and give talks clearly

don't waste time teaching calculus

finding an advisor

finding a topic

finding a problem

doing research: good practices

Areas of topology

My interests

**Research resources**

It seems like a good idea to try to put together a batch of useful library-type resources. Play with the following things so that you know what's out there and available! (Thanks to Li Yu and Orest Bucicovschi for providing lots of these links.)

**UCSD Library**

The Roger UCSD library catalogue. Sometimes a little tricky to use, especially for conference proceedings edited by bunches of authors. Some journals are available online directly through the library.

The UCSD Library Avanti service They send you pdf files of papers on demand, so that you don't have to go to the library at all! Go to the website to register and they will set up an account for you. Be sure to use the email address of Math department. Then every time you need a paper, you just send them an request email with the full description of the paper you need. They will send the paper you need to your account and give a notice to your email address.

**Electronically available journals**

How to become a pure mathematician A really useful looking compilation of links, texts and so on by someone in Hong Kong.

Online texts If you want to look up some mathematical question online it's often easiest to just try to find someone's lecture notes covering the appropriate stuff. Google is pretty good at this since it can read postscript and pdf files. But here's someone's list.

A pretty good list of online books and notes by "Alex Stef"

Another online booklist

Numdam A site where you can get old French journals, in particular Publications IHES.

The front This is a nice front page (done by Greg Kuperberg) for the "xxx archive server" where people have been sticking their preprints since about 1992. If you want to find a paper from the last ten years or so, try here first. The official front page is here; the search engine is sort of crappy, make sure you turn on "all years"!

J-Stor The J-stor archive has online copies of various maths journals including the Annals and AMS Proceedings and Transactions going back quite a long way.

Digital Mathematics Library A list of online retrodigitized journals including the J-Stor and Numdam ones.

Goettingen I'm not quite sure what this does! German index of digitised papers.

Electronic Library of Mathematics Another list of digitally available journals

Grenoble Some kind of search engine I haven't tried yet!

Allen Knutson's advice Allen is a symplectic geometer; I think his thoughts about doing Phds and student/advisor interaction are really good.

Allen Hatcher's recommended books This is a helpful list of good books on topology and related subjects.

Geometry and Topology Top quality online journal based at Warwick University.

Algebraic and Geometric Topology The sister publication (of which I'm an editor!)

If you know any other good things that should be here, send me an email! justin@math.ucsd.edu