Title

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1.1 Review

Let $k = \bar{k}$, we're setting up correspondences

$$\begin{aligned}
\text{Ring Theory} &\qquad \text{Geometry/Topology of Affine Varieties} \\
\text{Polynomial functions} &\qquad \text{Affine space} \\
k[x_1, \cdots, x_n] &\qquad \mathbb{A}^n/k \coloneqq \{[a_1, \cdots, a_n] \in k^n\} \\
\text{Maximal ideals } \langle x_1 - a_1, \cdots, x_n - a_n \rangle &\qquad \text{Points } [a_1, \cdots, a_n] \in \mathbb{A}^n/k \\
\text{Radical ideals } I \trianglelefteq k[x_1, \cdots, x_n] &\qquad \text{Affine varieties } X \subset \mathbb{A}^n/k, \text{ vanishing locii of polynomials} \\
&I \mapsto V(I) \coloneqq \left\{ a \;\middle|\; f(a) = 0 \forall f \in I \right\} \\
I(X) \coloneqq \left\{ f \;\middle|\; f|_X = 0 \right\} \mapsfrom X & \\
\text{Radical ideals containing } I(X) \text{, i.e. ideals in } A(X) &\qquad \text{closed subsets of } X \text{, i.e. affine subvarieties} \\
A(X) \text{ is a domain} &\qquad X \text{ irreducible} \\
A(X) \text{ is not a direct sum} &\qquad X \text{ connected} \\
\text{Prime ideals in } A(X) &\qquad \text{Irreducible closed subsets of } X \\
\text{Krull dimension } n \text{ (longest chain of prime ideals)} &\qquad \dim X = n, \text{ (longest chain of irreducible closed subsets).}
\end{aligned}$$

> Recall that we defined the coordinate ring $A(X) \coloneqq k[x_1, \cdots, x_n]/I(X)$, which contained no nilpotents.

We had some results about dimension

1. $\dim X < \infty$ and $\dim \mathbb{A}^n = n$.
2. $\dim Y + \operatorname{codim}_X Y = \dim X$ when $Y \subset X$ is irreducible.
3. Only over $\bar{k} = k$, $\operatorname{codim}_X V(f) = 1$.

Example 1.1.
Take $V(x^2 + y^2) \subset \mathbb{A}^2/\mathbb{R}$

Definition 1.0.1 (?).
An affine variety Y of

- $\dim Y = 1$ is a **curve**,
- $\dim Y = 2$ is a **surface**,
- $\mathrm{codim}_X Y = 1$ is a **hypersurface in** X

Question: Is every hypersurface the vanishing locus of a *single* polynomials $f \in A(X)$.