Title

D. Zack Garza

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Questions to look at for next Tuesday:

Exercise 1.1 (?).
Show that the 3 natural coordinate charts on $\mathbb{CP}^2$ given by e.g. $\varphi_{U_0}([z_0 : z_1 : z_2]) = \left[\frac{z_1}{z_0}, \frac{z_2}{z_0}\right]$ yield a smooth atlas.

Exercise 1.2 (?).
Consider the map

$$\begin{aligned}
\pi : \mathbb{CP}^2 &\to \mathbb{R}^2 \\
[z_0 : z_1 : z_2] &\mapsto \left[\frac{|z|_1^2}{|z|_0^2 + |z|_1^2 + |z|_2^2}, \frac{|z|_2^2}{|z|_0^2 + |z|_1^2 + |z|_2^2}\right].
\end{aligned}$$

Show that π is smooth and $\mathrm{im}\pi = \{p_1, p_2 \geq 0, p_1 + p_2 \leq 1\}$.

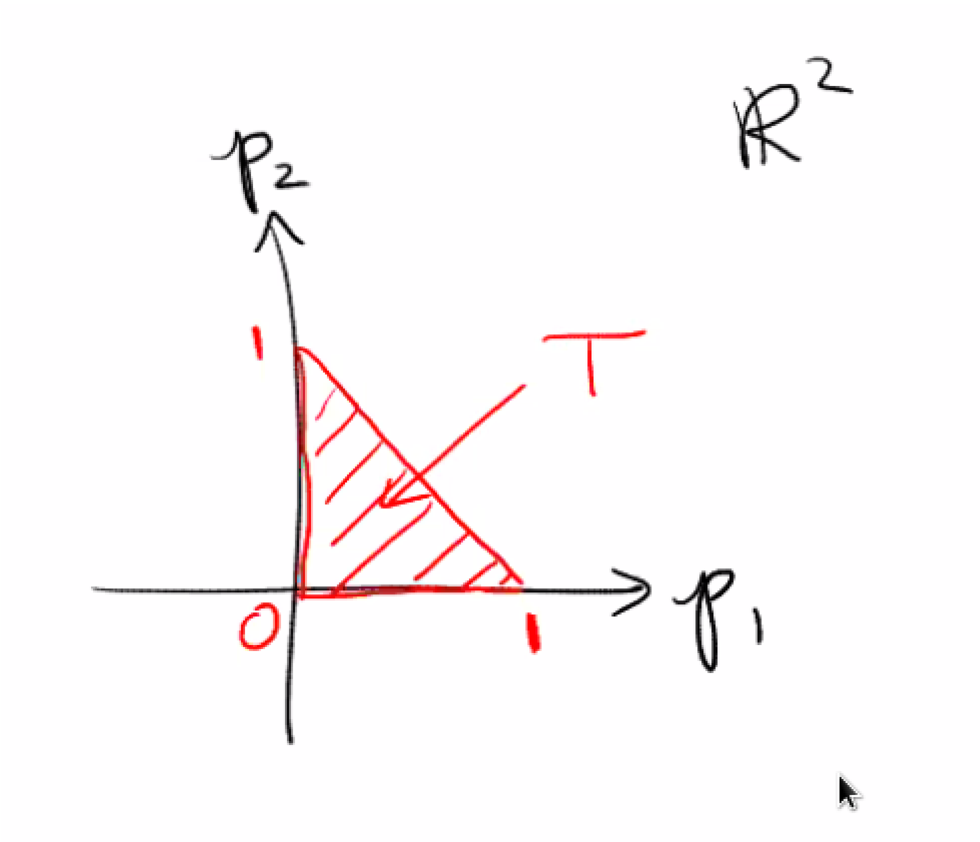

Figure 1: O

Exercise 1.3 (?)**.**
Show that

- If $[p_1, p_2] \in T^\circ$ is in the interior of the above triangle, then $\pi^{-1}(p_1, p_2) \cong S^1 \times S^1$ is diffeomorphic to a torus.
- If the point is on an edge, the fiber is diffeomorphic to S^1,
- If the point is on a vertex, the fiber is a single point.

Exercise 1.4 (?)**.**
Find a vector field V on some maximal subset of $\mathbb{CP}^2$ such that $D\pi(V) = p_1\partial_{p_1} + p_2\partial_{p_2}$ (the radial vector field).

I.e., for all $q \in \mathbb{CP}^2$, we have a map

$$D_1\pi : T_1\mathbb{CP}^2 \to T_{\pi(q)}\mathbb{R}^2$$

and $V(q) \in T_q\mathbb{CP}^2$, so we want $D_q\pi(V(q)) = p_1\partial_{p_1} + p_2\partial_{p_2}$.

Note that there will be a problem defining V on the fiber over the hypotenuse of T.

Theorem 1.1 ***(Collar Neighborhood).***
For all manifolds with boundary X, there exists an open neighborhood N of ∂X which is diffeomorphic to $(-\varepsilon, 0] \times \partial X$.

Proof strategy: construct a vector field pointing outward and flow it backward. Construct by forming local vector fields on open sets, then patch together using a partition of unity.

Definition 1.1.1 (Partition of Unity)**.**
A collection $\left\{\varphi_i : M \to \mathbb{R} \;\middle|\; i \in I\right\}$ such that

1. $\{\mathrm{supp}\varphi_i\}$ is locally finite, i.e. for all p, we have $\left|\left\{i \;\middle|\; p \in \mathrm{supp}(\varphi_i)\right\}\right| < \infty$.
2. $\varphi(p) \geq 0$ for all $p \in X$
3. For all $p \in X$, the sum $\sum_{i \in I} \varphi_i(p) = 1$.