Problem Set 5

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October 21, 2019

Contents

1 Problem 1

We first make the following definitions:

$$\begin{aligned}
S &\coloneqq \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} a_{jk} = \sup \left\{ \sum_{(j,k)\in B} a_{jk} \ni B \subset \mathbb{N}^2,\ |B| < \infty \right\} \\
T &\coloneqq \sum_{k=1}^{\infty}\sum_{j=1}^{\infty} a_{kj} = \sup \left\{ \sum_{(k,j)\in C} a_{kj} \ni C \subset \mathbb{N}^2,\ |B| < \infty \right\}.
\end{aligned}$$

We will show that $S = T$ by showing that $S \leq T$ and $T \leq S$.

1.1 $S \leq T$:

Let $B \subset \mathbb{N}^2$ be finite, so $B \subseteq [0, I] \times [0, J] \subset \mathbb{N}^2$. Now letting $R > \max(I, J)$, we can define $C = [0, R]^2$, which satisfies $B \subseteq C \subset \mathbb{N}^2$ and $|C| < \infty$. Moreover, since $a_{jk} \geq 0$ for all pairs (j, k), we have the following inequality:

$$\sum_{(j,k)\in B} a_{jk} < \sum_{(k,j)\in C} a_{jk} \leq \sum_{(j,k)\in C} a_{jk} \leq T,$$

since T is a supremum over *all* such sets C. But since this holds for every B, we this inequality also holds for the supremum of the smaller term by order-limit laws, and so

$$S \coloneqq \sup_{B} \sum_{(j,k)\in B} a_{jk} \leq T.$$