

# Title

D. Zack Garza

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Recall the definition of a presheaf: a sheaf of rings on a space is a contravariant functor from its category of open sets to ring, such that

1.  $F(\emptyset) = 0$
2. The restriction from  $U$  to itself is the identity,
3. Restrictions compose.

Examples:

- Smooth functions on  $\mathbb{R}^n$
- Holomorphic functions on  $\mathbb{C}$

Recall the definition of sheaf: a presheaf satisfying *unique* gluing: given  $f_i \in \mathcal{F}(U_i)$ , such that  $f_i|_{U_i \cap U_j} = f_j|_{U_i \cap U_j}$  implies that there exists a unique  $f \in \mathcal{F}(\cup U_i)$  such that  $f|_{U_i} = f_i$ .

Question: Are the constant functions on  $\mathbb{R}$  a presheaf and/or a sheaf?

Answer: This is a presheaf but not a sheaf. Set  $\mathcal{F}(U) = \{f : U \rightarrow \mathbb{R} \mid f(x) = c\} \cong \mathbb{R}$  with  $\mathcal{F}(\emptyset) = 0$ . Can check that restrictions of constant functions are constant, the composition of restrictions is the overall restriction, and restriction from  $U$  to itself gives the function back.

Given constant functions  $f_i \in \mathcal{F}(U_i)$ , does there exist a unique constant function  $\mathcal{F}(\cup U_i)$  restricting to them? No: take  $f_1 = 1$  on  $(0, 1)$  and  $f_2 = 2$  on  $(2, 3)$ . Can check that they both restrict to the zero function on the intersection, since these sets are disjoint.

How can we make this into a sheaf? One way: weaken the topology. Another way: define another presheaf  $\mathcal{G}$  on  $\mathbb{R}$  given by *locally* constant function