Linearization Continued

Section 8.4 Follow-Up

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Review

Definitions

- The Floer equation is given by

$$\frac{\partial u}{\partial s} + J(u)\frac{\partial u}{\partial t} + \operatorname{grad} H_t(u) = 0.$$

- We fixed a solution and lifted it to a sphere:

$$u \in C^\infty(S^1 \times \mathbb{R}; W) \quad \mapsto \quad \tilde{u} \in C^\infty(S^2; W)$$

- We use the assumption:
  For every $w \in C^\infty(S^2, W)$ there exists a symplectic trivialization of the fiber bundle $w^* TW$, i.e. $\langle c_1(TW),\ \pi_2(W)\rangle = 0$ where c_1 denotes the first Chern class of the bundle TW.

- We use this trivialize the pullback $\tilde{u}^* TW$ to obtain an orthonormal unitary frame

$$\{Z_i\}_{i=1}^{2n} \subset T_{u(s,t)} W$$

Order 0 Part is Symmetric in the Limit

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Theorem (8.4.4, CR + Symmetric in the Limit)

If u solves Floer's equation, then

$$(d\mathcal{F})_u = \bar{\partial} + S(s,t)$$

where

1. *S is linear*
2. *S tends to a symmetric operator as $s \longrightarrow \pm\infty$, and*
3. *We have the limiting behavior*

$$\frac{\partial S}{\partial s}(s,t) \xrightarrow{s \longrightarrow \pm\infty} 0 \quad \text{uniformly in } t$$

Proof

Collect terms in the order zero part:

Linearization of Hamilton's Equation

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Recall

$$(d\mathcal{F})_u = \bar{\partial} Y + SY = \left(\bar{\partial} + S\right) Y$$

Now think of S as a map $Y \mapsto S \cdot Y$, so $S \in C^\infty(\mathbb{R} \times S^1; \mathrm{End}(\mathbb{R}^{2n}))$ and define the symmetric operators

$$S^\pm := \lim_{s \longrightarrow \pm\infty} S(s, \cdot) \quad \text{respectively}$$

Theorem

The equation

$$\partial_t Y = J_0 S^\pm Y$$

is a linearization of Hamilton's equation

$$\frac{\partial z}{\partial t} = X_t(z) \quad at \quad \begin{cases} x = \lim_{s \longrightarrow -\infty} u & \text{for } S^- \\ y = \lim_{s \longrightarrow \infty} u & \text{for } S^+ \end{cases} \quad \textit{respectively.}$$