

# Title

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Question: how do we define  $h_{V,D}$ ?

Answer: write  $D = D_1 - D_2$  which are (very) ample divisors and basepoint free. We then obtain embeddings

$$\begin{aligned}\varphi_1 : V &\hookrightarrow \mathbb{P}_K^{n_1} \\ \varphi_2 : V &\hookrightarrow \mathbb{P}_K^{n_2}.\end{aligned}$$

So write

$$h_{V,D}(p) = h(\varphi_1(p)) - h(\varphi_2(p)) + O(1)$$

#### Example 1.1.

For  $E/K$  an elliptic curve,

- $2[0]$  is an ample divisor
- $3[0]$  is a very ample divisor.

Let  $K$  be a local field (i.e.  $\mathbb{C}, \mathbb{R}$ , a  $p$ -adic field, or  $\mathbb{F}_q((t))$  formal Laurent series) and  $A/K$  be an abelian variety; we want to understand  $A(K)$ . We know this has the structure of compact abelian  $K$ -analytic Lie group.

- Question 1: What does Lie theory say?
- Question 2: What extra information comes from  $A/K$  being a  $g$ -dimensional abelian variety?

If  $K = \mathbb{C}$ , then  $A(K) \cong (\mathbb{R}/\mathbb{Z})^{2g}$ . If  $K = \mathbb{R}$ , then  $A(K) \cong (\mathbb{R}/\mathbb{Z})^g \oplus \prod_{i=1}^d \mathbb{Z}/2\mathbb{Z}$  where  $0 \leq d \leq g$ .