# Title

D. Zack Garza

# **Table of Contents**

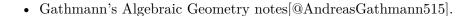
## **Contents**

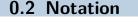
Table o	of Contents	2
Prologi	Je	3
0.1	References	3
	Notation	
0.3	Polynomial ring in $n$ indeterminates	3
0.4	Summary of Important Concepts	4
0.5	Useful Examples	5
	0.5.1 Varieties	
	0.5.2 Presheaves / Sheaves	
0.6	The Algebra-Geometry Dictionary	5

Table of Contents

### **Prologue**







• If a property P is said to hold **locally**, this means that for every point p there is a neighborhood  $U_p \ni p$  such that P holds on  $U_p$ .

Notation Definition  $k[\mathbf{x}] = k[x_1, \cdots, x_n] \\ k(\mathbf{x}) = k(x_1, \cdots, x_n)$   $\mathbf{0.3 \ Polynomial \ ring \ in \ } n \ \mathbf{indeterminates}$ Rational function field in n indeterminates

0.2 Notation 3

$$\left\{ x \in \mathbb{A}^n \mid f(x) \neq 0 \right\} \mid +$$

Fruit	Price	Advantages
Bananas	\$1.34	<ul><li>built-in wrapper</li><li>bright color</li></ul>
Oranges	\$2.10	<ul><li>cures scurvy</li><li>tasty</li></ul>

#### 0.4 Summary of Important Concepts

- What is an affine variety?
- What is the coordinate ring of an affine variety?
- What are the constructions  $V(\cdot)$  and  $I(\cdot)$ ?
- What is the Nullstellensatz?
- What are the definitions and some examples of:
  - The Zariski topology?
  - Irreducibility?
  - Connectedness?
  - Dimension?
- What is the definition of a presheaf?
  - What are some examples and counterexamples?
- What is the definition of sheaf?
  - What are some examples?
  - What are some presheaves that are not sheaves?
- What is the definition of  $\mathcal{O}_X$ , the sheaf of regular functions?
  - How does one compute  $\mathcal{O}_X$  for X = D(f) a distinguished open?
- What is a morphism between two affine varieties?
- What is the definition of separatedness?
  - What are some examples of spaces that are and are not separated?
- What is a projective space?
- What is a projective variety?
- What is the projective coordinate ring?
- How does one take the closure of an affine variety X in projective space?
- What is completeness?
  - What are some examples and counterexamples of complete spaces?

#### 0.5 Useful Examples



#### 0.5.1 Varieties

- $V(xy-1) \subseteq \mathbb{A}^2$  a hyperbola
- V(x) a coordinate axis
- V(x-p) a point.

#### 0.5.2 Presheaves / Sheaves

- $C^{\infty}(\cdot, \mathbb{R})$ , a sheaf of smooth functions
- $C^0(\cdot,\mathbb{R})$ , a sheaf of continuous functions
- $\mathcal{O}_X(\,\cdot\,)$ , the sheaf of regular functions on X
- $\underline{\mathbb{R}}(\cdot)$ , the constant sheaf associated to  $\mathbb{R}$  (locally constant real-valued functions)
- $\operatorname{Hol}(\cdot,\mathbb{C})$ , a sheaf of holomorphic functions
- $K_p$  the skyscraper sheaf:

$$K_p(U) \coloneqq \begin{cases} k & p \in U \\ 0 & \text{else.} \end{cases}$$

### 0.6 The Algebra-Geometry Dictionary

Let  $k = \bar{k}$ , we're setting up correspondences

Algebra	Geometry
$\frac{1}{k[x_1,\cdots,x_n]}$	$\mathbb{A}^n_{/k}$
Maximal ideals $\mathfrak{m} = x_1 - p_1, \cdots, x_n - p_n$	Points $[a_1, \cdots, a_n]$
Radical ideals $J = \sqrt{J} \le k[x_1, \cdots, x_n]$	V(J) the zero locus
Prime ideals $\mathfrak{p} \in \operatorname{Spec}(k[x_1, \cdots, x_n])$	Irreducible subsets
I(S) the ideal of a set	$S \subseteq \mathbb{A}^n$ a subset
I + J	$V(I) \cap V(J)$
$\sqrt{I(V) + I(W)}$	$V \cap W$
$I \cap J, IJ$	$V(I) \cup V(J)$
$I(V) \cap I(W), \sqrt{I(V)I(W)}$	$V \cup W$
I(V):I(W)	$\overline{V\setminus W}$
$k[x_1,\cdots,x_n]/I(X)$	A(X) (Functions on $X$ )
A(X) a domain	X is irreducible
A(X) indecomposable	X is connected
Krull dimension $n$ (chaints of primes)	Topological dimension $n$ (chains of irreducibles)