

# Title

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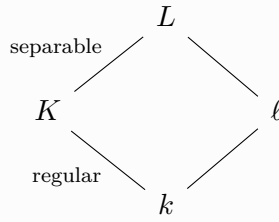
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# 1 | Lecture 25: Differential Pullback Theorem

This will recover the Riemann-Hurwitz formula by taking degrees.

## Lemma 1.0.1(?).

Let  $K/k \subset L/\ell$  be a finite degree extension of function fields, and suppose  $K/k$  is regular and  $L/K$  is separable. Then  $\ell/k$  and  $L/\ell$  are separable and  $L\ell$  is regular.



[Link to diagram](#)

Recall some facts/definitions:

- The **adele ring** of  $K$  is defined as

$$\mathcal{A}_K := \prod'_{v \in \Sigma(K/k)} K$$

which is a *restricted direct product*, i.e. each element  $\alpha \in \mathcal{A}_K$  has the property that for almost every  $p$ , the  $p$ -adic valuation of the  $p$ th coordinate  $v_p(\alpha_p) \geq 0$ . There is a diagonal embedding

$$\begin{aligned} K &\hookrightarrow \mathcal{A}_K \\ f &\mapsto (f, f, \dots). \end{aligned}$$

- For any divisor  $D \in \text{Div } K$ , define

$$\mathcal{A}_K(D) := \left\{ \alpha \in \mathcal{A}_K \mid v_p(\alpha_p) \geq -v_p(D) \ \forall p \right\},$$

the adelic analog of the Riemann-Roch space.

- A space of linear forms

$$\Omega(D) := \left\{ \omega : \mathcal{A}_K \rightarrow A \mid \ker \omega \supseteq K + \mathcal{A}_K(D) \right\}$$

where  $D_1 \leq D_2 \implies \Omega_K(D_2) \leq \Omega_K(D_1)$ .

- $\Omega_K := \varinjlim_D \Omega_K(D)$ .

- For any  $\omega \in \Omega_K^\bullet$ ,  $(\omega) := \max \left\{ D \mid \omega = 0 \text{ on } \mathcal{A}_K(D) + K \right\}$ .

- $\mathcal{A}_{L/K} = \left\{ \alpha \in \mathcal{A}_L \mid \alpha q_1 = \alpha q_2 \text{ if } Q_1, Q_2/p \right\} \leq_{\text{Vect}_\ell} \mathcal{A}_L$

- The **adelic trace map**

$$\begin{aligned} \text{Tr}_{L/K} : \mathcal{A}_{L/K} &\rightarrow \mathcal{A}_K \\ \alpha &\mapsto \text{Tr}_{L/K}(\alpha)/p = \text{Tr}_{L/K}(\alpha_Q) \end{aligned} \quad \text{for any } Q/p.$$

**Theorem 1.0.2(?)**.

Let  $\omega \in \Omega_K$ , then there exists a unique  $\omega^* \in \mathcal{A}_L$  such that

- For all  $\alpha \in \mathcal{A}_{L/K}$ , we have  $\text{Tr}_{\ell/k} \omega^*(\alpha) = \omega(\text{Tr}_{L/K}(\alpha))$ .

$\omega^*$  is formally denoted  $\text{Cotr}_{L/K}(\omega)$  and called the **cotrace** of  $\omega$ .