

# Title

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Friday 21<sup>st</sup> August, 2020

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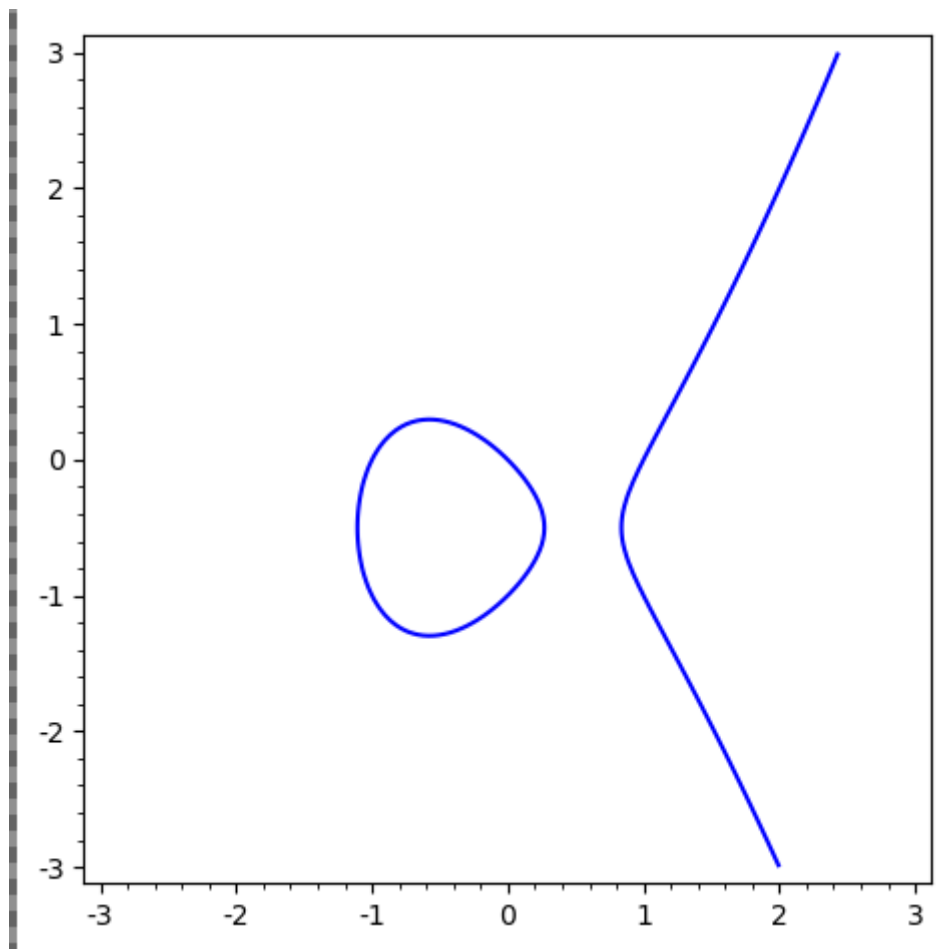
## 1 Friday, August 21

Reference:  
<https://www.mathematik.uni-kl.de/~gathmann/class/alggeom-2019/alggeom-2019.pdf>

General idea: functions a coordinate ring  $R[x_1, \dots, x_n]/I$  will correspond to the geometry of the variety cut out by  $I$ .

### Example 1.1.

- $x^2 + y^2 - 1$  defines a circle, say, over  $\mathbb{R}$
- $y^2 = x^3 - x$  gives an elliptic curve:



- $x^n + y^n = 1$ : does it even contain a  $\mathbb{Q}$ -point? (Fermat's Last Theorem)
- The variety  $\langle x^2 + 1 \rangle$ , which has no  $\mathbb{R}$ -points.
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**Theorem 1.1 (Harnack Curve Theorem).**

If  $f \in \mathbb{R}[x, y]$  is of degree  $d$ , then

$$\pi_1 V(f) \subseteq \mathbb{R}^2 \leq 1 + \frac{(d-1)(d-2)}{2}$$

Actual statement: the number of connected components is bounded above by this quantity.