Real Analysis

D. Zack Garza

August 15, 2019

Contents

---

1 Lecture 1 (Thu 15 Aug 2019 11:04)

> See Folland's Real Analysis, definitely a recommended reference.

Possible first day question: how can we "measure" a subset of $\mathbb{R}$? We'd like bigger sets to have a higher measure, we wouldn't want removing points to increase the measure, etc. This is not quite possible, at least something that works on *all* subsets of $\mathbb{R}$. We'll come back to this in a few lectures.

Notions of "smallness" in $\mathbb{R}$

Definition: Let E be a set, then E is *countable* if it is in a one-to-one correspondence with $E' \subseteq \mathbb{N}$, which includes $\emptyset, \mathbb{N}$.

Definition: E is *meager* (or of *1st category*) if it can be written as a countable union of **nowhere dense** sets.

Intuitively, a set is *nowhere dense* if it is full of holes. Recall that a $X \subseteq Y$ is dense in Y iff the closure of X is all of Y. So we'll make the following definition.

Definition: A set $A \subseteq \mathbb{R}$ is *nowhere dense* if every interval I contains a subinterval $S \subseteq I$ such that $S \subseteq A^c$.

Note that a finite union of nowhere dense sets is also nowhere dense, which is why we're giving a name to such a countable union above.

Equivalently, - A^c contains a dense, open set.