

Title

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Recommended exercises:

- 0.9
- 0.5 (easy)
- 0.10

Taken:

- 0.11
- 0.3
- 0.4

Exercise 1.1 (0.9).

Let R_1, R_2 be two k -algebras that are also domains with fraction fields K_i .

Show that $R_1 \otimes_k R_2$ is a domain $\iff K_1 \otimes_k K_2$ is a domain.

Exercise 1.2 (0.9).

Let k be a field and $d \geq 2$ with $4 \nmid d$ and $p \in k[x]$ a polynomial of positive degree.

Factor p in $\bar{k}[x]$ as $\prod_{i=1}^r (x - a_i)^{e_i}$, and suppose there is some i such that $d \nmid e_i$. Show that

$$f(x, y) := y^d - p(x) \in k[x, y]$$

is geometrically irreducible.

Conclude that

$$ff(k[x, y]/\langle f \rangle).$$

is a regular one-variable function field over k .

Solution:

Recall:

- For L/K ,
- A polynomial $f \in k[t_i]$ is *geometrically irreducible* iff $f \in \bar{k}[t_i]$ is irreducible as a polynomial, i.e. if $f = pq \implies p = 1$ or $q = 1$.
- A field extension L/k is *regular* iff any of the following conditions hold:
 - $\kappa(L) = k$ and L/k is separable, where $\kappa(L)$ is the field of elements of L algebraic over k
 - $L \otimes_k \bar{k}$ is a domain or a field.
 - For all L'/k , $L \otimes_k L'$ is a domain.