Goal: Compute $\widehat{HF}(Y)$ for $Y \in \mathrm{Mfd}^3_{C^\infty(\mathbb{R})}$

$\{$Admissible diagrams$\}$ $\rightsquigarrow$ $\{$ Nice diagrams$\}$

- isotopies
- handleslides

For every generator x,

$\phi \in \pi_2(x,x) \Rightarrow \phi$ is trivial

Domain

Notation: $\alpha\beta := \{\alpha_1, \dots, \alpha_g, \beta_1, \dots, \beta_g\}$

$\Sigma_{\alpha\beta} := \Sigma \setminus \alpha\beta$

Defs

- $(\Sigma, \vec{\alpha}, \vec{\beta}, \vec{z})$ a pointed Heegaard diagram
- $C_1 := \mathbb{Z}\langle \vec{x}_i \rangle$ where $\vec{x}_i := \sum_{j=1}^{n} x_j$ is a divisor on Σ, $n = g+k-1$, all coefs 1?, $\hookrightarrow \mathbb{Z}\langle \text{Points of } \Sigma \rangle$
- Region: $R_i \in \pi_0 \Sigma_{\alpha\beta}$ a connected component
- $C_2 := \mathbb{Z}\langle R_i \in \pi_0 \Sigma_{\alpha\beta} \rangle$ 2-chains
- For $\vec{x}, \vec{y} \in C_1$, $\pi_2(x,y) := \{ \phi \in C_2 \mid \partial(\partial(\phi)|_\alpha) = \vec{y} - \vec{x} \} \in C_2$ domains.
- $E : \mathbb{N} \times C_2 \to \mathbb{Z}$ $\rightsquigarrow$ $n_p : C_2 \to \mathbb{Z}$

$(j, R = \sum_{j=1}^{|\pi_0\Sigma_{\alpha\beta}|} r_j R_j) \mapsto r_j$ $\qquad$ $R = \sum_{j=1}^{|\pi_0 \Sigma_{\alpha\beta}|} r_j R_j \mapsto E(j(p), \phi)$

How are the coefs determined?

In words: the coef of the region containing p.

where $j(p) : \text{Points}(\Sigma) \to \mathbb{N}$
$p \mapsto j$, where $R_j \ni p$ is the region containing p.

- $R \geq 0$ positive $\Leftrightarrow$ $n_p(R) \geq 0$ $\forall p \in \Sigma_{\alpha\beta}$
- $\pi_2^0(\vec{x}, \vec{y}) := \{ \phi \in \pi_2(x,y) \mid n_{p_{\vec{z}_i}}(\phi) = 0 \ \forall i \}$
- Admissible $\{ R \geq 0 \in C_2 \} \cap \pi_2^0(x,y) \} = \{$ Trivial domains? $\}$
- $\partial \vec{x} = \sum_{\vec{y}} \sum_{\substack{\phi \in \pi_2^0(\vec{x},\vec{y}) \\ \mu(\phi)=1}} c(\phi) \cdot \vec{y}$ $\qquad$ $c(\phi) := \mathcal{M}(\phi)/\mathbb{R}$

The Algorithm

- "Good": Bigon & square regions

1) Annihilate non-disc regions
   - "Finger moves" on β_i $\rightsquigarrow$ new intersections with α_i
     ↳ Enough times makes every R_i a disc.
   - Very short procedure.

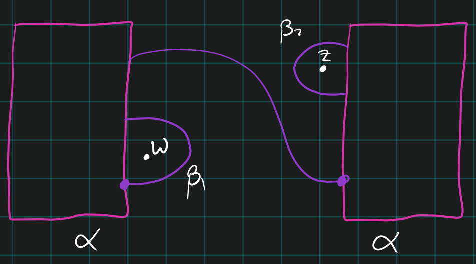
2) Convert all but one region to "good"
   - Now only have disc regions
   - Define: distance fn on regions $d: C_2 \longrightarrow \mathbb{Z}^{\geq 0}$
     - badness $f_n(R) = \max\{n-2, 0\}$, where R is a $2n$-gon
     - dist. fn. on diagram $\mathcal{H}$: $d(\mathcal{H}) := \max\{d(R) \mid R \in C_2 \text{ bad}\}$
     - complexity fn on $\mathcal{H}$ $\rightsquigarrow$ tuple $c_d(\mathcal{H})$
   - Lem: $\exists \mathcal{H}'$, $\mathcal{H} \underset{\substack{\text{handleslides}\\ \text{isotopies}}}{\rightsquigarrow} \mathcal{H}'$, $d(\mathcal{H}') \leq d(\mathcal{H})$
     $c_d(\mathcal{H}') < c_d(\mathcal{H})$] lex order

     ↳ Casework to prove
       1) Reach a bigon
       2) Reach a smaller dist. region
       3) Reach a same dist bad region
       4) ? D_m ? Coming back
          4.1) via adjacent edge
          4.2) via non-adjacent edge

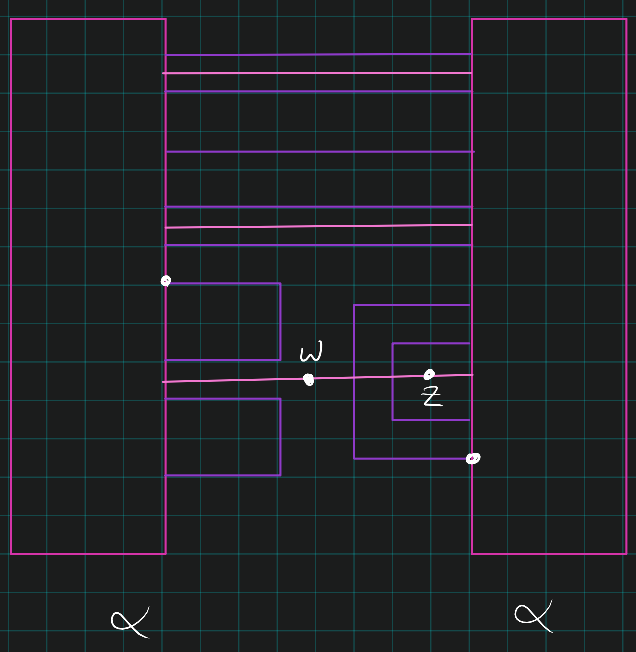
   - Repeat until $c_d = (0)$.

Examples

- Trefoil  $\rightsquigarrow$



Identify same labels by matching pts



Trefoil?

9 generators

$\Sigma(2,3,5)$