Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

Intro

Outline

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x)-\mu(y)$.

What we have so far:

- Define

$$L : W^{1,p}\left(\mathbb{R}\times S^1;\mathbb{R}^{2n}\right) \longrightarrow L^p\left(\mathbb{R}\times S^1;\mathbb{R}^{2n}\right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0\frac{\partial Y}{\partial t} + S(s,t)Y$$

where

$$S : \mathbb{R}\times S^1 \longrightarrow \mathrm{Mat}(2n;\mathbb{R})$$
$$S(s,t) \overset{s\longrightarrow\pm\infty}{\longrightarrow} S^{\pm}(t).$$

Outline

- Took $R^{\pm} : I \longrightarrow \mathrm{Sp}(2n; \mathbb{R})$: symplectic paths associated to $S^{\pm}$
- These paths defined $\mu(x), \mu(y)$
- Section 8.7:

$$R^{\pm} \in \mathcal{S} := \left\{ R(t) \;\middle|\; R(0) = \mathrm{id},\ \det(R(1) - \mathrm{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\mathrm{Ind}(L) \overset{\text{Thm?}}{=} \mu(R^{-}(t)) - \mu(R^{+}(t)) = \mu(x) - \mu(y).$$

From Yesterday

- Han proved 8.8.2 and 8.8.4.
  - So we know $\mathrm{Ind}(L) = \mathrm{Ind}(L_1)$
- Today: 8.8.5 and 8.8.3:
  - Computing $\mathrm{Ind}(L_1)$ by computing kernels.

$$
\begin{tikzcd}
\text{8.8.4: } \mathrm{Ind}(L_0) = \mathrm{Ind}(L) \arrow[d] & & \text{8.8.5: } \dim\ker F, F^* \arrow[d] \\
\text{8.8.2: } \mathrm{Ind}(L_1) = \mathrm{Ind}(L) \arrow[dr] & & \text{8.8.3: } \mathrm{Ind}(L_1) = k^- - k^+ \arrow[dl] \\
& \text{8.8.1: } \mathrm{Ind}(L) = k^- - k^+ = \mu(x) - \mu(y) &
\end{tikzcd}
$$

Section 1

Outline

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Section 2

Outline

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