

# Title

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## 1.1 Review

Let  $k = \bar{k}$ , we're setting up correspondences

Ring Theory	Geometry/Topology of Affine Varieties
Polynomial functions	Affine space
$k[x_1, \dots, x_n]$	$\mathbb{A}^n/k := \{[a_1, \dots, a_n] \in k^n\}$
Maximal ideals $\langle x_1 - a_1, \dots, x_n - a_n \rangle$	Points $[a_1, \dots, a_n] \in \mathbb{A}^n/k$
Radical ideals $I \subseteq k[x_1, \dots, x_n]$	Affine varieties $X \subset \mathbb{A}^n/k$ , vanishing loci of polynomials
	$I \mapsto V(I) := \{a \mid f(a) = 0 \forall f \in I\}$
	$I(X) := \{f \mid f _X = 0\} \leftarrow X$
Radical ideals containing $I(X)$ , i.e. ideals in $A(X)$	closed subsets of $X$ , i.e. affine subvarieties
$A(X)$ is a domain	$X$ irreducible
$A(X)$ is not a direct sum	$X$ connected
Prime ideals in $A(X)$	Irreducible closed subsets of $X$
Krull dimension $n$ (longest chain of prime ideals)	$\dim X = n$ , (longest chain of irreducible closed subsets).

Recall that we defined the coordinate ring  $A(X) := k[x_1, \dots, x_n]/I(X)$ , which contained no nilpotents.

We had some results about dimension

1.  $\dim X < \infty$  and  $\dim \mathbb{A}^n = n$ .
2.  $\dim Y + \text{codim}_X Y = \dim X$  when  $Y \subset X$  is irreducible.
3. Only over  $\bar{k} = k$ ,  $\text{codim}_X V(f) = 1$ .

**Example 1.1.**

Take  $V(x^2 + y^2) \subset \mathbb{A}^2/\mathbb{R}$

**Definition 1.0.1** (?).

An affine variety  $Y$  of

- $\dim Y = 1$  is a **curve**,
- $\dim Y = 2$  is a **surface**,
- $\operatorname{codim}_X Y = 1$  is a **hypersurface in  $X$**