Linearization and Transversality

D. Zack Garza

Review 8.2

Space of Perturbations of H

Section 8.4: Linearizing the Floer Equation: The Differential of F

Linearization and Transversality

Sections 8.3 and 8.4

D. Zack Garza

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Section 8.3: The Space of Perturbations of

Linearizing the Floer Equation: The Differential

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Goal

Linearization and Transversality

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Section 8.3: The Space of Perturbations of H

Section 8.4: Linearizing the Floer Equation: The Differential of F **Goal**: Given a fixed Hamiltonian $H \in C^{\infty}(W \times S^1; \mathbb{R})$, perturb it (without modifying the periodic orbits) so that $\mathcal{M}(x, y)$ are manifolds of the expected dimension.

Goal

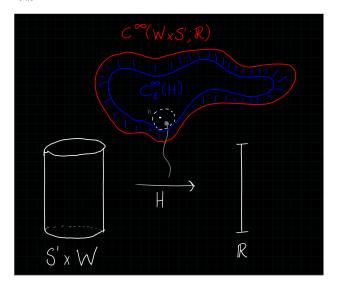
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Section 8.3: The Space of Perturbations of

Section 8.4: Linearizing the Floer Equation: The Differential of F Start by trying to construct a subspace $\mathcal{C}^{\infty}_{\mathbb{C}}(H) \subset \mathcal{C}^{\infty}(W \times S^1; \mathbb{R})$, the space of perturbations of H depending on a certain sequence $\varepsilon = \{\varepsilon_k\}$, and show it is a dense subspace.



Define an Absolute Value

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Section 8.3: The Space of Perturbations of H

Section 8.4: Linearizing the Floer Equation: The Differential of F Idea: similar to how you build $L^2(\mathbb{R})$, define a norm $\|\cdot\|_{\varepsilon}$ on $C_{\varepsilon}^{\infty}(H)$ and take the subspace of finite-norm elements.

- Let $h(\mathbf{x}, t) \in C_{\varepsilon}^{\infty}(H)$ denote a perturbation of H.
- Fix $\varepsilon = \left\{ \varepsilon_k \mid k \in \mathbb{Z}^{\geq 0} \right\} \subset \mathbb{R}^{>0}$ a sequence of real numbers, which we will choose carefully later.
- For a fixed $\mathbf{x} \in W$, $t \in \mathbb{R}$ and $k \in \mathbb{Z}^{\geq 0}$, define

$$|d^k h(\mathbf{x}, t)| = \max \{d^{\alpha} h(\mathbf{x}, t) \mid |\alpha| = k\},$$

the maximum over all sets of multi-indices α of length k. Note: I interpret this as

$$d^{\alpha_1,\alpha_2,\cdots,\alpha_k}h=\frac{\partial^k h}{\partial x_{\alpha_1}\,\partial x_{\alpha_2}\cdots\partial x_{\alpha_k}},$$

the partial derivatives wrt the corresponding variables.

Define a Norm

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Section 8.3: The Space of Perturbations of

Section 8.4: Linearizing the Floer Equation: The Differential of F – Define a norm on $C^{\infty}(W \times S^1; \mathbb{R})$:

$$||h||_{U} = \sum_{k\geq 0} \varepsilon_k \sup_{(x,t)\in W\times S^1} |d^k h(x,t)|.$$

– Since $W \times S^1$ is assumed compact (?), fix a finite covering $\{B_i\}$ of $W \times S^1$ such that

$$\bigcup_{i} B_{i}^{\circ} = W \times S^{1}.$$

Choose them in such a way we obtain charts

$$\Psi_i: B_i \longrightarrow \overline{B(0,1)} \subset \mathbb{R}^{2n+1}$$
 (?).

Obtain the computable form

$$||h||_{U} = \sum_{k>0} \varepsilon_k \sup_{(x,t)\in W\times S^1} \sup_{i,z\in B(0,1)} |d^k(h\circ \Psi_i^{-1})(z)|.$$

Define a Banach Space

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Section 8.3: The Space of Perturbations of

Section 8.4: Linearizing the Floer Equation: The Differentia of F Define

$$C_{\varepsilon}^{\infty} = \left\{ h \in C^{\infty}(W \times S^{1}; \mathbb{R}) \mid \|h\|_{\varepsilon} < \infty \right\} \subset C^{\infty}(W \times S^{1}; \mathbb{R}),$$

which is a Banach space (normed and complete).

– Show that the sequence $\{\varepsilon_k\}$ can be chosen so that C_{ε}^{∞} is a dense subspace for the C^{∞} topology, and in particular for the C^1 topology.

Theorem

Such a sequence $\{\varepsilon_k\}$ can be chosen.

Lemma

 $C^{\infty}(W \times S^1; \mathbb{R})$ with the C^1 topology is separable as a topological space (contains a countable dense subset).

Sketch Proof of Theorem

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Section 8.4: Linearizing the Floer Equation: The Differential of F

- By the lemma, produce a sequence $\{f_n\} \subset C^{\infty}(W \times S^1; \mathbb{R})$ dense for the C^1 topology.
- Using the norm on $C^n(W \times S^1; \mathbb{R})$ for the f_n , define

$$\frac{1}{\varepsilon_n} = 2^n \max \left\{ \|f_k\| \mid k \le n \right\} \implies \varepsilon_n \sup |d^n f_k(x, t)| \le 2^{-n}$$

which is summable.

Linearization and Transversality

Section 8.4: Linearizing the Floer Equation:

The Differential

Section 8.4: Linearizing the Floer Equation: The Differential of F