

Problem Set 5

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We first make the following definitions:

$$S := \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} a_{jk} = \sup \left\{ \sum_{(j,k) \in B} a_{jk} \mid B \subset \mathbb{N}^2, |B| < \infty \right\}$$
$$T := \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} a_{kj} = \sup \left\{ \sum_{(j,k) \in C} a_{kj} \mid C \subset \mathbb{N}^2, |C| < \infty \right\}.$$

We will show that $S = T$ by showing that $S \leq T$ and $T \leq S$.

Let $B \subset \mathbb{N}^2$ be finite, so $B \subseteq [0, I] \times [0, J] \subset \mathbb{N}^2$.

Now letting $R > \max(I, J)$, we can define $C = [0, R]^2$, which satisfies $B \subseteq C \subset \mathbb{N}^2$ and $|C| < \infty$.

Moreover, since $a_{jk} \geq 0$ for all pairs (j, k) , we have the following inequality:

$$\sum_{(j,k) \in B} a_{jk} < \sum_{(k,j) \in C} a_{jk} \leq \sum_{(k,j) \in C} a_{kj} \leq T,$$

since T is a supremum over *all* such sets C , and the terms of any finite sum can be rearranged.

But since this holds for every B , we this inequality also holds for the supremum of the smaller term by order-limit laws, and so

$$S := \sup_B \sum_{(k,j) \in B} a_{jk} \leq T.$$

An identical argument shows that $T \leq S$, yielding the desired equality. \square