

# 8.8 Part 2, Computing the Index of $L$

D. Zack Garza

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What we're trying to prove:

- 8.1.5:  $(d\mathcal{F})_u$  is a Fredholm operator of index  $\mu(x) - \mu(y)$ .
- Define

$$L : W^{1,p}(\mathbf{R} \times S^1; \mathbf{R}^{2n}) \longrightarrow L^p(\mathbf{R} \times S^1; \mathbf{R}^{2n})$$

$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y$$

where

$$S : \mathbb{R} \times S^1 \longrightarrow \text{Mat}(2n; \mathbb{R})$$

$$S(s, t) \xrightarrow{s \rightarrow \pm\infty} S^\pm(t).$$

- 8.7: Shows  $L$  is Fredholm
- By the end of 8.8: replace  $L$  by  $L_1$  with the same *index*
  - (not the same kernel/cokernel)
- Compute  $\text{Ind } L_1$ : explicitly describe  $\ker L_1, \text{coker } L_1$ .
- Replace in two steps:
  - $L \rightsquigarrow L_0$ , modified outside  $B_{\sigma_0}(0)$  in  $s$ .
    - \* Replace  $S(s, t)$  by a matrix

$$\tilde{S}(s, t) = \begin{cases} S^-(t) & s \leq -\sigma_0 \\ S^+(t) & s \geq \sigma_0 \end{cases}.$$

- \* Idea: approximate by cylinders at infinity.
- \* Use invariance of index under small perturbations.
- $L_0 \rightsquigarrow L_1$  by a homotopy, where  $S_\lambda : S \rightsquigarrow S(s)$  a diagonal matrix that is a constant matrix *outside*  $B_\varepsilon(0)$ .
  - \* Use invariance of index under homotopy.

**0.1 Main Results**

- Theorem 8.8.1:

$$\text{Ind}(L) = \mu(R^-(t)) - \mu(R^+(t)) = \mu(x) - \mu(y).$$

- Prop 8.8.2: Construct an operator

$$L_1 : W^{1,p}(\mathbf{R} \times S^1; \mathbf{R}^{2n}) \longrightarrow L^p(\mathbf{R} \times S^1; \mathbf{R}^{2n})$$

$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s)Y$$

where  $S : \mathbb{R} \longrightarrow \text{Mat}(2n; \mathbb{R})$  is a path of diagonal matrices depending on  $\text{Ind}(R^\pm(t))$ ; then

$$\text{Ind}(L) = \text{Ind}(L_1) = \text{Ind}(R^-(t)) - \text{Ind}(R^+(t)).$$

- Han's Talk:
  - Prop 8.8.3, using Lemma 8.8.5
- Me
  - Proof of 8.8.5

**0.2 8.8.5:**