Title

D. Zack Garza

Contents

L	Monday, November 09	2
	1.1 Chapter 1	2

Monday, November 09

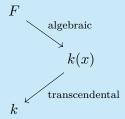
1.1 Chapter 1



Let k be a field, not necessarily algebraically closed.

Definition 1.1.1 (Algebraic Function Field).

An one variable algebraic function field F/K is a field extension F of K which factors as



where $x \in \bar{k}$ is some element that is not algebraic over k.

Definition 1.1.2 (Field of Constants).

The subfield

$$\tilde{k} := \left\{ z \in F \cap K^{\text{alg}} \right\} \le F,$$

consisting of elements that are algebraic over F is denoted the **field of constants**.

Definition 1.1.3 (Algebraically Closed).

If $\tilde{k} = k$, we say that k is algebraically closed in F.

Definition 1.1.4 (Rational Function Field).

An extension F/k is **rational** iff F = k(y) for some $y \in k^{\text{transc}}$ which is transcendental over k.

Definition 1.1.5 (Valuation Ring).

A ring $\mathcal{O} \subseteq F$ is a valuation ring for F iff $k \subset \mathcal{O} \subseteq F$ and $z \in F \implies z \in \mathcal{O}$ or $z^{-1} \in \mathcal{O}$.

Contents 2

Definition 1.1.6 (Discrete Valuation Ring).

A ring local R (thus with a unique maximal ideal) which is a PID but not a field is a **discrete** valuation ring.

Definition 1.1.7 (Place).

A **place** of a function field F/K is the maximal ideal of a valuation ring of F/K.

Definition 1.1.8 (Discrete Valuation).

A discrete valuation of F/k is a function

$$v: F \to \mathbb{Z} \cup \{\infty\}$$

that is

- 1. Nondegenerate: $v(x) = \infty$ iff x = 0.
- 2. Multiplicative: v(xy) = v(x) + v(y).
- 3. Ultrametric triangle inequality: $v(x+y) \ge \min(v(x), v(y))$.
- 4. Fiber over one: there exist a $z \in F$ with v(z) = 1.
- 5. $v|_k = 0$.

Definition 1.1.9 (Rational Place).

A place of degree one is said to be a rational place.

Definition 1.1.10 (Valuation Ring of a Place).

The valuation ring of a place is defined by

$$\mathcal{O}_p \coloneqq \left\{ z \in F \mid z^{-1} \notin P \right\}.$$

Definition 1.1.11 (Degree of a Place).

The **degree** of a place P is defined by

$$deg(P) := [F_p : k],$$

where $F_p = \mathcal{O}_P/P$.

Definition 1.1.12 (Discrete Valuation of a Place).

To any place P we associate the function

$$v_p: F \to \mathbb{Z} \cup \{\infty\}$$

defined by choosing any prime $t \in P$, writing any $x \in F$ as $x = t^n u$ with $u \in \mathcal{O}_P^{\times}$, and setting

$$v_p(x) = \begin{cases} n & \text{if } x = t^n u \\ 0 & x = \infty. \end{cases}$$

Note: from now on we assume $\tilde{K} = K$

Definition 1.1.13 (Divisor).

The **divisor group** of F/K is the free abelian group on the set of places of F/K, i.e. a formal sum

$$D = \sum_{\text{Places}p} b_p P \qquad n_p \in \mathbb{Z}m$$

where cofinitely many n_p are zero.

Definition 1.1.14 (Degree of a Divisor).

Definition 1.1.15 (Principle Divisors).

The set of divisors

$$\operatorname{Princ}(F) := \left\{ (x) \mid 0 \neq x \in F \right\}.$$

Definition 1.1.16 (Divisor Class Group).

TRh

Definition 1.1.17 (Riemann-Roch Space).

For a divisor $A \in \text{Div}(F)$, the **Riemann-Roch** space is defined as

$$\mathcal{L}(A) := \left\{ x \in F \mid (x) \ge -A \right\} \cup \{0\}.$$

4

1.1 Chapter 1