

Homotopy Groups of Spheres

Graduate Student Seminar

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Summary

Outline

Homotopy
Groups of
Spheres

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Summary

Examples

- Homotopy as a means of classification somewhere between homeomorphism and cobordism
- Comparison to homology
- Higher homotopy groups of spheres exist
- Homotopy groups of spheres govern gluing of CW complexes
- CW complexes fully capture that homotopy category of spaces
- There are concrete topological constructions of many important algebraic operations at the level of spaces (quotients, tensor products)
- Relation to framed cobordism?
- “Measuring stick” for current tools, similar to special values of L-functions
- Serre’s computation

Classification

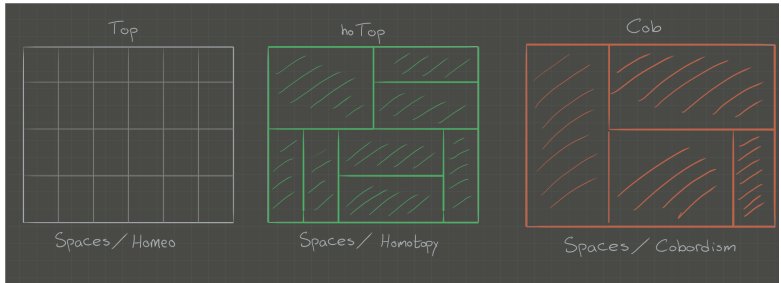
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- Holy grail: understand the topological category completely
 - I.e. have a well-understood geometric model one space of each homeomorphism type



Also have the derived category $DTop$, its interplay with $hoTop$ is the subject of e.g. the Poincare conjecture(s).

Point 1

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Point 2

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Sphere 1

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