Problem Set 10

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1 Problem 1

Let ϕ be an n-form. If suffices to show these statements for $n = 2$.

$\implies$: Suppose ϕ is alternating, then $\phi(b, b) = 0$ for all $b \in B$.

Letting $a, b \in B$ be arbitrary, we then have

$$\begin{aligned}
\phi(a+b, a+b) &= \phi(a, a+b) + \phi(b, a+b) \\
&= \phi(a, a) + \phi(a, b) + \phi(b, a) + \phi(b, b) \\
&= \phi(a, b) + \phi(b, a) \\
&\implies \phi(a, b) = -\phi(b, a),
\end{aligned}$$

which shows that ϕ is skew-symmetric.

$\impliedby$ Suppose ϕ is skew-symmetric, so $\phi(a, b) = -\phi(b, a)$ for all $a, b \in B$. Then $\phi(b, b) = -\phi(b, b)$ by transposing the terms, which says that $\phi(b, b) = 0$ for all $b \in B$ and thus ϕ is alternating.