Math 174 HW #4

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1) $p_1 = (-1, 2)$
$p_2 = (1, 3)$
$p_3 = (2, -2)$

b)

• $x = -1 \rightarrow$

$$\frac{a(-2)(-3)}{6} + \cancel{\frac{b(0)(-3)}{-2}} + \cancel{\frac{c(0)\sim}{\sim}} = 2$$

$\rightarrow -a = 2 \rightarrow \underline{a = -2}$

• $x = 1 \rightarrow$

$$\frac{b(2)(-1)}{-2} = 3$$

$\rightarrow \underline{b = 3}$

• $x = 2 \rightarrow \frac{c(3)(1)}{3} = -2$

$\rightarrow \underline{c = -2}$

$$\boxed{\text{So } f(x) = \frac{(-2)(x-1)(x-2)}{6} + \frac{3(x+1)(x-2)}{-2} + \frac{(-2)(x+1)(x-1)}{3}. ■}$$

c) • $x = -1 \rightarrow 2 = a + \cancel{b(0)} + \cancel{c(0)(-2)}$
$\rightarrow \boxed{2 = a}$

• $x = 1 \rightarrow 3 = a + b(2) + \cancel{c(2)(0)}$
$= a + 2b$
$\rightarrow 3 = 2 + 2b$
$\rightarrow \frac{3}{2} = 1 + b$
$\rightarrow \boxed{b = \frac{1}{2}}$

• $x = 2 \rightarrow -2 = a + b(3) + c(3)(1)$
$= a + 3b + 3c$
$2 \quad + 3(\frac{1}{2}) + 3c$
$\rightarrow c = -2 - 2 - \frac{3}{2}$
$\rightarrow \boxed{c = -11/2}$

$$\boxed{\text{So } f(x) = 2 + \tfrac{1}{2}(x+1) - \tfrac{11}{2}(x+1)(x-1)}$$

2)

· Degree 2: $f(x) = ax^2 + bx + c$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad A\bar{c} = \bar{b}$$

$$\longrightarrow \quad \bar{c} = A^{-1}\bar{b} = \left[\tfrac{9}{2}, \tfrac{-39}{2}, 19\right]$$

$$\longrightarrow f_1(x) = \tfrac{9}{2}x^2 - \tfrac{39}{2}x + 19$$

· degree 3:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \end{bmatrix} \overset{RREF}{\sim} \left[\begin{array}{c|c} & -1/6 \\ I & -1 \\ & 11/6 \end{array}\right] \rightarrow \begin{cases} a = \tfrac{1}{6}d \\ b = d \\ c = \tfrac{-11}{6}d \end{cases}$$

so let $d=6$, then $[a\,b\,c\,d] = [1, -6, -11, 6]$

$$\longrightarrow f_2(x) = x^3 - 6x^2 - 11x + 6$$

· degree 4:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 \\ 81 & 27 & 9 & 3 & 1 \end{bmatrix} \overset{RREF}{\sim} \left[\begin{array}{c|cc} & \frac{1}{6} & \frac{11}{36} \\ I & -1 & \frac{-5}{3} \\ & \frac{11}{6} & \frac{85}{36} \end{array}\right] \rightarrow \begin{cases} a = (-6d - 11e)/36 \\ b = (d + 5e)/3 \\ c = (-11d - 85e)/36 \end{cases}$$

$d = e = 36$, then

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -17 \\ 72 \\ -96 \end{bmatrix}$$

$$\longrightarrow f_3(x) = -17x^4 + 72x^3 - 96x^2 + 36x + 36.$$

3) $$L_{n,k}(x) = \prod_{i \neq k} \frac{(x - x_i)}{(x_k - x_i)}$$

$$f(x) = \frac{(x-x_0)(x-(x_0+h))}{((x_0-h)-x_0)((x_0-h)-(x_0+h))} f(x_0-h) + \frac{(x-(x_0-h))(x-(x_0+h))}{(x_0-(x_0-h))(x_0-(x_0+h))} f(x_0) + \frac{(x-x_0)(x-(x_0-h))}{(x_0+h-x_0)(x_0+h-(x_0-h))} f(x_0+h)$$

$$= \frac{(x-x_0)(x-x_0-h)}{2h^2} f(x_0-h) +$$

$$= \frac{(x-x_0)(x-x_0-h)}{2h^2} f(x_0-h) +$$

a) $$\boxed{\to f(x) = \frac{f(x_0-h)}{2h^2}(x-x_0)(x-(x_0+h)) - \frac{f(x_0)}{h^2}(x-(x_0-h))(x-(x_0+h)) + \frac{f(x_0+h)}{2h^2}(x-x_0)(x-(x_0-h))}$$

b) $$\to f'(x) = \frac{f(x_0-h)}{2h^2}[2x-2x_0-h] - \frac{f(x_0)}{h^2}[2x-2x_0] + \frac{f(x_0+h)}{2h^2}[2x-2x_0+h]$$

$$\to \underline{f'(x_0)} = \frac{f(x_0-h)}{2h^2}(-h) - 0 + \frac{f(x_0+h)}{2h^2}(h)$$

$$\boxed{= \frac{1}{2h}(f(x_0-h)+f(x_0+h)).} \ \square$$

c) $$f'(x) = \frac{2f(x_0-h)}{2h^2} - \frac{2f(x_0)}{h^2} + \frac{2f(x_0+h)}{2h^2}$$

$$\boxed{\to f''(x_0) = \frac{1}{h^2}(f(x_0-h)-2f(x_0)+f(x_0+h)).} \ \square$$

4) a - $x_0 = -1$
$x_1 = 0$
$x_2 = 1$

x_0	$f(x_0)$	1^{st}	2^{nd}
-1	-2		
		2	
0	0		1/2
		3	
1	3		

b) $$P_n(x) = \sum f[x_0 \cdots x_i] \prod_{k=1}^{i-1}(x-x_k)$$

$$= f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$= f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$\boxed{\rightarrow f(x) = -2 + 2(x+1) + \tfrac{1}{2}(x+1)(x-0)}$$

c)

x_i	$f(x_i)$	Δ^1	Δ^2	Δ^3
-1	-2			
		2		
0	0		1/2	
		3		-3/8
1	3		-1	
		0		
3	3			

$$\boxed{\rightarrow f(x) = -2 + 2(x+1) + \tfrac{1}{2}(x+1)(x-0) - \tfrac{3}{8}(x+1)(x-0)(x-1)}$$