Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

Section 1

Section 2

Intro

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x) - \mu(y)$.

What we have so far:

Define

$$L: W^{1,p}\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right) \longrightarrow L^p\left(\mathbb{R} \times S^1; \mathbb{R}^{2n}\right)$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s,t)Y$$

where

$$S: \mathbb{R} \times S^1 \longrightarrow \operatorname{Mat}(2n; \mathbb{R})$$
$$S(s, t) \stackrel{s \longrightarrow \pm \infty}{\longrightarrow} S^{\pm}(t).$$



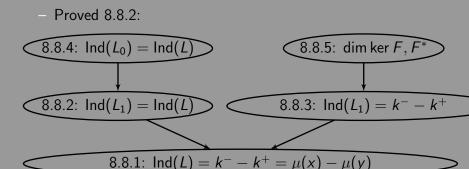
- Took $R^{\pm}: I \longrightarrow \operatorname{Sp}(2n; \mathbb{R})$: symplectic paths associated to S^{\pm}
- These paths defined $\mu(x), \mu(y)$
- Section 8.7:

$$R^{\pm} \in \mathcal{S} := \Big\{ R(t) \ \Big| \ R(0) = \mathsf{id}, \ \mathsf{det}(R(1) - \mathsf{id}) \neq 0 \Big\} \implies \textit{L} \ \mathsf{is} \ \mathsf{Fredholm}.$$

- WTS 8.8.1:

$$\operatorname{Ind}(L) \stackrel{\mathsf{Thm?}}{=} \mu(R^{-}(t)) - \mu(R^{+}(t)) = \mu(x) - \mu(y).$$

From Yesterday



Section 1

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Section 2

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