Title

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Contents

1 Thursday, August 20

References: `https://www.daniellitt.com/tale-cohomology`

Prerequisites:

- Homological Algebra
  - Abelian Categories
  - Derived Functors
  - Spectral Sequences (just exposure!)
- Sheaf theory and sheaf cohomology
- Schemes (Hartshorne II and III)

Outline/Goals:

- Basics of etale cohomology
  - Etale morphism
  - Grothendieck topologies
  - The etale topology
  - Etale cohomology and the basis theorems
  - Etale cohomology of curves
  - Comparison theorems to singular cohomology
- Prove the Weil Conjectures (more than one proof)
  - Proving the Riemann Hypothesis for varieties over finite fields

    One of the greatest pieces of 20th century mathematics!
- Topics
  - Weil 2 (Strengthening of RH, used in practice)
  - Formality of algebraic varieties (topological features unique to varieties)
  - Other things (monodromy, refer to Kass' AWS notes)

What is Etale Cohomology? Suppose $X/\mathbb{C}$ is a quasiprojective variety: a finite type separated integral $\mathbb{C}$-scheme.

If you take the complex points, it naturally has the structure of a complex analytic space $X(\mathbb{C})^{\mathrm{an}}$: you can give it the Euclidean topology, which is much finer than the Zariski topology. For a nice topological space, we can associate the singular cohomology $H^i(X(\mathbb{C})^{\mathrm{an}}, \mathbb{Z})$.