Title

D. Zack Garza

Tuesday 1st September, 2020

Contents

1 Tuesday, September 01

Last time: $V(I) = \left\{ x \in \mathbb{A}^n \;\middle|\; f(x) = 0 \, \forall x \in I \right\}$ and $I(X) = \left\{ f \in k[x_1, \cdots, x_n] \;\middle|\; f(x) = 0 \, \forall x \in X \right\}$. We proved the Hilbert Nullstellensatz $I(V(J)) = \sqrt{J}$, defined the coordinate ring of an affine variety X as $A(X) \coloneqq k[x_1, \cdots, x_n]/I(X)$, the ring of "regular" (polynomial) functions on X.

Recall that a *topology* on X can be defined as a collection of "closed" subsets of X that are closed under arbitrary intersections and finite unions. A subset $Y \subset X$ inherits a subspace topology with closed sets of the form $Z \cap Y$ for $Z \subset X$ closed.

Definition 1.0.1 (Zariski Topology)**.**
Let X be an affine variety. The closed sets are affine subvarieties $Y \subset X$.

We have $\emptyset, X$ closed, since

1. $V_X(1) = \emptyset$,
2. $V_X(0) = X$

Closure under finite unions: Let $V_X(I), V_X(J)$ be closed in X with $I, J \subset A(X)$ ideals. Then $V_X(IJ) = V_X(I) \cup V_X(J)$.

Closure under intersections: We have $\bigcap_{i \in \sigma} V_X(J) = V_X(\sum_{i \in \sigma} J_i)$.

Remark 1.
There are few closed sets, so this is a "weak" topology.