

# Title

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## 1 | Monday, November 09

### 1.1 Chapter 1

Let  $k$  be a field, not necessarily algebraically closed.

**Definition 1.1.1** (Algebraic Function Field).

An one variable **algebraic function field**  $F/K$  is a field extension  $F$  of  $K$  which factors as



where  $x \in \bar{k}$  is some element that is not algebraic over  $k$ .

**Definition 1.1.2** (Field of Constants).

The subfield

$$\tilde{k} := \{z \in F \cap K^{\text{alg}}\} \leq F,$$

consisting of elements that are algebraic over  $F$  is denoted the **field of constants**.

**Definition 1.1.3** (Algebraically Closed).

If  $\tilde{k} = k$ , we say that  $k$  is **algebraically closed** in  $F$ .

**Definition 1.1.4** (Rational Function Field).

An extension  $F/k$  is **rational** iff  $F = k(y)$  for some  $y \in k^{\text{transc}}$  which is transcendental over  $k$ .

**Definition 1.1.5** (Valuation Ring).

A ring  $\mathcal{O} \subseteq F$  is a **valuation ring** for  $F$  iff  $k \subset \mathcal{O} \subseteq F$  and  $z \in F \implies z \in \mathcal{O}$  or  $z^{-1} \in \mathcal{O}$ .

**Definition 1.1.6** (Discrete Valuation Ring).

A ring local  $R$  (thus with a unique maximal ideal) which is a PID but not a field is a **discrete valuation ring**.

**Definition 1.1.7** (Place).

A **place** of a function field  $F/K$  is the maximal ideal of a valuation ring of  $F/K$ .

**Definition 1.1.8** (Discrete Valuation).

A **discrete valuation** of  $F/K$  is a function

$$v : F \rightarrow \mathbb{Z} \cup \{\infty\}$$