Miscellaneous Notes

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Contents

- Canonical forms:
  - Rational Canonical Form: Invariant Factor Decomposition of T
  - Jordan Canonical Form: Elementary Divisor Decomposition of T
- For characteristic polynomials $p(x) = \det(A - x1)) = \det(SNF(A - x1))$.
- Writing

$$\begin{aligned}\text{char }_A(x) &= \prod (x - \lambda_i)^{a_i} \\ \min_A(x) &= \prod (x - \lambda_i)^{b_i}\end{aligned}$$

  then $b_i \leq a_i$, and
  - b_i tells you the size of the largest Jordan block associated to λ_i,
  - a_i is the sum of sizes of all Jordan blocks associated to λ_i
  - $\dim E_{\lambda_i}$ is the number of Jordan blocks associated to λ_i
- Given an invariant factor decomposition

$$V = \bigoplus_{j=1}^{n} \frac{k[x]}{(f_j)}$$

  then
  - $f_n(x) = \min_T(x)$ (i.e., the minimal polynomial is the invariant factor of highest degree)
  - $\prod_{j=1}^{n} f_j(x) = \text{char }_T(x)$.

1 Some Matrix Examples

1.

$$M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \sim \left(\begin{array}{r|r} -1\sqrt{-1} & 0 \\ \hline 0 & 1\sqrt{-1} \end{array}\right).$$

- Not diagonalizable over $\mathbb{R}$, diagonalizable over $\mathbb{C}$
- No eigenvalues in $\mathbb{R}$, distinct eigenvalues over $\mathbb{C}$
- $\min_M(x) = \text{char }_M(x) = x^2 + 1$

2.

$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

- Not diagonalizable over $\mathbb{C}$
- Eigenvalues $[1, 1]$ (repeated, multiplicity 2)
- $\min_M(x) = \text{char }_M(x) = x^2 - 2x + 1$