

Title

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Question: Let $f \in L^1([0, 1])$ and $F(x) = \int_a^x f(y) \, dy$ – is F differentiable a.e. and $F' = f$?

If f is continuous, then absolutely yes.

Otherwise, we are considering

$$\frac{f(x+h) - F(x)}{h} = \frac{1}{h} \int_x^{x+h} f(y) \, dy \rightarrow? f(x)$$

so the more general question is

$$\lim_{m(I) \rightarrow 0, x \in I} \frac{1}{m(I)} \int_I f(y) \, dy =? f(x) \text{ a.e.}$$