## Title

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# Monday, November 09

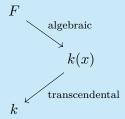
#### 1.1 Chapter 1



Let k be a field, not necessarily algebraically closed.

**Definition 1.1.1** (Algebraic Function Field).

An one variable algebraic function field F/K is a field extension F of K which factors as



where  $x \in \bar{k}$  is some element that is not algebraic over k.

**Definition 1.1.2** (Field of Constants).

The subfield

$$\tilde{k} := \left\{ z \in F \cap K^{\text{alg}} \right\} \le F,$$

consisting of elements that are algebraic over F is denoted the **field of constants**.

**Definition 1.1.3** (Algebraically Closed).

If  $\tilde{k} = k$ , we say that k is algebraically closed in F.

**Definition 1.1.4** (Rational Function Field).

An extension F/k is **rational** iff F = k(y) for some  $y \in k^{\text{transc}}$  which is transcendental over k.

**Definition 1.1.5** (Valuation Ring).

A ring  $\mathcal{O} \subseteq F$  is a valuation ring for F iff  $k \subset \mathcal{O} \subseteq F$  and  $z \in F \implies z \in \mathcal{O}$  or  $z^{-1} \in \mathcal{O}$ .

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#### **Definition 1.1.6** (Discrete Valuation Ring).

A ring local R (thus with a unique maximal ideal) which is a PID but not a field is a **discrete** valuation ring.

#### Definition 1.1.7 (Place).

A **place** of a function field F/K is the maximal ideal of a valuation ring of F/K.

#### **Definition 1.1.8** (Discrete Valuation).

A discrete valuation of F/k is a function

$$v: F \to \mathbb{Z} \cup \{\infty\}$$

that is

- 1. Nondegenerate:  $v(x) = \infty$  iff x = 0.
- 2. Multiplicative: v(xy) = v(x) + v(y).
- 3. Ultrametric triangle inequality:  $v(x+y) \ge \min(v(x), v(y))$ .
- 4. Fiber over one: there exist a  $z \in F$  with v(z) = 1.
- 5.  $v|_k = 0$ .

#### **Definition 1.1.9** (Rational Place).

A place of degree one is said to be a **rational place**.

#### **Definition 1.1.10** (Valuation Ring of a Place).

$$\mathcal{O}_p := \left\{ z \in F \mid z^{-1} \notin P \right\}.$$

#### **Definition 1.1.11** (Degree of a Place).

The **degree** of a place P is defined by

$$\deg(P) := [F_p : k],$$

where  $F_p = \mathcal{O}_P/P$ , with  $\mathcal{O}_P$  the valuation ring of the place P.