

8.8 Part 2, Computing the Index of L

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Contents

What we're trying to prove:

- 8.1.5: $(d\mathcal{F})_u$ is a Fredholm operator of index $\mu(x) - \mu(y)$.
- Define

$$L : W^{1,p}(\mathbf{R} \times S^1; \mathbf{R}^{2n}) \longrightarrow L^p(\mathbf{R} \times S^1; \mathbf{R}^{2n})$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y$$

where

$$S : \mathbf{R} \times S^1 \longrightarrow \text{Mat}(2n; \mathbf{R})$$
$$S(s, t) \xrightarrow{s \rightarrow \pm\infty} S^\pm(t).$$

- 8.7: Shows L is Fredholm
- By the end of 8.8: replace L by L_1 with the same *index*
 - (not the same kernel/cokernel)
- Compute $\text{Ind } L_1$: explicitly describe $\ker L_1, \text{coker } L_1$.
- Replace in two steps:
 - $L \rightsquigarrow L_0$, modified outside $B_{\sigma_0}(0)$ in s .
 - * Replace $S(s, t)$ by a matrix

$$\tilde{S}(s, t) = \begin{cases} S^-(t) & s \leq -\sigma_0 \\ S^+(t) & s \geq \sigma_0 \end{cases}.$$

- * Idea: approximate by cylinders at infinity.
 - * Use invariance of index under small perturbations.
- $L_0 \rightsquigarrow L_1$ by a homotopy, where $S_\lambda : S \rightsquigarrow S(s)$ a diagonal matrix that is a constant matrix *outside* $B_\varepsilon(0)$.
 - * Use invariance of index under homotopy.