Title

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1 Lecture 11

Last time: we saw the Leray spectral sequence, but no examples yet, so that's what we'll do now. We had $X \xrightarrow{f} Y \xrightarrow{g} Z$ to which we associated the spectral sequence $R^i f_* R^j f_*(\cdot) \Rightarrow R^{i+j}(g \circ f)_*(\cdot)$. To deduce existence we used that pushforwards preserve injectives, and we looked at some E_2 differentials.

Example 1.0.1 ***(?):*** Let $X \xrightarrow{\pi} Z \coloneqq \operatorname{Spec} k$, where $k \neq \overline{k}$ necessarily. The spectral sequence for the functors π_*, Γ yields the Leray spectral sequence $H^i(k, R^j \pi_* \mathcal{F}) \Rightarrow H^i(X_{\text{ét}}, \mathcal{F})$. The LHS is the étale cohomology of $\operatorname{Spec} k$, i.e. Galois cohomology. The Galois module corresponding to $R^j \pi_* \mathcal{F}$ is $H^j(X_{k^{\text{sep}}}, \mathcal{F})$ by taking the $\overline{k}$ points of this functor So the Leray spectral sequence yields