Title

D. Zack Garza

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Recall that we had a dominant weight $\lambda \in X(T)_+$ with

$$\begin{tikzcd}
 & V(\lambda) \arrow[dl, "\twoheadrightarrow"'] \arrow[dr, "\hookrightarrow"] & \\
L(\lambda) \arrow[dr, "\hookrightarrow"] & & T(\lambda) \arrow[dl, "\twoheadrightarrow"] \\
 & H^0(\lambda) &
\end{tikzcd}$$

where we have a module with both a *good* and a *Weyl* filtration.

If $B \subseteq P \subseteq G$ with P parabolic and $M \in \operatorname{Mod}(G)$, we have a "transfer theorem": maps

$$H^n(G; M) \xrightarrow{\text{Res}} H^n(P; M) \xrightarrow{\text{Res}} H^n(B; M)$$

induced by restrictions which are isomorphisms.

Proposition 1.1 ***(?).***
Let $M \in \operatorname{Mod}(P)$ with $P \supseteq B$.

a. If $\dim M < \infty$ then $\dim H^n(P; M) < \infty$.
b. If $H^j(P; M) \neq 0$ then there exists a weight λ of M such that $-\lambda \in \mathbb{N}\Phi^+$ and $\operatorname{ht}(-\lambda) \geq j$.

Part (a) is proved in the book, we won't show it here.

Proof (of part b).
Suppose $H^j(P; M) \neq 0$, then we have an injective resolution I_* for k. Tensoring with M yields an injective resolution for M,

$$0 \to M \to I_0 \otimes M \to I_1 \otimes M \to \cdots.$$

Since $H^j(B; M) \neq 0$, we know that the cocycles $\hom_B(k, I_j \otimes M) \neq 0$ and thus $\hom_T(k, I_j \otimes M) \neq 0$.

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