Title

Contents

- Stuff I often get wrong:
  - $-x^{-2} \neq \int x^{-1} = \int \frac{1}{x} = \ln x$
  - $\frac{1}{x} \neq \int \ln x = x \ln x - x$
  - $\int x^{-k} = \frac{1}{-k+1} x^{-k+1} \neq \frac{1}{-(k+1)} x^{-(k+1)}$
    - e.g. $\int x^{-2} = -x^{-1} \neq -\frac{1}{3} x^{-3}$
  - $\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$
- $\frac{\partial}{\partial x} a^x = \frac{\partial}{\partial x} e^{x \ln a} = e^{x \ln a} \ln a = a^x \ln a.$
- Exponentials: when in doubt, write $a^b = e^{b \ln a}$
- $\frac{\partial}{\partial x} x^{f(x)} = ?$
- $\sum x^k = \frac{1}{1-x} \neq \frac{1}{1+x} = \sum (-1)^k x^k$

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	sin	cos	tan
0	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{4}}{2}$	0
$\frac{\pi}{6}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{3}}{1}$
$\frac{\pi}{2}$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{0}}{2}$	∞

$$
\begin{aligned}
(a+b)^2 &= a^2+b^2+2ab \\
(a-b)^2 &= a^2+b^2-2ab \\
a^2+b^2 &= (a+b)^2+2ab \\
a^2-b^2 &= (a+b)(a-b) \\
(a+b)^3 &= a^3+b^3+3(a^2b+ab^2) \\
(a-b)^3 &= a^3-b^3+3(-a^2b+ab^2) \\
a^3+b^3 &= (a+b)(a^2+b^2-ab) \\
a^3-b^3 &= (a-b)(a^2+b^2+ab) \\
(\sqrt{a}+\sqrt{b})^2 &= a+b+2\sqrt{ab} \\
(\sqrt{a}-\sqrt{b})^2 &= a+b-2\sqrt{ab} \\
(a+\sqrt{b})(a-\sqrt{b}) &= a^2-b \\
(a+i\sqrt{b})(a-i\sqrt{b}) &= a^2+b \\
(a+b)(a-b) &= a^2+b^2
\end{aligned}
$$

$\frac{\partial f}{\partial x} \Leftarrow$	f	$\Rightarrow \int f dx$
$\frac{1}{2\sqrt{x}}$	$\sqrt{x}$	$\frac{2}{3}x^{\frac{3}{2}}$
nx^{n-1}	$x^n, n \neq -1$	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$\ln(x)$	$x\ln(x) - x$
$a^x \ln(a)$	a^x	$\frac{a^x}{\ln a}$
$\cos(x)$	$\sin(x)$	$-\cos(x)$
$-\sin(x)$	$\cos(x)$	$\sin(x)$
$2\sec^2(x)\tan(x)$	$\sec^2(x)$	$\tan(x)$
$2\csc^2(x)\cot(x)$	$\csc^2(x)$	$-\cot(x)$
$\sec^2(x)$	$\tan(x)$	$\ln\lvert\sec(x)\rvert$
$\sec(x)\tan(x)$	$\sec(x)$	$\ln\lvert\sec(x) + \tan(x)\rvert$
$-\csc(x)\cot(x)$	$\csc(x)$	$\ln\lvert\csc(x) - \cot(x)\rvert$
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$	$x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$	$x\sin^{-1}x + \sqrt{1-x^2}$
$-\frac{1}{\sqrt{1-x^2}}$	$\cos^{-1}(x)$	$x\cos^{-1}x - \sqrt{1-x^2}$
$\frac{1}{\sqrt{x^2+a}}$	$\ln\left\lvert x + \sqrt{x^2+a}\right\rvert$	.
$-\csc^2(x)$	$\cot(x)$	?
?	$\cos^2(x)$	?
?	$\sin^2(x)$	?
?	xe^{ax}	$\frac{1}{a^2}(ax-1)e^{ax}$
?	$e^{ax}\sin(bx)$	$\frac{1}{a^2+b^2}e^{ax}(a\sin bx - b\cos bx)$
?	$e^{ax}\cos(bx)$	$\frac{1}{a^2+b^2}e^{ax}(a\sin bx + b\cos bx)$
?	?	?

$\sqrt{a^2-x^2}$	$\Rightarrow$	$x = a\sin(\theta)$	$dx = a\cos(\theta)\ d\theta$
$\sqrt{a^2+x^2}$	$\Rightarrow$	$x = a\tan(\theta)$	$dx = a\sec^2(\theta)\ d\theta$
$\sqrt{x^2-a^2}$	$\Rightarrow$	$x = a\sec(\theta)$	$dx = a\sec(\theta)\tan(\theta)\ d\theta$

$p(y)y' = q(x)$	separable
$y' + p(x)y = q(x)$	integrating factor
$y' = f(x,y), f(tx,ty) = f(x,y)$	$y = xV(x)$ COV reduces to separable
$y' + p(x)y = q(x)y^n$	Bernoulli, divide by y^n and COV $u = y^{1-n}$
$M(x,y)dx + N(x,y)dy = 0$	$M_y = N_x : \varphi(x,y) = c(\varphi_x = M, \varphi_y = N)$
$P(D)y = f(x,y)$	$x^k e^{rx}$ for each root