Title

D. Zack Garza

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Contents

[prob:1.1] If $\sigma = (i_1 i_2 \cdots i_r) \in S_n$ and $\tau \in S_n$, then show that $\tau\sigma\tau^{-1} = (\tau(i_1)\tau(i_2)\cdots\tau(i_r))$.

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[prob:1.2] Show that $S_n \cong \langle (12), (123\cdots n)\rangle$ and also that $S_n \cong \langle (12), (23\cdots n)\rangle$

[prob:1.3] Let G be a finite abelian group that is not cyclic. Show that G contains a subgroup isomorphic to $\mathbb{Z}_p \oplus \mathbb{Z}_p$ for some prime p.

[prob:1.4] Determine all abelian groups of order n for $n \leq 20$.

[prob:1.5] Let G be a group and $A \trianglelefteq G$ be a normal abelian subgroup. Show that G/A acts on A by conjugation and construct a homomorphism $\varphi : G/A \to \mathrm{Aut}(A)$.

[prob:1.6] Let $Z(G)$ be the center of G. Show that if $G/Z(G)$ is cyclic, then G is abelian.

Note that Hungerford uses the notation $C(G)$ for the center.

[prob:1.7] Let G be a finite group and $H \trianglelefteq G$ a normal subgroup of order p^k. Show that H is contained in every Sylow p-subgroup of G.

[prob:1.8] Let $|G| = p^n q$ for some primes $p > q$. Show that G contains a unique normal subgroup of index q.

0.1 Qual Problems

[prob:1.9] Let G be a finite group and p a prime number. Let X_p be the set of Sylow-p subgroups of G and n_p be the cardinality of X_p. Let $\mathrm{Sym}(X)$ be the permutation group on the set X_p.

1. Construct a homomorphism $\rho : G \to \mathrm{Sym}(X_p)$ with image a transitive subgroup (i.e. with a single orbit).
2. Deduce that G is simple and the order of G divides $n_p!$.
3. Show that for any $1 \leq a \leq 4$ and any prime power p^k, no group of order ap^k is simple.

[prob:1.10] Let G be a finite group and $H < G$ a subgroup. Let n_H be the number of subgroups of G that are conjugate to H. Show that n_H divides the order of G.

[prob:1.11] Let $G = S_5$, the symmetric group on 5 elements. Identify all conjugacy classes of elements in G, provide a representative from each class, and prove that this list is complete.