Homework 6

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1 Homework Problems

1.1 Problem 1

Todo

1.2 Problem 2

We can note that since f has 4 roots, the Galois group G of its splitting field will be a subgroup of S_4. Moreover, G must be a *transitive subgroup* of S_4, i.e. the action of G on the roots of f should be transitive. This reduces the possibilities to $G = S^4, A^4, D^4, \mathbb{Z}_4, \mathbb{Z}_2^2$.

Since f has exactly 2 real roots and thus a pair of roots that are complex conjugates, the automorphism given by complex conjugation is an element of G. But this corresponds to a 2-cycle $\tau = (ab)$, and we can then make the following conclusions:

- Not A_4: A_4 contains only even cycles, and τ is odd.
- Not Z_4: This subgroup is generated by a single 4-cycle σ, which up to conjugacy is (1234), and σ^n is not a 2-cycle for any n.
- Not $\mathbb{Z}_2^2$: In order to be transitive, this subgroup must be $\{e, (12)(34), (13)(24), (14)(23)\}$, which does not contain τ.

The only remaining possibilities are S^4 and D^4. $\square$

1.3 Problem 3

1.4 Problem 4

1.5 Problem 5

1.6 Problem 6

2 Qual Problems

2.1 Problem 1

2.2 Problem 2

2.3 Problem 3