Title

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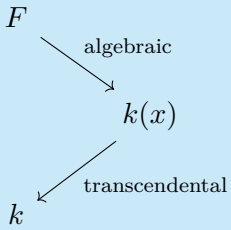
1.1 Chapter 1

Let k be a field, not necessarily algebraically closed.

Definition 1.1.1 (Algebraic Function Field)**.**
An one variable **algebraic function field** F/K is a field extension F of K which factors as



where $x \in \bar{k}$ is some element that is not algebraic over k.

Definition 1.1.2 (Field of Constants)**.**
The subfield

$$\tilde{k} \coloneqq \left\{ z \in F \cap K^{\text{alg}} \right\} \leq F,$$

consisting of elements that are algebraic over F is denoted the **field of constants**.

Definition 1.1.3 (Algebraically Closed)**.**
If $\tilde{k} = k$, we say that k is **algebraically closed** in F.

Definition 1.1.4 (Rational Function Field)**.**
An extension F/k is **rational** iff $F = k(y)$ for some $y \in k^{\text{transc}}$ which is transcendental over k.

Definition 1.1.5 (Valuation Ring)**.**
A ring $\mathcal{O} \subseteq F$ is a **valuation ring** for F iff $k \subset \mathcal{O} \subseteq F$ and $z \in F \implies z \in \mathcal{O}$ or $z^{-1} \in \mathcal{O}$.

Definition 1.1.6 (Discrete Valuation Ring)**.**
A ring local R (thus with a unique maximal ideal) which is a PID but not a field is a **discrete valuation ring**.

Definition 1.1.7 (Place)**.**
A **place** of a function field F/K is the maximal ideal of a valuation ring of F/K.

Definition 1.1.8 (Discrete Valuation)**.**
A **discrete valuation** of F/K is a function

$$v : F \to \mathbb{Z} \cup \{\infty\}$$

satisfying

1. Nondegeneracy: $v(x) = \infty$ iff $x = 0$.
2.