Interesting Topological Spaces in Algebraic Geometry

D. Zack Garza

Monday 27th July, 2020

Contents

1 Ideas for Spaces

- Curves
  - Elliptic Curves
  - Higher genus
  - Hyperelliptic curves
  - The modular curve
- Surfaces
  - Compact Riemann surfaces
    - Bolza Surface (Genus 2)
    - Klein Quartic (Genus 3)
    - Hurwizt Surfaces
  - Kummer surfaces
  - Del Pezzo surfaces
- Compact Complex Surfaces
  - Rational ruled
  - Enriques Surfaces
  - $K3$
    - Kahler Manifolds
  - Kodaira
  - Toric
  - Hyperelliptic
  - Properly quasi-elliptic
  - General type
  - Type VII
- Fake projective planes
- Conics
- Calabi-Yau manifolds
  - Dimension 1: All elliptic curves (up to homeomorphism)

  - Dimension 2: $K3$ surfaces
  - Dimension 3 (threefolds): 500 million +, unknown if infinitely many
  - The bananafold
  - Hyperkähler
- Hurwitz schemes
- Topological galois groups, e.g. $G(\overline{F}/F)$ for $F = \mathbb{Q}, \mathbb{F}_p$.
- Spec (R) for R a DVR (a Sierpinski space)
- Quiver Grassmannians
- Rigid analytic spaces
- Affine line with two origins
- Moduli stack of elliptic curves $\mathcal{M}_{1,1}$.
- Abelian Surface
- Fano Varieties
- Weighted projective space

Due to Kunihiko Kodaira's classification of complex surfaces, we know that any compact hyperkähler 4-manifold is either a K3 surface or a compact torus T^{4}. (Every Calabi–Yau manifold in 4 (real) dimensions is a hyperkähler manifold, because SU(2) is isomorphic to Sp(1).)

As was discovered by Beauville, the Hilbert scheme of k points on a compact hyperkähler 4-manifold is a hyperkähler manifold of dimension 4k. This gives rise to two series of compact examples: Hilbert schemes of points on a K3 surface and generalized Kummer varieties.