

Assignment 6 Qual Problems

D. Zack Garza

November 7, 2019

Contents

1 Problem 1	1
1.1 Part (a)	1
1.2 Part (b)	1
1.3 Part (c)	1

1 Problem 1

1.1 Part (a)

Definition: A field extension L/F is said to be a *splitting field* of a polynomial $f(x)$ if L contains all roots of f and thus decomposes as

$$f(x) = \prod_{i=1}^n (x - \alpha_i)^{k_i} \in L[x]$$

where α_i are the distinct roots of f and k_i are the respective multiplicities.

1.2 Part (b)

Let F be a finite field with q elements, where $q = p^k$ is necessarily a prime power, so $F \cong \mathbb{F}_{p^k}$. Then any finite extension of E/F is an F -vector space, and contains $q^n = (p^k)^n = p^{kn}$ elements. Thus $E \cong \mathbb{F}_{p^{kn}}$. Then if $\alpha \in E$, we have $\alpha^{p^{kn}} = \alpha$, so we can define

$$f(x) := x^{p^{kn}} - x \in F[x].$$

The roots of f are exactly the elements of E , so f splits in E .

1.3 Part (c)

The polynomial f is separable, since $f'(x) = p^{kn}x^{p^{kn}-1} - 1 = -1$ since $\text{char} E = p$. Since E is a finite extension, E is thus a separable extension. Then, since E is a separable splitting field, it is a Galois extension.