

# Title

D. Zack Garza

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## 1 Friday February 21st

### 1.1 Singularities

Recall that there are three types of singularities:

- Removable
- Poles
- Essential

#### **Theorem 1.1(3.2).**

An isolated singularity  $z_0$  of  $f$  is a pole  $\iff \lim_{z \rightarrow z_0} f(z) = \infty$ .

#### **Theorem 1.2(3.3, Casorati-Weierstrass).**

If  $f$  is holomorphic in  $D_r(z_0) \setminus \{z_0\}$  and has an essential singularity  $z_0$ , then there exists a radius  $r$  such that  $f(D_r(\{z_0\}) \setminus \{z_0\})$  is dense in  $\mathbb{C}$ .

*Proof.*

Proceed by contradiction. Suppose there exists a  $w \in \mathbb{C}$  and a  $\delta > 0$  such that

$$D_\delta(w) \cap f(D_r(\{z_0\}) \setminus \{z_0\}) = \emptyset.$$

If  $z \in D_r(w) \setminus z_0$ , then  $|f(z) - w| > \delta$ . Define  $g(z) = \frac{1}{f(z) - w}$  on  $D_r(z_0) \setminus \{z_0\}$ ; then  $|g(z)| < \frac{1}{\delta}$ . Then  $g(z)$  has a removable singularity at  $z = z_0$ .  
If  $g(z_0) \neq 0$ , then  $f(z) - w$  is holomorphic at  $z_0$ . If not, then  $z_0$  is a pole.

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