

CRAG

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Zeta  
Functions

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## The Weil Conjectures

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April 2020

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## Zeta Functions

# Definition of Zeta Function

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Fix  $q$  a prime and  $\mathbb{F} := \mathbb{F}_q$  the (unique) finite field with  $q$  elements, along with its (unique) degree  $n$  extensions

$$\mathbb{F}_{q^n} = \left\{ x \in \bar{\mathbb{F}}_q \mid x^{q^n} - x = 0 \right\} \quad \forall n \in \mathbb{Z}^{\geq 2}$$

## Definition (Zeta Function)

Let  $J = \langle f_1, \dots, f_M \rangle \trianglelefteq k[x_0, \dots, x_n]$  be an ideal, then a *projective algebraic* variety  $X \subset \mathbb{P}_{\mathbb{F}}^n$  can be described as

$$X = V(J) = \left\{ \mathbf{x} \in \mathbb{P}_{\mathbb{F}}^n \mid f_1(\mathbf{x}) = \dots = f_M(\mathbf{x}) = \mathbf{0} \right\}$$

where  $J$  is generated by *homogeneous* polynomials in  $n + 1$  variables, i.e. there is some fixed  $d \in \mathbb{Z}^{\geq 1}$  such that

$$f(\mathbf{x}) = \sum_{\substack{\mathbf{l}=(i_1, \dots, i_n) \\ \sum_j i_j = d}} \alpha_{\mathbf{l}} \cdot x_0^{i_1} \cdots x_n^{i_n} \quad \text{and} \quad f(\lambda \cdot \mathbf{x}) = \lambda^d f(\mathbf{x}), \lambda \in \mathbb{F}^{\times}.$$