Title

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Remark 1.
There is a natural action of $\mathrm{MCG}(\Sigma)$ on $H_1(\Sigma; \mathbb{Z})$, i.e. a *homology representation* of $\mathrm{MCG}(\Sigma)$:

$$\begin{aligned} \rho : \mathrm{MCG}(\Sigma) &\to \mathrm{Aut}_{\mathrm{Grp}}(H_1(\Sigma; \mathbb{Z})) \\ f &\mapsto f_*. \end{aligned}$$

Theorem 1.1 ***(Mapping Class Group of the Torus).***
The homology representation of the torus induces an isomorphism

$$\sigma : \mathrm{MCG}(\Sigma_2) \xrightarrow{\cong} \mathrm{SL}(2, \mathbb{Z})$$

Proof .

- For f any automorphism, the induced map $f_* : \mathbb{Z}^2 \to \mathbb{Z}^2$ is a group automorphism, so we can consider the map

$$\begin{aligned} \tilde{\sigma} : \mathrm{Map}(X, X) &\to \mathrm{GL}(2, \mathbb{Z}) \\ f &\mapsto f_*. \end{aligned}$$

- This will descend to the quotient $\mathrm{MCG}(X)$ iff $\tilde{\sigma}$ is constant

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