Title

D. Zack Garza

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1 Friday February 21st

Question: how do we define $h_{V,D}$?

Answer: write $D = D_1 - D_2$ which are (very) ample divisors and basepoint free. We then obtain embeddings

$$\begin{aligned}
\varphi_1 &: V \hookrightarrow \mathbb{P}_K^{n_1} \\
\varphi_2 &: V \hookrightarrow \mathbb{P}_K^{n_2}.
\end{aligned}$$

So write

$$h_{V,D}(p) = h(\varphi_1(p)) - h(\varphi_2(p)) + O(1)$$

Example 1.1.
For E/K an elliptic curve,

- $2[0]$ is an ample divisor
- $3[0]$ is a very ample divisor.

Let K be a local field (i.e. $\mathbb{C}, \mathbb{R}$, a p-adic field, or $\mathbb{F}_q((t))$ formal Laurent series) and A/K be an abelian variety; we want to understand $A(K)$. We know this has the structure of compact abelian K-analytic Lie group.

- Question 1: What does Lie theory say?
- Question 2: What extra information comes from A/K being a g-dimensional abelian variety?

If $K = \mathbb{C}$, then $A(K) \cong (\mathbb{R}/\mathbb{Z})^{2g}$. If $K = \mathbb{R}$, then $A(K) \cong (\mathbb{R}/\mathbb{Z})^g \oplus \prod_{i=1}^d \mathbb{Z}/2\mathbb{Z}$ where $0 \leq d \leq g$.

Fix d, then

- Let $E_1/\mathbb{R}$ with $\Delta > 0$ (and thus 3 real roots), then $E_1(\mathbb{R})[2] = (\mathbb{Z}/2\mathbb{Z})^2$.
- Let $E_2/\mathbb{R}$ with $\Delta < 0$ (and 1 real root), then $E_2(\mathbb{R})[2] = \mathbb{Z}/2\mathbb{Z}$.

By taking products of E_1 and E_2, i.e. $A = (E_1)^d \times (E_2)^{g-d}$.

Todo: find reference in Silverman?

Fact $A(K)$ is totally disconnected and homeomorphic to a Cantor set.

Fact (From Lie Theory, Serre p.116) There exists a filtration by open finite index subgroups

$$G = G^0 \supset G^1 \supset \cdots \supset G^n \supset \cdots$$

such that

1. The successive quotients are finite, and each G^i is *standard*, i.e. obtained by evaluating a formal group law on $\left(\mathfrak{m}^i\right)^g$.
2. $\bigcap_i G^i = (0)$.
3. G^i/G^{i+1} has exponent p, i.e. it is a finite dimensional $\mathbb{Z}/p\mathbb{Z}$-vector space.
4. $G'[\text{tors}] = G'[p^\infty]$.