

# Algebra

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Definition: A *group* is an ordered pair  $(G, \cdot : G \times G \rightarrow G)$  where  $G$  is a set and  $\cdot$  is a binary operation, which satisfies the following axioms:

1. Associativity:  $(g_1 g_2) g_3 = g_1 (g_2 g_3)$
2. Identity:  $\exists e \in G \ni ge = eg = g$
3. Inverses:  $g \in G \implies \exists h \in G \ni gh = gh = e$ .

Some examples of groups:

- $(\mathbb{Z}, +)$
- $(\mathbb{Q}, +)$
- $(\mathbb{Q}^\times, \times)$
- $(\mathbb{R}^\times, \times)$
- $(\text{GL}(n, \mathbb{R}), \times) = \{A \in \text{Mat}_n \ni \det(A) \neq 0\}$
- $(S_n, \circ)$

Definition: A subset  $S \subseteq G$  is a *subgroup* of  $G$  iff

1.  $s_1, s_2 \in S \implies s_1 s_2 \in S$
2.  $e \in S$
3.  $s \in S \implies s^{-1} \in S$

We denote such a subgroup  $S \leq G$ .

Examples:

- $(\mathbb{Z}, +) \leq (\mathbb{Q}, +)$
- $\text{SL}(n, \mathbb{R}) \leq \text{GL}(n, \mathbb{R})$ , where  $\text{SL}(n, \mathbb{R}) = \{A \in \text{GL}(n, \mathbb{R}) \ni \det(A) = 1\}$

Definition: A group  $G$  is cyclic iff  $G$  is generated by a single element.

Exercise: Show  $\langle g \rangle = \{g^n \ni n \in \mathbb{Z}\} \cong \bigcap \{H \leq G \ni g \in H\}$ .