

## Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

# Summary/Outline

# Outline

What we're trying to prove:

- 8.1.5:  $(d\mathcal{F})_u$  is a Fredholm operator of index  $\mu(x) - \mu(y)$ .

What we have so far:

- Define

$$L : W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \longrightarrow L^p(\mathbb{R} \times S^1; \mathbb{R}^{2n})$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y$$

where

$$S : \mathbb{R} \times S^1 \longrightarrow \text{Mat}(2n; \mathbb{R})$$
$$S(s, t) \xrightarrow{s \rightarrow \pm\infty} S^\pm(t).$$

# Outline

- Took  $R^\pm : I \longrightarrow \text{Sp}(2n; \mathbb{R})$ : symplectic paths associated to  $S^\pm$
- These paths defined  $\mu(x), \mu(y)$
- Section 8.7:

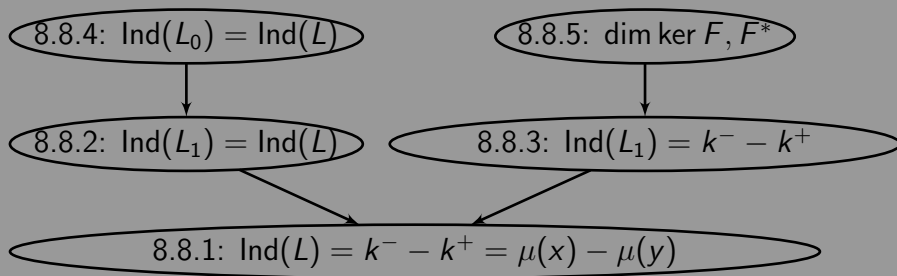
$$R^\pm \in \mathcal{S} := \left\{ R(t) \mid R(0) = \text{id}, \det(R(1) - \text{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\text{Ind}(L) \stackrel{\text{T hm?}}{=} \mu(R^-(t)) - \mu(R^+(t)) = \mu(x) - \mu(y).$$

# From Yesterday

- Han proved 8.8.2 and 8.8.4.
  - So we know  $\text{Ind}(L) = \text{Ind}(L_1)$
- Today: 8.8.5 and 8.8.3:
  - Computing  $\text{Ind}(L_1)$  by computing kernels.



$$8.8.3: \text{Ind}(L_1) = k^- - k^+$$

# Setup

Used in the proof of 8.8.3 ( $\text{Ind}(L_1) = k^- - k^+$ ).

Recall

$$L : W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \longrightarrow L^p(\mathbb{R} \times S^1; \mathbb{R}^{2n})$$

$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y$$

$$L_1 : W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \longrightarrow L^p(\mathbb{R} \times S^1; \mathbb{R}^{2n})$$

$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s)Y$$

Define its adjoint

$$L_1^* : W^{1,q}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \longrightarrow L^q(\mathbb{R} \times S^1; \mathbb{R}^{2n})$$

$$Z \longmapsto -\frac{\partial Z}{\partial s} + J_0 \frac{\partial Z}{\partial t} + {}^t S(s)Z$$

# Setup

We have

$$J_0^1 := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \implies J_0 = \begin{bmatrix} J_0^1 & & \\ & J_0^1 & \\ & & \ddots \\ & & & J_0^1 \end{bmatrix} \in \bigoplus_{i=1}^n \text{Mat}(2; \mathbb{R}).$$



# Outline

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## 8.8.5: $\dim \ker F, F^*$

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