

# Section 8.6 - 8.8: Setup for Computing the Index

May 27, 2020

# Intro

# Outline

What we're trying to prove:

- 8.1.5:  $(d\mathcal{F})_u$  is a Fredholm operator of index  $\mu(x) - \mu(y)$ .

What we have so far:

- Define

$$L : W^{1,p}(\mathbb{R} \times S^1; \mathbb{R}^{2n}) \longrightarrow L^p(\mathbb{R} \times S^1; \mathbb{R}^{2n})$$
$$Y \longmapsto \frac{\partial Y}{\partial s} + J_0 \frac{\partial Y}{\partial t} + S(s, t)Y$$

where

$$S : \mathbb{R} \times S^1 \longrightarrow \text{Mat}(2n; \mathbb{R})$$
$$S(s, t) \xrightarrow{s \rightarrow \pm\infty} S^\pm(t).$$

# Outline

- Took  $R^\pm : I \longrightarrow \mathrm{Sp}(2n; \mathbb{R})$ : symplectic paths associated to  $S^\pm$
- These paths defined  $\mu(x), \mu(y)$
- Section 8.7:

$$R^\pm \in \mathcal{S} := \left\{ R(t) \mid R(0) = \mathrm{id}, \det(R(1) - \mathrm{id}) \neq 0 \right\} \implies L \text{ is Fredholm.}$$

- WTS 8.8.1:

$$\mathrm{Ind}(L) \stackrel{\mathrm{Thm?}}{=} \mu(R^-(t)) - \mu(R^+(t)) = \mu(x) - \mu(y).$$

# From Yesterday

– Proved 8.8.2:

$$8.8.4: \text{Ind}(L_0) = \text{Ind}(L)$$



$$8.8.2: \text{Ind}(L_1) = \text{Ind}(L)$$

$$8.8.5: \dim \ker F, F^*$$



$$8.8.3: \text{Ind}(L_1) = k^- - k^+$$



$$8.8.1: \text{Ind}(L) = k^- - k^+ = \mu(x) - \mu(y)$$

# Section 1

# Outline

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## Section 2



# Outline

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