Title

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1.1 Review and Proposition

From last time: Steinberg's tensor product.

Let G be a reductive algebraic group scheme over k with char $(k) > 0$. We have a Frobenius $F : G \to G$, we iterate to obtain F^r and examine the Frobenius kernels $G_r := \ker F^r$.

If we have a representation $\rho : G \to \mathrm{GL}(M)$, we can "twist" by F^r to obtain $\rho^{(r)} : G \to \mathrm{GL}(M^{(r)})$. We have

$$G \xrightarrow{\rho} \mathrm{GL}(M)$$

Here $M^{(r)}$ has the same underlying vector space as M, but a new module structure coming from $\rho^{(r)}$. Note that G_r acts trivially on $M^{(r)}$.

- $\left\{ L(\lambda) \;\middle|\; \lambda \in X(T)_+ \right\}$ are the simple G-modules,
- $\left\{ L_r(\lambda) \;\middle|\; \lambda \in X_r(T)_+ \right\}$ are the simple G_r-modules,

Note that $L(\lambda) \downarrow_{G_r}$ is semisimple, equal to $L_r(\lambda)$ for $\lambda \in X_r(T)$.

1960's, Curtis and Steinberg.

Proposition 1.1 ***(?).***
Let $\lambda \in X_r(T)$ and $\mu \in X(T)_+$. Then

$$L(\lambda + p^r \mu) \cong L(\lambda) \otimes L(\mu)^{(r)}.$$

Recall that socle formula: letting M be a G-module, we have an isomorphism of G-modules:

$$\operatorname{Soc}_{G_r} \cong \bigoplus_{\lambda \in X_r(T)} L(\lambda) \otimes \hom_{G_r}(L(\lambda), M).$$

1.2 Proof

Let $M = L(\lambda + p^r \mu)$.