

Problem Set 2

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Wednesday 16th September, 2020

1 | Exercises

Exercise 1.1 (Gathmann 2.17).

Find the irreducible components of

$$X = V(x - yz, xz - y^2) \subset \mathbb{A}^3/\mathbb{C}.$$

Solution:

Since $x = yz$ for all points in X , we have

$$X = V(x - yz, yz^2 - y^2) = V(x - yz, y()).$$

Exercise 1.2 (Gathmann 2.18).

Let $X \subset \mathbb{A}^n$ be an arbitrary subset and show that

$$V(I(X)) = \overline{X}.$$

Solution:

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Exercise 1.3 (Gathmann 2.21).

Let $\{U_i\}_{i \in I} \rightrightarrows X$ be an open cover of a topological space with $U_i \cap U_j \neq \emptyset$ for every i, j .

a. Show that if U_i is connected for every i then X is connected.

b. Show that if U_i is irreducible for every i then X is irreducible.

Solution:

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Exercise 1.4 (Gathmann 2.22).

Let $f : X \rightarrow Y$ be a continuous map of topological spaces.

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- a. Show that if X is connected then $f(X)$ is connected.
 - b. Show that if X is irreducible then $f(X)$ is irreducible.

Solution:

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Definition 1.0.1 (Ideal Quotient).

For two ideals $J_1, J_2 \subseteq R$, the *ideal quotient* is defined by

$$J_1 : J_2 := \left\{ f \in R \mid fJ_2 \subseteq J_1 \right\}.$$

Solution:

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Exercise 1.5 (Gathmann 2.23).

Let X be an affine variety.

- a. Show that if $Y_1, Y_2 \subset X$ are subvarieties then

$$I(\overline{Y_1 \setminus Y_2}) = I(Y_1) : I(Y_2).$$

- b. If $J_1, J_2 \subseteq A(X)$ are radical, then

$$\overline{V(J_1) \setminus V(J_2)} = V(J_1 : J_2).$$

Solution:

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Exercise 1.6 (Gathmann 2.24).

Let $X \subset \mathbb{A}^n$, $Y \subset \mathbb{A}^m$ be irreducible affine varieties, and show that $X \times Y \subset \mathbb{A}^{n+m}$ is irreducible.

Solution:

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