Title

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August 17, 2019

Contents

1 List of Topics

Chapters 1-9 of Dummit and Foote

- Left and right cosets
- Lagrange's theorem
- Isomorphism theorems
- Group generated by a subset
- Structure of cyclic groups
- Composite groups
  - HK is a subgroup iff $HK = KH$
- Normalizer
  - $HK \leq H$ if $H \leq N_G(K)$
- Symmetric groups
  - Conjugacy classes are determined by cycle types
- Group actions
  - Actions of G on X are equivalent to homomorphisms from G into Sym(X)
- Cayley's theorem
- Orbits of an action
- Orbit stabilizer theorem
- Orbits act on left cosets of subgroups
- Subgroups of index p, the smallest prime dividing $|G|$, are normal
- Action of G on itself by conjugation
- Class equation
- p-groups
  - Have non trivial center
- p^2 groups are abelian
- Automorphisms, the automorphism group
  - Inner automorphisms
  - $Inn(G) \cong Z/Z(G)$
  - $Aut(S_n) = Inn(S_n)$ unless $n = 6$
  - $Aut(G)$ for cyclic groups

  - $G \cong Z_p^n$, then $Aut(G) \cong GL_n(Z_p)$
- Proof of Sylow theorems
- A_n is simple for $n \geq 5$
- Recognition of internal direct product
- Recognition of semi-direct product
- Classifications:
  - pq
- Free group & presentations
- Commutator subgroup
- Solvable groups
  - S_n is solvable for $n \leq 4$
- Derived series
  - Solvable iff derived series reaches e
- Nilpotent groups
  - Nilpotent iff all sylow-p subgroups are normal
  - Nilpotent iff all maximal subgroups are normal
- Upper central series
  - Nilpotent iff series reaches G
- Lower central series
  - Nilpotent iff series reaches e
- Fratini's argument
- Rings
  - I maximal iff R/I is a field
  - Zorn's lemma
  - Chinese remainer theorem
  - Localization of a domain
  - Field of fractions
  - Factorization in domains
  - Euclidean algorithm
  - Gaussian integers
  - Primes and irreducibles
  - Domains
    - Primes are irreducible
  - UFDs
    - Have GCDs
    - Sometimes PIDs
  - PIDs
    - Noetherian
    - Irreducibles are prime
    - Are UFDs
    - Have GCDs
  - Euclidean domains
    - Are PIDs
  - Factorization in $Z[i]$
  - Polynomial rings
  - Gauss' lemma
  - Remainder and factor theorem
  - Polynomials

– Reducibility
– Rational root test
– Eisenstein's criterion