Title

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Next topic: Kempf's Vanishing Theorem. Proof in Jantzen's book involving ampleness for sheaves.

Setup:

We have

$$\begin{array}{cc} G & \text{a reductive algebraic group over } k = \bar{k} \\ \uparrow\subseteq & \\ B & \text{the Borel subgroup} \\ \uparrow\subseteq & \\ T & \text{its maximal torus} \end{array}$$

along with the weights $X(T)$.

We can consider derived functors of induction, yielding $R^n \operatorname{Ind}_B^G \lambda = \mathcal{H}^n(G/B, \mathcal{L}(\lambda)) \coloneqq H^n(\lambda)$ where $\mathcal{L}(\lambda)$ is a line bundle and G/B is the flag variety.

Recall that

- $H^0(\lambda) = \operatorname{Ind}_B^G(\lambda)$,
- $\lambda \notin X(T)_+ \implies H^0(\lambda) = 0$
- $\lambda \in X(T)_+ \implies L(\lambda) = \operatorname{Soc}_G H^0(\lambda) \neq 0$.

Theorem 1.1 ***(Kempf).***
If $\lambda \in X(T)_+$ a dominant weight, then $H^n(\lambda) = 0$ for $n > 0$.

Remark 1.
In char $(k) = 0$, $H^n(\lambda)$ is known by the Bott-Borel-Weil theorem. In positive characteristic, this is not know: the characters char $H^n(\lambda)$ is known, and it's not even known if or when they vanish. Wide open problem!

Could be a nice answer when $p > h$ the Coxeter number.