# Title

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Tuesday $25^{\text{th}}$ August, 2020

## Contents



## 1 Tuesday, August 25

Let $k = \bar{k}$ and $R$ a ring containing ideals $I, J$.

> **Definition 1.0.1** (Radical)**.**
> Recall that the *radical* of $I$ is defined as
>
> $$\sqrt{I} = \left\{ r \in R \;\middle|\; r^k \in I \text{ for some } k \in \mathbb{N} \right\}.$$

**Example 1.1.**
Let $I = (x_1, x_2^2) \subset \mathbb{C}[x_1, x_2]$, so $I = \left\{ f_1 x_1 + f_2 x_2 \;\middle|\; f_1, f_2 \in \mathbb{C}[x_1, x_2] \right\}$. Then $\sqrt{I} = (x_1, x_2)$, since $x_2^2 \in I \implies x_2 \in \sqrt{I}$.

Given $f \in k[x_1, \cdots, x_n]$, take its value at $a = (a_1, \cdots, a_n)$ and denote it $f(a)$. Set $\deg(f)$ to be the largest value of $i_1 + \cdots + i_n$ such that the coefficient of $\prod x_j^{i_j}$ is nonzero.

**Example 1.2.**
$\deg(x_1 + x_2^2 + x_1 x_2^3 = 4)$

> **Definition 1.0.2** (Affine Variety)**.**
>
> 1. Affine $n$-space $\mathbb{A}^n = \mathbb{A}_k^n$ is defined as $\left\{ (a_1, \cdots, a_n) \;\middle|\; a_i \in k \right\}$.
>
>    Remark: not $k^n$, since we won't necessarily use the vector space structure (e.g. adding points).
>
> 2. Let $S \subset k[x_1, \cdots, x_n]$ to be a set of polynomials. Then define $V(S) = \left\{ x \in \mathbb{A}^n \;\middle|\; f(x) = 0 \right\} \subset$.