Title

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1.1 Setup

- (M, ω) a symplectic manifold, $H \in ?$ a Hamiltonian, X_H its ?
- $\displaystyle\int_{S^2} u^* \omega = \sigma_1$ where $u \in C^\infty(S^2, W)$.
- $\langle c_1(TW),\ \pi_2(TW) \rangle = 0$?
- $C_k(H) \coloneqq \mathbb{Z}/2\mathbb{Z}[S]$ where S is the set of periodic orbits of X_H of Maslov index k.
- x, y critical points of $\mathcal{A}_H$ with $\mathcal{M}(x, y)$ the moduli space of contractible solutions of finite energy connecting x, y.

1.2 Review Last Time

- $\mathbb{R} \curvearrowright \mathcal{M}_{x,y}$, so we quotient to define $\mathcal{L}(x, y) =\coloneqq \mathcal{M}_{x,y}/\mathbb{R}$ with the quotient topology.
- Topology defined by when sequences converge:

$$\tilde{u}_n \overset{n\to\infty}{\to} \tilde{u} \iff \exists \{s_n\} \subseteq \mathbb{R} \text{ such that } u_n(s_n + s, \,\cdot\,) \overset{n\to\infty}{\to} u(s, \cdot).$$

Proposition 1.1 ***(?).***
$\mathcal{L}(x, y)$ is Hausdorff.

- Want to show $\mathcal{L}(x, y)$ is a compact 0-dimensional manifold.
- Have a differential

$$\partial : C_k(H) \longrightarrow C_{k-1}(H)$$
$$\partial(x) = \sum_{\mathrm{Ind}(y)=k-1} n(x,y)y.$$

with $n(x,y)$ the number (mod 2) of trajectories of grad $\mathcal{A}_H$ connecting x,y, i.e solutions to the Floer equation.

- Want to prove that the following is a 1-dimensional manifold:

$$M := \overline{\mathcal{L}}(x,z) = \mathcal{L}(x,z) \cup_{\mu(y)=\mu(x)+1} \mathcal{L}(x,y) \times \mathcal{L}(y,z).$$

  and show that M is compact with ∂M equal to the last union.

- Last time: closure of space of trajectories connecting x,y contains "broken" trajectories.
- Last time: toward proving that M is compact

1.3 Upcoming

- Wanted to compactify $\mathcal{L}(x,y)$, needed to go to space of broken trajectories.
- Main theorem of chapter 9: 9.2.1.

Theorem 1.2 ***(9.2.1).***
Let (H,J) be a regular pair with H nondegenerate.
Let x,z be two periodic trajectories of H such that $\mu(x)=\mu(z)+2$.
Then $\overline{\mathcal{L}}(x,y)$ is a compact 1-manifold with boundary satisfying

$$\partial\overline{\mathcal{L}}(x,y) = \bigcup_{\mu(x)<\mu(y)<\mu(z)} \mathcal{L}(x,y) \times \mathcal{L}(y,z).$$

As a corollary, $\partial^2 = 0$.

- Know $\overline{\mathcal{L}}(x,y)$ is compact and $\mathcal{L}(x,y)$ is a 1-manifold
- Now suffices to study in a neighborhood of boundary points ("gluing theorem")

Three steps to gluing theorem:

1. Pre-gluing: Get a function w_p which interpolates between u and v (not exactly a solution itself, but will be approximated by one later).
2. Constructing ψ a "true solution" from w_p using the Newton-Picard method. We'll have

$$\psi(p) = \exp_{w_p}(\gamma(p)) \qquad \gamma(p) \in W^{1,p}(w_p^* TW) = T_{w_p}\mathcal{P}(x,z).$$

   where $\mathcal{P} = ?$.

3. Get a lift $\widehat{\psi} = \pi \circ \psi$ where $\pi = ?$ satisfying

- $\widehat{\psi}(p) \overset{n\to\infty}{\to} (\widehat{u}, \widehat{v})$
- $\widehat{\varphi}$ is an embedding
- $\widehat{\psi}$ is unique in the following sense:

Theorem 1.3*(9.2.3).*
Let x, y, z be critical points of the action functional $\mathcal{A}_H$ such that $\mu(x) = \mu(y) + 1 = \mu(z) + 2$. Let $(u, v) \in \mathcal{M}(x, y) \times \mathcal{M}(y, z)$ be trajectories, inducing $(\bar{u}, \bar{v}) \in \mathcal{L}(x, y) \times \mathcal{L}(y, z)$.

- There exist a differentiable map $\psi : \rho_0, \infty) \to \mathcal{M}(x, z)$ for some $\rho > 0$ such that
- $\pi \circ \psi : (\rho_0, \infty) \to \mathcal{L}(x, z)$ is an embedding
- $\widehat{\psi} \overset{\rho\to\infty}{\to} (\bar{u}, \bar{v}) \in \overline{\mathcal{L}(x, z)}$.
- If $\ell_n \in \mathcal{L}(x, z)$ with $\ell_n \overset{n\to\infty}{(} \quad \bar{u}, \bar{v})$, then for $n \gg 1$ we have $\ell \in\in \Im(\widehat{\psi})$.