

# Problem Set 7

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## 1 Regular Problems

### 1.1 Problem 1

We want to show that  $G$  has a non-trivial normal subgroup, noting that  $\#G = p^2q$  where  $p = 2, q = 3$ , we will proceed by showing that either  $n_p$  or  $n_q$  must be 1.

We immediate note that we have

$$\begin{array}{ll} n_p \equiv 1 \pmod{p} & n_q \equiv 1 \pmod{q} \\ n_p \mid q & n_q \mid p^2, \end{array}$$

which immediately forces

$$n_p \in \{1, q\}, \quad n_1 \in \{1, p, p^2\}.$$

Proceeding by cases:

#### 1.1.1 Case 1: $p = q$ .

Then  $\#G = p^3$  and  $G$  is a  $p$ -group. But every  $p$ -group has a non-trivial center  $Z(G) \leq G$ , and the center is always a normal subgroup.

#### 1.1.2 Case 2: $p > q$ .

In this case we have

## 2 Qual Problems