# Title

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## Contents



## 1 Fall 2014

### 1.1 Fall 2014 # 2

Let $I$ be an index set and $\alpha : I \longrightarrow (0, \infty)$.

1. Show that

$$\sum_{i \in I} a(i) := \sup_{\substack{J \subset I \\ J \text{ finite}}} \sum_{i \in J} a(i) < \infty \implies I \text{ is countable.}$$

2. Suppose $I = \mathbb{Q}$ and $\sum_{q \in \mathbb{Q}} a(q) < \infty$. Define

$$f(x) := \sum_{\substack{q \in \mathbb{Q} \\ q \leq x}} a(q).$$

   Show that $f$ is continuous at $x \iff x \notin \mathbb{Q}$.

### 1.2 Fall 2014 # 3

Let $f \in L^1(\mathbb{R})$. Show that

$$\forall \varepsilon > 0 \ \exists \delta > 0 \text{ such that } m(E) < \delta \implies \int_E |f(x)| dx < \varepsilon$$

## 1.3 Fall 2014 # 4

Let $g \in L^\infty([0,1])$ Prove that

$$\int_{[0,1]} f(x)g(x)dx = 0 \quad \text{for all continuous } f : [0,1] \longrightarrow \mathbb{R} \implies g(x) = 0 \text{ almost everywhere.}$$

## 1.4 Fall 2014 # 5

1. Let $f \in C_c^0(\mathbb{R}^n)$, and show

$$\lim_{t \to 0} \int_{\mathbb{R}^n} |f(x+t) - f(x)| dx = 0.$$

2. Extend the above result to $f \in L^1(\mathbb{R}^n)$ and show that

$$f \in L^1(\mathbb{R}^n), \ g \in L^\infty(\mathbb{R}^n) \implies f * g \text{ is bounded and uniformly continuous.}$$