Study Guide for Complex Analysis Exam

I. Calculus and Undergraduate Analysis

Continuity and differentiation in one and several real variables

Inverse and implicit function theorems

Compactness and connectedness in analysis

Uniform convergence and uniform continuity

Riemann integrals

Contour integrals and Green's theorem

Reference: [3].

II. Preliminary Topics in Complex Analysis

Complex arithmetic

Analyticity, harmonic functions, and the Cauchy-Riemann equations

Contour Integration in C

References: [1] Chapters 1, 2; [2] Chapters 1, 2, 4; [4] Chapter 1.

III. Cauchy's Theorem and its consequences

Cauchy's theorem and integral formula, Morera's theorem, Schwarz reflection

Uniform convergence of analytic functions

Taylor and Laurent expansions

Maximum modulus principle and Schwarz's lemma

Liouville's theorem and the Fundamental theorem of algebra

Residue theorem and applications

Singularities and meromorphic functions, including the Casorati-Weierstrass theorem

Rouche's theorem, the argument principle, and the open mapping theorem

Estimates using Cauchy Integral Formula: Cauchy inequalities and, more generally,

bounds on holomorphic functions and their derivatives on compact sets

References: [1] Chapters 4, 5, 6; [2] Chapters 5, 7, 8, 9; [4] Chapters 2, 3, 5, 8 (Â§2,3).

IV. Conformal Mapping

General properties of conformal mappings

Analytic and mapping properties of linear fractional transformations

Automorphisms of the disk, plane, and Riemann sphere

References: [1] Chapters 3, 8; [2] Chapters 3, 4; [4] Chapter 8 (Â§1,2).

References

[1] L. Ahlfors, Complex Analysis, Third Edition, McGraw-Hill.

[2] E. Hille, Analytic Function Theory, Vol. 1, Ginn and Company.

[3] W. Rudin, Principles of Mathematical Analysis, Third Edition, McGraw-Hill.

[4] E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press.

[Revised June 2007]