Title

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1 Modules

1.1 General Questions

1.1.1 Fall 2019 Final #2

Consider the $\mathbb{Z}$-submodule N of $\mathbb{Z}^3$ spanned by $f_1 = [-1, 0, 1], f_2 = [2, -3, 1], f_3 = [0, 3, 1], f_4 = [3, 1, 5]$. Find a basis for N and describe $\mathbb{Z}^3/N$.

1.1.2 Spring 2018 #6.

Let

$$M = \{(w, x, y, z) \in \mathbb{Z}^4 \mid w + x + y + z \in 2\mathbb{Z}\},$$

and

$$N = \{(w, x, y, z) \in \mathbb{Z}^4 \mid 4 \mid (w - x), \ 4 \mid (x - y), \ 4 \mid (y - z)\}.$$

(a) Show that N is a $\mathbb{Z}$-submodule of M .

(b) Find vectors $u_1, u_2, u_3, u_4 \in \mathbb{Z}^4$ and integers d_1, d_2, d_3, d_4 such that

$$\{u_1, u_2, u_3, u_4\}$$

is a free basis for M, and

$$\{d_1 u_1, \ d_2 u_2, \ d_3 u_3, \ d_4 u_4\}$$

is a free basis for N .

(c) Use the previous part to describe M/N as a direct sum of cyclic $\mathbb{Z}$-modules.