Title

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1 | Saturday, November 28: Introduction to ∞-categories

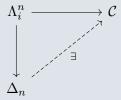Dealing with size issues: take a Grothendieck Universe $\mathcal{U}$: sets whose subsets are closed under all of the usual set operations (small).

> **Definition 1.0.1** (∞-Category)
> An ∞-category $\mathcal{C}$ is a (large) simplicial set $\mathcal{C}$ such that any diagram of the form
>
> $$\begin{array}{ccc} \Lambda_i^n & \longrightarrow & \mathcal{C} \\ \downarrow & \nearrow & \\ \Delta_n & \exists & \end{array}$$
>
> admits the indicated lift, where Λ_i^n is an i-horn (a simplex missing the ith face) for $0 < i < n$.

Remark 1.0.2: All inner horns are fillable, i.e. simplicial sets are *inner* Kan complexes. Different to Kan complexes, which include all i.

> **Definition 1.0.3** (Functors between ∞-categories)
> A functor between two ∞-categories is a map between simplicial sets.

> **Definition 1.0.4** (Nerve of a category)
> Given an ordinary category $\mathcal{C}$, define the **nerve** of $\mathcal{C}$ to be the simplicial set given by
>
> $$N(\mathcal{C})_n := \{\text{Functors } F : [n] \to \mathcal{C}\}$$
>
> where $[n]$ is the poset category on $\{1, 2, \cdots, n\}$. So an n-simplex is a diagram of objects $X_0, \cdots, X_n \in \text{Ob}(\mathcal{C})$ and a sequence of maps. This defines an $