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Taut Sutured handlebodies as Twisted Homology Products

$(N, \mathcal{F})$ a 3-mfd with a foliation, $\mathcal{F}$ a codim 1 (co)oriented is taut if there is a properly embedded 1-mfd $\in \partial N$

When is $\mathcal{F}$ taut?

- $\mathcal{F}$ has no compact leaves, or
- $\mathcal{F}$ has compact leaves which are top. minimal

Cut N along some compact leaf $\to$ sutured mfd

γ = collection in ∂M that separates ∂M

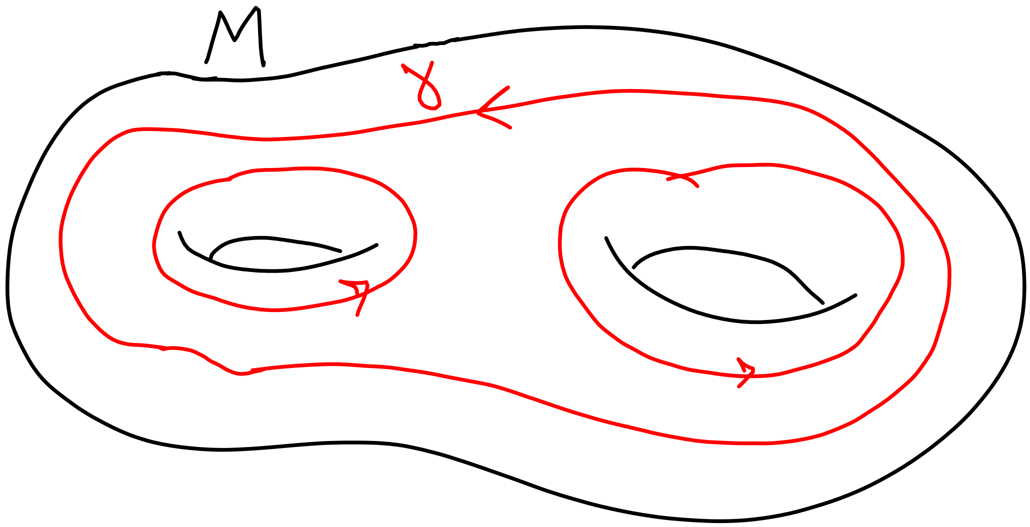
so $\partial M = R_+ \bigsqcup_{\gamma_i} R_-$.

Define the complexity $\chi_N(M) = -1 \cdot \max(\chi(M), 0)$

Thurston Norm: given $\varphi \in H^2(M, \partial M; \mathbb{Z})$

$$\|\varphi\|_T = \min_{[S]=\varphi}$$

Defn: A sutured mfd is taut when irreducible and $R_\pm$ is incompressible and $\|\cdot\|_T$ minimizing



How can we know when M is taut?

If $M = R_+ \times I$ is a product, M is taut

Can weaken to just "looks like a product in homology"

$$H^*(R_+;\mathbb{Q}) \xrightarrow{\cong} H^*(M;\mathbb{Q})$$

Not quite enough structure so allow twisted coefs

$$\alpha : \pi_1 M \to GL(n,\mathbb{C})$$

Note: euler char gets multiplied by $\dim \alpha$,
other thms for homology carry over

<u>Thm</u>: If M is taut, it is a homology product

Just need to produce the right rep, but
a catch - it uses virtual fibering

How can we assign a complexity to such a certifying representation?

Ex: Genus of R_+, # sutures, hyperbolic structures, sutured mfd decomp.

Thm: For M a book of I-bundles, a rep of dim $\underbrace{n=2 \text{ works.}}$

Conjecture: this always works

Restrict to handlebodies, then to solvable reps

Let complexity = length of derived series

$\rightsquigarrow$ Unbounded complexity!