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Taut Foliations on surgered 3-mfds & The Euler class

Motivation: L-space conjecture

M a QHS 3-mfd   Rational homology sphere

$\begin{cases} \pi_1 M \text{ left orderable iff} \\ M \text{ admits a co-orientable taut foliation} \end{cases}$

G is left orderable iff it admits a total order

and a left action $(\cdot c)$: $a \to ca$

where $a \leq b \Rightarrow ca \leq cb$

(Note: forces $|G| = \infty$

Thm: M irreducible, orientable

$\pi_1 M$ is LO iff $\exists\ \rho: \pi_1 M \to \mathrm{Homeo}(\mathbb{R})$

Given $(M, \mathcal{F})$, produce $\pi_1 M \curvearrowright S^1$; does it lift to $\mathbb{R}$?

Uses universal circle action (Thurston)

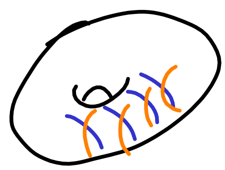
Answer: yes iff $e(\underbrace{T\mathcal{F}}_{\text{Tangent foliation}}) = 0 \in H^2(M)$

$(\Rightarrow \pi_1 M$ is LO$)$

Some Notation

Y a QHS, $K \subseteq Y$ a null-homologous knot

$X = Y \setminus \nu(K)$, $\mu, \lambda \in \partial X$ meridian & longitude

$X(p/q) = X \cup_f$  slope p/q

Thm: $\mathcal{F}$ a $\underbrace{\text{co-oriented taut foliation}}_{\text{CTF}}$ on $X(p/q)$,

$\mathcal{F} \pitchfork$ the core of f surgery solid torus, let

$g = \text{genus}(K)$; $p/q \in M_g \Rightarrow e(T\mathcal{F}) \neq 0$

where $M_g = \mathbb{Z} \cup \{ \bigcup_{n \geq 1} M_{g,n} \}$

$M_{g,1} = \{ -a + 1/k \mid k+a, a \text{ odd } \& \ |a| \leq 2g-1 \}$

$M_{g,n} = \frac{1}{n} M_{g,1}$

Note $M_g \subseteq \mathbb{R}$ is nowhere dense (ie $(\overline{M})^\circ = \emptyset$)

<u>Thm</u>: Let $\mathcal{F}$ be a CTF on X, where $X \pitchfork \partial X$ in simple closed curves of slope p/q. Then $e(T\hat{\mathcal{F}}) = 0 \in H^2(X(p/q))$ where

- $\hat{\mathcal{F}}$ = extension of $\mathcal{F}$ on $X(p/q)$,
- σ = Section of $T\mathcal{F}$ pointing inward on ∂X

iff

① $PD(e_\sigma(T\mathcal{F})) = \underline{\underline{u}}[\mu]$

② $\underline{\underline{u}}(g) = 1 \bmod p$

<u>Pf</u>: Look at SES

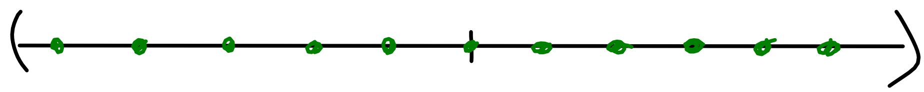
$$0 \to H^1(\partial X) \to \overbrace{H^2(X, \partial X) \oplus H^2(N, \partial N)}^{= H^2(X(p/q), x)} \to \cdots$$

$\downarrow$ restrict $\downarrow$ foliation

Ex: figure 8

Green: Included
Red: Excluded

M_g $\mathbb{Z}$



$M_{g,1}$