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Homology directions & veering triangulations

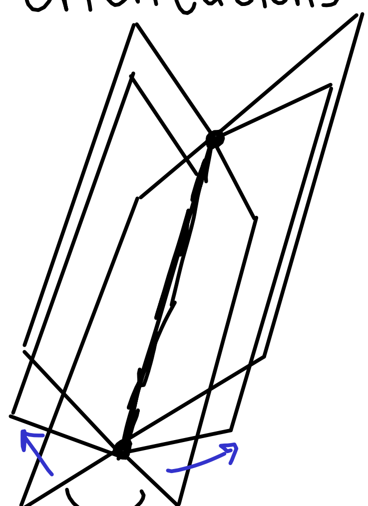
Let M be a compact 3-mfd

An ideal triangulation of $M - \partial M$ satisfies

~ Each face has a co-orientation

- For each 3-cell, 2 faces point inward and 2 out

- Around each edge, faces occur in pairs with opposite orientations



This gives a triangulation of ∂M by flat triangles ($\sum \theta = 2\pi$)

↳ Actually yields a nonvanishing vector field

Defn: A ladder

Defn: Pseudohyperbolic splits into ladders of alternating type

Defn: Veering if induced flat triangulations are all pseudohyperbolic

Thm: Let M be hyperbolic, σ a fibred free of $B_1(M)$ in the Thurston norm.

$\Rightarrow$ There exists a canonical pseudo-Anisov flow φ s.t.

A) $\alpha \in H_2(M;\mathbb{R})$ is represented by a cross section iff $\alpha \in \mathrm{Cone}(\sigma)$

$\rightsquigarrow$ First return map

B) $C_\varphi \in H_1 = \mathrm{cone}^\vee$ where C_φ is the smallest closed cone in the same hty class as φ

Taut branched surfaces: The cone in H^2 carried by a TBS dies inside a cone in B_1; if these cones are equal, say the TBS spans a face

Q (86): Given a face $\sigma \in B_1(M)$, does it have a spanning TBS?

<u>Thm</u>: There exists a canonical veering triangulation X of $\mathrm{int}(M)$ where $X^{(2)}$ spans $\underbrace{\sigma_*}_{\text{fibred face}}$

<u>Application</u>: Transverse Surface Theorem

$\alpha \in H^2$ integral, then $\alpha \in \mathrm{cone}(\sigma)$ iff

iff α represented by a surface almost transverse to φ

(ie φ can be perturbed to be traverse)

Application: Classifying C_φ's (hence cone(σ)'s)

Look at "stable train tracks"

Thm: C_φ is the smallest convex cone in $H_1(M)$ containing the class of each MSL. ∎