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Genus 2 Heegaard Splitting & Dehn Surgery on Tunnel number

Motivation: Let $K \subseteq S^3$ be a knot, H a handlebody. $K \subseteq H$ is <u>primitive</u> if H has a disc D s.t. $D \cap K = \{pt\}$

If S ($g(S) = 2$) Heeg. surface, K is <u>doubly primitive</u> if K is primitive on both sides

$\rightsquigarrow$ K admits a lens space surgery

<u>Berge's Conj</u>: Converse of this

K admits a lens space surgery

$\overset{?}{\Rightarrow}$ K is d.p. wrt a genus 2 H. splitting of S^3

Can generalize to replace $K \subseteq M^3$

Conj: 1) Yes when $M = S^3$

2) Yes when $M = S^2 \times S^1$ $\square$

No for some lens spaces, yes for $M = S^2 \times S^1 \# L(p,q)$

Pushing a knot into a handlebody $\longrightarrow$ S becomes a Heegaard surface for K^c

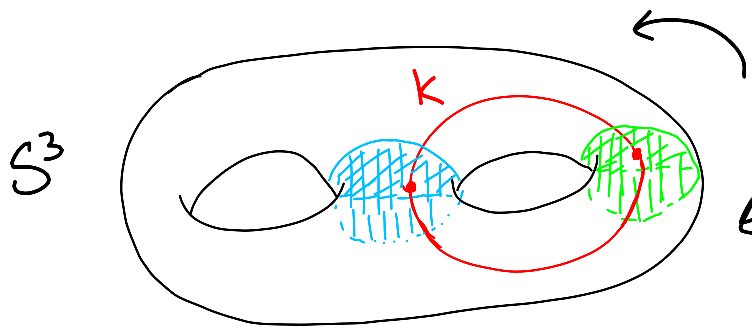
Conj:

1) Tunnel # 1 + admits surgery $\Rightarrow$ double prim

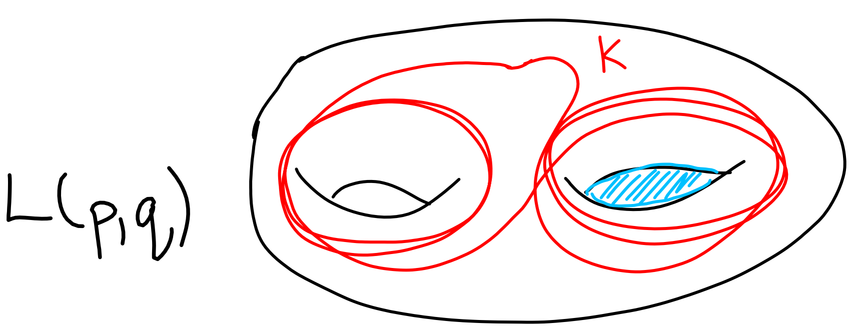
2) Admits surgery $\Rightarrow$ Tunnel # 1

Let $K(s)$ = Dehn surgery with slope s

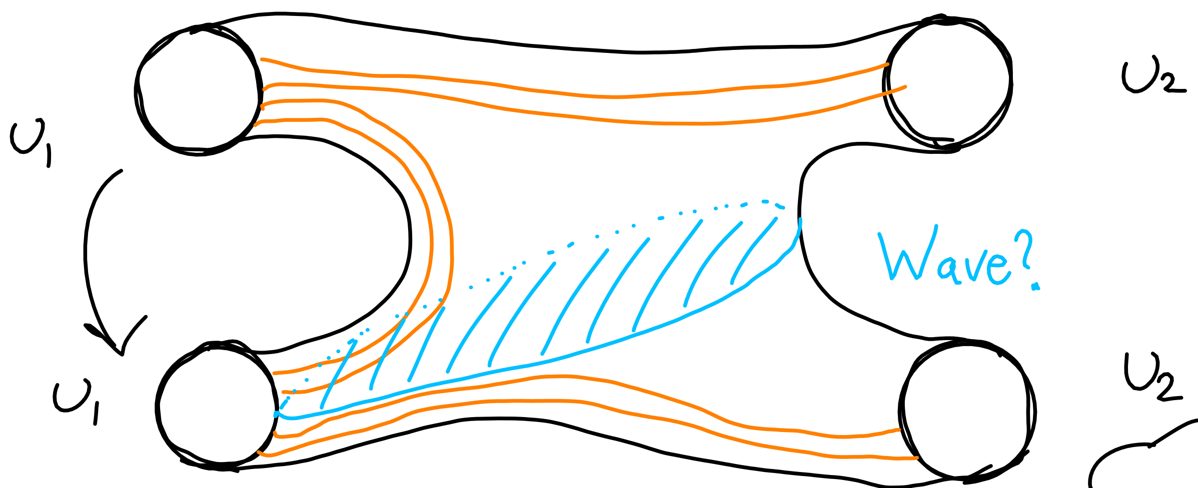
Sp. $K(s) = L(p,q)$



Two annuli, yielding a Heegaard diagram

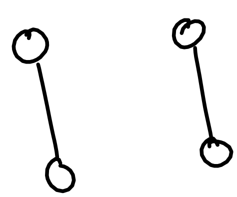


Thm: A genus 2 Heeg. diagram of S^3, it contains a wave



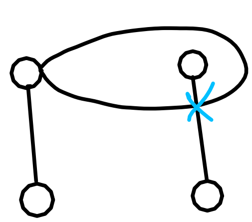
"See 'wave move'"

Correspond to Whitehead graphs



Waves correspond to intersecting arcs

$L(p,q)$



Thm: A genus 2 Heeg diag of $S^2 \times S^1 \# L(p,q)$

is standard, or has a wave

Pf: Any 2 diags. are related by band moves

Def: k-reducible = reduced to standard in k moves

Induct on K, look at diags and see

$S^2 \times S^1$ obstruction when taking band sums

Work in progress

Have (D, P) pair (disc + planar surface)

α in surface, ∂P divides α into segments

Let $c = \# \alpha_i \cap D$; if $c = 0, 1$ then

K is doubly primitive $\rightsquigarrow$ a new Heeg. surface