Taut Foliations and Seiberg-Witten Eqns.

Let

Y = smooth 3-mfd, oriented

F = codim 1, oriented

F is <u>taut</u> if $\forall p \in Y, \exists S^1 \in Y$ s.t.

S^1 passes through p and is transverse to F

<u>Thm</u>: $Y = S^2 \times S^1$ and Y support a taut foliation

$\Rightarrow Y$ is irreducible

<u>Thm</u>: If $b_1(Y) \geq 1$, the converse holds

If $b(Y) = 0$, $\underbrace{Y \text{ is not an L-space.}}_{HM^{Red}(Y) \neq 0}$ See L-Space conjecture

(reduced Floer homology)

Question - If F_1, F_2 are homotopic, are they homotopic through foliations?

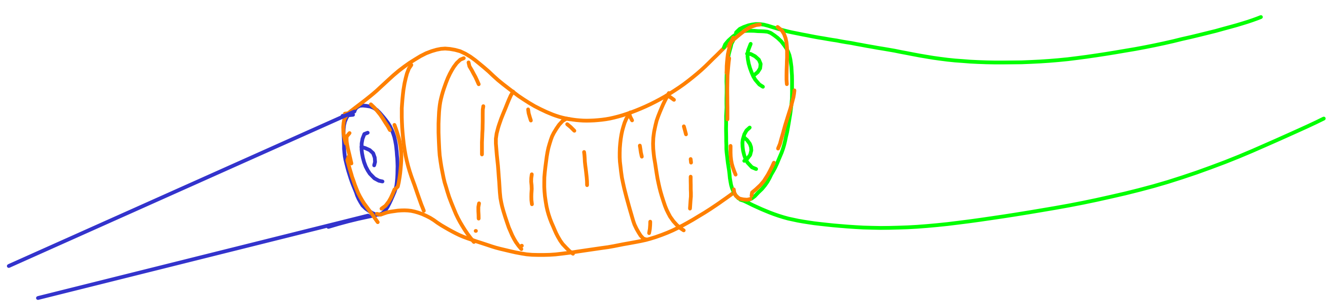
Seiberg-Witten Theory

Essentially the Morse theory of a certain functional on a manifold?

Critical pts $\longrightarrow$ Solns to SW-eqns on Y

Flow lines $\longrightarrow$ " " " " $Y \times \mathbb{R}$

Can count over cobordism



Can use this idea to construct an invariant

↳ Allows reproving old results without heavy theorems

↳ Possible new results, can look at

$\mathcal{M}_J$ = Moduli space of J-holomorphic curves

its cobordism class is a foliation invariant

(Can control non-compact parts when there's a group action.)