

Nathan Dunfield

An $SL(2, \mathbb{R})$
Invariant

If K is a knot in the 3-sphere, then
there exists an irr. rep $\pi_1 M \rightarrow SU_2$

Let

$$\star: h_{SU_2}^0(M) + h_{SL_2 \mathbb{R}}^0(M) = h_{\mathbb{R}}(M)$$

Q: Is this also true for $SL_2 \mathbb{R}$

If $\sigma_K \neq \text{const}$, then we have irr $SL_2 \mathbb{R}$ rep
and by \star , $h_{SU_2}^0 = \frac{1}{2} \sigma_M$, so $h_{SL_2 \mathbb{R}}^0 \neq \text{const}$ and is
thus 0 is non-zero somewhere.

$h_{SL_2 \mathbb{R}}^0 =$ signed count of irr reps $\pi_1(\cdot) \rightarrow SL_2 \mathbb{R}$ where
 $\rho(v)$ is conjugate to $\begin{pmatrix} 1 & \\ 0 & 1 \end{pmatrix}$ (\sim parabolic)

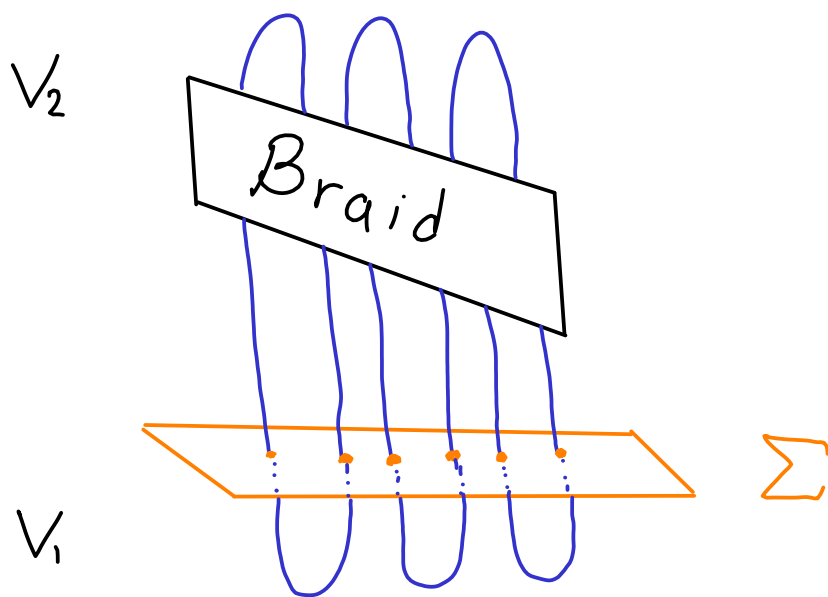
Conj: For K a 2-bridge knot, $\star X_{SL_2 \mathbb{R}}^0 \geq \frac{1}{2} \underbrace{|\sigma_K(0 = \frac{\pi}{2})|}_{\text{signature}}$

Proved, Gordon 2016. Does it hold for all K ?

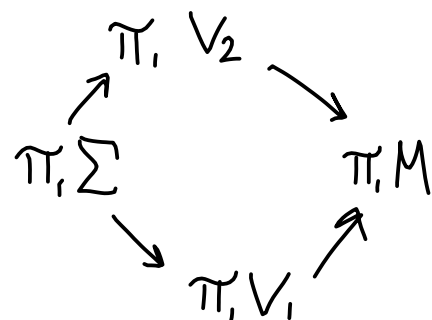
Since 2-fold branched cover is a lens space, where
 π_1 is cyclic and thus has only trivial reps.

How is this invariant defined?

$X_G^\circ \approx \text{character variety}$



V_i : genus n handlebodies



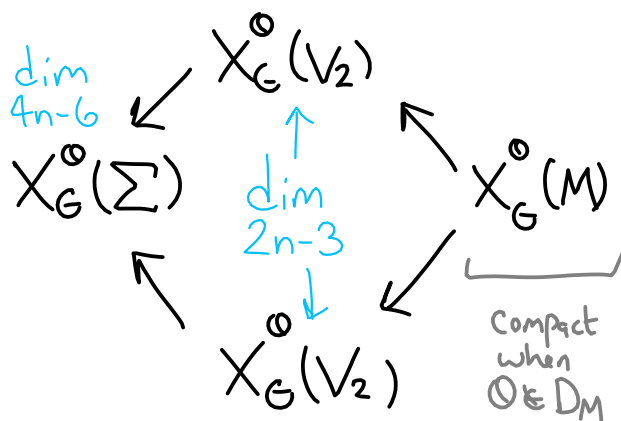
Apply X_G° contravariant

$$SU_2, SL(2, \mathbb{R}) \leq SL(2, \mathbb{C})$$

$$\rightarrow \overline{X}_{SU_2}^\circ \cup \overline{X}_{SL(2, \mathbb{R})}^\circ = \overline{X}_{SL(2, \mathbb{C})}^\circ$$

and

$$\overline{X}_{SU_2}^\circ \cap \overline{X}_{SL(2, \mathbb{R})}^\circ = \underbrace{\overline{X}_{SL(2, \mathbb{C})}^\circ}_{\text{A highly singular subvariety}}^{\text{reducible}}$$



Smooth mfd's, oriented, but not compact

Key idea: blow up along singular locus to produce a smooth manifolds

$$X_{\mathbb{R}}^\circ \xrightarrow[\rho]{\text{smooth}} \overline{X}_{SU_2}^\circ \cup \overline{X}_{SL(2, \mathbb{R})}^\circ, \quad \rho^{-1} \in \mathbb{C}^\infty, \text{ diffeo for reducible reps}$$