Cameron Gordon

ADE Links & Cyclic Branched Covers

FSQP Links

An oriented link $L \subseteq S^3$ is <u>strongly quasiprojective</u> if $L = \hat{\beta}$ where $\beta \in B^n$ is a product of $a_{r,s}$ $(1 \le r \le s \le n)$

$\beta \mapsto$ Seifert surface $F(\beta)$ for L

<u>Rmks</u>

- Push out $F(\beta)$ to B^4, then $F(\beta) = B^4 \cap C$ for $C \subseteq \mathbb{C}^2$ a nonsingular alg. curve
- If L is fibred, say L is FSQP

<u>Question</u>: If L is FSQP, when is some $\underbrace{\Sigma_n L}_{\text{n-fold branched cover}}$ an L-Space?

Examples: Plumbing using certain Dynkin diagrams

$\longrightarrow$ ADE links ($E_{6,7,8}$, A_n, B_n)

Σ_n an L-Space iff

$|\pi_1| < \infty$ iff

$n=2$ or iff

$$L = T(2, m+1) \begin{cases} m=1 - \text{all } n \geq 2 \\ m=2 - 2 \leq n \leq 5 \\ m=3,4 - n=2,3 \end{cases}$$

<u>Conjecture</u>: L is FSQP, $\exists n$ s.t. $\Sigma_n L$ is an L-Space <u>iff</u> L is an ADE link.

$|L| = \#$ components of L

$\sigma(L) =$ signature (comes from quadratic form arising from symmetrizing Seifert surface)

$g(L) =$ genus

$$|\sigma(L)| \leq 2g(L) + |L| - 1$$

<u>Thm</u>: L is FSQP, some Σ_n an L-Space

If ① L is definite

② $\Delta_L(t) \not\simeq (t-1)^{\sigma(L)}$

Then $n \geq 5$. $\blacksquare$

<u>Thm</u>: Conjecture true for positive braids

II. BKL filtration & baskets

Let $\delta = \prod_{i=1}^{n-1} a_i \in B^n$, then every $\beta \in B^n$ can be written as $\beta = \delta^k P$ where P is a positive word in a_{rs}. Define $\mathcal{K}(L) = \max_P (k)$. Then

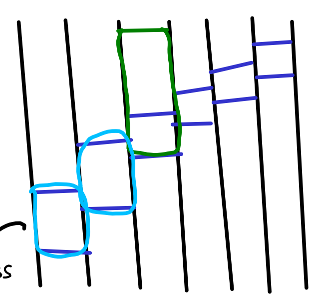
$$L \text{ is FSQP iff } \mathcal{K}(L) \geq 0$$

A <u>basket</u> is a surface $\subseteq S^3$ obtained by plumbing positive Hopf bands onto a disc.

$L = \partial F$ is a <u>basket link</u>

<u>Thm</u>: L is definite and $\mathcal{K}(L) \leq 2$
iff
L is ADE

easily id'd cycles give info about definite-ness



Cyclic Branched Covers

For $m \geq 3$ arcs, basket $F(m,p)$ is determined by P, $1 \leq p \leq m-1$, $L(m,p) = \partial F(m,p)$

Thm

1. $L(m,p)$ positive & def iff p odd
2. ADE iff $p=1$
3. $p > 1 \Rightarrow \Sigma_n$ is not an L-Space

Problems

1) Determine the definite basket links
2) Prove the conjecture for them
3) Prove for FSQP links where $\varkappa(L) = 0$.