Nathan Dunfield

If K is a knot in the 3-sphere, then there exists an irr. rep $\pi_1 M \to SU_2$

Let

$$\star: h^{0}_{SU_2}(M) + h^{0}_{SL_2\mathbb{R}}(M) = h_{\mathbb{R}}(M)$$

Q: Is this also true for $SL_2\mathbb{R}$

If $\sigma_K \neq$ const, then we have irr $SL_2\mathbb{R}$ rep and by $\star$, $h^0_{SU_2} = \frac{1}{2}\sigma_M$, so $h^0_{SL_2\mathbb{R}} \neq$ const and is thus 0 is nonzero somewhere.

$h^0_{SL_2\mathbb{R}} =$ signed count of irr reps $\pi_1(\cdot) \to SL_2\mathbb{R}$ where

$\rho(u)$ is conjugate to $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ ($\sim$parabolic)

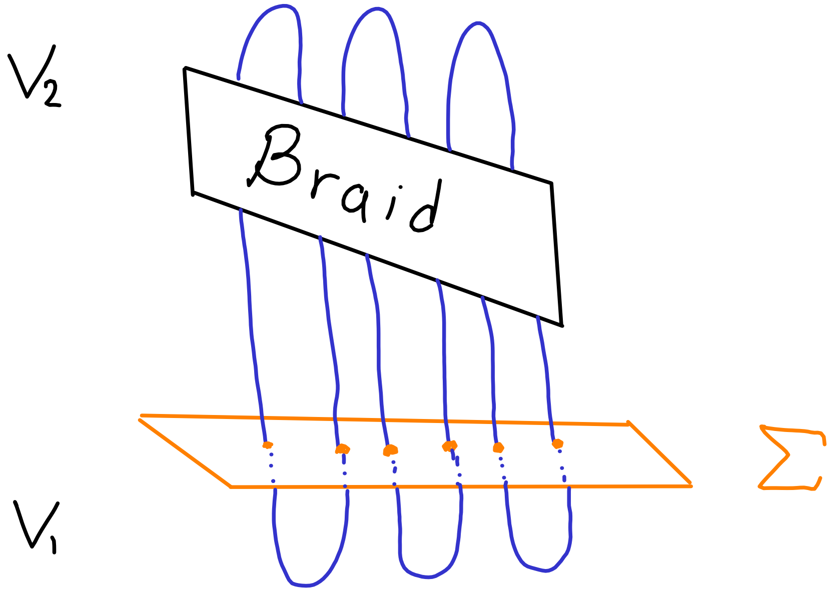
Conj: For K a 2-bridge knot, $\# X^0_{SL_2\mathbb{R}} \geq \underbrace{\frac{1}{2}\left|\sigma_K\left(\theta = \frac{\pi}{2}\right)\right|}_{\text{signature}}$

Proved, Gordon 2016. Does it hold for all K?

Since 2-fold branched cover is a lens space, where π_1 is cyclic and thus has only trivial reps.

How is this invariant defined?

$X_G^{\mathcal{O}} \approx$ character variety



$$\begin{tikzcd} & \pi_1 V_2 \arrow[dr] & \\ \pi_1 \Sigma \arrow[ur] \arrow[dr] & & \pi_1 M \\ & \pi_1 V_1 \arrow[ur] & \end{tikzcd}$$

$\downarrow$ Apply $X_G^{\mathcal{O}_1}$ contravariant

V_i: genus n handlebodies

$$\begin{tikzcd} & X_G^{\mathcal{O}}(V_2) \arrow[dl, "\text{dim } 4n-6"'] & \\ X_G^{\mathcal{O}}(\Sigma) & \text{dim } 2n-3 & X_G^{\mathcal{O}}(M) \arrow[ul] \arrow[dl] \\ & X_G^{\mathcal{O}}(V_2) \arrow[ul] & \end{tikzcd}$$

$X_G^{\mathcal{O}}(M)$: Compact when $\mathcal{O} \in D_M$

Smooth mfds, oriented, but not compact

$SU_2, SL(2,\mathbb{R}) \leq SL(2,\mathbb{C})$

$\rightarrow \overline{X}^{\mathcal{O}}_{SU_2} \cup \overline{X}^{\mathcal{O}}_{SL(2,\mathbb{R})} \subseteq \overline{X}^{\mathcal{O}}_{SL(2,\mathbb{C})}$

and

$$\overline{X}^{\mathcal{O}}_{SU_2} \cap \overline{X}^{\mathcal{O}}_{SL(2,\mathbb{R})} = \underbrace{\overline{X}^{\mathcal{O},\text{ reducible}}_{SL(2,\mathbb{C})}}_{\text{A highly singular subvariety}}$$

Key idea: blow up along singular locus to ~~produce~~ a smooth manifolds

$$X_{\mathbb{R}}^{\mathcal{O}} \xrightarrow[p]{\text{smooth}} \overline{X}^{\mathcal{O}}_{SU_2} \cup \overline{X}^{\mathcal{O}}_{SL(2,\mathbb{R})}, \quad p^{-1} \in C^\infty, \text{ diffeo for reducible reps}$$