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Taut Foliations, Positive 3-braids, and the L-Space Conjecture

L-Space Conjecture: TFAE

1. M admits a taut foliation (geom.)
2. M is a non L-Space (HFH)
3. $\pi_1 M$ is left-orderable (alg.)

Ex to keep in mind: a fibred 3-mfd

$\phi: S \circlearrowleft$ a surface diff, $M_\phi = S \times I / \phi$ @ 0,1

L-space: M^3 closed/connected/oriented/irr.

L-space iff $\operatorname{rank}(\widehat{HF}(M, \mathbb{F}_2)) = |H_1(M, \mathbb{Z})| < \infty$

Ex: An L space = Lens Spaces $\subseteq$ mfds with elliptic geom

Poincaré HS = non L space

<u>Thm</u>: ① $\Rightarrow$ ②

<u>Thm</u>: LSC true for graph mfds

For ② $\Rightarrow$ ①, need to id non L spaces + build T.F.s

Look at L-space Knots: admit surgeries to L-spaces

Ex: torus knots

All Knots admit surgeries to non Lspaces

K an L Space Knot $\Rightarrow$ $\underbrace{S_r(K)}_{\text{Surgery}} = \begin{cases} NLS, & r \in (-\infty, 2g-1) \\ LS, & \text{else} \end{cases}$

<u>Thm:</u> K an NLSK $\Rightarrow$ $S^3_r(K)$ admits a TF.

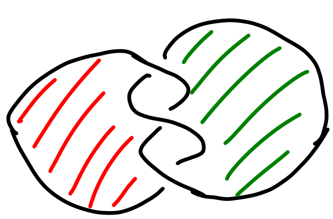
$P(-2,3,7)$ famous!

$\hookrightarrow$ LSK, $g=5$, S^3_r an NLS iff $r \in (-\infty, 9)$

$K = \hat{\beta} \in B^3$ (closure of a 3-braid)

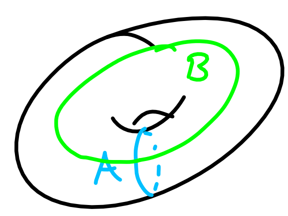
<u>Thm:</u> ① iff ② for mfds obtained from surgery along a ?. Knot (can obtain TFs)

Ex: RH Trefoil

K= 

Seifert surface

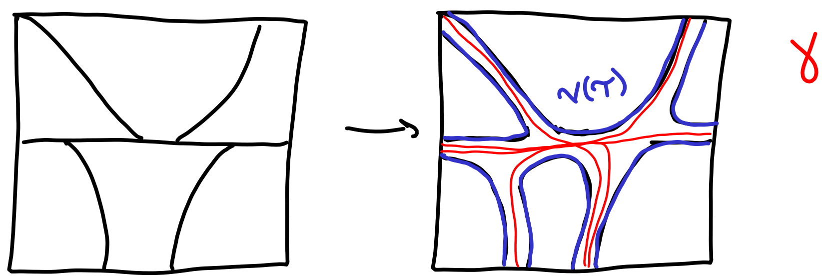
K is fibred in S^3



$X_K = S^3 - \nu(K) = F \times I/\varphi$

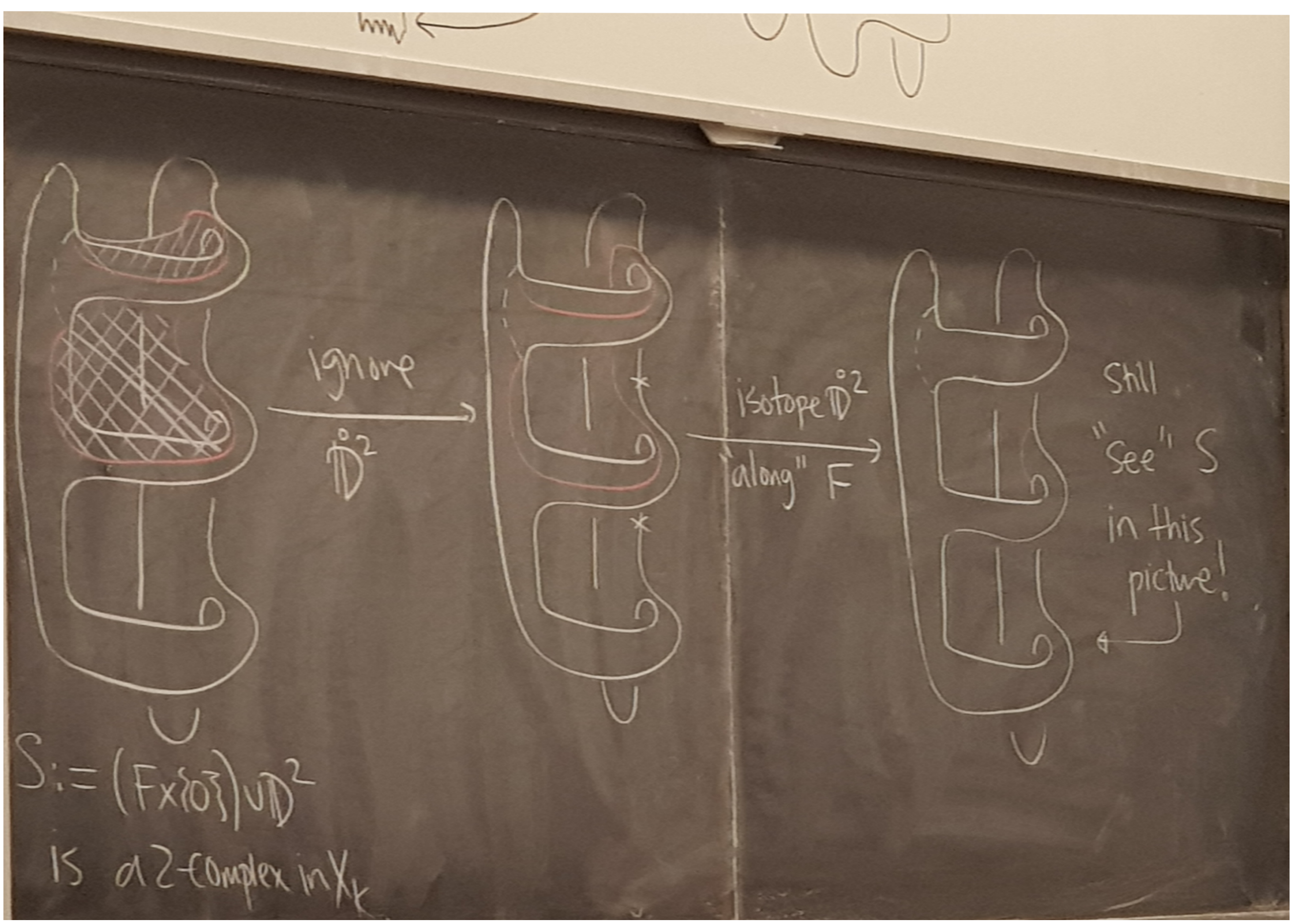
where $\varphi = \overbrace{\tau_B \circ \tau_A}^{\text{Dehn Twists}}$

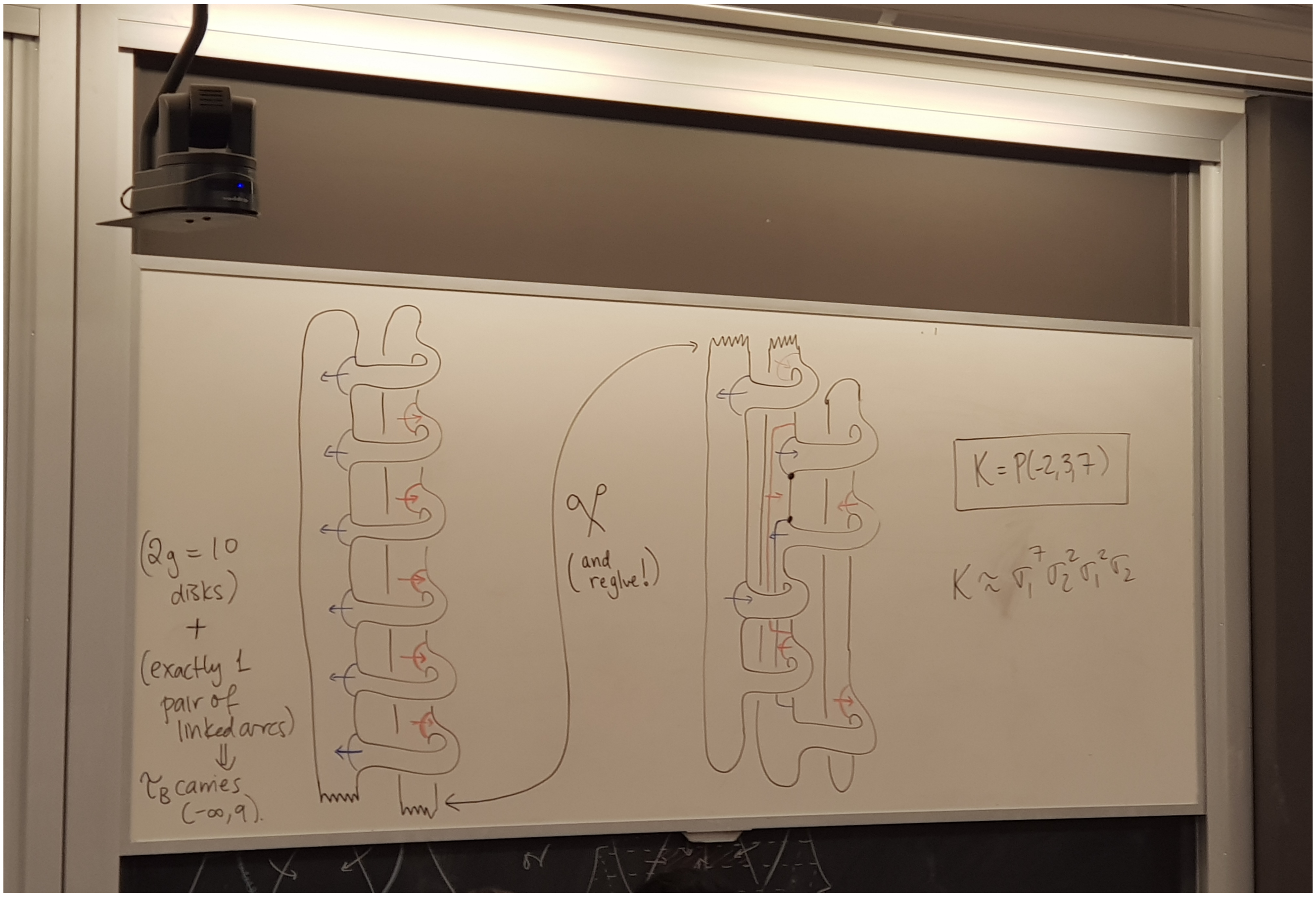
A train track τ carries a slope γ if $\gamma \in r(\tau)$



Let $B = F \times \{0\} \underbrace{\bigcup_{i=1}^{k} \mathbb{D}_i^2}_{\text{co-oriented disks}}$

then B <u>sink disk</u> free $\Rightarrow B$ laminar




(2g = 10 disks)
+
(exactly 1 pair of linked arcs)
⇓
τ_B carries $(-\infty, 9)$.
φ
(and reglue!)
$K = P(-2,3,7)$
$K \approx \sigma_1^7 \sigma_2^2 \sigma_1^2 \sigma_2$