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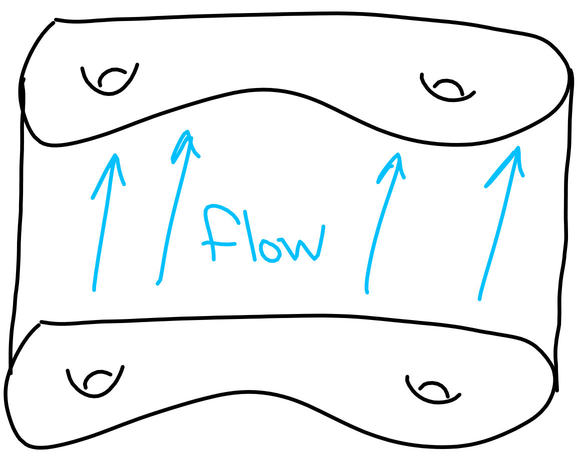
Cannon-Thurston

M^3 a closed hyp. mfd

We'll get a fibration $\rightsquigarrow$

$\pi_1 M \curvearrowright S^1 \quad \longrightarrow \quad \partial \mathbb{H}^3 \circlearrowleft_{\pi_1 M}$

Consider the mapping torus



Σ genus ≥ 2, $\varphi : \Sigma \circlearrowleft$

$M_\varphi = \Sigma \times I / \varphi(0) = \varphi(1)$

There is a pseudo-Anisov flow - has expanding and contracting flows (look at singular foliations, stable & unstable) transverse to the fibration.

Blue & Red Leaves

Note: since Σ is hyperbolic, $\underbrace{\widetilde{\Sigma}}_{\text{universal cover}} = \mathbb{H}^3$

With some work, we can extend (a lift of the flow to int $\mathbb{H}$) to $\partial\mathbb{H}$ due to quasideocity.

<u>Thm</u>: • Φ pseudo-Anisov on M^3 closed

$$\Rightarrow \pi_1 \curvearrowright S^1_U \quad \text{(Universal circle)}$$

• Φ quasi-geodesic on M^3 closed hyperbolic

$$\Rightarrow \pi_1 \curvearrowright S^1_U \xrightarrow{F} S^2_{\infty} \circ_{\pi_1 M} \qquad \blacksquare$$

Not all groups can act on S^1, yields a way to show manifolds don't have p.A/QG flows.

Let $\Gamma \leq PSL(2,\mathbb{R}) \curvearrowright S^1$, then fixed points have attracting/repelling behavior.

Convergence group: every sequence $g_1 g_2 \cdots$ distinct $\in \Gamma$, $\exists$ a subsequence converging to an attractor.

Canon Conjecture: $\Gamma \curvearrowright_{+} S^2$ (* Thm: Convergence $\Rightarrow$ conj to some $\Gamma \leq PSL(2,\mathbb{R})$)

+uniform convergence?

$\Rightarrow \Gamma = \pi_1(\text{Some closed hyp. } M^3)$

Conjectures

1) QG flows have closed orbits

2) QG flow $\Rightarrow$ p.A. flow

Would like analogs of above theorems in the presence of these S^1 actions.

Conjectured corollaries

- Canon true if $\exists$ a Canon-Thurston curve
- Every Γ hyperbolic with $\partial\Gamma = S^2$ is virtually cyclically orderable