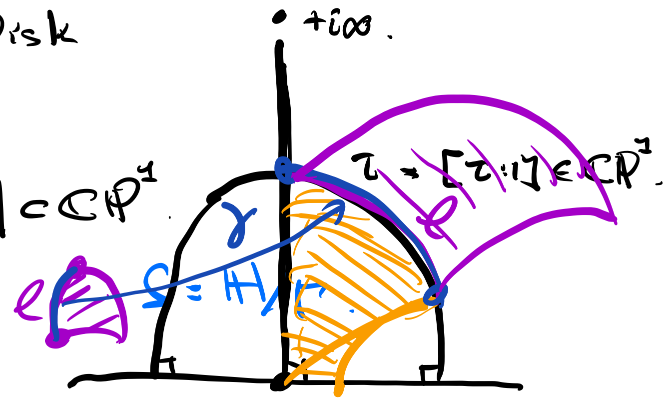
Hyperbolic Disk



$\mathbb{D} \cong \mathbb{H} \subset \mathbb{CP}^1$.

$\frac{\tau - i}{\tau + i} \leftarrow\!\shortmid \tau$

$$\frac{r\,dr\,d\theta}{(1-r^2)^2}$$

(or.-preserving) Isometries : conf(H). $\frac{(dx)^2+(dy)^2}{y^2}$ Γ discrete.

$= PSL_2(\mathbb{R}) \supset \Gamma$.

$\curvearrowright \mathbb{H}$. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \tau = \frac{a\tau+b}{c\tau+d}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \tau \\ 1 \end{pmatrix} = \begin{pmatrix} a\tau+b \\ c\tau+d \end{pmatrix}$.

$\mathbb{C}^2$ $\measuredangle$

$dz_1 d\bar{z}_1 - dz_2 d\bar{z}_2$

$\mathbb{P}(\mathbb{C}^2) = \mathbb{P}^1 \supset \mathbb{P}\{v \in \mathbb{C}^2 \mid \|v\| > 0\}$.

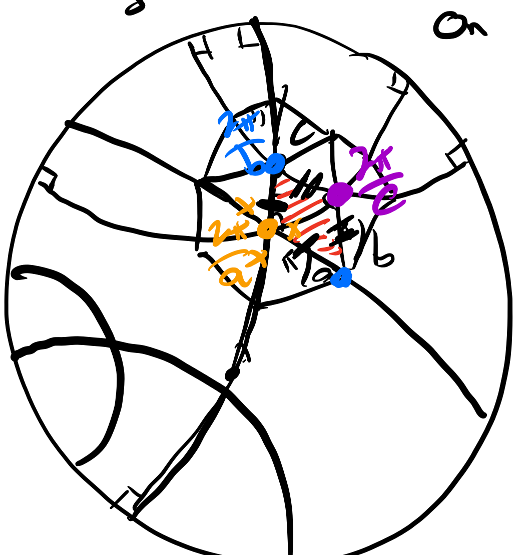
$\alpha \in S^1$ Isometry $\Rightarrow$ $\alpha \in \mathbb{C}^*$ conformal.

$f : \mathbb{C} \to \mathbb{C}$.

On $T_p\mathbb{C} \to T_{f(p)}\mathbb{C}$.

$\mathbb{C} \xrightarrow{df_p} \mathbb{C}$ $\forall p$ $\mathbb{C}$-linear $\forall f$ holo.

$a, b, c \in \mathbb{N}$. $z \mapsto \alpha z$ $\alpha \in \mathbb{C}$.

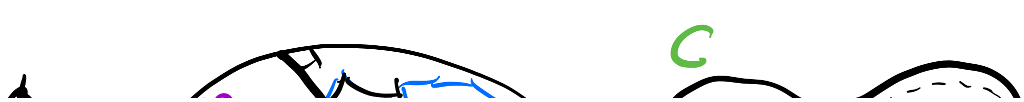


$\Gamma_{a,b,c}$ triangle group.

$\subset PGL_2(\mathbb{R})$.

$\Gamma_{a,b,c} \xrightarrow{\phi} \mathbb{Z}/2\mathbb{Z}$.

$\mathbb{P}^1 \cong \mathbb{H} / \ker\phi$



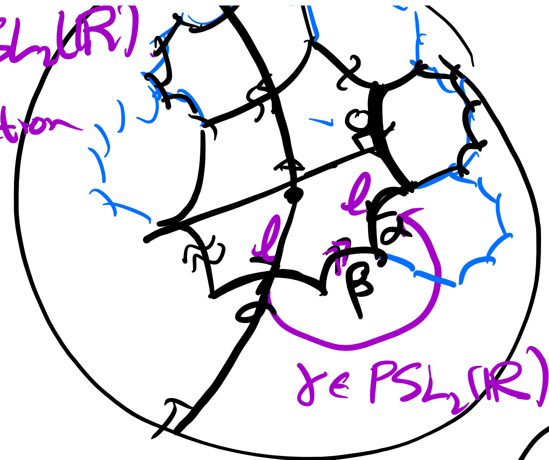
$\pi_1(C,p) \xrightarrow{\varphi} PSL_2(\mathbb{R})$

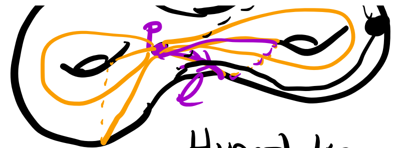
deck action

$\varphi(\beta)$.

$\tilde{C} \to C$

$\tilde{C} = \mathbb{H}$.

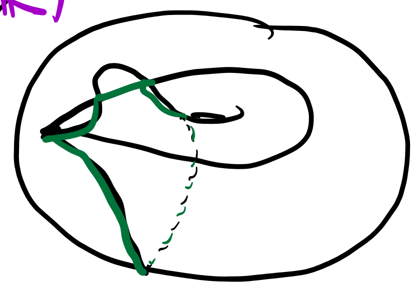




Hyperbolic

metric const curv. -1.

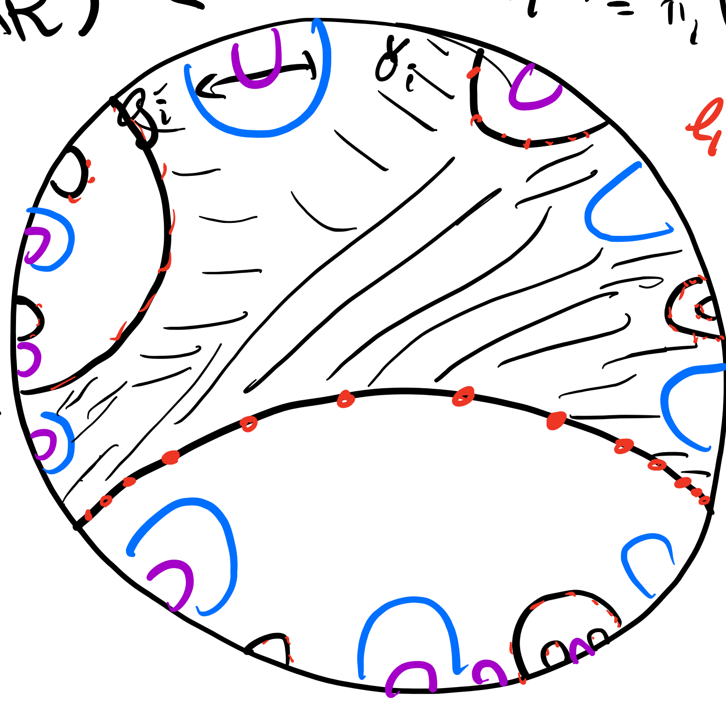
Geodesics never bound bigons.

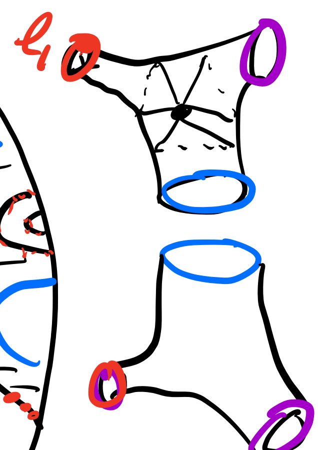


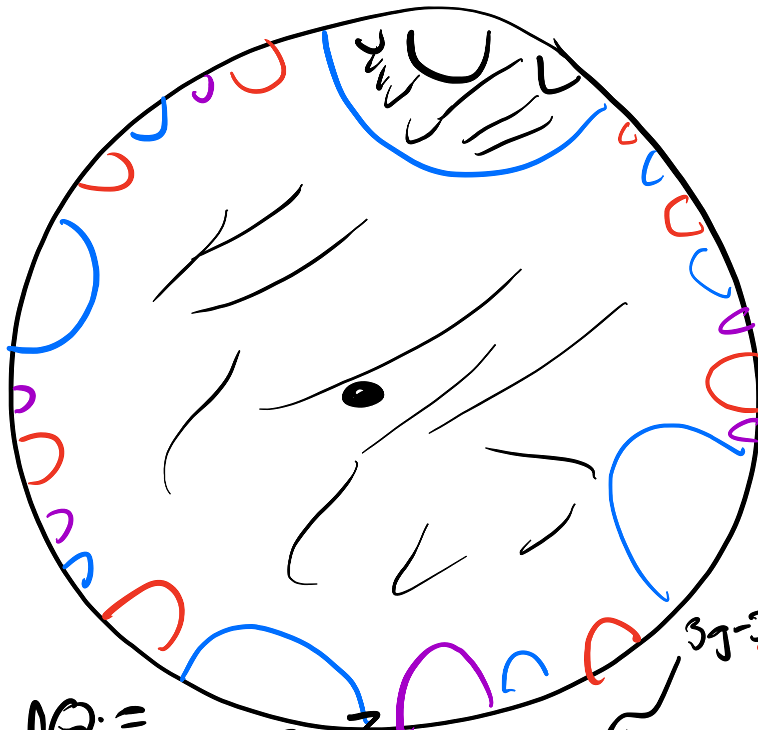
$PSL_2(\mathbb{R}) \leftarrow \{\gamma_1, \gamma_2, \gamma_3 \mid \gamma_1\gamma_2\gamma_3 = 1\} = \pi_1(\text{Pair of Pants})$

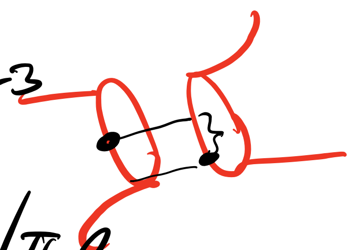
g_1, g_2, g_3

$g_1 g_2 g_3 = 1$.







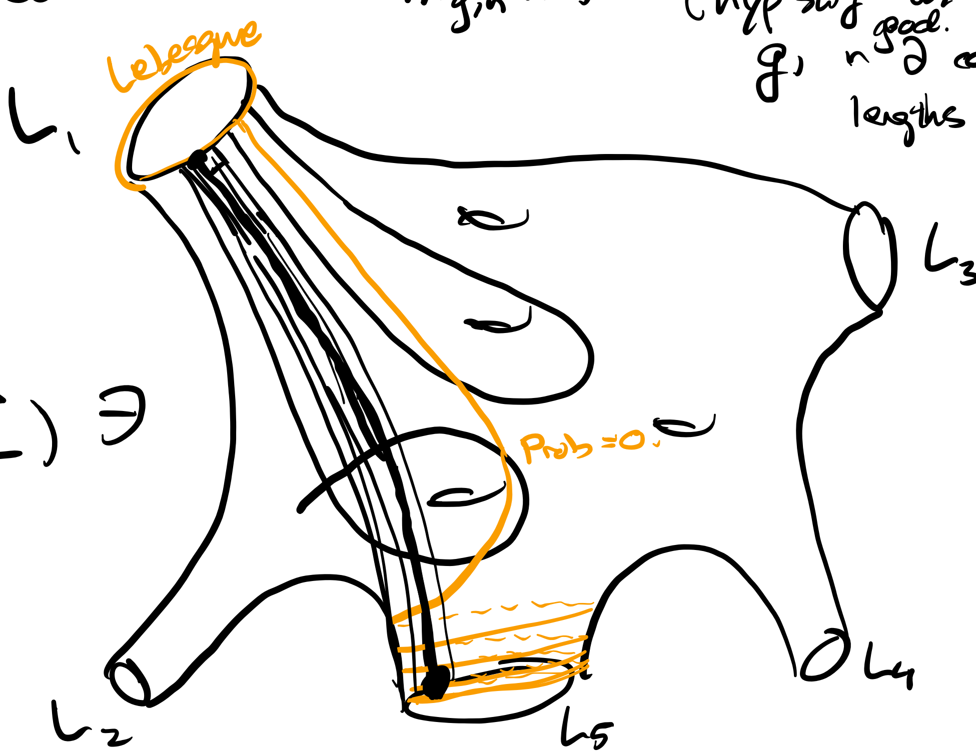


$$\sum_{i=1}^{3g-3} dl_i \wedge d\theta_i = \omega$$

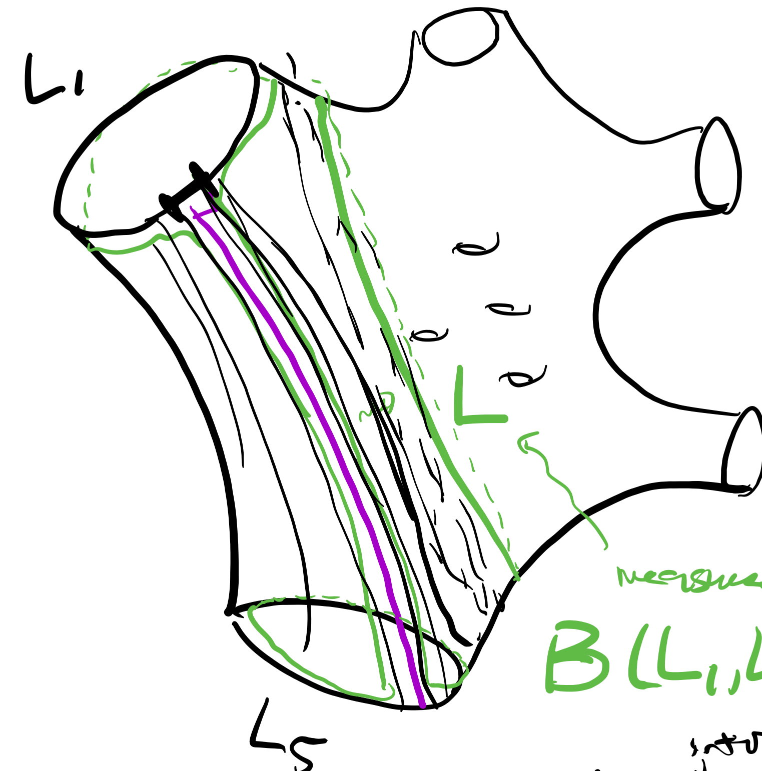
coords $\xrightarrow{3g-3}$ l_i's, $\underbrace{\theta_i}_{3g-3} \in \mathbb{R}/\mathbb{Z}\, l_i$.

$\mathcal{M}_{g,n}(L) = \{$hyp surf w/ geod. genus g, n ∂ comp lengths $L_i\}$.

$\mathcal{M}_{4,5}(\vec{L}) \ni$

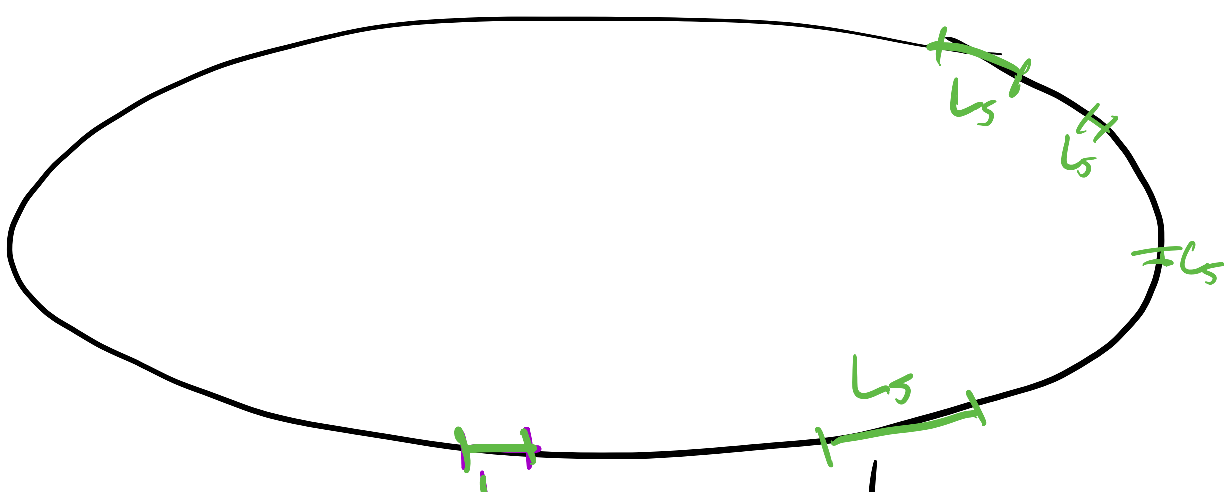


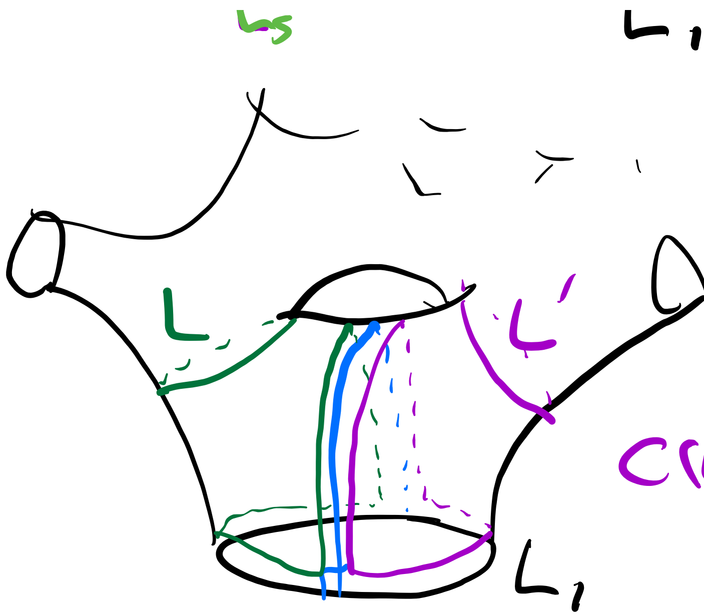
$L_1 = \sum$ (all measures all events).



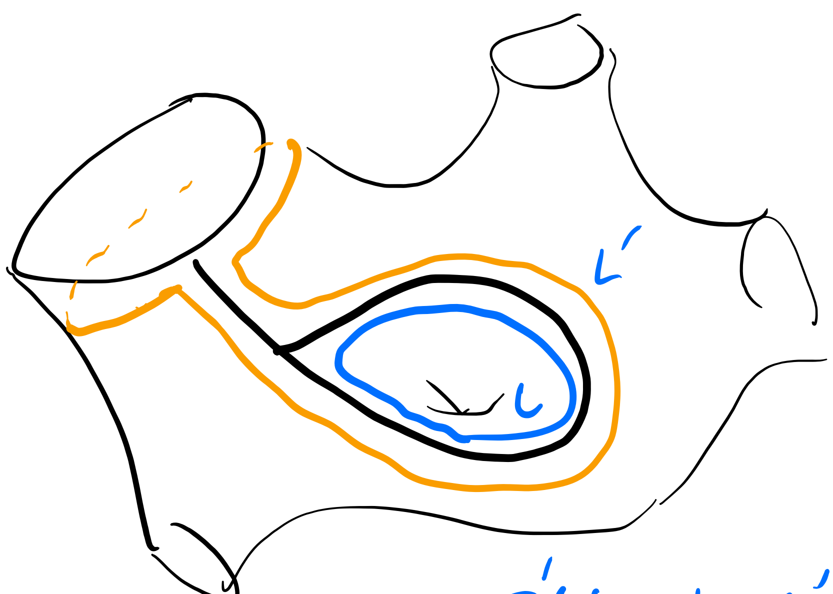
$B(L_1, L_5, L)$

$:=$ Length interval hits L_5.





$C(L_1, L, L')$.



$C'(L_1, L, L')$